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# Deterministic and stochastic methods for sensitivity analysis of neutron noise

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ABSTRACT

Neutron noise calculated from the neutron noise equation in the frequency domain is governed by the cross section and kinetic parameters. Deterministic and stochastic methods to obtain the sensitivity coefficient of neutron noise with respect to the abovementioned parameters are proposed. As a deterministic method, a diffusion equation for the first derivative of neutron noise with respect to a cross section or kinetic parameter is derived by differentiating the neutron noise diffusion equation. As a stochastic method, the differential operator sampling method, which is a well-established Monte Carlo technique, is applied to calculate the sensitivity coefficient. Neither method requires adjoint mode calculations and can be expanded to higher-order derivatives. Based on verifications performed in this study, it is discovered that these techniques yield accurate sensitivity coefficients. The methods developed in this study eliminates a large number of calculations that need to be performed in the random sampling method.

## 1. Introduction

Neutron noise in an operating power reactor core, i.e., inherent fluctuations in neutron flux detected by in-core or ex-core instrumentation, provides useful information for the early detection of anomalies, such as abnormal vibrations of core internals and flow blockage (Seidl et al., 2015; Torres et al., 2019; Chionis et al., 2020). Numerous studies and developments regarding core monitoring techniques using neutron noise have been pursued for many decades. From 2017 to 2021, the CORTEX project, a research and innovation action aiming to develop an innovative core monitoring technique using the fluctuations in neutron flux, was performed by a multinational consortium under the auspices of the European Commission in the Euratom 2016–2017 work program (Demazière et al., 2018). The calculation methods employed in the project for neutron noise propagation analyses in reactor cores were classified primarily into two categories: the time-domain method (Vidal-Ferràndis et al., 2020) and frequency-domain method (Pázsit and Demazière, 2010; Demazière, 2011; Mylonakis et al., 2021; Zoia et al., 2021). In the time-domain method, a time-varying neutron flux is explicitly calculated by solving the time-dependent neutron diffusion equation, where the cross sections change with time owing to the anomaly in a reactor. The frequency-domain method directly calculates the neutron noise by solving the frequency-domain diffusion or transport equation for neutron noise. The frequency-domain method is typically limited to diffusion approximation. A Monte Carlo method was developed to solve the neutron noise transport equation in the frequency domain (Yamamoto, 2013, 2018b; Rouchon et al., 2017; Zoia et al., 2021). Recently, Yi et al. (2021) developed a deterministic neutron noise transport solver that uses the discrete ordinates method (Sn method).

In addition to obtaining an accurate solution of the neutron noise equation, the uncertainty of the neutron noise calculated by these calculation methods must be quantified. The uncertainties of the calculated neutron noise originate from the uncertainties of the constants used in the diffusion or transport equation (e.g., nuclear data, group constants, and kinetic parameters), as well as the theoretical and numerical approximations for the neutron noise. If variables that affect the neutron noise are identified using the sensitivity and uncertainty analysis (SUA) method, then the accuracy of the calculation results can be improved by reducing the uncertainty of the variables. The SUA method comprises two primary approaches (Rochman et al., 2011; Chiba et al., 2015; Endo et al., 2018). In the conventional SUA method, the sensitivity coefficients and covariance of the input variables are combined to obtain the uncertainty of the output variables. Another approach is random-sampling-based uncertainty quantification, where a

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large number of calculations are performed using input parameters that are randomly sampled from the probability distribution of the parameters. Although the random sampling method is a robust technique that does not require sensitivity coefficients, it is computationally inefficient. The random-sampling-based approach was adopted in the CORTEX project to estimate the uncertainty of neutron noise (Yum et al., 2019).

In this study, the first approach was selected to quantify the sensitivity coefficients of neutron noise with respect to the group constants and kinetic parameters. In deterministic methods, the sensitivity coefficient is typically obtained by combining the solutions of forward and adjoint equations. A new deterministic method was developed in this study to calculate sensitivity coefficients without solving the adjoint equation. This new method is an extension of the "perturbation source method," which was developed for the Monte Carlo perturbation method (Sakamoto and Yamamoto, 2017; Yamamoto and Sakamoto, 2020a, 2021). In addition to the deterministic method for sensitivity analysis, a Monte Carlo method is introduced herein to perform the sensitivity analysis of neutron noise. In this regard, the differential operator sampling method (Rief, 1984; Nagaya and Mori, 2011; Yamamoto, 2018a; Yamamoto and Sakamoto, 2019, 2020b) can be applied straightforwardly. This paper addresses the first attempt to calculate the sensitivity coefficients of neutron noise using the frequency-domain diffusion or transport equation.

The remainder of this paper is organized as follows: In Section 2, a formula for the sensitivity coefficient of neutron noise, which is directly calculated from the neutron noise diffusion equation, is derived, and its verification is presented. In Section 3, the differential operator sampling method applied to the neutron noise transport equation in the frequency domain is discussed. The sensitivity coefficients obtained from the Monte Carlo method are compared with those obtained using the diffusion method. Finally, Section 4 presents the conclusions of the study.

## 2. Sensitivity coefficient of neutron noise based on diffusion theory

### 2.1. Formulation of sensitivity coefficient

First, the "neutron noise" addressed in this study is defined. The neutron noise in the time domain is expressed by the difference between the time-dependent neutron flux and its mean value, as follows:

$$\delta\varphi_g(\boldsymbol{r},t)\equiv\varphi_g(\boldsymbol{r},t)-\varphi_{0g}(\boldsymbol{r}), \tag{1}$$

where $\varphi_g(\boldsymbol{r},t)$ is the neutron flux of the $g$th energy group at position $\boldsymbol{r}$ and time $t$, and $\varphi_{0g}(\boldsymbol{r})$ is the mean value of the neutron flux. A Fourier transformation of the neutron noise in the time domain yields the neutron noise in the frequency domain, as follows:

$$\delta\varphi_g(\boldsymbol{r},\omega)\equiv\int_{-\infty}^{+\infty}\delta\varphi_g(\boldsymbol{r},t)e^{-i\omega t}dt, \tag{2}$$

where $\omega$ is the angular frequency, and $i=\sqrt{-1}$. Because the derivation of the multigroup diffusion equation for the neutron noise in the frequency domain has been presented in many studies (e.g., Pázsit and Demazière, 2010), the typically used formulas are first presented in this section, as follows:

$$\mathbf{A}\delta\varphi_g(\boldsymbol{r},\omega)=S_g(\boldsymbol{r},\omega). \tag{3}$$

The left-hand side of Eq. (3) is defined as follows:

$$\mathbf{A}\delta\varphi_g(\boldsymbol{r},\omega)\equiv-\nabla D_g\cdot\nabla\delta\varphi_g(\boldsymbol{r},\omega)+\Sigma_{ag}(\boldsymbol{r})\delta\varphi_g(\boldsymbol{r},\omega)+\sum_{\substack{g'=1\\ g\neq g'}}^{G}\Sigma_s^{g\to g'}(\boldsymbol{r})\delta\varphi_g(\boldsymbol{r},\omega)-\frac{\chi_g}{k_{eff}}\left(1-\frac{i\omega\beta}{i\omega+\lambda}\right)\sum_{g'=1}^{G}\nu\Sigma_{fg'}(\boldsymbol{r})\delta\varphi_{g'}(\boldsymbol{r},\omega)-\sum_{\substack{g'=1\\ g\neq g'}}^{G}\Sigma_s^{g'\to g}(\boldsymbol{r})\delta\varphi_{g'}(\boldsymbol{r},\omega)+\frac{i\omega}{v_g}\delta\varphi_g(\boldsymbol{r},\omega), \tag{4}$$

where $\lambda$ is the time decay constant of the delayed neutron precursors, $v_g$ is the neutron velocity, $S$ is the noise source induced by the fluctuation of the cross sections, and the other notations are standard in nuclear engineering. The delayed neutrons are represented by one group, and the energy spectra of the delayed neutrons are assumed to be identical to those of the prompt neutrons.

The sensitivity coefficient of the neutron noise is obtained from the first derivative of the neutron noise with respect to a variable, such as the group constant or kinetic parameter. The source term $S_g(\boldsymbol{r},\omega)$ in the neutron noise equation is typically composed of $\delta\Sigma_\alpha(\boldsymbol{r},\omega)\varphi_0(\boldsymbol{r})$, where $\delta\Sigma_\alpha(\boldsymbol{r},\omega)$ is the Fourier transform of the perturbed term of the cross section for reaction $\alpha$ and $\varphi_0(\boldsymbol{r})$ is the steady state neutron flux (Pázsit and Demazière, 2010). Thus, the source term actually depends on the variable such as the group constant. However, this study does not envisage any kind of anomaly that causes the noise source. Hence, throughout this study, we assume that the source term is invariant with respect to the variables. The first derivative of the neutron noise is derived by differentiating Eq. (3) with variable $\alpha$ as follows:

$$\mathbf{A}\delta\varphi'_{g,\alpha}(\boldsymbol{r},\omega)+\mathbf{A}'_\alpha\delta\varphi_g(\boldsymbol{r},\omega)=0. \tag{5}$$

The first derivative $\delta\varphi'_g(\boldsymbol{r},\omega)$ is defined as follows:

$$\delta\varphi'_{g,\alpha}(\boldsymbol{r},\omega)\equiv\frac{\partial\delta\varphi_g(\boldsymbol{r},\omega)}{\partial\alpha}, \tag{6}$$

where $\alpha$ is a variable in Eq. (4), such as the cross-section $\Sigma_{ag}$ or parameter $\omega$. The second term on the left-hand side of Eq. (5) represents the source term of the equation. This term is presented by excluding the terms with $\delta\varphi'_{g,\alpha}(\boldsymbol{r},\omega)$ from the first derivative of Eq. (4) with respect to $\alpha$. Specifically, this term is presented for each variable in Eq. (4) as follows:

$$\mathbf{A}'_\alpha\delta\varphi_g(\boldsymbol{r},\omega)\equiv-\nabla D'_{g,\alpha}\cdot\nabla\delta\varphi_g(\boldsymbol{r},\omega)+\delta\varphi_g(\boldsymbol{r},\omega),\quad\text{for }\alpha=\Sigma_{cg}\text{ or }\Sigma_s^{g\to g'}, \tag{7}$$

$$\mathbf{A}'_\alpha\delta\varphi_g(\boldsymbol{r},\omega)\equiv-\nabla D'_{g,\alpha}\cdot\nabla\delta\varphi_g(\boldsymbol{r},\omega)-\frac{\chi_g}{k_{eff}}\nu\left(1-\frac{i\omega\beta}{i\omega+\lambda}\right)\delta\varphi_g(\boldsymbol{r},\omega),\quad\text{for }\alpha=\Sigma_{fg}, \tag{8}$$

$$\mathbf{A}'_\alpha\delta\varphi_g(\boldsymbol{r},\omega)\equiv\frac{i}{v_g}\delta\varphi_g(\boldsymbol{r},\omega)+\frac{\chi_g}{k_{eff}}\frac{i\beta\lambda}{(i\omega+\lambda)^2}\sum_{g'=1}^{G}\nu\Sigma_{fg'}(\boldsymbol{r})\delta\varphi_{g'}(\boldsymbol{r},\omega),\quad\text{for }\alpha=\omega, \tag{9}$$

$$\mathbf{A}'_\alpha\delta\varphi_g(\boldsymbol{r},\omega)\equiv\frac{\chi_g}{k_{eff}}\frac{i\omega}{i\omega+\lambda}\sum_{g'=1}^{G}\nu\Sigma_{fg'}(\boldsymbol{r})\delta\varphi_{g'}(\boldsymbol{r},\omega),\quad\text{for }\alpha=\beta, \tag{10}$$

$$\mathrm{A}'_{\alpha}\delta\varphi_g(\boldsymbol{r},\omega) \equiv -\frac{\chi_g}{k_{eff}}\frac{i\omega\beta}{(i\omega+\lambda)^2}\sum_{g'=1}^{G}\nu\Sigma_{fg'}(\boldsymbol{r})\delta\varphi_{g'}(\boldsymbol{r},\omega),\quad \text{for}\quad \alpha=\lambda, \tag{11}$$

where $\Sigma_{cg}$ is the capture cross-section. We assume that the diffusion coefficient $D_g$ is expressed as $1/(3\Sigma_{tg})$ and that it changes accordingly with $\Sigma_{cg}$, $\Sigma_s^{g\to g'}$ and $\Sigma_{fg}$. $D'_{g,\alpha}$ in Eqs. (7) and (8) are expressed as follows:

$$D'_{g,\alpha} \equiv \frac{\partial D_g}{\partial \alpha} = \frac{\partial}{\partial \alpha}\left(\frac{1}{3\Sigma_{tg}}\right) = -\frac{1}{3\Sigma_{tg}^2}\frac{\partial \Sigma_{tg}}{\partial \alpha} = -\frac{1}{3\Sigma_{tg}^2}, \tag{12}$$

where $\partial\Sigma_{tg}/\partial\alpha = 1$. As indicated in Eq. (5), $\delta\varphi_g(\boldsymbol{r},\omega)$ is necessitated to solve Eq. (5) such that the first derivative $\delta\varphi'_{g,\alpha}(\boldsymbol{r},\omega)$ can be obtained. Hence, the first derivative is calculated in two steps. In the first step, the neutron noise is calculated using Eq. (3). Subsequently, the first derivative is calculated.

### 2.2. Numerical examples of sensitivity coefficient

Numerical tests for the formulation of the first derivatives in Section 2.1 were performed for a cylinder of infinite height. The cylinder was composed of an inner fuel region with a radius of 36 cm; it was surrounded by an annular light-water reflector with an outer radius of 72 cm. The inner fuel region comprised a homogenized $UO_2$ fuel rod array. The group constants were prepared using the standard thermal reactor analysis code SRAC (Okumura et al., 2007). Table 1 lists the two-group constants used in the numerical tests. The diffusion equations were solved using the finite-difference scheme. A vacuum boundary condition ($-D_g\dot{\varphi}_g/\varphi_g = 0.4692$) was imposed on the outer surface of the light-water reflector. The value of $k_{eff}$ used in Eqs. (4) and (5) was 1.001453. The noise source of the second energy group was placed at the center of the cylinder. The frequency and intensity of the noise source were 1 Hz and $1-i$, respectively. The neutron noise distributions were calculated by solving Eq. (3) and are shown in Fig. 1, where the Monte Carlo results, which will be presented later in Section 3, are shown as well. The diffusion results were primarily consistent with the Monte Carlo results, except near the noise source, where the diffusion approximation was not necessarily appropriate.

The approximate reference values of the first derivatives were obtained from the difference in the neutron noise caused by an infinitesimally small perturbation of parameter $\alpha$ in the fuel region. The real parts of the first derivatives of the fast energy group at $r =$ 18.09 cm were compared with the reference values listed in Table 2. The first derivative with respect to the neutron velocity $v_g$ is omitted because it is negligibly small. The approximate reference values were obtained as follows:

**Table 1**
Group constants of fuel region and water reflector.

| | Fuel region | Water reflector |
|---|---|---|
| $D_1$ (cm) | 1.45293 | 0.905414 |
| $D_2$ (cm) | 0.197177 | 0.125647 |
| $\Sigma_{t1}$ (cm$^{-1}$) | 0.229420 | 0.368156 |
| $\Sigma_{t2}$ (cm$^{-1}$) | 1.69053 | 2.65294 |
| $\nu\Sigma_{f1}$ (cm$^{-1}$) | 0.00643278 | – |
| $\nu\Sigma_{f2}$ (cm$^{-1}$) | 0.155089 | – |
| $\Sigma_{c1}$ (cm$^{-1}$) | 0.00696154 | 0.000460058 |
| $\Sigma_{c2}$ (cm$^{-1}$) | 0.0527943 | 0.0188813 |
| $\Sigma_s^{1\to1}$ (cm$^{-1}$) | 0.1972572 | 0.3092749 |
| $\Sigma_s^{1\to2}$ (cm$^{-1}$) | 0.0225208 | 0.0584208 |
| $\beta$ | 0.007 | – |
| $\lambda$ (s$^{-1}$) | 0.08 | – |
| $v_1$ (cm/s) | $2.8\times10^7$ | $2.8\times10^7$ |
| $v_2$ (cm/s) | $3\times10^5$ | $3\times10^5$ |

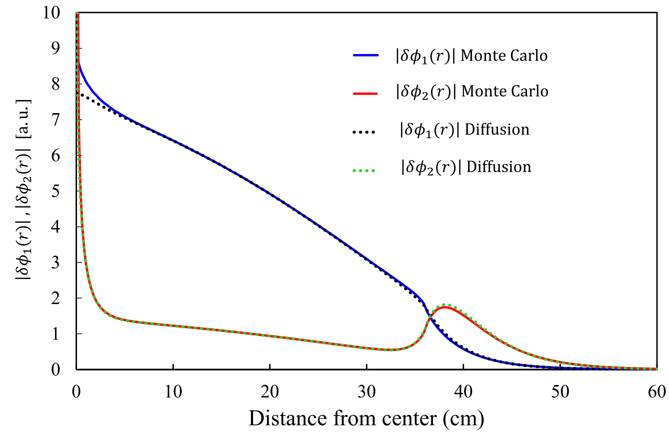


**Fig. 1.** Neutron noise radial distribution obtained using Monte Carlo and diffusion methods.

**Table 2**
Relative first derivative for $f$ = 1 Hz and $r$ = 18.09 cm

| $\alpha$ | $\frac{\mathrm{Re}[\delta\varphi'_{1,\alpha}]}{\mathrm{Re}[\delta\varphi_1]}$ | $\frac{\mathrm{Re}[\delta\varphi_{1,\alpha+\Delta\alpha}]-\mathrm{Re}[\delta\varphi_{1,\alpha}]}{\mathrm{Re}[\delta\varphi_1]\Delta\alpha}$ | $\Delta\alpha$ |
|---|---|---|---|
| $\Sigma_{c1}$ | −2620 | −2619 | $1\times10^{-7}$ (cm$^{-1}$) |
| $\Sigma_{c2}$ | −685.9 | −685.6 | $1\times10^{-6}$ (cm$^{-1}$) |
| $\Sigma_{f1}$ | 3682 | 3682 | $1\times10^{-7}$ (cm$^{-1}$) |
| $\Sigma_{f2}$ | 571.6 | 571.6 | $1\times10^{-6}$ (cm$^{-1}$) |
| $\Sigma_s^1$ | 109.5 | 109.6 | $1\times10^{-5}$ (cm$^{-1}$) |
| $\beta$ | −100.2 | −100.2 | $1\times10^{-6}$ |
| $\lambda$ | −0.1191 | −0.1191 | $1\times10^{-4}$ (s$^{-1}$) |
| $\omega$ | −0.001373 | −0.001378 | 0.02 (s$^{-1}$) |

$$\frac{\mathrm{Re}\left[\delta\varphi_{1,\alpha+\Delta\alpha}\right]-\mathrm{Re}\left[\delta\varphi_{1,\alpha}\right]}{\Delta\alpha}, \tag{13}$$

where $\delta\varphi_{1,\alpha}$ is the unperturbed neutron noise, $\delta\varphi_{1,\alpha+\Delta\alpha}$ the neutron noise that is slightly perturbed by $\Delta\alpha$, and Re[ ] the real part. Table 2 lists the relative first derivatives calculated using Eq. (5), and they precisely reproduced the reference values. Although the results of the thermal energy group are not presented, the same thing went for the thermal energy group.

The sensitivity coefficient of the amplitude of the neutron noise is defined as follows:

$$S(|\delta\varphi_g|;\alpha) \equiv \frac{\alpha}{|\delta\varphi_g|}\frac{\partial|\delta\varphi_g|}{\partial\alpha}, \tag{14}$$

where $|\delta\varphi_g|$ is the neutron-noise amplitude. The partial derivative of Eq. (14) is expressed as follows:

$$\frac{\partial|\delta\varphi_g|}{\partial\alpha} = \frac{\mathrm{Re}\left[\delta\varphi_g\right]\mathrm{Re}\left[\delta\varphi'_{g,\alpha}\right]+\mathrm{Im}\left[\delta\varphi_g\right]\mathrm{Im}\left[\delta\varphi'_{g,\alpha}\right]}{|\delta\varphi_g|}, \tag{15}$$

where Im[ ] denotes the imaginary part. The sensitivity coefficient of the neutron noise phase is defined as follows:

$$S(\theta;\alpha) \equiv \frac{\alpha}{\theta}\frac{\partial\theta}{\partial\alpha}, \tag{16}$$

where $\theta$ is the neutron-noise phase. The partial derivative of Eq. (16) is expressed as follows:

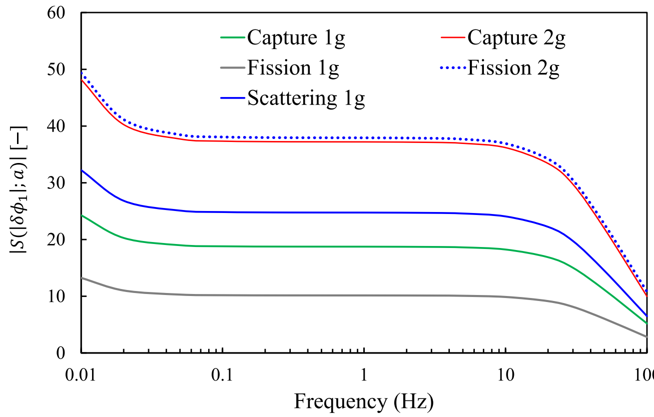


**Fig. 2.** Sensitivity coefficients of neutron noise amplitude with respect to group constant vs. frequency.

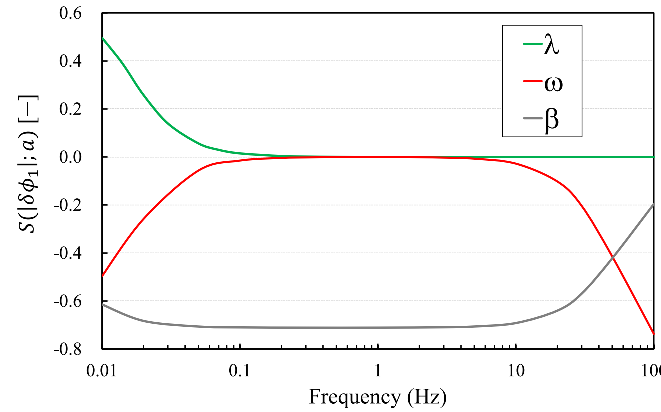


**Fig. 4.** Sensitivity coefficients of neutron noise amplitude with respect to kinetic parameter vs. frequency.

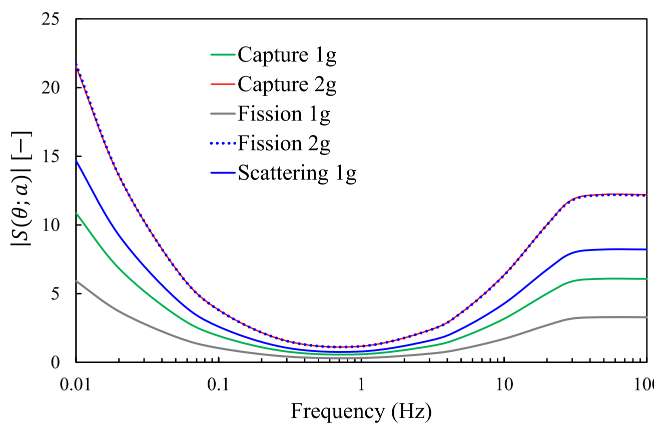


**Fig. 3.** Sensitivity coefficients of neutron noise phase with respect to group constant vs. frequency.

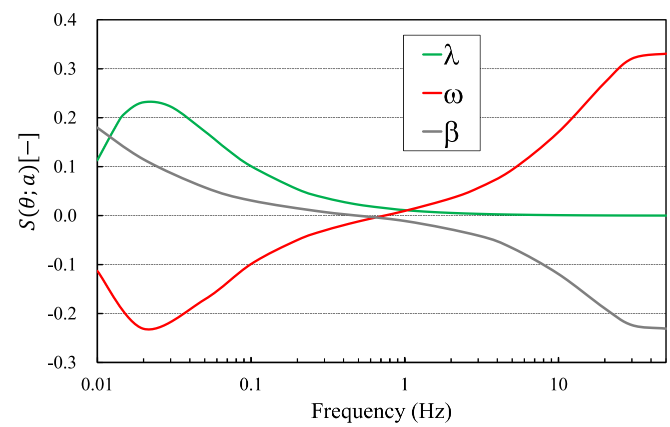


**Fig. 5.** Sensitivity coefficients of neutron noise phase with respect to kinetic parameter vs. frequency.

$$\frac{\partial\theta}{\partial\alpha}=\frac{\mathrm{Re}\left[\delta\varphi_g\right]\mathrm{Im}\left[\delta\varphi'_{g,\alpha}\right]-\mathrm{Im}\left[\delta\varphi_g\right]\mathrm{Re}\left[\delta\varphi'_{g,\alpha}\right]}{\left|\delta\varphi_g\right|^2}. \tag{17}$$

The sensitivity coefficients defined in Eqs. (14) and (16) are shown as a function of frequency in Figs. 2–5. As shown in Fig. 4, whereas the uncertainty of $\lambda$ was sensitive to the neutron noise at lower frequencies, it did not affect the accuracy of the neutron noise whose frequency exceeded 0.1 Hz. The effect of the uncertainty of $\omega$ was significant below 0.1 Hz and above 10 Hz. Between 0.1 Hz and 10 Hz, it exerted only a slight effect on the accuracy of the neutron noise.

The advantage of this proposed method is to avoid adjoint mode calculations. The adjoint calculation needs to be performed for each response that is sought to be evaluated. The number of the adjoint calculations to be performed increases with the number of responses. The proposed method needs to be performed only once for each parameter of interest. The alternative approximate method, as shown in Eq. (13), where the parameter of interest is slightly perturbed needs to adjust the quantity of perturbation in such a way that the perturbation is small enough for accurate estimation of sensitivity. The proposed method can omit this adjustment.

## 3. Sensitivity coefficient of neutron noise obtained using Monte Carlo method

### 3.1. Differential operator sampling method

This section presents a Monte Carlo method for calculating the sensitivity coefficient of neutron noise. The multigroup transport equation of the neutron noise used in this study is as follows (Yamamoto, 2018b):

$$\boldsymbol{\Omega}\cdot\nabla\delta\varphi_g(\boldsymbol{r},\boldsymbol{\Omega},\omega)+\Sigma_{tg}(\boldsymbol{r})\delta\varphi_g(\boldsymbol{r},\boldsymbol{\Omega},\omega)=\frac{1}{4\pi}\int_{4\pi}d\boldsymbol{\Omega}'\sum_{g'=1}^{G}\Sigma_s^{g'\to g}(\boldsymbol{r})\delta\varphi_{g'}(\boldsymbol{r},\boldsymbol{\Omega}',\omega)+\frac{\chi_g}{4\pi k_{eff}}\left(1-\frac{i\omega\beta}{i\omega+\lambda}\right)\int_{4\pi}d\boldsymbol{\Omega}'\sum_{g'=1}^{G}\nu\Sigma_{fg'}(\boldsymbol{r},E')\delta\varphi_{g'}(\boldsymbol{r},\boldsymbol{\Omega}'\omega)-\frac{i\omega}{v_g}\delta\varphi_g(\boldsymbol{r},\boldsymbol{\Omega},\omega)$$
$$+S_g(\boldsymbol{r},\boldsymbol{\Omega},\omega), \tag{18}$$

where isotropic scattering is assumed. The Monte Carlo algorithm for solving this transport equation has already been developed (Yamamoto, 2013, 2018b; Rouchon et al., 2017). The differential operator sampling (DOS) method is a well-established technique that can yield the first derivative of the neutron flux with respect to a cross section (Rief, 1984;

McKinney and Iverson, 1996; Densmore et al., 1997; Nagaya and Mori 2011; Yamamoto and Sakamoto, 2020b). The first-order DOS method was applied to the sensitivity coefficient of the $k_{eff}$ eigenvalue (Yamamoto, 2018a). In this study, the DOS was implemented in the Monte Carlo algorithm to solve the neutron noise transport equation in the frequency domain. Because the detailed description of the DOS has been presented in previous publications, the minimum steps for calculating the sensitivity coefficient of neutron noise flux are presented here.

During the random walk process of the Monte Carlo calculation to obtain the neutron noise, the following estimate for the first derivative of the neutron noise $\delta\varphi_{g,j}$ within region $j$ with respect to parameter $\alpha$ is scored from the birth of the $m$th noise source particle until its death (including all resulting progenies):

$$\frac{\partial \delta\varphi_{g,j,m}}{\partial \alpha} = \sum_i \frac{1}{\Sigma_{tg,j}} w_i P_i(\alpha), \tag{19}$$

where the summation is performed at each collision within region $j$, and $w_i$ is the complex-valued particle weight of the $i$th collision within region $j$. $P_i(\alpha)$ is the score accumulated until the $i$th collision and is defined for each group constant as follows:

$$P_i(\Sigma_{cg}) = -\sum_k s_k, \tag{20}$$

$$P_i(\Sigma_s^g) = \sum_{l=1}^{i-1} \frac{1}{\Sigma_s^g} - \sum_k s_k, \tag{21}$$

$$P_i(\Sigma_{fg}) = -\sum_{l=1}^{i-1} \frac{1}{\Sigma_{tg}} + \sum_{n=1}^{i-1} \frac{1}{\Sigma_{fg}} - \sum_k s_k, \tag{22}$$

where the summation for $k$ is performed over all tracks until the $i$th collision, the summation for $l$ in Eq. (21) is performed over all collisions until the $(i-1)$th collision, and the summation for $n$ in Eq. (22) is performed over all the fissions until the $(i-1)$th collision. These summations are performed within the region where the group constant $\alpha$ exists.

The DOS method to obtain the sensitivity with respect to the kinetic parameters ($\omega$, $\beta$, and $\lambda$) is devised in this study. When a particle propagates from position $\boldsymbol{r}$ to collision point $\boldsymbol{r}'$, the transport kernel for the neutron noise transport equation (Eq. (18)) is expressed as follows:

$$T(\boldsymbol{r} \rightarrow \boldsymbol{r}') = \Sigma_{tg} \exp(-\Sigma_{tg} s) exp\left(-\frac{i\omega}{v_g} s\right), \tag{23}$$

where $s = |\boldsymbol{r}' - \boldsymbol{r}|$. The weighting coefficient of this transport kernel with respect to the frequency $\omega$ is scored during the random walk process and is expressed as

$$\frac{1}{T(\boldsymbol{r}\rightarrow\boldsymbol{r}')} \cdot \frac{\partial}{\partial \omega} T(\boldsymbol{r} \rightarrow \boldsymbol{r}') = -\frac{is}{v_g}. \tag{24}$$

When a fission occurs, the following weight is assigned to each fission neutron:

$$W_f = \frac{1}{k_{eff}} \left(1 - \frac{i\omega\beta}{i\omega + \lambda}\right). \tag{25}$$

Hence, the weighting coefficient of the $n$th fission reaction with respect to $\omega$ is expressed as

$$Q_{\omega n} = \frac{1}{W_f} \frac{\partial W_f}{\partial \omega} = -\frac{i\beta\lambda}{(i\omega + \lambda)(\lambda + i\omega(1-\beta))}. \tag{26}$$

Similarly, the weighting coefficient of the $n$th fission reaction with respect to $\beta$ or $\lambda$ is expressed as follows:

$$Q_{\beta n} = \frac{1}{W_f} \frac{\partial W_f}{\partial \beta} = -\frac{i\omega}{\lambda + i\omega(1-\beta)}, \tag{27}$$

$$Q_{\lambda n} = \frac{1}{W_f} \frac{\partial W_f}{\partial \lambda} = \frac{i\omega\beta}{(i\omega + \lambda)(\lambda + i\omega(1-\beta))}. \tag{28}$$

The term $P_i(\alpha)$ in Eq. (19) for $\alpha = \omega$, $\beta$, and $\lambda$ are expressed as follows:

$$P_i(\omega) = \sum_{n=1}^{i-1} Q_{\omega n} - \sum_k \frac{is_k}{v_g}, \tag{29}$$

$$P_i(\beta) = \sum_{n=1}^{i-1} Q_{\beta n}, \tag{30}$$

$$P_i(\lambda) = \sum_{n=1}^{i-1} Q_{\lambda n}, \tag{31}$$

where the summation for $k$ is performed throughout the entire region, and the summation for $n$ is performed over all fissions until the $(i-1)$th collision. After the source particle and all resulting progenies are killed, the next source particle is re-emitted. This process is repeated many times until the desired statistics are obtained. The mean value of the first derivative of the neutron noise $\delta\varphi_{g,j}$ within region $j$ with respect to parameter $\alpha$ is expressed as follows:

$$\frac{\partial \delta\varphi_{g,j}}{\partial \alpha} = \frac{1}{M} \sum_{m=1}^{M} \frac{\partial \delta\varphi_{g,j,m}}{\partial \alpha}, \tag{32}$$

where $M$ is the total number of source particles.

The advantages of the DOS method used in this study are as follows. For Monte Carlo sensitivity analyses, the approximate method where the quantity of interest is slightly perturbed is almost impossible due to the statistical uncertainties involved in Monte Carlo calculations. Alternative Monte Carlo sensitivity analysis methods other than the DOS are adjoint-based methods (Kiedrowski, 2017); however, the adjoint based methods such as the iterated fission probability method are more cumbersome than the DOS.

### 3.2. Numerical examples of DOS

The algorithm of the DOS presented in Section 3.1 was applied to the problem discussed in Section 2. Because the diffusion method to obtain the sensitivity coefficients has been verified, the Monte Carlo algorithm was verified through a comparison with results obtained using the diffusion method. The neutron noise distributions were obtained by solving Eq. (18) using an in-house Monte Carlo calculation code. The Monte Carlo results are shown along with the results obtained using the diffusion method presented in Fig. 1. The proposed DOS algorithm was implemented into the in-house code, and the sensitivity coefficients were obtained using the code. The sensitivity coefficients of the neutron

**Table 3**
Sensitivity coefficients of neutron noise amplitude in fast energy group at $f = 1$ Hz, $r = 18.09$ cm

| $\alpha$ | $S(\lvert\delta\varphi_1\rvert;\alpha)$ | |
|---|---|---|
| | Diffusion | Monte Carlo |
| $\Sigma_{c1}$ | −18.74 | −18.82 ± 0.06 |
| $\Sigma_{c2}$ | −37.21 | −37.33 ± 0.19 |
| $\Sigma_{f1}$ | 10.14 | 10.16 ± 0.03 |
| $\Sigma_{f2}$ | 37.93 | 38.03 ± 0.09 |
| $\Sigma_s^1$ | 24.75 | 24.37 ± 0.05 |
| $\beta$ | −0.7108 | −0.7075 ± 0.0020 |
| $\lambda$[a] | 0.01471 | 0.01475 ± 0.00030 |
| $\omega$[a] | −0.1486 | −0.1476 ± 0.0025 |

[a] These values are for 0.1 Hz instead of 1 Hz because they are approximately zero for 1 Hz.

noise amplitude in the fast energy group defined in Eq. (14) (as shown in Table 3) obtained using the DOS and diffusion methods were compared. The position for the sensitivity coefficient was $r = 18.09$ cm. This position was sufficiently remote from the noise source and material boundary, where the diffusion approximation was relatively appropriate. The frequency of the sensitivity coefficients was 1 Hz, except for $\alpha = \omega$ and $\lambda$ because the sensitivity coefficients of these two parameters were extremely small at 1 Hz. In fact, the sensitivity coefficients for $\omega$ and $\lambda$ were 0.1 Hz. The DOS results of the sensitivity coefficients were consistent with the diffusion results for all the parameters. Although the results of the thermal energy group are not presented, the DOS results of the thermal energy group were also consistent with the diffusion results. The sensitivity analysis method for neutron noise using DOS was verified through a comparison with the diffusion method.

## 4. Conclusions

Both deterministic and stochastic neutron noise analysis methods have been developed hitherto. A new method for calculating the sensitivity coefficient of neutron noise with respect to the cross section and kinetic parameters was presented herein. A formula that yields the first derivative of neutron noise was derived by differentiating the neutron noise diffusion equation with respect to a parameter. This method was verified through a comparison with accurate direct perturbation results. As a stochastic method, the DOS method was applied to the neutron noise transport equation in the frequency domain. An algorithm for kinetic parameters was newly proposed, whereas the DOS algorithm for a cross section was identical to the conventional method. The DOS method was verified through a comparison with the diffusion method.

The technique developed in this study can be applied to existing codes without requiring significant modifications. Furthermore, the methods developed in this study do not require adjoint mode calculations. The formula for the first derivative derived in this study can be expanded to higher-order, cross-term analyses. The newly developed method can expand the availability of neutron noise techniques for ensuring nuclear reactor safety.

Throughout this study, the noise source term in the neutron noise equation was assumed to be invariant with respect to the parameters. Future work includes considering the dependence of the noise source term on the parameters.

## Declaration of competing interest

The authors declare no conflicts of interest associated with this manuscript.

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