Journal of Public Economics 211 (2022) 104589

Contents lists available at ScienceDirect

Journal of Public Economics

journal homepage: www.elsevier.com/locate/jpube

Pass-through, welfare, and incidence under imperfect competition ☆

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ARTICLE INFO

Article history:
Received 15 February 2021
Revised 28 December 2021
Accepted 28 December 2021
Available online 3 June 2022

JEL classification numbers:
D43
H22
L13

Keywords:
Imperfect Competition
Pass-through
Marginal Value of Public Funds
Incidence
Sufficient Statistics

ABSTRACT

This paper provides a comprehensive framework to study welfare effects of multiple policy interventions and other external changes under imperfect competition with emphasis on specific and ad valorem taxation as a leading case. Specifically, in relation to tax pass-through, we provide "sufficient statistics" formulas for two welfare measures under a fairly general class of demand, production cost, and market competition. The measures are (i) marginal value of public funds (i.e., the marginal change in consumer and producer surplus relative to an increase in the net cost to the government), and (ii) incidence (i.e., the ratio of a marginal change in consumer surplus to a marginal change in producer surplus). We begin with the case of symmetric firms facing both unit and ad valorem taxes to derive a simple and empirically relevant set of formulas. Then, we provide a substantial generalization of these results to encompass firm heterogeneity by using the idea of tax revenue that is specified as a general function parameterized by a vector of policy instruments including government and non-government interventions and costs other than taxation.

1. Introduction

In thinking of market intervention such as taxation, it is essential to understand how such a policy change distorts economic welfare. In addition, policymakers may also be concerned about distributional consequences, i.e., how the tax burden is borne by consumers and producers subject to such a change in tax policy. Generally, this issue is related to *pass-through*—a measure of how an infinitesimal change in the surrounding environment that firms face affects final prices—and *incidence*—a measure of how it impacts economic agents differently. Pass-through is important because it is closely related to (i) how much of the burdens or benefits for a society accrues due to a change in the environment that surrounds firms, and (ii) how such burdens or benefits are divided between the demand side (consumers) and the supply side (firms). Examples other than taxation include a change in the exchange rate and a technological improvement that causes reduction of costs in production, to name a few.

The importance of pass-through has been recognized since, at least, Cournot (1838, Ch. 6) in the context of monopoly. A convenient framework that extends to imperfect competition in general is presented by Weyl and Fabinger (2013) who stress the price theory tradition since Marshall (1890) in recognizing the importance of pass-through and incidence in a range of economic questions. This generalization is important because many industries are characterized as oligopolies where a small number of firms are dominant. However, Weyl and Fabinger (2013) focus only on the case of *uni-dimensional* shift such as

☆ We are very grateful to Nathan Hendren for a number of encouraging comments that have greatly contributed to the improvement of the paper as well as to two anonymous referees for insightful suggestions. We also thank Yong Chao, Naoshi Doi, Germain Gaudin, Jonas Häckner, Makoto Hanazono, Chiaki Hara, Masayoshi Hayashi, Mathias Herzing, Hiroaki Ino, Jakub Kastl, Konstantin Kucheryavyy, Laurent Linnemer, Carol McAusland, Babu Nataha, Hikaru Ogawa, Robert Ritz, Tadashi Sekiguchi, Yasuhiro Shirata, and Glen Weyl as well as conference and seminar participants for helpful discussions. An earlier version of the paper was circulated under the title of "Multi-Dimensional Pass-Through and Welfare Measures under Imperfect Competition." Adachi and Fabinger acknowledge a Grant-in-Aid for Scientific Research (C) (15K03425; 18K01567; 21K01440) and a Grant-in-Aid for Young Scientists (A) (26705003) from the Japan Society for the Promotion of Science, respectively. Adachi also thanks financial support from the Japan Economic Research Foundation. Any remaining errors are solely ours.

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https://doi.org/10.1016/j.jpubeco.2021.104589
0047-2727/

a constant change in marginal cost precipitated by the introduction of a *unit/specific* tax.[1]

Extending their framework, this paper provides a generalization of Weyl and Fabinger's (2013) model to encompass the case of *multi-dimensional* interventions, including non-governmental external changes, under general forms of market demand, production cost, and imperfect competition. As in Weyl and Fabinger (2013) who consider a specific tax as an example of a *uni*-dimensional intervention, this paper studies a canonical problem of *specific and ad valorem* taxation as a leading example of a multi-dimensional intervention.[2]

Our main contributions relative to Weyl and Fabinger (2013) are threefold. First, we study the welfare consequences of taxation more broadly than they do: in particular, we also consider the "marginal value of public funds" (MVPF; see below). Second, we, as stated above, include ad valorem taxes, whereas they only consider specific taxes. We also stress (in Appendix C) that our analysis of two-dimensional taxation opens up a methodology for encompassing more general cases of multiple interventions such as combinations of taxation and non-tax costs such as market regulations. Lastly, we allow for pre-existing (i.e., non-zero) taxes of either type, whereas Weyl and Fabinger (2013) consider specific taxation in previously untaxed markets. Notably, our framework is readily extendible to the case of heterogeneous firms.[3]

As stated above, our arguments are best understood in the case of two-dimensional taxation in which symmetric firms face both unit and ad valorem taxes, an argument that generalizes Anderson et al. (2001a) and Häckner and Herzing's (2016) analyses of taxation under imperfect competition to derive "sufficient statistics" formulas expressed in terms of observable and estimable variables such as elasticities (Chetty, 2009; Kleven, 2021).[4] These formulas relate pass-through of the taxes to (i) *marginal value of public funds* (MVPF) and (ii) *incidence*, i.e., the ratio of a marginal change in consumer surplus to a marginal change in producer surplus. We also generalize Weyl and Fabinger's (2013) analysis in this dimension because they do not focus on the MVPF aspects of taxation. Here, the MVPF is a simple benefit/cost ratio that measures the marginal change in aggregate surplus in the private sector—consumer surplus as well as corporate surplus that is positive under imperfect competition—relative to a marginal change in the net cost to the government (Hendren, 2016; Hendren and Sprung-Keyser, 2020): in the analysis below, a higher MVPF corresponds to a greater welfare *cost* imposed per dollar revenue raised because we solely focus on the revenue side of the government (i.e., taxation) without considering any beneficial effects of government spending on consumers as well as firms by way of public policy or public goods provision (e.g., Lockwood, 2003).[5] In addition, we complement Weyl and Fabinger's (2013) analysis by providing graphical illustrations to facilitate an intuitive understanding of welfare effects of specific and ad valorem taxation under firm symmetry (see Subsection 2.2).

The welfare aspects of taxation have been extensively studied since, at least, Pigou (1928). A majority of existing studies simply assume perfect competition (together with zero pre-existing taxes).[6] As is widely known, unit and ad valorem taxes are equivalent in achieving the same level of revenue under this situation, and whether consumers or producers bear more is determined by the relative elasticities of demand and supply (Weyl and Fabinger, 2013, p. 534). Relaxing the assumption of perfect competition was initially attempted by the studies of homogeneous-product oligopoly under quantity competition, i.e., Cournot oligopoly. Notably, Delipalla and Keen (1992), Skeath and Trandel (1994), Hamilton (1999), Anderson et al. (2001b) compare unit and ad valorem taxes in such a setting.[7] Anderson et al. (2001a), then, extend these results to the case of differentiated oligopoly under price competition. Specifically, they find that whether the after-tax price for firms and their profits rise by a change in ad valorem tax depends importantly on the ratio of the curvature of the firm's own demand to the elasticity of market demand. Miravete et al. (2018) also stress the importance of imperfect competition in considering policy recommendations: they find empirical relevance of firms' strategic responses in pricing when evaluating the effect of taxation, implying the necessity of considering imperfect competition for policy evaluation.

In contrast to these previous studies, one appealing feature of our framework is that—as in Weyl and Fabinger (2013) and Kroft et al. (2020), among others—we introduce the *conduct index,by which we mean* the conduct paramete*r* that is not necessarily constant across the level of output. The conduct index measures the degree of market monopolization and hence nesting a variety of market structures. It allows us to work with a

[1] The importance of (uni-dimensional) pass-through has also been recognized in empirical studies of homogeneous or differentiated product markets in oligopoly, see, e.g., Bonnet and Réquillart, 2013; Campos-Vázquez and Medina-Cortina, 2019; Conlon and Rao, 2020; Duso and Szücs, 2017; Ganapati et al., 2020; Griffith et al., 2018; Jametti et al., 2013; Kim and Cotterill, 2008; Miller et al., 2017; Muehlegger and Sweeney, 2022; Shrestha and Markowitz, 2016; Stolper, 2021; Conlon. and Rao, 2019; Genakos and Pagliero, 2022. See also Ritz (2018) and references therein for theoretical studies on pass-through and pricing under imperfect competition, including monopolistic competition.

[2] However, readers should be reminded that our analysis is not only confined to specific and ad valorem taxation: interested readers are encouraged to consult Appendix C for our general framework of *multi-dimensional pass-through* that our analysis is based on as well as Online Appendix B for some examples of outside of taxation.

[3] From the viewpoint of optimal taxation without entry/exit, it is well known that ad valorem taxes are more efficient (i.e., less welfare distorting) than unit taxes in raising the same amount of tax revenue (see Wicksell (1896); Suits and Musgrave (1953); and Delipalla and Keen (1992) for earlier studies), which implies that no unit taxes should be used (however, in the presence of negative externalities such as pollution, unit taxes can be superior to ad valorem taxes; see Pirttilä, 2002). In our generalized framework, this is also verified (see the last paragraph of Subsection 2.5 below). It appears that this generally holds, as a numerical analysis in Section 4 argues, even if cost heterogeneity between firms is allowed, though Anderson et al. (2001b) find a counter example: a further investigation is left for future research. In reality, specific and ad valorem taxes are often used together for commodity taxation (e.g., gasoline, alcohols, tobaccos, sodas, etc), which suggests the relevance of studying both taxes in a unified framework (see, e.g., Keen, 1998).

[4] Adachi (2022) also shares the same spirit with the sufficient statistics approach in that they also provide welfare formulas for oligopolistic third-degree price discrimination based on sufficient statistics, including pass-through, under fairly general conditions.

[5] Therefore, the net cost to the government in this study is a negative of an increase in government revenue. Conversely, for expenditure or subsidy policies, a higher MVPF implies greater welfare *gain* per dollar spent. Note also that, in contrast to the traditional definition of the marginal cost of public funds or the MCPF (Stiglitz and Dasgupta, 1971; Atkinson and Stern, 1974; Dahlby, 2008), the MVPF focuses directly on causal, not on compensated, effects of public policy, and hence is widely applicable in guiding cost-benefit analysis in a more systematic manner (Finkelstein and Hendren, 2020; Paradisi 2021). This Hendrenian MVPF is identical to the Marginal Excess Burden or the MEB by Mayshar (1990) and the MCPF by Slemrod and Yitzhaki (1996), Slemrod and Yitzhaki (2001) and Kleven and Kreiner (2006). We thank Nathan Hendren for making us realize this point.

[6] The early studies include Vickrey (1963), Buchanan and Tullock (1965), Johnson and Pauly (1969) and Browning (1976). See also Auerbach and Hines (2002) and Fullerton and Metcalf (2002) for comprehensive surveys of this field.

[7] More specifically, Delipalla and Keen (1992) firstly showed that ad valorem taxes are welfare superior to unit taxes with symmetric quantity-setting firms. Skeath and Trandel (1994) further strengthen Delipalla and Keen's (1992) results by showing the Pareto dominance: for a given level of unit tax under monopoly, there always exists an ad valorem tax that yields higher levels of all of consumer surplus, firm profits, and tax revenue. Under Cournot oligopoly, Skeath and Trandel (1994) show the same result holds if the required amount of tax revenue is sufficiently large, and this requirement depends on the demand curve and the number of firms in the market.

fairly general mode of market competition and to capture its complicated nature in reality: both from a theoretical and an empirical standpoint, it is desirable to understand the welfare properties of oligopolistic markets for a fairly general class of competition.[8] In real-world situations, firms' conduct might not simply be categorized into either idealized price or quantity competition, and may include the possibility of collusive behavior as well.

Furthermore, we are able to provide sufficient statistics formulas for the welfare measures that are useful for empirical study because we also accommodate *firm heterogeneity*, which cannot be neglected in almost any data. When it is considered in Section 4, we introduce the *pricing strength index* that is firm-specific and measures the degree of the firm's market power: this concept is related to the conduct index, but is much better to work with when firms are not identical. It turns out that our characterization of the two welfare measures is readily extendible to this case of firm heterogeneity.

In this sense, this paper aims to be a response to a commonly held view, particularly in the field of public finance, exemplified by the following quotations from three representative textbooks (in chronological order; emphasis added):

> "Unfortunately, there is *no well-developed theory of tax incidence in oligopoly*. [...] As economic behavior under oligopoly becomes better understood, improved models of incidence will be developed" (Rosen and Gayer, 2014, pp. 310-311).
>
> "There is no widely accepted theory of firm behavior in oligopoly, so it is *impossible to make any definite predictions about the incidence of taxation in this case*" (Stiglitz and Rosengard, 2015, p. 556).
>
> "Although there are widely accepted models for how competitive and monopolistic markets work, there is much less consensus on models for oligopolistic markets. As a result, economists tend to assume that the same rules of tax incidence apply in these markets as well, but *there is more work to do to understand the burden of taxes in oligopoly markets*" (Gruber, 2019, p. 601).[9]

In a similar vein, Kroft et al. (2020) also consider a comparison of ad valorem and unit taxes, and derive a sufficient statistics formula for the welfare burden of commodity taxation as well as its incidence under imperfect competition, especially in consideration of the possibility of "behavioral" consumers having misconceptions about whether the price is tax inclusive. Specifically, they parameterize the degree of how accurately consumers attribute a change in consumer price to the change in tax behind and calibrate the marginal excess burden of commodity taxation by maintaining firm symmetry. In contrast, we aim to provide general formulas for welfare measures that allow for firm heterogeneity. In this sense, their study and ours are complementary in providing structural frameworks that are useful for welfare evaluation in consideration of a variety of important policy issues under imperfect competition.[10]

The remainder of this paper is organized as follows. In the next section, we construct our model of specific and ad valorem taxation under symmetric imperfect competition, and present general formulas for marginal value of public funds as well as incidence in relation to tax pass-through, elasticity of market demand, and the market conduct, among others. In Section 3, we conduct a numerical analysis for these formulas by employing three representative classes of market demand. Then, Section 4 further generalizes our formulas to include heterogeneous firms. Finally, Section 5 concludes the paper. Note that some detailed arguments are delegated to the appendices. In particular, Appendix C provides a more general framework, which Section 4 is based on, than simply (two-dimensional) specific and ad valorem taxes to accommodate multi-dimensional interventions including additional changes (such as a change in exchange rate) that accrue outside of government activities. Lastly, among other interesting generalizations, Online Appendix B illustrates some applications other than taxation such as tax evasion, and a sales restriction due to, for instance, the outbreak of a pandemic.

2. Specific and Ad Valorem Taxation under Symmetric Imperfect Competition

In this section, we study symmetric oligopoly. Before we start, let us point out that the formulas we derive are not much longer than the corresponding formulas for the special case of monopoly. We keep our derivations explicit to emphasize the logical flow, which generalizes beyond specific and ad valorem taxes and beyond symmetric firm oligopoly (see Appendix C). We use figures as visual anchors to help the reader clearly understand the many welfare component changes and many forces that play a role in the discussion (see Subsection 2.2 below).

This section generalizes the results of Anderson et al. (2001a) (ADKa) in several important directions. First, we consider a fairly general class of market competition, captured by the *conduct index* (see below), including both quantity and price competition. Second, we provide a complete characterization of welfare measures that enables one to quantitatively compare consumers' burden with producers' burden, whereas ADKa focus only on the effective prices for consumers and producers' profits. Third, while ADKa assume constant marginal cost, we permit non-constant marginal cost and show how this generalization makes a difference in our generalized formulas. Fourth, we further generalize the initial tax level. When they analyze the effects of a unit tax, ADKa assume that ad valorem tax is zero, and vice versa. In contrast, we allow non-zero initial taxes in both dimensions. Overall, it turns out that generalizing the ADKa results of the two-dimensional tax problem is suggestive in studying a much wider range of

[8] The interested reader should refer to Weyl and Fabinger (2013) for examples of market structures that are nested by the conduct index approach. The concept of conduct parameter has been developed mainly in the empirical industrial organization literature (see, e.g., Bresnahan, 1989; Delipalla and O'Donnell, 2001), and has also been successfully applied to such issues as selection markets (Mahoney and Weyl, 2017), supply chains (Gaudin, 2018; Adachi, 2020) and two-sided markets (Adachi and Tremblay, 2020). It also has pedagogical usefulness (Menezes and Quiggin, 2020). See Footnote 15 below for more details.

[9] On the other hand, Hindriks and Myles (2013) and Tresch (2015), which are two other representative textbooks, have no explicit reference to tax incidence in oligopoly.

[10] Note that Kroft et al. (2020) allow consumer heterogeneity whereas we maintain the representative consumer assumption. In other contexts, for example, Atkin and Donaldson (2016) study the pass-through rate provides a sufficient statistic for welfare implications of intra-national trade costs in low-income countries, without the need for a full demand estimation. Hollenbeck and Uetake (2021) also use the idea of sufficient statistics in their study of tax revenue in the legalized cannabis market to estimate tax incidence and social cost of tax, based on the estimation of the conduct parameter as well as cost pass-through under firm symmetry and find that there is significant room for climbing up the Laffer curve from the left side, i.e., for raising a higher amount of tax revenue by an increase in the tax rate. Alternatively, it is also effective to strengthen the intensity of competition by deregulating the license cap for a higher amount of tax revenue. In a different vein, Montag et al. (2021) propose a search model of heterogeneous consumers, where different consumers incur different costs of searching sellers and their prices and find that the tax pass-through is higher if the search cost becomes lower.

interventions and taxes to characterize welfare measures in terms of sufficient statistics.

Below, we employ the standard assumption that the representative consumer has quasi-linear utility, $U(\mathbf{q}, y) = u(\mathbf{q}) + y$, where $\mathbf{q} \equiv (q_1, \ldots, q_n)$ is their consumption bundle from n single-product firms in the industry, and $y > 0$ is a numeraire outside good with no taxes. In effect, we assume that all markets outside this industry are perfectly competitive to isolate this particular market from such feedback effects as income effects that may arise in a general-equilibrium framework. A full-fledged analysis of "imperfect competition in general equilibrium" awaits further research in this direction (see, e.g., d'Aspremont and Dos Santos Ferreira, 2021). We hereafter use t for specific taxes (unit taxes) and v for ad valorem taxes. In most applications, these would be non-negative.[11]

Then, following Häckner and Herzing (2016), Kroft et al. (2020), and many others, we define social welfare W as $W = CS + PS + R$, where CS, PS, and R denote consumer surplus, producer surplus (corporate profit), and tax revenue, respectively. The main task of this paper is to characterize two important measures for the welfare effects of commodity taxation: (i) the marginal value of public funds

$$MVPF_T \equiv \left(-\frac{\partial R}{\partial T}\right)^{-1}\left(\frac{\partial CS}{\partial T} + \frac{\partial PS}{\partial T}\right),$$

and (ii) the incidence

$$I_T \equiv \left(\frac{\partial PS}{\partial T}\right)^{-1}\left(\frac{\partial CS}{\partial T}\right)$$

for $T \in \{t, v\}$.[12]

2.1. Setup

Here we study an oligopolistic market with n symmetric firms and a general mode of competition, and consider the resulting symmetric equilibrium. Formally, the demand for firm i's product $q_i = q_i(p_1, \ldots, p_n) \equiv q_j(\mathbf{p})$ depends on the vector of prices, $\mathbf{p} \equiv (p_1, \ldots, p_n)$, charged by the individual firms. The demand system is symmetric and the cost function $c(q_i)$ is the same for all firms. We assume that $q_i(\cdot)$ and $c(\cdot)$ are twice differentiable and the conditions for the uniqueness of equilibrium as well as the associated second-order conditions are satisfied. The marginal cost of production is defined by $mc(q) \equiv c'(q)$.

We denote by $q(p)$ per-firm industry demand under symmetric prices: $q(p) \equiv q_i(p, \ldots, p)$. The elasticity of this function, defined as $\epsilon(p) \equiv -pq'(p)/q(p) > 0$ and referred to as the *price elasticity of industry demand,* should not be confused with the elasticity of the residual demand that any of these firms faces.[13] We also define by $\eta(q) \equiv 1/\epsilon(p)|_{q(p)=q}$ the reciprocal of this elasticity as a function of q. When we do not need to specify explicitly their dependence on either q or p in the following analysis, we use η interchangeably with $1/\epsilon$. In addition, we define the *industry inverse demand function* p(q) as the inverse of $q(p)$, which satisfies $\eta(q) = -qp'(q)/p(p)$.[14]

As mentioned above, we introduce two types of taxation: a specific tax (unit tax) t and an ad valorem tax v, with firm i's profit being $\pi_i = (1-v)p_i(\mathbf{q})q_i - tq_i - c(q_i)$. At symmetric output q, the government tax revenue per firm is $R(q) \equiv tq + vp(q)q$, which we can separate into the specific tax part and the ad valorem part: $R(q) = R_t(q) + R_v(q), R_t(q) = tq, R_v(q) = vp(q)q$. We denote by $\tau(q)$ the fraction of firm's pre-tax revenue that is collected by the government in the form of taxes: $\tau(q) \equiv R(q)/pq = v + t/p(q)$, as this notation makes many expressions simpler.

In the special case of monopoly, the first-order condition for the equilibrium would be $(1-v)mr(q) - t = mc(q)$ with $mr(q) = p(q) + qp'(q) = p(q) - \eta(q)p(q)$ and $mc(q) = c'(q)$.

This condition can be rearranged as

$$\frac{1}{\eta(q)p(q)}\left(p(q) - \frac{t + mc(q)}{1-v}\right) = 1.$$

Intuitively, the left-hand side measures a degree of departure from competitive pricing, which would have $p(q) - [t + mc(q)]/(1-v) = 0$. We use this intuition to write a more general form of the first order condition that applies to oligopoly.

For oligopoly, we introduce the *conduct index* $\theta(q)$, which measures the degree of market monopolization and is determined *independently* of the cost side. The conduct index $\theta(q)$ is defined by the requirement that the symmetric equilibrium condition takes the form

$$\frac{1}{\eta(q)p(q)}\left(p(q) - \frac{t + mc(q)}{1-v}\right) = \theta(q), \tag{1}$$

where $mc(q) \equiv c'(q)$ is the marginal cost of production.[15] Perfect competition corresponds to $\theta(q) = 0$ and monopoly to $\theta(q) = 1$.[16] With a little abuse of notation, we denote the equilibrium price by p, and assume that any equilibrium is symmetric. We further impose a condition on the functions in Eq. (1) to ensure that any equilibrium is necessarily unique.[17]

We denote by θ the functional value of $\theta(q)$ at the equilibrium quantity. We can think of it as an *elasticity-adjusted Lerner index*. The Lerner index $[p - (t + mc)/(1-v)]/p$ multiplied by the industry demand elasticity $\epsilon = 1/\eta$ equals θ. Here the Learner index is based on an (effective) marginal cost $(t + mc)/(1-v)$.[18] We emphasize that once the conduct index is introduced, it becomes

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[11] One may wonder if the welfare distortion in this market can be eliminated if the unit tax is not constrained to be non-negative. This is because, starting from any combination of taxes t and v, it is possible to keep the same level of government revenue but unambiguously lower the deadweight loss by raising v just enough to generate a marginal unit of revenue, and simultaneously lowering t just enough give back that marginal unit of revenue. Extending this reasoning, Myles (1999) finds that the optimal combination entails a positive ad valorem tax and a negative unit tax, although in reality the feasibility of this method would be very limited.

[12] Note that Häckner and Herzing (2016) call

$$\left(-\frac{\partial R}{\partial T}\right)^{-1}\left(\frac{\partial CS}{\partial T} + \frac{\partial PS}{\partial T} + \frac{\partial R}{\partial T}\right)$$

the Marginal *Cost* of Public Funds. Similarly, Kroft, Laliberté, Leal-Vizcaíno, and Notowidigdo (2020) focus on $\partial W/\partial T$ as the marginal excess burden as a welfare measure. This is equal to $(1 - MVPF_T)(\partial R/\partial T)$.

[13] The elasticity ϵ here corresponds to ϵ_D in Weyl and Fabinger (2013, p. 542). Note that

$$q'(p) = \frac{\partial q_i(\mathbf{p})}{\partial p_i} + (n-1)\frac{\partial q_i(\mathbf{p})}{\partial p_j}|_{\mathbf{p}=(p,\ldots,p)}$$

for any two distinct indices i and j. We define the firm's elasticity and other related concepts in Appendix B.

[14] In the case of monopoly, there is no distinction between the industry demand and the demand for the monopolist's good. Then $q(p)$ is the monopolist's demand curve, ϵ is its elasticity, and η is the reciprocal of the elasticity.

[15] As already noted in Footnote 8 above, $\theta(q)$ is a generalization of conduct parameter in the sense that it is a function of q rather than a constant for any q. Hence, Eq. (1) *should not be interpreted as an equation that defines* $\theta(q)$. For our analysis, we can just introduce $\theta(q)$ in an implicit manner: $\theta(q)$ is a function independent of the cost side of the problem, in which Eq. (1) is the symmetric first-order condition of the equilibrium. Note that $\theta(q) > 1$ is not necessarily excluded, although in most interesting cases, it lies in $[0, 1]$.

[16] Symmetric Cournot oligopoly also corresponds to a constant conduct index, which in this case takes the value of $1/n$, where n is the number of firms. But more generally, $\theta(q)$ depends on q.

[17] The condition is as follows. Eq. (1) may be rearranged as

$$(1 - \eta(q)\theta(q))p(q) - \frac{1}{1-v}(t + mc(q)) = 0.$$

We require that the left-hand size be a decreasing function q. For constant marginal cost, this translates to the requirement that $(1 - \eta(q)\theta(q))p(q)$ be a decreasing function of q. In the special case of monopoly, $\theta(q) = 1$, this reduces to the requirement of decreasing marginal revenue.

[18] The tax-adjusted Lerner rule $[p - (t + mc)/(1-v)]/p = \eta\theta$ implies the restriction on θ, namely $\theta \leqslant \epsilon$.

possible to describe oligopoly in a unified manner, without specifying whether it is price or quantity setting, or whether it exhibits strategic substitutability or complementarity.[19]

Finally, we define the specific tax pass-through rate ρ_t and the ad valorem pass-through semi-elasticity ρ_v as

$$\rho_t = \frac{\partial p}{\partial t}$$

and

$$\rho_v = \frac{1}{p}\frac{\partial p}{\partial v},$$

respectively, where the equilibrium price p is considered as a function of the tax levels. Both ρ_t and ρ_v are dimensionless. The reason for considering semi-elasticity for the ad valorem tax becomes clear in the next subsection, where several results take the same form for both taxes and differ just by the presence of ρ_t or ρ_v.[20] They are also non-negative because otherwise second-order conditions for the equilibrium would be violated.[21]

2.2. Welfare components

Now, we develop how a tax reform affects government revenue, consumer surplus, producer surplus, and social welfare as the sum of these. The derivations in the current and next subsections provide the building blocks for the propositions in the subsequent subsections.

More specifically, the welfare characteristics we study are related to the following four welfare components: (i) consumer surplus per firm $CS = \int_0^q p(\tilde{q})d\tilde{q} - pq$, (ii) ad valorem tax revenue per firm $R_v = vpq$, (iii) specific tax revenue per firm $R_t = tq$, and (iv) producer surplus per firm $PS = (1-v)pq - tq$. These are depicted in Fig. 1.[22] The points $A_0, B_0, C_0, D_0, E_0, F_0$ are at $q = 0$ and the points A, B, C, D, E are at the equilibrium quantity for a given value of the taxes t and v. Total cost (per firm) $c(q) = \int_0^q mc(\tilde{q})d\tilde{q}$ corresponds to B_0BAA_0, producer surplus to C_0CBB_0, specific tax revenue to D_0DCC_0, ad valorem tax revenue to E_0EDD_0, and consumer surplus to the area F_0EE_0. The total (per firm) welfare $W = PS + R_t + R_v + CS$ is represented by the area F_0EBB_0. The point O is at the socially optimal quantity, and the area EOB represents the deadweight loss.

This figure shows five generally non-linear functions: $mc(q)$, $(1-v)[1-\theta(q)\eta(q)]p(q) - t$, $(1-v)p(q) - t$, $(1-v)p(t)$, and $p(q)$ that determine the boundaries of the regions. In the special case of monopoly, the figure would look almost the same, except that $(1-v)[1-\theta(q)\eta(q)p(q)] - t$ would be replaced by $(1-v)[1-\eta(q)]p(q) - t$. Then, Figs. 2 and 3 indicate how the diagram would change if we increase the specific tax and the ad valorem tax, respectively. This graphical illustration is helpful for considering the changes in the welfare components if we infinitesimally change the taxes (note that the changes shown in the figures are non-infinitesimal, though).

As the taxes infinitesimally change, $t \to t + dt$, $v \to v + dv$, the areas corresponding to a welfare component change due to a horizontal movement of the regions' right borders (points A, B, C, D, E) and due to a vertical movement of the top and bottom borders of the regions. We call these "quantity effects" ($\leftrightarrow$) and "value effects" ($\updownarrow$), respectively.[23]

For example, the specific tax revenue is tq, and the corresponding infinitesimal change $d(tq) = t\,dq + q\,dt$, consists of a quantity effect $t\,dq$ and a value effect $q\,dt$ because the right border of the region shifts by dq and the vertical height of the region changes by dt. We introduce the following expressions for infinitesimal changes in the welfare components:

$$dPS = dPS_{\leftrightarrow} + dPS_{\updownarrow}$$

$$dR_t = dR_{t\leftrightarrow} + dR_{t\updownarrow}$$

$$dR_v = dR_{v\leftrightarrow} + dR_{v\updownarrow}$$

$$dCS = dCS_{\leftrightarrow} + dCS_{\updownarrow}$$

$$dW = dW_{\leftrightarrow} + dW_{\updownarrow}.$$

First, the change in tax revenue $R = tq + vpq$ is given by

$$dR = dR_t + dR_v = dR_{t\leftrightarrow} + dR_{v\leftrightarrow} + dR_{t\updownarrow} + dR_{v\updownarrow} \quad (2)$$

with

$$dR_{t\leftrightarrow} = t\,dq, \quad dR_{v\leftrightarrow} = vp\,dq, \quad dR_{t\updownarrow} = q\,dt,$$
$$dR_{v\updownarrow} = qv\,dp + qp\,dv. \quad (3)$$

Note here that the two quantity effects, $dR_{t\leftrightarrow}$ and $dR_{v\leftrightarrow}$, result from the "behavioral" change in output: they are presumably negative for a positive change in t and v because $dq < 0$ ("fiscal externality"). The value effect for the specific tax change, $dR_{t\updownarrow}$ purely reflects the "mechanical" change in government revenue with no behavioral response included. In this way, the quantity and value effects for a change in specific tax separately correspond to the behavioral and mechanical changes, respectively. The value effect for the ad valorem tax change, $dR_{v\updownarrow}$, however, includes another behavioral change through the firms' pricing p that affects the infra-marginal consumers as well.

Next, the change in producer surplus, $PS = (1-v)qp(q) - tq - c(q)$, is written as

$$dPS = dPS_{\leftrightarrow} + dPS_{\updownarrow}, \quad (4)$$

and the quantity and value effects are

$$dPS_{\leftrightarrow} = [(1-v)p - (t + mc)]dq, \quad dPS_{\updownarrow} = (1-v)q\,dp - q\,dt - pq\,dv,$$

respectively. Given $dq < 0$, the first term in the bracket of the quantity effect captures loss from a reduction in production, multiplied by the adjusted unit price $(1-v)p$, whereas t and mc express the gains from unit tax saving and from cost savings by the output reduction, respectively. In the value effect $dPS_{\updownarrow}$, the first term corresponds to the (direct) gain from the associated price increase, mitigated by $(1-v)$, due to the ad valorem tax, multiplied by the output q: the firms' behavioral response to $dt > 0$ and $dv > 0$ contributes positively to their profits. However, the mechanical change, $q\,dt + pq\,dv$, has a negative effect: the firms incur the (direct) loss from an increase in unit or ad valorem tax: this is captured by the second and the third terms, respectively. After substituting for $t + mc = (1-v)p(1-\eta\theta)$ from Eq. (1), they are alternatively expressed as

$$dPS_{\leftrightarrow} = (1-v)p\eta\theta\,dq, \qquad dPS_{\updownarrow} = (1-v)q\,dp - q\,dt - pq\,dv, \quad (5)$$

respectively. The first expression implies that the quantity effect is zero under perfect competition (i.e., $\theta = 0$).

For consumer surplus CS, the quantity effect is zero, $dCS_{\leftrightarrow} = 0$, and the value effect is $dCS_{\updownarrow} = -q\,dp$, so that

$$dCS = -q\,dp. \quad (6)$$

Finally, for social welfare W, the value effect is zero, $dW_{\updownarrow} = 0$, because the curves $mc(q)$ and $p(q)$ do not move in response to a tax change, whereas the quantity effect is $dW_{\leftrightarrow} = (p - mc)dq$, implying that $dW = (p - mc)dq$. Substituting for mc using Eq. (1) gives $dW = [t + vp + (1-v)p\eta\theta]dq$, or using our definition $\tau = v + t/p$,

[19] Of course, it is possible to build oligopoly models with even more complicated interactions between firms that would be outside of the scope of the present analysis.

[20] Note that Häckner and Herzing (2016) use the symbol ρ_v for the ad valorem tax pass-through rate $\partial p/\partial v$, which corresponds to $p\rho_v$ in our notation.

[21] This follows using the requirement in Footnote 17 and by totally differentiating the equality there.

[22] In the discussion that follows, we will not say "per firm" explicitly, although we will continue to think about welfare on a per-firm basis. Also, we assume that the producer surplus is finite.

[23] These are analogous to an extensive and an intensive margin, respecitively, but are distinct.

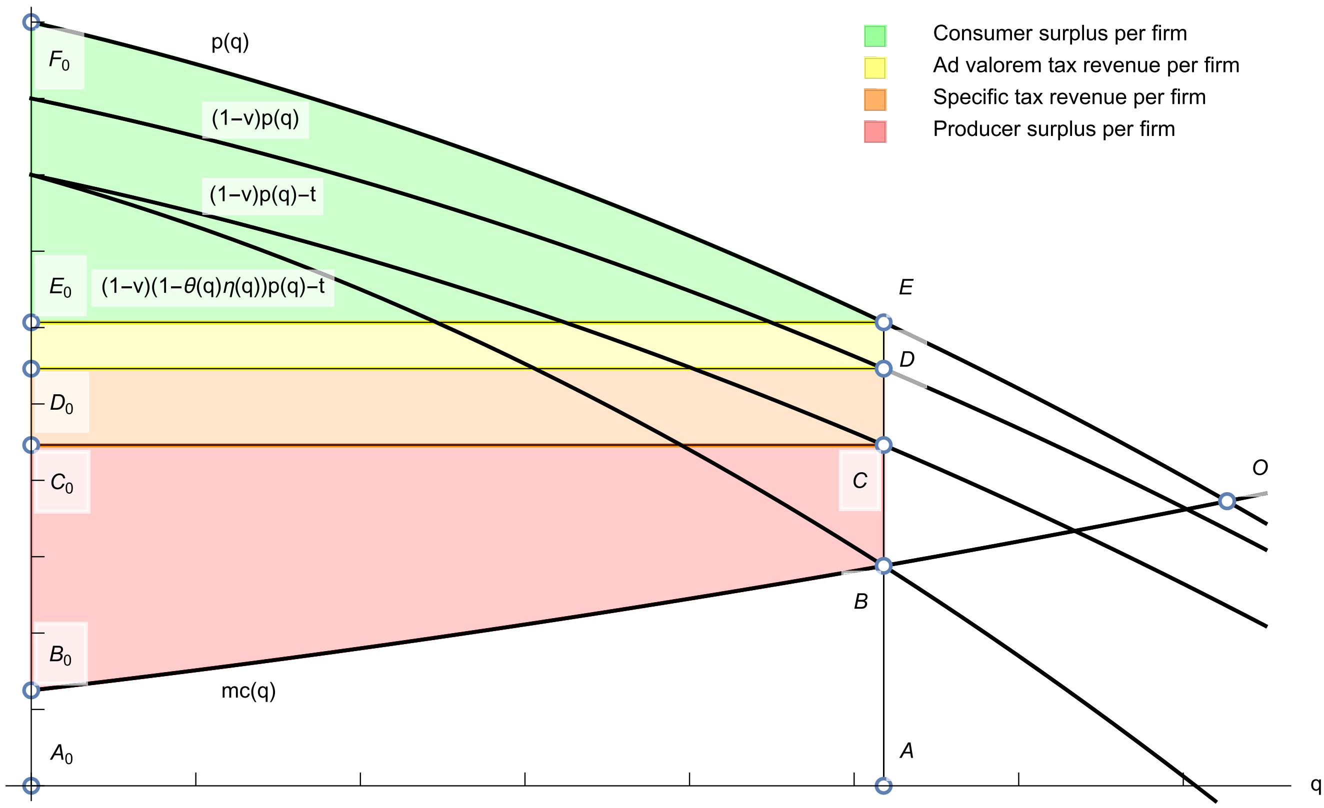

Fig. 1. Welfare components at tax levels $t = 0.1$ and $v = 0.1$ for a chosen case of oligopoly.

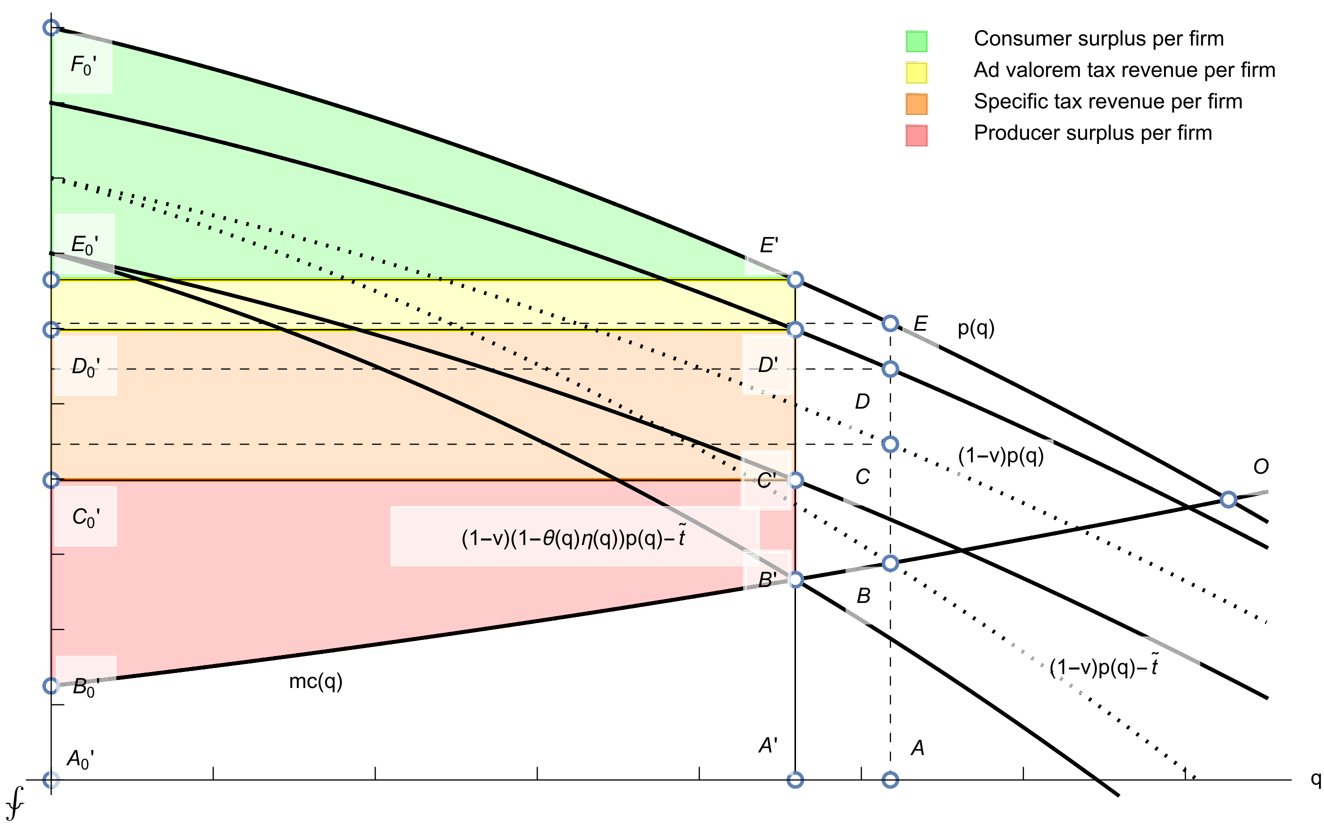

Fig. 2. Visualization of oligopoly welfare components after an increase of the specific tax from $t = 0.1$ to $\tilde{t} = 0.2$, with $v = 0.1$ and $p(0) = 1$, starting from the situation in Fig. 1. In this figure, PS, R_v, and CS decrease, whereas R_t increases. See Appendix A.1 for details.

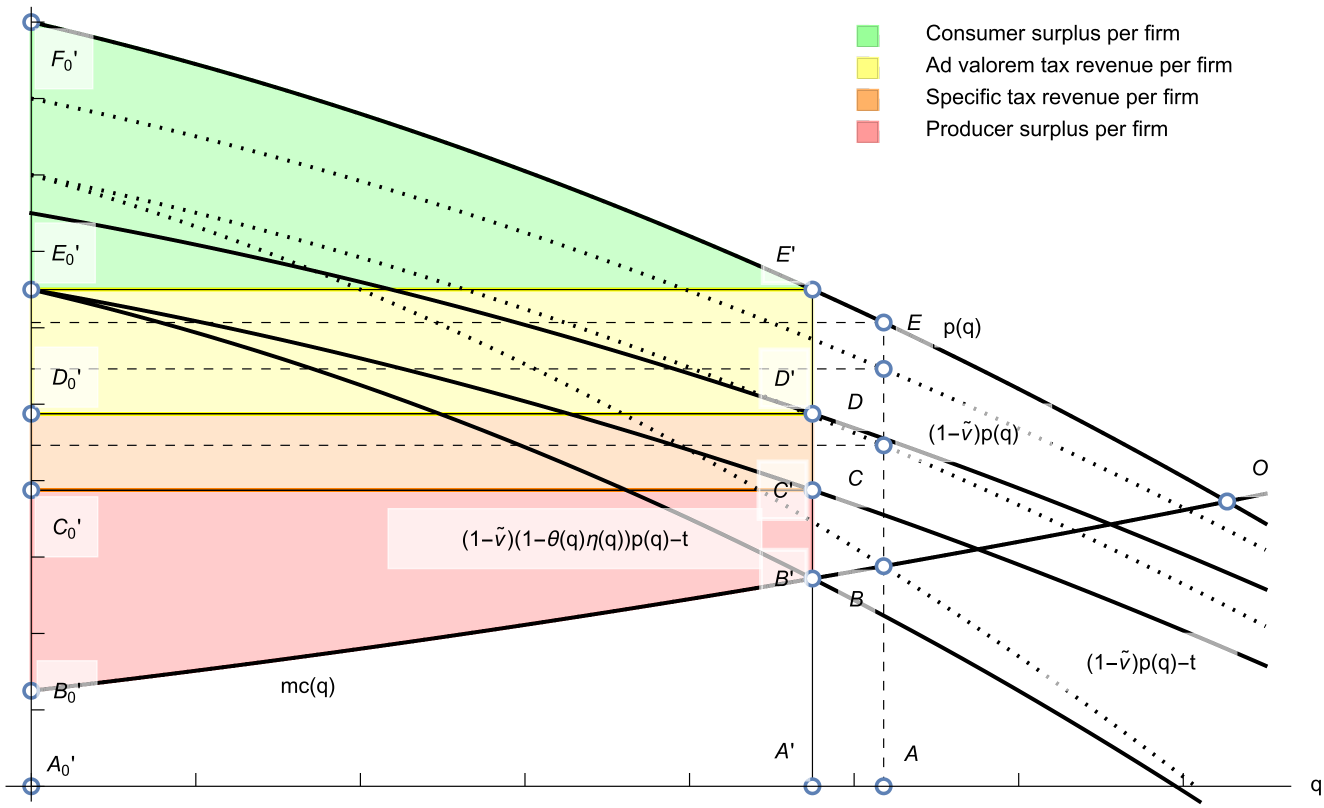

Fig. 3. Visualization of oligopoly welfare components after an increase of the ad valorem tax from $v = 0.1$ to $\tilde{v} = 0.2$, with $t = 0.1$ and $p(0) = 1$, starting from the situation in Fig. 1. In this figure, PS, R_t, and CS decrease, whereas R_v increases. See Appendix A.2 for details.

$$dW = [(1 - v)\eta\theta + \tau]p\,dq. \tag{7}$$

2.3. Changes in equilibrium prices and quantities

It is useful to express infinitesimal price changes and tax changes in terms of infinitesimal quantity changes. In the case of a change in specific tax dt, the price changes by $dp = \rho_t dt$, and the quantity changes by $dq = -q\epsilon\, dp/p$. These relationships imply

$$dt = -\frac{\eta p}{q\rho_t}dq, \qquad dp = -\frac{\eta p}{q}dq. \tag{8}$$

Here, the first relationship states how the mechanical effect, dt, affects the behavioral effect, dq, whereas how the latter is related to the firms' pricing response, dp, is described in the second relationship.

In the case of a change in ad valorem tax dv, the price changes by $dp = \rho_v p\, dv$, while the quantity changes by $dq = -q\epsilon\, dp/p$. Therefore

$$dv = -\frac{\eta}{q\rho_v}dq, \qquad dp = -\frac{\eta p}{q}dq. \tag{9}$$

Note the difference between Eqs. (8) and (9): the mechanical changes, dt and dv, affect the behavioral effect, dq, differently. However, given this behavioral effect, how the firms adjust in their pricing is identical.

2.4. Tax pass-through

First, we show how two pass-throughs *between the ad-valorem and per-unit tax*, ρ_t and ρ_v, are related. This result is interesting for its own sake because it shows that ρ_v is no greater than ρ_t in a general manner.

Proposition 1. *Under symmetric oligopoly with a possibly non-constant marginal cost, the pass-through semi-elasticity ρ_v of an ad valorem tax may be expressed in terms of the unit tax pass-through rate ρ_t, the conduct index θ, and the industry demand elasticity ϵ as*

$$\rho_v = \left(1 - \frac{\theta}{\epsilon}\right)\rho_t. \tag{10}$$

The proof is provided in Appendix A.3.[24]

To understand this proposition intuitively, note that to keep prices and quantities constant, Δt and Δv must satisfy:

$$\frac{t + \Delta t + mc}{1 - (v + \Delta v)} = \frac{t + mc}{1 - v}.$$

Thus, the relative Δt that must be offset by a reduction $-\Delta v$ is equal to $(t + mc)/(1 - v) : \Delta t = -(t + mc)\Delta v/(1 - v)$, which, along with $\rho_t dt + \rho_v p\, dv = 0$, leads to $(t + mc)\rho_t/[(1 - v)p] = \rho_v$. Now, recall the Lerner rule:

$$1 - \frac{t + mc}{(1 - v)p} = \eta\theta,$$

which indicates that $(1 - \eta\theta)\rho_t = \rho_v$, as Proposition 1 claims. Here, $\theta/\epsilon = 1 - \rho_v/\rho_t$ implies that $\rho_v \le \rho_t \le (1 - 1/\epsilon)\rho_v$.

Next, the following proposition shows how the two forms of pass-through are characterized.

Proposition 2. *Under symmetric oligopoly with a general mode of competition and a possibly non-constant marginal cost, the unit tax pass-through is characterized by:*

[24] Under Cournot competition, Eq. (6.13) of Auerbach and Hines (2002) coincides with Eq. (10) above. Proposition 1 implies that their equation holds more generally. We thank Germain Gaudin for pointing this out.

$$\rho_t = \frac{1}{1-v} \cdot \frac{1}{\left[1 + \frac{1-\tau}{1-v}\epsilon\chi\right] - (\eta+\chi)\theta + \epsilon q(\theta\eta)'}$$

where the derivative is taken with respect to q and $\chi \equiv mc'q/mc$ is the elasticity of the marginal cost with respect to quantity. Similarly, the ad valorem tax pass-through is characterized by:

$$\rho_v = \frac{\epsilon-\theta}{(1-v)\epsilon} \cdot \frac{1}{\left[1 + \frac{1-\tau}{1-v}\epsilon\chi\right] - (\eta+\chi)\theta + \epsilon q(\theta\eta)'}.$$

Moreover, in Weyl and Fabinger's (2013) notation, they are expressed as:

$$\rho_t = \frac{1}{1-v} \cdot \frac{1}{1 + \left(\frac{1-\tau}{1-v}\epsilon - \theta\right)\chi + \frac{\theta}{\epsilon_\theta} + \frac{\theta}{\epsilon_{ms}}}$$

and

$$\rho_v = \frac{\epsilon-\theta}{(1-v)\epsilon} \cdot \frac{1}{1 + \left(\frac{1-\tau}{1-v}\epsilon - \theta\right)\chi + \frac{\theta}{\epsilon_\theta} + \frac{\theta}{\epsilon_{ms}}},$$

respectively, where $\epsilon_\theta \equiv \theta/(\theta)'q$ and $\epsilon_{ms} \equiv ms/[ms'q]$ are the inverses of the quantity elasticities of θ, and $ms \equiv -p'q$, which is the "negative of marginal consumer surplus" (Weyl and Fabinger 2013, p. 538), respectively.

This proposition is proved in Appendix A.4.[25]

Let us provide a brief discussion of these results. In the case of prefect competition ($\theta = 0$) and zero initial taxes ($t = 0$ and $v = 0$), the pass through is given by $\rho_t = 1/(1+\epsilon\chi)$ (see Weyl and Fabinger 2013, p. 534) and $\rho_v = 1/(1+\epsilon\chi)$. With non-zero initial taxes, $(t, v) \geq 0$, there are adjustment factors, but the nature of the formulas is similar. More specifically, as in Weyl and Fabinger's (2013, p. 549) explanation, suppose that "θ is invariant to changes in q," i.e., $1/\epsilon_\theta = 0$, and "costs are linear," i.e., $\chi = 0$ (the case of constant marginal cost). Then, the only difference between our

$$\rho_t = \frac{1}{1-v} \cdot \frac{1}{1 + \frac{\theta}{\epsilon_{ms}}}$$

and Weyl and Fabinger's (2013)

$$\rho_t = \frac{1}{1 + \frac{\theta}{\epsilon_{ms}}}$$

is that our pass-through must be larger to adjust to the deducted price $(1-v)p$ due to a positive ad valorem tax $v > 0$. The positive unit tax $t > 0$ works separately as an addition to the cost function: once non-constant marginal cost is allowed ($\chi \neq 0$), the adjustment term $(1-\tau)/(1-v)$ in our formula.

More interesting interpretations can be provided by considering our own expressions of (the first equation in Proposition 2). With imperfect competition, the term in the denominator $-\eta\theta$ is negative and leads to higher pass-through. This is intuitive because in less competitive markets, firms have the ability to reflect higher costs in their prices to a larger extent. The term in the denominator $-\chi\theta$ has a sign opposite to that of $\chi = mc'q/mc$. For increasing marginal costs, χ is positive and $-\chi\theta$ negative, which leads to higher pass-through, especially if θ is large.

Further, with imperfect competition, the term in the denominator $\epsilon q(\theta\eta)'$ may be split into two parts: $\epsilon q(\theta\eta)' = q\theta' + q\epsilon\theta\eta'$. If at lower quantities the market is less competitive, then $\theta' < 0$ and $q\theta' < 0$, which leads to higher pass-through. Intuitively, in such situations, increasing taxes decreases the quantity provided, which in turn makes the market less competitive, leading to an even larger increase in prices than in the case of $\theta' = 0$. Similarly, if at lower quantities the industry demand elasticity, ϵ, is lower, then $\eta' < 0$ and $q\epsilon\theta\eta' < 0$, which leads to higher pass-through. Intuitively, in such situations, increasing taxes decreases the quantity provided, which in turn makes the industry demand more inelastic, leading to an even larger increase in prices than in the case of $\eta' = 0$. This effect is larger for a larger θ, which is consistent with the fact that in these situations the firms are more sensitive to the properties of the overall industry demand.

We extended these results on pass-through in several directions. In Online Appendix A, we show how our framework applies to the case of *multi-product* firms if intra-firm symmetry is guaranteed. In Online Appendix B, we present generalizations that go beyond the case taxation and include other market changes.[26]

2.5. Marginal value of public funds

We now define the *marginal value of public funds $MVPF_t$* of the *specific tax t* and the *marginal value of public funds $MVPF_v$* of the *ad valorem tax v* as the ratio of the change in consumer and producer surplus to a marginal change in the net cost to the government (which is, in our focus of taxation, the associated change in tax revenue induced by an infinitesimal increase the corresponding tax):

$$MVPF_t \equiv \left(-\frac{\partial R}{\partial t}\right)^{-1}\left(\frac{\partial CS}{\partial t} + \frac{\partial PS}{\partial t}\right),$$
$$MVPF_v \equiv \left(-\frac{\partial R}{\partial v}\right)^{-1}\left(\frac{\partial CS}{\partial v} + \frac{\partial PS}{\partial v}\right).$$

Note that $MVPF_T, T \in \{t, v\}$, in this study measures welfare *loss* because no beneficial effects of government spending are explicity modeled: incorporating such effects into our framework is left for future research.

We are now able to provide a sufficient statistics formula of $MVPF_T$, in terms of the pass-thorough that we have characterized above, the reciprocal of the price elasticity, and the conduct index as well as the other observable variables such as v and τ.

Proposition 3. *Under symmetric oligopoly with a possibly non-constant marginal cost, the marginal value of public funds (MVPF) associated with a change in the specific tax t and the ad valorem tax v is characterized by:*

$$MVPF_t = \frac{\frac{1}{\rho_t} + v + (1-v)\theta}{\frac{1}{\rho_t} + v - \tau\epsilon}, \qquad MVPF_v = \frac{\frac{1}{\rho_v} + v + (1-v)\theta}{\frac{1}{\rho_v} + v - \tau\epsilon},$$

respectively.

Proof. First let us consider the marginal value of public funds $MVPF_t$ for changes in the specific tax, $dt \neq 0, dv = 0$. Using Eqs. (2), (3), and (7), we have

$$MVPF_t = \frac{dCS + dPS}{-dR} = \frac{[(1-v)\eta\theta + \tau]p\,dq - (t\,dq + vp\,dq + q\,dt + qv\,dp)}{-(t\,dq + vp\,dq + q\,dt + qv\,dp)}.$$

In order to cancel the infinitesimal changes on the right-hand side, we substitute for dp and dt in terms of dq using Eq. (8),

[25] Weyl and Fabinger (2013) use ϵ_D and ϵ_S for our ϵ and $1/\chi$, respectively, and call the latter the "elasticity of supply" (p.535). However, although the mathematical definition can be extended to imperfect competition, this concept itself is only meaningful under perfect competition. Therefore, this paper introduces χ and calls it the elasticity of the marginal cost with respect to quantity rather than the elasticity of supply.

[26] While many issues in public economics entail small changes such as a shift in tax rate, it would also be interesting to consider expressions for global changes in the surplus measures: see Online Appendix C. Furthermore, free entry is analyzed in Online Appendix D as an additional extension. In addition, Online Appendix E discusses the relationship with the concept of aggregative games.

$$MVPF_t = \frac{[(1-v)\eta\theta + \tau]p\,dq - (t+vp)dq + q\left(\frac{\eta p}{q\rho_t}dq\right) + qv\left(\frac{\eta p}{q}dq\right)}{-\left[t\,dq + vp\,dq + q\left(-\frac{\eta p}{q\rho_t}dq\right) + qv\left(-\frac{\eta p}{q}dq\right)\right]}$$
$$= \frac{[(1-v)\eta\theta+\tau]p - (t+vp) + \frac{\eta p}{\rho_t} + v\eta p}{-\left(t+vp - \frac{\eta p}{\rho_t} - v\eta p\right)}.$$

Dividing the numerator and denominator by p and using $\eta = 1/\epsilon$ yields:

$$MVPF_t = \frac{\frac{1}{\rho_t} + v + (1-v)\theta}{\frac{1}{\rho_t} + v - \tau\epsilon}.$$

We proceed in a similar fashion for changes in the ad valorem tax, $dv \neq 0, dt = 0$. The marginal value of public funds $MVPF_v$ is

$$MVPF_v = \frac{dCS + dPS}{-dR}$$
$$= \frac{[(1-v)\eta\theta+\tau]p\,dq - (t\,dq + vp\,dq + qv\,dp + qp\,dv)}{-(t\,dq + vp\,dq + qv\,dp + qp\,dv)}.$$

We substitute for dp and dv in terms of dq using Eq. (9),

$$MVPF_v = \frac{[(1-v)\eta\theta + \tau]p\,dq - (t+vp)dq + qv\left(\frac{\eta p}{q}dq\right) + qp\left(\frac{\eta}{q\rho_v}dq\right)}{-\left[t\,dq + vp\,dq + qv\left(-\frac{\eta p}{q}dq\right) + qp\left(-\frac{\eta}{q\rho_v}dq\right)\right]}$$
$$= \frac{[(1-v)\eta\theta+\tau]p - (t+vp) + v\eta p + \frac{\eta p}{\rho_v}}{-\left(t+vp - v\eta p - \frac{\eta p}{\rho_v}\right)}.$$

Dividing the numerator and denominator by p and using $\eta = 1/\epsilon$ yield:

$$MVPF_v = \frac{\frac{1}{\rho_v} + v + (1-v)\theta}{\frac{1}{\rho_v} + v - \tau\epsilon},$$

which completes the proof. □

The intuition behind Proposition 3 for the case of unit taxation can be explained as follows. The argument for ad valorem taxation is analogous. First, the tax revenue increase by a tax reform $dt > 0$ has the mechanical change given by the current output q. However, it is also associated with the behavioral change with respect to pricing ($dp > 0$) as well as production/consumption ($dq < 0$): from Eqs. (2) and (3), it is expressed as

$$dR = \underbrace{q\,dt}_{(+)} + \underbrace{vq\,dp}_{(+)} + \underbrace{(t+vp)\,dq}_{(-)},$$

where the first term of the right hand side expresses direct (mechanical) gains, multiplied by the output q, the second term shows indirect (behavioral) gains, due to the associated price increase, multiplied by vq, and the third term is the part that exhibits another indirect (behavioral) effect that is a loss in government revenue due to the output reduction. Owing to Eq. (8), the net cost to the government is given by

$$-dR = -[-(p\eta/\rho_t)dq - (vp\eta)dq + (t+vp)dq]$$
$$= \left[\left(\frac{1}{\rho_t} + v\right)\eta - \tau\right](p\,dq),$$

where the first term in the bracket exhibits gains in the government revenue, and the second term the loss.

Now, for the denominator, $dCS + dPS$, we make use of the relationship, $dCS + dPS = dW - dR$, to treat $dCS + dPS$ as the private surplus as a whole: the next subsection studies how dCS and dPS are affected by the tax reform differently. As discussed in the last part of Subsection 2.2, the effects of an increase in unit tax, $dt > 0$, on the social welfare under imperfect competition can be written as $dW = -(p - mc)(-dq)$, which implies that *the firm's per-output profit margin serves as a measure for welfare change*. Then, the firm's per-output profit margin is decomposed into two parts: (a) surplus from imperfect competition, $(1-v)p\eta\theta$, and (b) tax payment, $t + vp = p\tau$, as Eq. (7) indicates. Therefore,

$$dCS + dPS = dW - dR$$
$$= [(1-v)\eta\theta + \tau](p\,dq) + \left[\left(\frac{1}{\rho_t} + v\right)\eta - \tau\right](p\,dq)$$
$$= \left[\left(\frac{1}{\rho_t} + v\right)\eta + (1-v)\eta\theta\right](p\,dq),$$

which implies that the ratio of the loss incurred in the private sector to the total gain for the government revenue is given by

$$MVPF_t = \frac{\underbrace{\left(\frac{1}{\rho_t} + v\right)\eta + (1-v)\eta\theta}_{\text{private welfare loss}}}{\underbrace{\left(\frac{1}{\rho_t} + v\right)\eta}_{\text{gov. revenue gain}} + \underbrace{(-\tau)}_{\text{gov. revenue loss}}}$$
$$= \frac{\left(\frac{1}{\rho_t} + v\right) + (1-v)\theta}{\left(\frac{1}{\rho_t} + v\right) - \left(v + \frac{t}{p}\right)\epsilon}.$$

This latter expression for $MVPF_t$ has some intuitive properties. If we think of $MVPF_t$ as a function of t, keeping all other variables in the expression fixed, we see that it is an increasing function of t. That is intuitive: The tax is more distortionary on the margin if the initial tax level is already high. Since t in the expression is multiplied by ϵ/p, the dependence of $MVPF_t$ on t will be stronger if ϵ/p is large. This is also intuitive: (a) for a low price p, t is sizable relative to the price, and (b) for a large elasticity ϵ of the industry demand, an increase in t may have a larger effect on the quantity supplied. In both cases we would expect the initial tax level t to have a strong influence on how distortionary the tax is on the margin. Similarly, if we think of $MVPF_t$ as a function of θ, keeping all other variables in the expression fixed, we see that it is an increasing function of θ, the conduct index. This is consistent with the intuition that when the market is very competitive, with a small θ, the tax should not be as distortionary on the margin as when the market is non-competitive.[27]

For $MVPF_v$, the expression is the same, except that ρ_t is replaced by ρ_v. The intuition regarding the pass-through and market competitiveness applies for $MVPF_v$ as well. The dependence on v is more complicated, though, than the dependence on t.

Next, by combining Propositions 1 and 3, we find that $MVPF_t$ and $MVPF_v$ can be expressed *in terms of estimable elasticities without the conduct index,* θ. The reasoning is simple: Proposition 1 allows us to express the conduct index θ as $\theta = (1 - \rho_v/\rho_t)\epsilon$. Substituting this into the relationships in Proposition 3 then gives the desired result.

Corollary 1. *Under symmetric oligopoly with a possibly non-constant marginal cost, the unit pass-through rate ρ_t, the ad valorem pass-through semi-elasticity ρ_v, and the elasticity of industry demand ϵ (along with the tax rates and the fraction τ of the firm's pre-tax revenue collected by the government in the form of taxes) serve as sufficient statistics for the marginal value of public funds both with respect to unit taxes and ad valorem taxes. Specifically,*

$$MVPF_t = \frac{1 + v\rho_t + (1-v)(\rho_t - \rho_v)\ \epsilon}{1 + (v - \epsilon\tau)\rho_t},$$

and

$$MVPF_v = \frac{1 + v\rho_v + (1-v)(\rho_t - \rho_v)\ \frac{\rho_v}{\rho_t}\epsilon}{1 + (v - \epsilon\tau)\rho_v}.$$

[27] Unfortunately, it is not unambiguous to determine whether $MVPF_t$ is an increasing or decreasing function of ρ_t in this way.

This corollary is consistent with the well-known result that unit tax and ad valorem tax are equivalent in the welfare effects under perfect competition: if $\theta = 0$, then $\rho_t = \rho_v$, and under imperfect competition, $\rho_t > \rho_v$, and $MVPF_t > MVPF_v$. This provides another look of the result of Anderson et al. (2001b) (ADKb) that specific taxes are welfare-inferior to ad valorem taxes.

2.6. Incidence

Having considered the welfare consequences of specific and ad valorem taxation by comparing the economy's surplus, $CS + PS$, to the government revenue, R, we now study the distributional aspects: how the consumers and the firms are differently affected by a tax reform. To do so, we introduce our second welfare measures, the *incidence* I_t of the specific tax t and the *incidence* I_v of the ad valorem tax v as the ratio of (a) the change in consumer surplus induced by an infinitesimal increase the corresponding tax, and (b) the associated change in producer surplus, i.e.,[28]

$$I_t \equiv \left(\frac{\partial PS}{\partial t}\right)^{-1}\left(\frac{\partial CS}{\partial t}\right), \qquad I_v \equiv \left(\frac{\partial PS}{\partial v}\right)^{-1}\left(\frac{\partial CS}{\partial v}\right).$$

The following proposition shows how the incidence is characterized in terms of our sufficient statistics such as pass-through and market conduct.

Proposition 4. *Under symmetric oligopoly with a general type of competition and with a possibly non-constant marginal cost, the incidence of the specific tax t and the ad valorem tax v is characterized by:*

$$\frac{1}{I_t} = \frac{1}{\rho_t} - (1-v)(1-\theta), \qquad \frac{1}{I_v} = \frac{1}{\rho_v} - (1-v)(1-\theta),$$

respectively.

Proof. For a specific tax change $dt \neq 0, dv = 0$, we get, using Eqs. (6), (4) and (5),

$$I_t = \frac{dCS}{dPS} = \frac{-q\,dp}{(1-v)p\eta\theta\,dq + (1-v)q\,dp - q\,dt}$$
$$= \frac{-q\left(-\frac{\eta p}{q}dq\right)}{(1-v)p\eta\theta\,dq + (1-v)q\left(-\frac{\eta p}{q}dq\right) - q\left(-\frac{\eta p}{q\rho_t}dq\right)},$$

where we eliminated dp and dt using Eq. (8). After a simplification,

$$I_t = \frac{1}{\frac{1}{\rho_t} - (1-v)(1-\theta)}.$$

For an ad valorem tax change, $dv \neq 0, dt = 0$, we obtain, again using Eqs. (6), (4) and (5),

$$I_v = \frac{dCS}{dPS}$$
$$= \frac{-q\,dp}{(1-v)p\eta\theta\,dq + (1-v)q\,dp - pq\,dv}$$
$$= \frac{-q\left(-\frac{\eta p}{q}dq\right)}{(1-v)p\eta\theta\,dq + (1-v)q\left(-\frac{\eta p}{q}dq\right) - pq\left(-\frac{\eta}{q\rho_v}dq\right)},$$

where we substituted for dp and dv from Eq. (9). This simplifies to

$$I_v = \frac{1}{\frac{1}{\rho_v} - (1-v)(1-\theta)},$$

which completes the proof. □

Note that in the case of zero ad valorem tax, the expression for I_t reduces to Weyl and Fabinger's (2013, p. 548) Principle of Incidence 3, that states $1/I_t = 1/\rho_t - (1-\theta)$. In this way, we are able to generalize Weyl and Fabinger's (2013) formula for incidence, and respond to the statements by Rosen and Gayer (2014), Stiglitz and Rosengard (2015), and Gruber (2019) mentioned in the Introduction.

To provide intuitive reasoning behind Proposition 4, recall from Eq. (5), that

$$dPS = \underbrace{-q\,dt + (1-v)q\,dp}_{\text{value effect}} + \underbrace{(1-v)p\eta\theta\,dq}_{\text{quantity effect}},$$

and from Eq. (9) that the behavioral responses (pricing and production) are expressed in terms of the mechanical change, dt, by using the tax pass-through, ρ_t:

$$dp = \rho_t\,dt, \qquad dq = -\frac{q\rho_t}{\eta p}dt,$$

respectively. Therefore, the above equation can also be interpreted as

$$dPS = \underbrace{-q\,dt+}_{\text{mechnical}} + \underbrace{(1-v)q\,dp + (1-v)p\eta\theta\,dq}_{\text{behavioral}}$$
$$= [\underbrace{-1}_{\text{mechnical}} + \underbrace{(1-v)\rho_t - (1-v)\theta\rho_t}_{\text{behavioral}}](q\,dt),$$

which implies that the per-unit loss in producer surplus due to a specific tax reform is $-1 + (1-v)(1-\theta)\rho_t$. Interestingly, producers can be better off ($dPS > 0$) by the tax reform if the second term dominates; this would be more likely if v is small, the market is more competitive, and the specific tax pass-through is large. Similarly, the per-unit loss in consumer surplus is simply the tax pass-through itself because from Eq. (6),

$$dCS = -q\,dp$$
$$= -\rho_t(q\,dt),$$

which implies that consumers can never be better off by the tax reform.[29] Hence, the specific tax incidence is simply the ratio of ρ_t to $1 - (1-v)(1-\theta)\rho_t$. A similar argument can also be developed for ad valorem tax.[30]

To conclude this section, we briefly describe how one can go beyond the two-dimensional case of specific and ad valorem taxation, while still preserving the simplicity for general forms of multi-dimensional interventions. First, the specific and ad valorem tax payment of a (symmetric) firm is expressed as

[28] One could also define *social incidence* by $SI_T \equiv dW/dPS$ in association with a small change in $T \in \{t, v\}$ (see Weyl and Fabinger 2013, p. 538). In this paper, we focus on $MVPF_T$ as a measure of welfare burden in society, and I_T as a measure of loss in consumer welfare because once $MVPF_T \equiv -(dW - dR)/dR$ and $I_T \equiv dCS/dPS$ are obtained, $SI_T = (dCS + dPS + dR)/dPS = -(1 + I_T)/(1 - 1/MVPF_T)$ can be readily calculated.

[29] Note here that economic agents in the private sector (i.e., consumers and producers) as a whole can never be better off because

$$dCS + dPS = -\rho_t - 1 + (1-v)(1-\theta)\rho_t$$
$$= -1 - \left[\underbrace{1 - (1-v)(1-\theta)}_{>0}\right]\rho_t < 0.$$

[30] Similar to Corollary above, the incidence of a unit tax is expressed as

$$\frac{1}{I_t} = \frac{1}{\rho_t} - (1-v)\left[(1-\epsilon) + \frac{\rho_v}{\rho_t}\epsilon\right],$$

and analogously for the case of an ad valorem tax.

$\phi(p, q, \mathbf{T}) = tq + vpq$, where $\mathbf{T}$ is a vector of (multi-dimensional) interventions, in this case $\mathbf{T} = (t, v)$. To generalize this, the key is to ask what the analog of such a pair of t and v might be. It turns out that in general we can write $\phi(p, q, \mathbf{T}) = \bar{t}q + \bar{v}pq$, where $\bar{t}$ and $\bar{v}$ are the averages of appropriately defined functions t and v over the ranges $(0, q)$ and $(0, pq)$. In the special case of specific and ad valorem taxes, these simply reduce to constants t and v. We are able to achieve this generalization by decomposing $\phi(p, q, \mathbf{T})$ into infinitesimal contributions, each of which resembles specific and ad valorem taxes. Using these functions t and v gives rise to a simple way of analyzing the welfare consequences of government interventions and non-governmental external changes. The resulting relationships are almost as simple as those in the two-dimensional case of specific and ad valorem taxes. Appendix C formalizes this idea and allows for firm heterogeneity.

3. Numerical Analysis of Parametric Examples

Although our formulas are presented in a general form, it would be illustrative to work through some parametric examples. Below we consider three demand specifications with n symmetric firms and constant marginal cost: $\chi = 0$. We define the *own-price elasticity* $\epsilon_{own}(p)$ *of the firm's direct demand and* the *own quantity elasticity* $\eta_{own}(q)$ *of the firm's inverse demand by*

$$\epsilon_{own}(p) \equiv -\frac{p}{q(p)} \cdot \frac{\partial q_i(\mathbf{p})}{\partial p_i}\Big|_{\mathbf{p}=(p,\ldots,p)}$$

and

$$\eta_{own}(q) \equiv -\frac{q}{p(q)} \cdot \frac{\partial p_i(\mathbf{q})}{\partial q_i}\Big|_{\mathbf{q}=(q,\ldots,q)},$$

respectively. Similarly, the *curvature of the industry's direct demand* $\alpha(p)$ and the *curvature of the industry's inverse demand* $\sigma(q)$ are defined as follows:

$$\alpha(p) \equiv \frac{-pq''(p)}{q'(p)}$$

and

$$\sigma(q) \equiv \frac{-qp''(q)}{p'(q)}.$$

Then, the results derived in Appendix B indicate that in this case, the pass-through expressions become

$$\rho_t = \frac{1}{(1-v)\left[1 + \left(1 - \frac{\alpha}{\epsilon_{own}}\right)\theta\right]},$$

$$\rho_v = \frac{\epsilon_{own} - 1}{\epsilon_{own}\left\{(1-v)\left[1 + \left(1 - \frac{\alpha}{\epsilon_{own}}\right)\theta\right]\right\}}$$

under price competition, where $\theta = \epsilon/\epsilon_{own}$, and

$$\rho_t = \frac{1}{(1-v)\left[1 + \left(1 - \frac{\sigma}{\theta}\right)\theta\right]}, \qquad \rho_v = \frac{1 - \eta_{own}}{(1-v)\left[1 + \left(1 - \frac{\sigma}{\theta}\right)\theta\right]}$$

under quantity competition, where $\theta = \eta_{own}/\eta$.

Below, we consider three classes of demand specification: linear, constant elasticity of substitution (CES), and multinomial logit, and we assume that the marginal cost is constant.

3.1. Linear demand

The first one is the case wherein each firm faces the followinglinear demand, $q_i(\mathbf{p}) = b - \lambda p_i + \mu \sum_{i' \neq i} p_{i'}$, where $\lambda > (n-1)\mu$ and $0 \leqslant mc < b/[\lambda - (n-1)\mu]$, implying that all firms produce substitutes and μ measures the degree of *substitutability* (firms are effectively monopolists when $\mu = 0$).[31,32] Under symmetric pricing, the industry's demand is thus given by $q(p) = b - [\lambda - (n-1)\mu]p$. The inverse demand system is given by

$$p_i(\mathbf{q}) = \frac{\lambda - (n-2)\mu}{(\lambda + \mu)[\lambda - (n-1)\mu]}(b - q_j) + \frac{\mu}{(\lambda + \mu)[\lambda - (n-1)\mu]}\left[\sum_{i' \neq i}(b - q_{i'})\right],$$

implying that $p(q) = (b - q)/[\lambda - (n-1)\mu]$ under symmetric production. Obviously, both the direct and the indirect demand curvatures are zero: $\alpha = 0, \sigma = 0$. Under price competition, the pass-through expressions are

$$\rho_t = \frac{1}{(1-v)(1+\theta)}, \qquad \rho_v = \frac{\epsilon_{own} - 1}{\epsilon_{own}(1-v)(1+\theta)},$$

where $\theta = [\lambda - (n-1)\mu]/\lambda$, and $\epsilon_{own} = \lambda(p/q)$. Under quantity competition,

$$\rho_t = \frac{1}{(1-v)(1+\theta)}, \qquad \rho_v = \frac{1 - \eta_{own}}{(1-v)(1+\theta)},$$

where $\theta = [\lambda - (n-2)\mu]/(\lambda + \mu)$ and $\eta_{own} = \{[\lambda - (n-2)\mu](q/p)\}/\{(\lambda + \mu)[\lambda - (n-1)\mu]\}$.

Under price competition, the marginal value of public funds and the incidence, discussed in Propositions 3 and 4, respectively, are given by

$$MVPF_t = \frac{1 + 2(1-v)\theta}{1 + (1-v)\theta - \epsilon\tau}, \qquad MVPF_v = \frac{v + (1-v)\left(\theta + \frac{1+\theta}{\epsilon_{own} - 1}\right)}{\frac{(1-v)(1+\theta)}{\epsilon_{own}-1} + v - \epsilon\tau},$$

$$I_t = \frac{1}{2(1-v)[1 - (n-1)(\mu/\lambda)]}, \qquad I_v = \frac{\epsilon_{own} - 1}{(1-v)[2 - \epsilon_{own}(1-\theta)]},$$

with $\epsilon = [\lambda - (n-1)\mu](p/q)$. Under quantity competition, they are

$$MVPF_t = \frac{1 + 2(1-v)\theta}{1 + (1-v)\theta - \frac{1}{\eta}\tau}, \qquad MVPF_v = \frac{v + (1-v)\left(\theta + \frac{1+\theta}{1 - \eta_{own}}\right)}{\frac{(1-v)(1+\theta)}{1 - \eta_{own}} + v - \frac{1}{\eta}\tau},$$

$$I_t = \frac{\lambda + \mu}{2(1-v)[\lambda - (n-2)\mu]}, \qquad I_v = \frac{1 - \eta_{own}}{(1-v)[\eta_{own} + (2 - \eta_{own})\theta]},$$

with $1/\eta = [\lambda - (n-1)\mu](p/q)$. Thus, in both cases, it suffices to solve for the equilibrium price and output to compute the pass-through and the marginal value of public funds.

Table 1 (a) summarizes the key variables that determine these values for the case of linear demand. It is verified that under both price and quantity competition, θ is a decreasing function of n and μ. To focus on the role of these two parameters, n and μ, which directly affect the intensity of competition, we employ the following simplification to compute the ratio p/q in equilibrium: $b = 1, mc = 0$, and $\lambda = 1$. (See Online Appendix H for the expres-

[31] This linear demand is derived by maximizing the representative consumer's net utility, $U(q_1, \ldots, q_n) - \sum_{i=1}^{n} pq_i$, with respect to $q_1, \ldots,$ and q_n. See Vives (2000, pp. 145–6) for details.

[32] In our notation below, the demand in symmetric equilibrium is given by $q_i(p_i, p_{-i}) = b - \lambda p_i + \mu(n-1)p_{-i}$, whereas it is written as

$$q_i(p_i, p_{-i}) = \frac{\alpha}{1 + \gamma(n-1)} - \frac{1 + \gamma(n-2)}{(1-\gamma)[1 + \gamma(n-1)]}p_i + \frac{\gamma(n-1)}{(1-\gamma)[1 + \gamma(n-1)]}p_{-i}$$

in Häckner and Herzing's (2016) notation, in which $\gamma \in [0, 1]$ is the parameter that measures substitutability between (symmetric) products. Thus, if our (b, λ, μ) is determined by $b = \alpha/[1 + \gamma(n-1)]$, $\lambda = [1 + \gamma(n-2)]/\{(1-\gamma)[1 + \gamma(n-1)]\}$, and $\mu = \gamma/\{(1-\gamma)[1 + \gamma(n-1)]\}$, given Häckner and Herzing's (2016) (α, γ), then our results below can be expressed by Häckner and Herzing's (2016) notation as well. Note here that our formulation is more flexible in the sense that the number of parameters is three. This is because the coefficient for the own price is normalized to one: $p_i(q_i, q_{-i}) = \alpha - q_i - \gamma(n-1)q_{-i}$, which is analytically innocuous, and Häckner and Herzing's (2016) γ is the normalized parameter (see also Häckner and Herzing, 2022).

Table 1
Sufficient Statistics: Elasticities, Conduct Indices, and Curvatures.

(a) Linear Demand	
Price setting	Quantity setting
$\epsilon = [\lambda - (n-1)\mu]\left(\frac{p}{q}\right)$	$\eta = \frac{1}{\lambda - (n-1)\mu}\left(\frac{q}{p}\right)$
$\epsilon_{\text{own}} = \lambda\left(\frac{p}{q}\right)$	$\eta_{\text{own}} = \frac{\lambda - (n-2)\mu}{(\lambda+\mu)[\lambda - (n-1)\mu]}\left(\frac{q}{p}\right)$
$\theta = \epsilon/\epsilon_{\text{own}} = 1 - (n-1)\left(\frac{\mu}{\lambda}\right)$	$\theta = \eta_{\text{own}}/\eta = \frac{\lambda - (n-2)\mu}{\lambda+\mu}$
$\alpha = 0$	$\sigma = 0$
(b) CES Demand	
Price setting	Quantity setting
$\epsilon = \frac{1}{1-\gamma\xi}$	$\eta = 1 - \gamma\xi$
$\epsilon_{\text{own}} = \frac{n - \gamma - (n-1)\gamma\xi}{n(1-\gamma)(1-\gamma\xi)}$	$\eta_{\text{own}} = \frac{\gamma(1-\xi) + (1-\gamma)n}{n}$
$\theta = \epsilon/\epsilon_{\text{own}} = \frac{n(1-\gamma)}{n - \gamma - (n-1)\gamma\xi}$	$\theta = \eta_{\text{own}}/\eta = \frac{\gamma(1-\xi) + (1-\gamma)n}{n(1-\gamma\xi)}$
$\alpha = \frac{2-\gamma\xi}{1-\gamma\xi}$	$\sigma = 2 - \gamma\xi$
(c) Multinomial Logit Demand	
Price setting	Quantity setting
$\epsilon = \beta(1-ns)p$	$\eta = \frac{1}{\beta(1-ns)p}$
$\epsilon_{\text{own}} = \beta(1-s)p$	$\eta_{\text{own}} = \frac{1-(n-1)s}{\beta(1-ns)p}$
$\theta = \epsilon/\epsilon_{\text{own}} = \frac{1-ns}{1-s}$	$\theta = \eta_{\text{own}}/\eta = 1 - (n-1)s$
$\alpha = \frac{(2ns-3)ns}{1-ns}p$	$\sigma = \frac{1-2ns}{1-ns}$

sions of the equilibrium prices and output levels under price and quantity competition).

The top two panels in Fig. 4 illustrate how ρ_t and ρ_v behave as we increase the number of firms (n, the left side) or the sustainability parameter (μ, the right side). The initial tax levels are $t = 0.05$ and $v = 0.05$. We distinguish price setting and quantity setting by superscripts P and Q, respectively. The middle panels show $MVPF_t$ and $MVPF_v$, while the bottom panels depict I_t and I_v. We observe that the ad valorem tax pass-through is close to zero because in this case both ϵ_{own} and η_{own} are close to 1. As competition becomes more intense, both ρ_t^P and ρ_t^Q become larger, and their difference also becomes larger. In the case of linear demand, the difference in the mode of competition does not yield a substantial difference in the three measures. As is verified by ADKb, the ad valorem tax is more efficient on the margin than the specific tax: the dashed lines in the two middle panels lie below the solid lines. This ranking is related inversely to pass-through and incidence: as pass-through or incidence increases, the marginal value of public funds decreases.

3.2. Constant elasticity of substitution (CES) demand

We next consider the market demand with constant elasticity of substitution given by

$$q_i(\mathbf{p}) = (\gamma\xi)^{\frac{1}{1-\gamma\xi}} \frac{p_i^{\frac{-1}{1-\gamma}}}{\left(\sum_{i'=1}^{n} p_{i'}^{\frac{-\gamma}{1-\gamma}}\right)^{\frac{1-\xi}{1-\gamma\xi}}},$$

where $0 < \gamma < 1$ and $0 < \xi < 1$.[33] Hence the direct demand under symmetric pricing is

$$q(p) = (\gamma\xi)^{\frac{1}{1-\gamma\xi}} n^{\frac{-(1-\xi)}{1-\gamma\xi}} p^{\frac{-1}{1-\gamma\xi}}$$

The elasticity of substitution, $1/(1-\gamma)$, is constant. Table 1 (b) shows the price elasticity of industry demand (ε), the own-price elasticity of a firm's demand (ε_{own}), the conduct index (θ), and the curvature of the industry's direct demand (α) are all independent of the equilibrium price.[34] This feature is in contrast to the linear demand above or the multinomial logit demand below.

Similarly, the inverse demand is given by

$$p_i(\mathbf{q}) = (\gamma\xi)\left(\sum_{i'=1}^{n} q_{i'}^{\gamma}\right)^{-(1-\xi)} q_i^{-(1-\gamma)}.$$

Hence the inverse demand under symmetric pricing is $p(q) = (\gamma\xi)n^{-(1-\xi)}q^{-(1-\gamma\xi)}$. Table 1 (b) indicates that for the case of quantity setting, η, η_{own}, θ, and σ are also independent of the equilibrium output or price.[35]

Note that for each tax $T \in \{t, v\}$, only ρ_T and θ, as well as the initial value of ad valorem tax v, are necessary to compute I_T, whereas the equilibrium price is necessary to compute $\tau = v + t/p$. With

[33] This CES demand is derived from $U(q_1, \dots, q_n) = \left(\sum_{i=1}^{n} q_i^{\gamma}\right)^{\xi}$ as the representative consumer's utility (Vives 2000, pp.147–8), where the elasticity of substitution between the firms is given by $1/(1-\gamma)$.

[34] We use the first-order derivative of

$q(p), q'(p) = -\left[n^{\frac{-(1-\xi)}{1-\gamma\xi}}(\gamma\xi)^{\frac{1}{1-\gamma\xi}}/(1-\gamma\xi)\right]p^{\frac{-(2-\gamma\xi)}{1-\gamma\xi}},$

and its second-order derivative,

$q''(p) = \left[n^{\frac{-(1-\xi)}{1-\gamma\xi}}(\gamma\xi)^{\frac{1}{1-\gamma\xi}}(2-\gamma\xi)/(1-\gamma\xi)^2\right]p^{\frac{-(3-2\gamma\xi)}{1-\gamma\xi}}$

for these derivations.

[35] Here, we use the first-order derivative of $p(q), p'(q) = -(1-\gamma\xi)(\gamma\xi)n^{-(1-\xi)}q^{-(2-\gamma\xi)}$, and its second-order derivative, $p''(q) = (2-\gamma\xi)(1-\gamma\xi)(\gamma\xi)n^{-(1-\xi)}q^{-(3-\gamma\xi)}$, for these derivations.

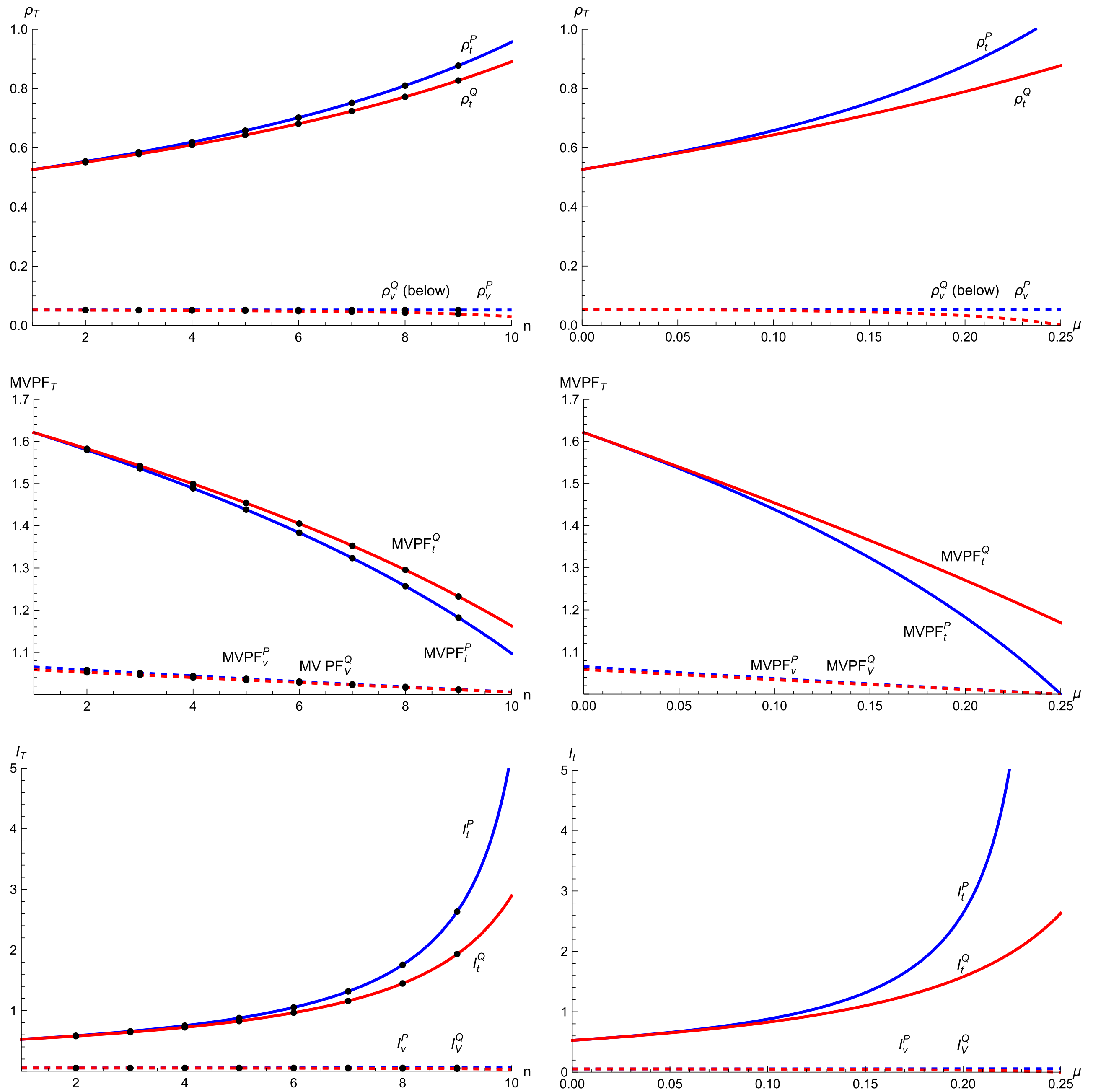

Fig. 4. Pass-through (top), marginal value of public funds (middle), and incidence (bottom) with linear demand. The horizontal axes on the left and the right panels correspond to the number of firms (n) with $\mu = 0.1$, and the substitutability parameter (μ) with $n = 5$, respectively, with the initial tax level, $(t, v) = (0.05, 0.05)$.

CES demand and a constant marginal cost mc, the equilibrium price under price competition is analytically solved as

$$p = \frac{n(1-\gamma\xi) - \gamma(1-\xi)}{\gamma n(1-\gamma\xi) - \gamma(1-\xi)} mc > mc,$$

and the equilibrium price under quantity competition is given by

$$p = \frac{n}{\gamma[n-(1-\xi)]} mc > mc.$$

More details on the equilibrium are included in Online Appendix H.

Fig. 5 depicts the differences across the competition-tax pairs regarding the pass-through value (top), the marginal value of public funds (middle), and the incidence (bottom) when $mc = 1, \xi = 0.9$, and $(t, v) = (0.05, 0.05)$. The left panel shows how ρ, $MVPF$, and I change in response to changes in the number of firms, and the right panel shows such changes in response to changes in γ.[36]

3.3. Multinomial logit demand

The last parametric example is the *multinomial logit demand.* Each firm $i = 1, \ldots, n$ faces the following demand: $s_i(\mathbf{p}) = \exp(\delta - \beta p_i)/\left[1 + \sum_{i'=1,\ldots,n} \exp(\delta - \beta p_{i'})\right] \in (0, 1)$, where δ is the (symmetric) product-specific utility and $\beta > 0$ is the *responsive-*

[36] Here we focus only on the intermediate values of γ (i.e., $\gamma \in [0.3, 0.7]$) to ensure that the elasticity of substitution is not close to zero or one.

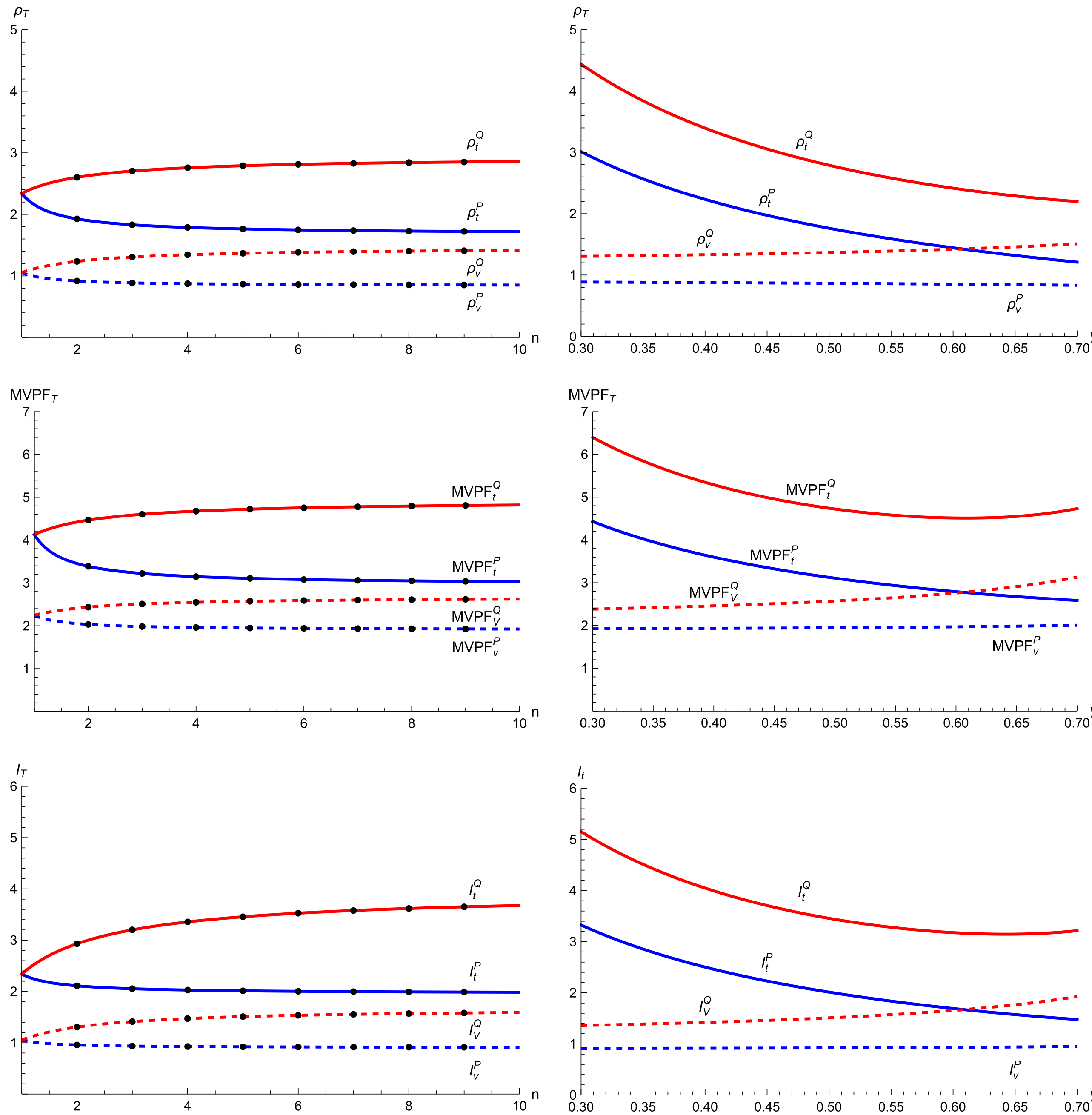

Fig. 5. Pass-through (top), marginal value of public funds (middle), and incidence (bottom) with constant elasticity of substitution (CES) demand. The horizontal axes on the left and the right panels are the number of firms (n) with $\gamma = 0.5$, and the substitution parameter (γ) with $n = 5$, respectively (with the initial tax level, $(t, v) = (0.05, 0.05)$).

ness to the price.[37] We define $s_0 = 1 - \sum_{i=1,\dots,n} s_i < 1$ as the share of all outside goods. Table 1 (c) summarizes the key variables that determine the pass-through, the marginal value of public funds, and the incidence. We need to numerically solve for the equilibrium price and market share under both settings to compute these values for all four cases. To focus on the two parameters, β and n, we assume that $\delta = 1$ and $mc = 0$. Because $\partial s_i(\mathbf{p})/\partial p_i|_{\mathbf{p}=(p,\dots,p)} = -\beta s(1-s)$, the first-order conditions for the symmetric equilibrium price and the market share satisfy $p - t/(1-v) = 1/[\beta(1-s)]$ and $s = \exp(1-\beta p)/[1 + n \cdot \exp(1-\beta p)]$. If p and s are solved numerically, then $\epsilon, \epsilon_{own}, \theta$ and α can also be numerically computed.[38]

[37] Here, $q_i(p_1, \dots, p_n)$ is derived by aggregating over individuals who choose product i (the total number of individuals is normalized to one): an individual's net utility from consuming i is given by $u_i = \delta - \beta p_i + \tilde{\varepsilon}_i$, whereas $u_0 = \tilde{\varepsilon}_0$ is the net utility from consuming nothing, and $\tilde{\varepsilon}_0, \tilde{\varepsilon}_1, \dots, \tilde{\varepsilon}_n$ are independently and identically distributed according to the Type I extreme value distribution for all individuals. See Anderson et al. (1992, pp. 39-45) for details. We work in terms of market share variables s_i and s, instead of q_i and q, which is consistent with the standard notation in the industrial organization literature.

[38] It can be verified that $s_i(\cdot; \mathbf{p}_{-i})$ is convex as long as $s_i < 1/2$ because $\partial^2 s_i/\partial p_i^2 = -\beta(\partial s_i/\partial p_i)(1 - 2s_i) > 0$. However, the second-order condition is always satisfied because $\partial^2 \pi_i/\partial p_i^2 = -\beta s_i < 0$. In symmetric equilibrium with $\delta = 1$ and $mc = 0$, the largest market share is attained as $1/(n+1)$ when the equilibrium price is zero, which implies that the market share of the outside goods s_0 is no less than each firm's market share: $s_0 > s$.

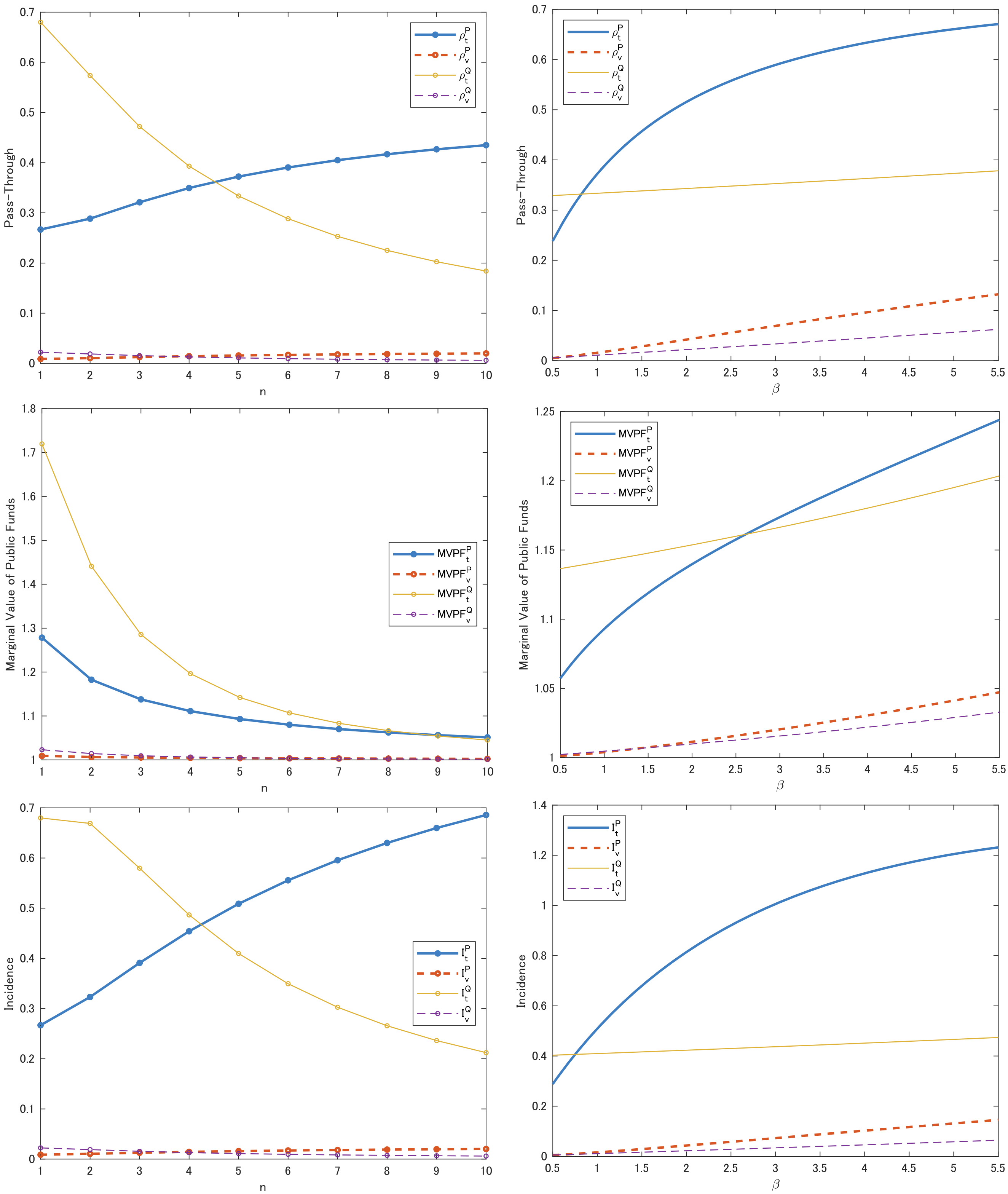

Fig. 6. Pass-through (top), marginal value of public funds (middle), and incidence (bottom) with multinomial logit demand. The horizontal axes on the left and the right panels are the number of firms (n) with $\beta = 1.0$, and the price coefficient (β) with $n = 5$, respectively (with the initial tax level, $(t, v) = (0.05, 0.05)$).

Next, we consider the inverse demands under quantity competition. Then, as in Berry (1994), firm i's inverse demand is given by $p_i(\mathbf{s}) = [\delta - \log(s_i/s_0)]/\beta$, where $\mathbf{s} = (s_1, \ldots, s_n)$, which implies that $\partial p_i(\mathbf{s})/\partial s_i|_{\mathbf{s}=(s,\ldots,s)} = -[1-(n-1)s]/[\beta s(1-ns)]$. Thus, the first-order conditions for the symmetric equilibrium price and the market share satisfy $p - t/(1-v) = [1-(n-1)s]/[\beta(1-ns)]$ and $p = [1 - \log(s/[1-ns])]/\beta$. Then, as above, η, η_{own}, θ and σ are computed by numerically solving the first-order conditions for p and s. Interestingly, it is verified that in symmetric equilibrium under quantity setting, $\partial p/\partial n = 0$: the equilibrium price is the same irrespective of the number of firms, whereas the individual market share is decreasing in the number of firms: $\partial s/\partial n < 0$. On the other hand, both the equilibrium price and market share are decreasing in the price coefficient, β.

Fig. 6 illustrates the pass-through, the marginal value of public funds, and the incidence, in analogy with Figs. 4 and 5. The right panels now show the variables' dependence on the price coefficient β. Overall, as in the case of the linear demand and the CES demand, an increase in the ad valorem tax has a small impact on these measures for each of n and β, whereas an increase in the unit tax has a large effect.

However, there are two important differences between linear and logit demands. First, the unit tax pass-through under quantity competition ρ_t^Q is *decreasing* in the number of firms. To understand this, compare the difference in the denominators of $\rho_t^P = 1/\{(1-v)[1+(1-\alpha/\epsilon_{own})\theta]\}$ and $\rho_t^Q = (1-v)[1+\theta-\sigma]$. As θ decreases (i.e., as competition becomes fiercer), the second term in the denominator of ρ_t^P decreases, and thereby ρ_t^P *increases* as n increases. However, $\theta - \sigma$ increases as θ decreases, and thus ρ_t^Q *decreases*. This difference in the denominators is also reflected in the fact that I_t^Q is decreasing in n as well. Naturally, $MVPF_t^Q$ is decreasing in n as in the case of linear demand because $1/\rho_t^Q$ becomes larger (see the formulas in Proposition 3). Second, while the pass-through and the incidence increase as β increases, the marginal value of public funds is also increasing in contrast to the case of linear demands. The reason is that the effect on the MVPF of decreases in θ is weaker than the effect of the increase in ϵ: the industry's demand becomes elastic quickly as consumers become more sensitive to a price increase.

4. Firm Heterogeneity

In this section, we extend our results to the case of n heterogeneous firms, where each firm $i = 1, 2, \ldots, n$ controls a strategic variable σ_i, which would be, for example, the price or quantity of its product. Appendix C presents the general version of multi-dimensional interventions and establishes some results on pass-through and welfare measures. In the following, p_i is the price of firm i's product, q_i is the quantity of the product sold by firm i. Then, firm i's profit function is written as

$$\begin{aligned}\pi_i &= (1-v)p_i(\mathbf{q})q_i - tq_i - c_i(q_i)\\ &= p_i(\mathbf{q})q_i - c_i(q_i) - R_i(\mathbf{q}),\end{aligned}$$

where $R_i(\mathbf{q}) = tq_i + vp_i(\mathbf{q})q_i$ is the (per-firm) tax revenue from firm i.

Under this firm heterogeneity, Eq. (1) is generalized as

$$\left[\left(1 - \frac{t}{p_i(\mathbf{q})} - v\right) - \psi_i(\mathbf{q})(1-v)\right]p_i(\mathbf{q}) = mc_i(q_i), \qquad (11)$$

for $i = 1, 2, \ldots, n$, where we call $\psi_i(\mathbf{q})$ firm i's *pricing strength index*. In the case of symmetric firms, the pricing strength index ψ is related to the conduct index $\theta(q)$ by $\theta = \epsilon\psi$.[39]

4.1. Pass-through

The pass-through matrix for the two-dimensional taxation (t and v) is defined as

$$\tilde{\boldsymbol{\rho}} \equiv \begin{pmatrix} \dfrac{\partial p_1}{\partial t} & \dfrac{\partial p_1}{\partial v} \\ \vdots & \vdots \\ \dfrac{\partial p_n}{\partial t} & \dfrac{\partial p_n}{\partial v} \end{pmatrix}.$$

Using the results from Proposition C.3 in Appendix C, the pass-through matrix is characterized as follows.

Proposition 5. *For heterogeneous firms with specific and ad valorem taxation, the pass-through matrix equals*

$$\tilde{\boldsymbol{\rho}} = \mathbf{b}^{-1}\begin{pmatrix} 1 & p_1\cdot(1-\psi_1) \\ \vdots & \vdots \\ 1 & p_n\cdot(1-\psi_n) \end{pmatrix},$$

where the (i,j) *element of the* $\mathbf{b}$ *matrix is given by*

$$b_{ij} = (1-v)\left[(1-\psi_i)\delta_{ij} - \psi_i\Psi_{ij}\right] + \left[(1-\tau_i) - (1-v)\psi_i\right]\chi_i\epsilon_{ij},$$

where δ_{ij} *is the Kronecker delta,*[40]

$$\epsilon_{ij} \equiv -\frac{p_i}{q_i}\frac{\partial q_i(\mathbf{p})}{\partial p_j},$$

$$\Psi_{ij} \equiv \frac{p_i}{\psi_i}\frac{\partial \psi_i(\mathbf{q}(\mathbf{p}))}{\partial p_j},$$

and

$$\tau_i = \frac{t}{p_i} + v.$$

Note that if all firms have constant marginal cost ($\chi_i = 0$ for all i), the expression for b_{ij} simplifies to $b_{ij} = (1-v)[(1-\psi_i)\delta_{ij} - \psi_i\Psi_{ij}]$.

4.2. Characterization of the two welfare measures

By using the results in Appendix C, we are also able to obtain the following proposition that characterizes the marginal value of public funds and the incidence for the case of heterogeneous firms, where we define $\boldsymbol{\epsilon}_i$, an n-dimensional row vector with its j-th component equal to ϵ_{ij} for each i, by $\boldsymbol{\epsilon}_i = (\epsilon_{i1}, \ldots, \epsilon_{ij}, \ldots \epsilon_{in})$.

Proposition 6. *Let* $\epsilon_{iT}^{\rho} \equiv \boldsymbol{\epsilon}_i\tilde{\boldsymbol{\rho}}_T/\tilde{\rho}_{iT} = \boldsymbol{\epsilon}_i\boldsymbol{\rho}_T/\rho_{iT}$ *for* $T \in \{t, v\}$. *Then, the marginal value of public funds associated with intervention T,* $MVPF_{iT} \equiv [(\nabla CS_i)_T + (\nabla PS_i)_T]/(-\nabla R_i)_T$, *is characterized by:*

$$MVPF_{iT} = \frac{\frac{1}{\epsilon_{iT}^{\rho}}\left(\frac{1}{\rho_{iT}} + v\right) + (1-v)\psi_i}{\frac{1}{\epsilon_{iT}^{\rho}}\left(\frac{1}{\rho_{iT}} + v\right) - \tau_i},$$

and the incidence of this intervention, $I_{iT} \equiv (\nabla CS_i)_T/(\nabla PS_i)_T$, *is characterized by:*

$$I_{iT} = \frac{1}{\frac{1}{\rho_{iT}} - (1-v)\left(1 - \psi_i\epsilon_{iT}^{\rho}\right)}.$$

Table 2 summarizes our characterization at each stage of generality.

[39] For clarity of intuition, suppose that $(t, v) = (0, 0)$. Then, Eq. (11) implies $p_i(\mathbf{q}) = mc_i(q_i)/[1-\psi_i(\mathbf{q})]$. If it was the case that $\psi_i(\mathbf{q}) = 0$ for any $\mathbf{q}$ and i, all firms would adopt marginal cost pricing. If ψ_i is sufficiently large, p_i can be substantially above the marginal cost. We find that with heterogeneous firms, it is significantly more convenient to use the pricing strength index than to use the conduct index when we characterize the marginal value of public funds and the incidence. Appendix D discusses the relationship between these two concepts.

[40] As usual, the Kronecker delta δ_{ij} is defined to be equal to 1 if its two indices are the same and zero otherwise.

Table 2
Summary of the Expressions for the Two Welfare Measures for $T \in \{t, v\}$ under Imperfect Competition.

	Symmetric firms		Heterogeneous firms
	No pre-existing taxes	With pre-existing taxes	
Marginal Value of Public Funds	$\theta\rho_T + 1$	$\dfrac{\frac{\frac{1}{\rho_T}+v}{\epsilon} + (1-v)(\theta/\epsilon)}{\frac{\frac{1}{\rho_T}+v}{\epsilon} - \tau}$	$\dfrac{\frac{\frac{1}{\rho_{iT}}+v}{\epsilon_{iT}^{\rho}} + (1-v)\psi_i}{\frac{\frac{1}{\rho_{iT}}+v}{\epsilon_{iT}^{\rho}} - \tau_i}$
Incidence	$\dfrac{1}{\frac{1}{\rho_T} - (1-\theta)}$	$\dfrac{1}{\frac{1}{\rho_T} - (1-v)(1-\theta)}$	$\dfrac{1}{\frac{1}{\rho_{iT}} - (1-v)\left(1-\psi_i\epsilon_{iT}^{\rho}\right)}$

Note: See the main text for the notations.

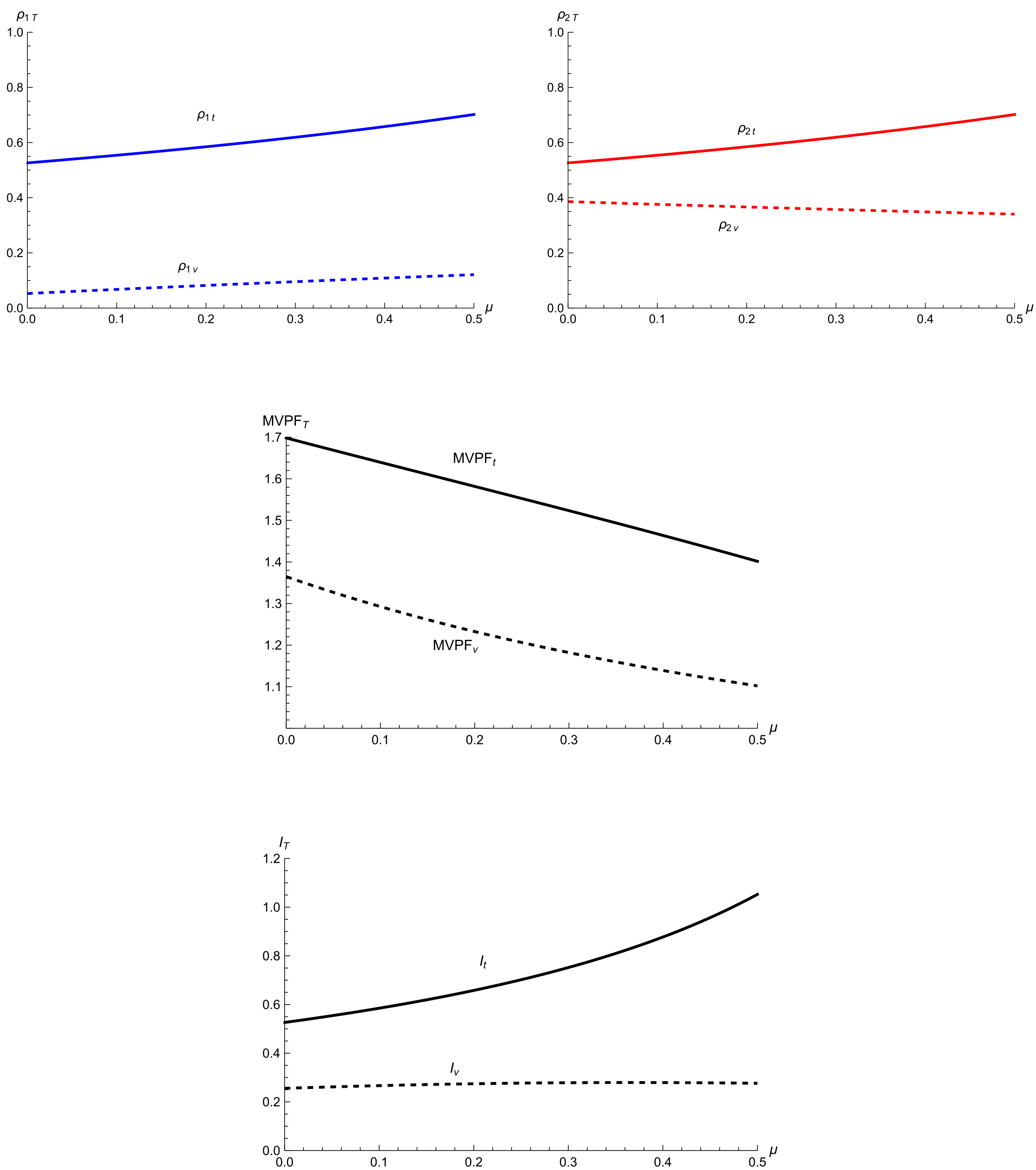

Fig. 7. Pass-through when Firm 1 (left) and Firm 2 (right) face an identical demand (linear) but have different marginal costs (top), and the associated market-level marginal value of public funds (middle) and incidence (bottom): The case of price competition.

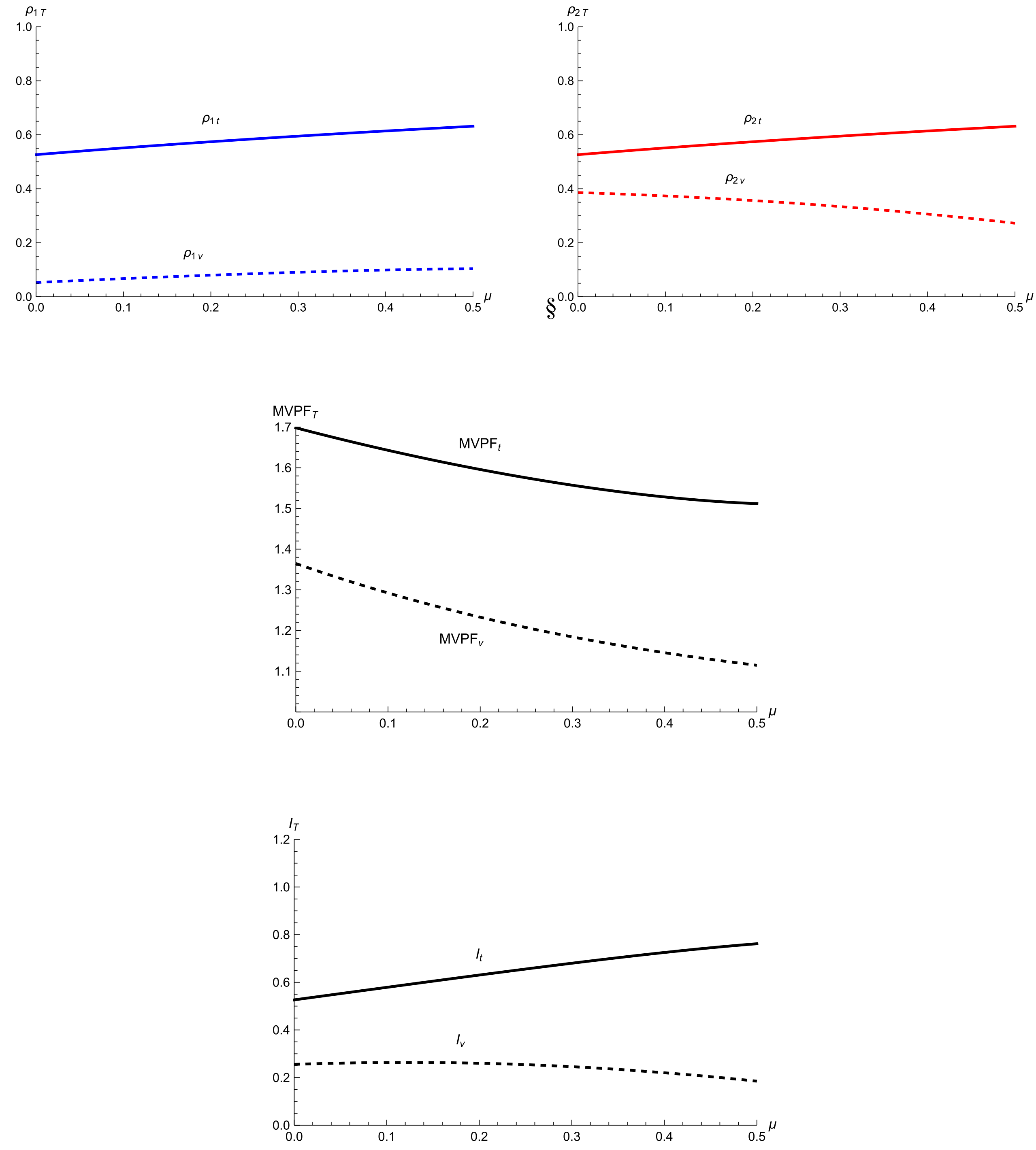

Fig. 8. Pass-through when Firm 1 (left) and Firm 2 (right) face an identical demand (linear) but have different marginal costs (top), and the associated market-level marginal value of public funds (middle) and incidence (bottom): The case of quantity competition.

4.3. Marginal value of public funds and incidence at the market-level

It is also useful to consider the *market-level* welfare measures in consideration of firm heterogeneity. More specifically, for $T \in \{t, v\}$, we define the market-level $MVPF_T$ and I_T by

$$MVPF_T = \frac{\left(\sum_{i=1}^{n} \partial CS_i/\partial T\right) + \left(\sum_{i=1}^{n} \partial PS_i/\partial T\right)}{\left(-\sum_{i=1}^{n} \partial R_i/\partial T\right)}$$

and

$$I_T = \frac{\left(\sum_{i=1}^{n} \partial CS_i/\partial T\right)}{\left(\sum_{i=1}^{n} \partial PS_i/\partial T\right)},$$

respectively,[41] where

[41] Note that the former is not necessarily equal to $\sum_{i=1}^{n} MVPF_{iT}$, and the latter is not necessarily equal to $\sum_{i=1}^{n} I_{iT}$ except for the case of symmetric firms.

$$\frac{\partial CS_i}{\partial T} = -\tilde{\rho}_{iT} \cdot q_i,$$

$$\frac{\partial PS_i}{\partial T} = (1-v)\left[\tilde{\rho}_{iT} - \psi_i \cdot \left(\sum_{j=1}^{n} \epsilon_{ij}\tilde{\rho}_{jT}\right)\right] \cdot q_i - \frac{\partial R_i}{\partial T},$$

and

$$\frac{\partial R_i}{\partial T} = \left[v - \tau_i \cdot \left(\sum_{j=1}^{n} \epsilon_{ij}\right)\right] \cdot \tilde{\rho}_{iT} \cdot q_i + F_{iT}$$

are the marginal per-firm changes in consumer surplus, producer surplus, and the government revenue, respectively (these can be derived by applying Proposition C.4.1 to this case of two-dimensional taxation), where

$$F_{iT} \equiv \begin{cases} q_i & \text{for } T = t \\ p_i q_i & \text{for } T = v. \end{cases}$$

To justify our definitions, recall that we adopt the representative consumer approach: ignoring y, the net utility, i.e., aggregate consumer surplus, is $CS = u(\mathbf{q}) - \mathbf{p}^{\mathrm{T}} \cdot \mathbf{q}$. Under firm symmetry, a change in (per-firm) consumer surplus is simply given by $dCS = -q\, dp$ (Eq. 6). Now, under firm heterogeneity, notice that $CS = u[\mathbf{q}(\mathbf{p})] - p_1 q_1(\mathbf{p}) - \cdots - p_n q_n(\mathbf{p})$ so that

$$\begin{aligned} dCS = &\left(\underbrace{\frac{\partial u}{\partial q_1}}_{=p_1}\frac{\partial q_1}{\partial p_1} + \cdots + \underbrace{\frac{\partial u}{\partial q_n}}_{=p_n}\frac{\partial q_n}{\partial p_1}\right)dp_1 \\ &\cdots + \left(\underbrace{\frac{\partial u}{\partial q_1}}_{=p_1}\frac{\partial q_1}{\partial p_n} + \cdots + \underbrace{\frac{\partial u}{\partial q_n}}_{=p_n}\frac{\partial q_n}{\partial p_n}\right)dp_n \\ &- \left(q_1 + p_1\frac{\partial q_1}{\partial p_1} + \cdots + p_n\frac{\partial q_n}{\partial p_1}\right)dp_1 \\ &\cdots - \left(p_1\frac{\partial q_1}{\partial p_n} + \cdots + q_n + p_n\frac{\partial q_n}{\partial p_n}\right)dp_n. \end{aligned}$$

Therefore, a change in aggregate consumer surplus is given by the following simple sum of each firm's contributions:

$$dCS_T = \underbrace{-q_1 dp_1}_{\equiv dCS_{1T}} - \cdots - \underbrace{-q_n dp_n}_{\equiv dCS_{nT}},$$

which justifies our definitions above.

4.4. Cost heterogeneity

To understand how firm heterogeneity is related to the welfare implications of taxation, we consider an example where two firms are symmetrically differentiated—hence facing an identical demand—but have different marginal costs. Specifically, firm $i = 1, 2$ faces the linear demand, $q_i(p_1, p_2) = b - \lambda p_i + \mu p_j, j \neq i$, $j = 1, 2$. Suppose that either firm's marginal cost of production is constant, $mc_i \geqslant 0$, and Firm 1 is a low-cost firm: $mc_1 < mc_2$.

In a similar vein, Anderson et al. (2001b) (ADKb) juxtapose price and quantity competition. However, their quantity competition assumes homogeneous products in the spirit of Cournot's (1838) original formulation. On the other hand, our formulation is more general than the ADKb setting because they only consider homogeneous-product quantity competition and one variant of the Hotelling (1929) competition. Instead, our formulation enables us to use the demand structure for both price and quantity competition that is derived from the same utility of the representative consumer. For the sake of exposition, we focus on the linear demand system.

First, suppose that these two firms complete in price. Then, the first-order conditions for firm i in this pricing game is expressed as:

$$\left[\left(1 - \frac{t}{p_i} - v\right) - \frac{q_i}{p_i \cdot \left(-\frac{\partial q_i}{\partial p_i}\right)}(1-v)\right]p_i = mc_i$$

in accordance with Eq. (12), where $-\partial q_i/\partial p_i = \lambda$. To compute the market-level welfare characteristics, we need the values for ψ_i (firm i's pricing strength index), ρ_{iT} (firm i's pass-through), and ϵ^{ρ}_{iT}, as well as v (ad valorem tax) and $\tau_i \equiv v + t/p_i$ (the government tax revenue divided by firm i's gross revenue). See Online Appendix G for these calculations.

The top panel of Fig. 7 depicts how the pass-through vary differently across the two firms (the left side is for Firm 1 and the right for Firm 2) when the degree of product differentiation changes (a higher μ indicates less differentiation), assuming $b = 1, (mc_1, mc_2) = (0, 0.5), \lambda = 1$ and $(t, v) = (0.05, 0.05)$.[42] As in Section 3, the market-level marginal value of public funds, and the the market-level incidence are displayed in the middle and bottom panels, respectively. It is observed that $MVPF_t$ is higher than $MVPF_v$, a result in accordance with the case of firm symmetry.

Similarly, the first-order conditions for firm i under quantity competition is given by:

$$\left[\left(1 - \frac{t}{p_i} - v\right) - \frac{q_i}{p_i \cdot [-1/(\partial p_i/\partial q_i)]}(1-v)\right]p_i = mc_i,$$

in accordance with Eq. (12), where

$$-\frac{1}{\partial p_i/\partial q_i} = \frac{(\lambda+\mu)(\lambda-\mu)}{\lambda}.$$

Fig. 8 exhibits the similarity to the case of price competition. In sum, it appears that whether firms price or quantity compete does not matter much to the determination of the MVPF and incidence in a general setting of product differentiation. More detailed analysis is left for future research.

5. Concluding Remarks

In this paper, we characterize the welfare measures of taxation under general specifications of market demand, production cost, and imperfect competition, encompassing a broader class of multiple policy interventions and other external changes other than taxation. For symmetric oligopoly, we first demonstrate how the unit tax pass-through rate ρ_t is be related to the ad valorem tax pass-through semi-elasticity ρ_v (i.e., Proposition 1). The pass-through is also characterized, generalizing Weyl and Fabinger's (2013) formula (i.e., Proposition 2). We then derive formulas for measuring marginal welfare losses resulting from unit and ad valorem taxation, $MVPF_t$ and $MVPF_v$, respectively (Proposition 3) as well as the formulas for tax incidence, I_t and I_v (Proposition 4). Section 3 computes these welfare measures using the representative classes of market demand.

We then introduce heterogeneous firms in Section 4 to generalize these formulas that can be understood as a natural extension of those obtained under firm symmetry. Our derivation is based on a general framework, illustrated in Appendix C, which uses the idea of tax revenue as a function parameterized by a vector of tax parameters and thus can allow multi-dimensional pass-through: the combination of specific and ad valorem taxes is interpreted as a special case of two-dimensional government intervention. In this way, we have provided a comprehensive framework for wel-

[42] Here, we consider the restriction, $\mu < \lambda < b/mc_2 + \mu$, for the range of μ. In Fig. 7, we highlight $\mu \in [0.0, 0.5]$.

fare evaluation of taxation under imperfect competition, which can also allow many applications in a variety of contexts other than taxation (see Online Appendix B).

In this paper, we seek for a general analysis of specific and ad valorem taxation under imperfect competition, assuming away any beneficial effects of government spending. How does the government raise its tax revenue and spend its expenditure in imperfectly competitive product, labor, and capital markets? Admittedly, our framework is, as in Weyl and Fabinger's (2013) analysis, limited to the Cournot-Marshall paradigm of partial equilibrium. Our study is a small step toward a more thorough understanding of the relationship between imperfectly competitive private markets and the role of public sector in such a framework as a general equilibrium model (e.g., Harberger, 1962; Azar and Vives, 2021).

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A. Proofs and further discussion for Section 2

A.1. Discussion of signs of changes in welfare components for a specific tax increase

Fig. 2 shows the effect of a specific tax increase in one case. Here we discuss the signs of welfare component changes in generality. It is helpful to work at the infinitesimal level, where such a tax change would correspond to $dt > 0$ and $dv = 0$. For the producer surplus, the quantity effect is negative $dPS_{\leftrightarrow} = (1-v)p\eta\theta\, dq < 0$, and the value effect,

$$dPS_{\updownarrow} = (1-v)q\, dp - q\, dt = -q(1-(1-v)\rho_t)\, dt\,,$$

is negative for $\rho_t < 1/(1-v)$ and positive for $\rho_t > 1/(1-v)$. The overall change is

$$\begin{aligned} dPS &= (1-v)p\eta\theta dq - q[1-(1-v)\rho_t]dt \\ &= \left[\tfrac{1}{\rho_t} - (1-v)(1-\theta)\right]\eta p\, dq, \end{aligned}$$

which is negative for $1/\rho_t > (1-v)(1-\theta)$ and positive for $1/\rho_t < (1-v)(1-\theta)$. For a sufficiently small value of pass-through, the firms' profit will decrease when t is increased. For the specific tax revenue, the quantity effect and the value effect have opposite signs: $dR_{t\leftrightarrow} = t\, dq < 0$, $dR_{t\updownarrow} = q\, dt > 0$. The overall change $dR_t = t\,dq + q\,dt = (t - \eta p/\rho_t)dq$ is positive for $t < \eta p/\rho_t$ and negative for $t > \eta p/\rho_t$. For the ad valorem tax revenue, the quantity effect and the value effect again have opposite signs: $dR_{v\leftrightarrow} = vp\, dq < 0$, $dR_{v\updownarrow} = qv\, dp > 0$. The overall change $dR = dR_{v\leftrightarrow} + dR_{v\updownarrow} = (1-\eta)vp\,dq$ is negative, if assume $\eta < 1$, as we typically do. The consumer surplus decreases, as $dCS_{\leftrightarrow}$ is zero, and $dCS_{\updownarrow} = -q\,dp$ is unambiguously negative for $dt > 0$.

A.2. Discussion of signs of changes in welfare components for an ad valorem tax increase

Fig. 3 show the effect of a specific tax increase in one case. Here we discuss the signs of welfare component changes in generality. It is helpful to work at the infinitesimal level where such a tax change would correspond to $dv > 0$ and $dt > 0$. For the producer surplus, the quantity effect is negative $dPS_{\leftrightarrow} = (1-v)p\eta\theta\, dq < 0$, and the value effect,

$$dPS_{\updownarrow} = (1-v)q\, dp - pq\, dv = -\left(1-(1-v)\rho_v\right)pq\, dv\,,$$

is negative for $\rho_t < 1/(1-v)$ and positive for $\rho_t > 1/(1-v)$. The overall change is

$$\begin{aligned} dPS &= (1-v)p\eta\theta dq - \left[1-(1-v)\rho_v\right]pq\, dv \\ &= \left[\tfrac{1}{\rho_v} - (1-v)(1-\theta)\right]\eta p\, dq, \end{aligned}$$

which is negative for $1/\rho_v > (1-v)(1-\theta)$ and positive for $1/\rho_v < (1-v)(1-\theta)$. For a sufficiently small value of pass-through, the firms' profit will decrease when v is increased. For the specific tax revenue, the quantity effect $dR_{t\leftrightarrow} = t\, dq$ is negative, while the value effect $dR_{t\updownarrow}$ is zero as the specific tax rate is unchanged. The overall change $dR_t = dR_{t\leftrightarrow} = t\, dq$ is therefore negative. For the ad valorem tax revenue, the quantity effect and the value effect again have opposite signs: $dR_{v\leftrightarrow} = vp\, dq < 0$, $dR_{v\updownarrow} = qv\, dp + qp\, dv > 0$. The overall change $dR = dR_{v\leftrightarrow} + dR_{v\updownarrow} = (1-\eta)vp\,dq$ is negative, if assume $\eta < 1$, as we typically do. The consumer surplus decreases, as $dCS_{\leftrightarrow}$ is zero, and $dCS_{\updownarrow} = -q\,dp$ is unambiguously negative for $dt > 0$.

A.3. Proof of Proposition 1

Let us consider a simultaneous infinitesimal change dt and dv in the taxes t and v that leaves the equilibrium price (and quantity) unchanged, which requires the "effective" marginal cost $(t+mc)/(1-v)$ in Eq. (1) to remain the same. This implies the following comparative statics relationship:

$$\begin{aligned} &\frac{\partial}{\partial t}\left(\frac{t+mc}{1-v}\right)dt + \frac{\partial}{\partial v}\left(\frac{t+mc}{1-v}\right)dv = 0 \\ &\Rightarrow \frac{dt}{1-v} + \frac{t+mc}{(1-v)^2}dv = 0 \\ &\Rightarrow dt = -\frac{t+mc}{1-v}dv. \end{aligned}$$

Note here that we do not need to take derivatives of mc even though it depends on q, simply because by assumption the quantity is unchanged. The total induced change in price, which is generally expressed as $dp = \rho_t dt + \rho_v p\, dv$, must equal zero in this case, implying the desired result:

$$\begin{aligned} &\rho_t dt + \rho_v p \cdot dv = 0 \\ &\Rightarrow -\frac{t+mc}{1-v}\rho_t dv + \rho_v p \cdot dv = 0 \\ &\Rightarrow \rho_v = (1-\eta\theta)\rho_t \\ &\Rightarrow \rho_v = \frac{\epsilon-\theta}{\epsilon}\rho_t. \end{aligned}$$

A.4. Proof of Proposition 2

Consider the comparative statics with respect to a small change dt in the per-unit tax t. Then, the Learner condition becomes:

$$\underbrace{p - \frac{t+mc}{1-v}}_{\text{markup}} = \theta \cdot ms.$$

Then, in equilibrium,

$$dp - \frac{dt + dmc}{1-v} = d(\theta \cdot ms)$$

$$\Longleftrightarrow \quad \underbrace{(1-v)\left[\underbrace{dp}_{>0} - \underbrace{d(\theta\cdot ms)}_{<0}\right]}_{\text{change in marginal benefit}} = \underbrace{\underbrace{dt}_{>0} + \underbrace{dmc}_{<0}}_{\text{change in specific-tax inclusive marginal cost}},$$

and thus, using $dt = dp/\rho_t$, the equation is rewritten as

$$\rho_t = \frac{1}{\underbrace{(1-v)[dp + (-d(\theta\cdot ms))]}_{\text{(A)>0:revenue increase}} + \underbrace{(-dmc)}_{\text{(B)>0:cost savings}}} dp.$$

Now, consider term (A) above. Note first $d(\theta\cdot ms) = (\theta\cdot ms)'dq$ so that $d(\theta\cdot ms) = -q\epsilon(\theta\cdot ms)'(dp/p)$, because by definition $dq = -q\epsilon\cdot(dp/p)$. Here, for a small increase $dt > 0$,

$$\underbrace{d(\theta\cdot ms)}_{<0} = \underbrace{-q\epsilon}_{>0}(\theta\cdot ms)'\underbrace{\frac{dp}{p}}_{>0}$$

so that $(\theta\cdot ms)' > 0$. By definition, $ms \equiv -p'q = \eta p$. Thus, $d(\theta\cdot ms) = -q\epsilon(\theta\eta p)'(dp/p)$. Now, note that $(\theta\eta p)' = (\theta\eta)'p + (\theta\eta)p'$. Hence,

$$d(\theta\cdot ms) = -q\epsilon\left[(\theta\eta)'p + (\theta\eta)p'\right]\frac{dp}{p}$$

$$\begin{aligned}\iff d(\theta\cdot ms) &= -q\epsilon(\theta\eta)'dp + [-q\epsilon(\theta\eta)p'\cdot(dp/p)] \\ &= \left[\theta\eta - q\epsilon(\theta\eta)'\right]dp > 0.\end{aligned}$$

Next, consider term (B). A change in marginal cost, dmc, is expressed in terms of dp by $dmc = -[(1-v)\theta\eta + 1 - \tau]\chi\epsilon\cdot dp < 0$. To see this, note first that $dmc = \chi mc\cdot(dq/q) = -(\chi\epsilon\cdot mc)(dp/p)$. Then, mc in this expression can be eliminated by rewriting $p - \theta\cdot ms = (mc+t)/(1-v) \Rightarrow mc = (1-v)(p+\theta qp') - t = (1-v)(1-\theta\eta)p - t$, which implies that $dmc = -[(1-v)(1+\theta\eta) - t/p]\chi\epsilon\cdot dp$. Then, in terms of the per-unit revenue burden, $\tau \equiv v + t/p$, that is, $dmc = -[(1-v)(1-\theta\eta) - \tau + v]\chi\epsilon dp = -[-(1-v)\theta\eta + 1 - \tau]\chi\epsilon dp$. Finally, using the expressions for dmc and $d(\theta\cdot ms)$, it is verified that

$$\begin{aligned}\rho_t &= \frac{dp}{(1-v)[dp - d(\theta\cdot ms)] - dmc} \\ &= \frac{1}{\underbrace{(1-v)\left[(1-\theta\eta) + (\theta\eta)'\epsilon q\right]}_{\text{revenue increase}} + \underbrace{(1-\tau)\epsilon\chi - (1-v)\theta\chi}_{\text{cost savings}}},\end{aligned}$$

$$\iff \rho_t = \frac{1}{1-v}\cdot\frac{1}{\underbrace{\left[(1-\theta\eta) + (\theta\eta)'\epsilon q\right]}_{\text{revenue increase}} + \underbrace{\left[-\theta + \frac{1-\tau}{1-v}\epsilon\right]\chi}_{\text{cost savings}}}.$$

Finally, ρ_v is obtained from this expression and Eq. (10).

Next, to express this formula in terms of Weyl and Fabinger's (2013, p. 548) notation, recall their Eq. (2):

$$\rho = \frac{1}{1 + (\epsilon - \theta)\chi + \theta/\epsilon_\theta + \theta/\epsilon_{ms}},$$

where their ϵ_D and ϵ_S are replaced by our ϵ and $1/\chi$, respectively. First, the denominator in our formula is rewritten as:

$$1 - (\eta+\chi)\theta + \epsilon q(\theta\eta)' + \frac{1-\tau}{1-v}\epsilon\chi = 1 + \left(\frac{1-\tau}{1-v}\epsilon - \theta\right)\chi + \frac{\theta}{\epsilon_\theta} + \theta\cdot\left(-\frac{1}{\epsilon} + \eta'\epsilon q\right)$$

because

$$(\theta\eta)'\epsilon q = (\theta'\eta + \theta\eta')\epsilon q = \left[\frac{\theta}{q\epsilon_\theta}\eta + \theta\eta'\right]\epsilon q = \frac{\theta}{\epsilon_\theta} + \theta\eta'\epsilon q.$$

Next, since $\eta = -qp'/p$, it is verified that $\eta' = -\{p'\ p + qpp'' - q\ [p']^2\}/p^2$, implying that

$$\eta'\epsilon q = \frac{p'p + qpp'' - q[p']^2}{p^2}\cdot\frac{p}{p'q}\cdot q = \frac{1}{\epsilon} + \left(1 + \frac{p''}{p'}q\right),$$

where $1 + p''q/p$ is replaced by $1/\epsilon_{ms}$ because $ms \equiv -p'q$ and thus $ms' = -(p''q + p')$. Then, it is readily verified that

$$1 - (\eta+\chi)\theta + \epsilon q(\theta\eta)' + \frac{1-\tau}{1-v}\epsilon\chi = 1 + \left(\frac{1-\tau}{1-v}\epsilon - \theta\right)\chi + \frac{\theta}{\epsilon_\theta} + \frac{\theta}{\epsilon_{ms}}.$$

In summary, Weyl and Fabinger's (2013, p. 548) original Eq. (2) is generalized to

$$\rho = \frac{1}{1-v}\cdot\frac{1}{1 + \left(\frac{1-\tau}{1-v}\epsilon - \theta\right)\chi + \frac{\theta}{\epsilon_\theta} + \frac{\theta}{\epsilon_{ms}}}$$

with non-zero initial ad valorem tax, which is equivalent to our formula for ρ_t:

$$\rho_t = \frac{1}{1-v}\cdot\frac{1}{1 + \frac{1-\tau}{1-v}\epsilon\chi - (\eta+\chi)\theta + \epsilon q(\theta\eta)'},$$

and from Proposition 1, it is readily observed that ρ_v can also be written in terms of Weyl and Fabinger's (2013) notation:

$$\rho_v = \frac{\epsilon - \theta}{(1-v)\epsilon}\cdot\frac{1}{1 + \left(\frac{1-\tau}{1-v}\epsilon - \theta\right)\chi + \frac{\theta}{\epsilon_\theta} + \frac{\theta}{\epsilon_{ms}}}.$$

Appendix B. Specifying the mode of imperfect competition under firm symmetry

In this appendix, we demonstrate that for a static game of price or quantity competition with no anti-competitive conduct, our general formulas of the marginal value of public funds and the pass-through derive the expressions in terms of demand primitives such as the *elasticities*, the *curvatures*, and the *marginal cost elasticity* χ.[43] Throughout this appendix, we assume that firms' conduct is simply described by one-shot Nash equilibrium, without any other further possibilities such as tacit collusion. As seen below, this assumption enables one to express the conduct index in terms of demand and inverse demand elasticities, using Eq. (1) directly (see Subsection B.2 below). Online Appendix F further investigates the relationship between elasticities and curvatures.

B.1. Elasticities and curvatures of the demand system

B.1.1. Direct demand

We additionally define the *cross-price elasticity* $\epsilon_{cross}(p)$ *of the firm's direct demand* by

$$\epsilon_{cross}(p) \equiv \frac{(n-1)p}{q(p)}\cdot\frac{\partial q_{i'}(\mathbf{p})}{\partial p_i}\Big|_{\mathbf{p}=(p,\ldots,p)},$$

where i and i' is an arbitrary pair of distinct indices. It is related to the industry demand elasticity $\epsilon(p)$ by $\epsilon_{own} = \epsilon + \epsilon_{cross}$.[44] Next, we define the *own curvature* $\alpha_{own}(p)$ *of the firm's direct demand* and the *cross curvature* $\alpha_{cross}(p)$ *of the firm's direct demand* by:[45]

[43] The question of whether quantity- or price-setting firms are more appropriate depends on the nature of competition. As Riordan (2008, p. 176) argues, quantity competition is a more appropriate model if one depicts a situation where firms determine the necessary production capacity. However, price-setting firms are more suitable if firms in the industry of focus can quickly adjust to demand by changing their prices.

[44] Holmes (1989) shows this for two symmetric firms, but it is straightforward to verify this relation more generally. See the equation in Footnote 13 above. Note that the equation $\epsilon_{own} = \epsilon + \epsilon_{cross}$ simply means that the percentage of consumers who cease to purchase firmi's product in response to its price increase is decomposed into (i) those who no longer purchase from any of the firms (ϵ) and (ii) those who switch to (any of) the other firms' products (ϵ_{cross}). Thus, ϵ_{own} measures the*firm's own competitiveness*, which is expressed in terms of the industry elasticity and the intensity of rivalry. In this sense, these three price elasticities characterize the "first-order" competitiveness, which determines whether the equilibrium price is high or low, but one of them is not independently determined from the other two elasticities.

[45] The curvature $\alpha_{own}(p)$ here corresponds to $\alpha(p)$ of Aguirre et al. (2010, p. 1603).

$$\alpha_{own}(p) \equiv -p \cdot \left(\frac{\partial q_i(\mathbf{p})}{\partial p_i}\right)^{-1} \cdot \frac{\partial^2 q_i(\mathbf{p})}{\partial p_i^2},$$

and

$$\alpha_{cross}(p) \equiv -(n-1)p \cdot \left(\frac{\partial q_i(\mathbf{p})}{\partial p_i}\right)^{-1} \cdot \frac{\partial^2 q_i(\mathbf{p})}{\partial p_i \partial p_{i'}},$$

respectively, where again the derivatives are evaluated at $\mathbf{p} = (p, \ldots, p)$, and i and i' is an arbitrary pair of distinct indices. These curvatures satisfy $\alpha = (\alpha_{own} + \alpha_{cross})\epsilon_{own}/\epsilon$ and are related to the elasticity of $\epsilon_{own}(p)$ by $p\epsilon_{own}'(p)/\epsilon_{own}(p) = 1 + \epsilon(p) - \alpha_{own}(p) - \alpha_{cross}(p)$ (see Online Appendix F.1 for the derivation and a related discussion).

B.1.2. Inverse demand

We introduce analogous definitions for inverse demand. First, we define *the cross quantity elasticity $\eta_{cross}(q)$ of the firm's inverse demand as*

$$\eta_{cross}(q) \equiv (n-1)\frac{q}{p(q)} \cdot \frac{\partial p_{i'}(\mathbf{q})}{\partial q_i}\Big|_{\mathbf{q}=(q,\ldots,q)}$$

for arbitrary distinct i and i'. It is verified that $\eta_{own} = \eta + \eta_{cross}$.[46] We furthermore define the *own curvature $\sigma_{own}(q)$ of the firm's inverse demand* and the *cross curvature $\sigma_{cross}(q)$ of the firm's inverse demand* by:

$$\sigma_{own}(q) \equiv -q \cdot \left(\frac{\partial p_i(\mathbf{q})}{\partial q_i}\right)^{-1} \cdot \frac{\partial^2 p_i(\mathbf{q})}{\partial q_i^2}$$

and

$$\sigma_{cross}(q) \equiv -(n-1)q \cdot \left(\frac{\partial p_i(\mathbf{q})}{\partial q_i}\right)^{-1} \cdot \frac{\partial^2 p_i(\mathbf{q})}{\partial q_i \partial q_{i'}},$$

respectively, where again the derivatives are evaluated at $\mathbf{q} = (q, \ldots, q)$ and the indices i and i' are distinct. These curvatures represent an oligopoly counterpart of monopoly $\sigma(q)$ of Aguirre et al. (2010, p. 1603). They satisfy the relationship $\sigma = (\sigma_{own} + \sigma_{cross})(\eta_{own}/\eta)$ and are related to the elasticity of $\eta_{own}(q)$ by $q\eta_{own}'(q)/\eta_{own}(q) = 1 + \eta(q) - \sigma_{own}(q) - \sigma_{cross}(q)$ (see Online Appendix F.2 for the derivation and a related discussion).

B.2. Expressions for pass-through and the conduct index

B.2.1. Price competition

In the case of price competition, the conduct index θ is $\theta = \epsilon/\epsilon_{own} = 1/(\eta\epsilon_{own})$, which is verified by comparing the firm's first-order condition with Eq. (1). The marginal change in deadweight loss and the incidence are obtained by substituting these expressions into those of Propositions 3 and 4.

Proposition B.2.1. *Under symmetric oligopoly with price competition and with a possibly non-constant marginal cost, the unit tax pass-through and the ad valorem tax pass-through are characterized by*

$$\rho_t = \frac{1}{1-v} \cdot \frac{1}{1 + \frac{(1-\alpha/\epsilon_{own})\epsilon}{\epsilon_{own}} + \left(\frac{1-\tau}{1-v} - \frac{1}{\epsilon_{own}}\right)\epsilon\chi}$$

and

$$\rho_v = \frac{1}{1-v} \cdot \frac{1}{\frac{1}{1-1/\epsilon_{own}} + \frac{(1-\alpha/\epsilon_{own})\epsilon}{\epsilon_{own}-1} + \left(\frac{1-\tau}{1-v} \cdot \frac{\epsilon_{own}}{\epsilon_{own}-1} - \frac{1}{\epsilon_{own}-1}\right)\epsilon\chi},$$

respectively.

Proof. Since in the case of price setting $\theta = \epsilon/\epsilon_{own} = 1/(\eta\epsilon_{own})$, we have $(\eta + \chi)\theta = (1 + \epsilon\chi)/\epsilon_{own}$ and

$$\begin{aligned}(\theta\eta)'\epsilon q &= \epsilon q(d(\theta\eta)/dq \\ &= \epsilon q(d(\epsilon_{own}^{-1}))/dq \\ &= -\epsilon_{own}^{-2}\epsilon q(d\epsilon_{own})/dq = \epsilon_{own}^{-2} p(d\epsilon_{own})/dp, \\ &= (1 + \epsilon - \alpha\epsilon/\epsilon_{own})/\epsilon_{own}\end{aligned}$$

where in the last equality we utilize the expression for the elasticity of $\epsilon_{own}(p)$ and $\alpha_{own} + \alpha_{cross} = \alpha\epsilon/\epsilon_{own}$ from Subsubsection B.1.1 above. Substituting these into the expression for ρ_t in Proposition 2 gives

$$\rho_t = \frac{1}{1-v} \cdot \frac{1}{1 - \frac{1}{\epsilon_{own}}(1+\epsilon\chi) + \frac{1}{\epsilon_{own}}\left(1 + \epsilon - \frac{\alpha\epsilon}{\epsilon_{own}}\right) + \frac{1-\tau}{1-v}\epsilon\chi},$$

which is equivalent to the expression for ρ_t in the proposition. Since for price setting $\theta = \epsilon/\epsilon_{own}$, the relationship in Proposition 1 implies $\rho_v = (\epsilon - \theta)\rho_t/\epsilon = (\epsilon_{own} - 1)\rho_t/\epsilon_{own}$, which leads to the desired expression for ρ_v. □

To understand this proposition, first recall from Proposition 2 that

$$\rho_t = \frac{1}{1-v} \quad \frac{1}{\underbrace{[(1-\theta\eta) + (\theta\eta)'\epsilon q]}_{\text{revenue increase}} + \underbrace{\left[\frac{1-\tau}{1-v}\epsilon - \theta\right]\chi}_{\text{cost savings}}}.$$

Then, with $\theta = \epsilon/\epsilon_{own}, 1 - \theta\eta = 1 - 1/\epsilon_{own}$, $(\theta\eta)'\epsilon q = (1 + \epsilon - \alpha\epsilon/\epsilon_{own})/\epsilon_{own}$, the equality above is rewritten as

$$\begin{aligned}\rho_t &= \frac{1}{1-v} \cdot \frac{1}{\underbrace{\left[\left(1 - \frac{1}{\epsilon_{own}}\right) + \frac{1 + \epsilon - \alpha\epsilon/\epsilon_{own}}{\epsilon_{own}}\right]}_{\text{revenue increase}} + \underbrace{\left[\frac{1-\tau}{1-v} - \frac{1}{\epsilon_{own}}\right]\epsilon\chi}_{\text{cost savings}}} \\ &= \frac{1}{1-v} \cdot \frac{1}{\underbrace{\left[1 + \frac{(1-\alpha/\epsilon_{own})\epsilon}{\epsilon_{own}}\right]}_{\text{revenue increase}} + \underbrace{\left[\frac{1-\tau}{1-v} - \frac{1}{\epsilon_{own}}\right]\epsilon\chi}_{\text{cost savings}}}.\end{aligned}$$

To further facilitate the understanding of the connection of this result to Proposition 2, consider the case of zero initial taxes ($t = v = \tau = 0$). Then, Proposition 2 claims that

$$\rho_t = \frac{1}{1 + \epsilon\chi - \theta\chi + [-\eta\theta + \epsilon q(\theta\eta)']},$$

whereas Proposition B.2.1 shows that

$$\begin{aligned}\rho_t &= \frac{1}{1 + \epsilon\chi - \theta\chi + \left[-\frac{1}{\epsilon} \cdot \frac{\epsilon}{\epsilon_{own}} + \frac{1 + (1-\alpha/\epsilon_{own})\epsilon}{\epsilon_{own}}\right]} \\ &= \frac{1}{1 + \epsilon\chi - \theta\chi + \left(1 - \frac{\alpha}{\epsilon_{own}}\right)\theta},\end{aligned}$$

because $\theta = \epsilon/\epsilon_{own}$. Here, the direct effect from $-\eta\theta$ is canceled out by the part of the indirect effect from $\epsilon q(\theta\eta)'$. The new term, which appears as the fourth term in the denominator, shows *how the industry's curvature affects the pass-through*: as the

[46] The identity $\eta_{own} = \eta + \eta_{cross}$ means that as a response to firm i's increase in its output, the industry as a whole reacts by lowering firm i's price (η). However, each firm (other than i) reacts to this firm i's output increase by reducing its own output. This counteracts the initial change in the price ($\eta_{cross} < 0$), and thus a percentage reduction in the price for firm i (η_{own}) is smaller thanη, which does not take into account strategic reactions. Note here that $1/\eta_{own}$, not η_{own}, measures the industry's competitiveness. Thus, as in the case of price competition, these three quantity elasticities characterize "first-order" competitiveness, which determines whether the equilibrium quantity is high or low.

demand curvature becomes larger (i.e., as the industry's demand becomes more convex), then the pass-through becomes higher, although this effect is mitigated by the intensity of competition, θ.

B.2.2. Quantity competition

Next, in the case of quantity competition, the conduct index θ is given by $\theta = \eta_{own}/\eta$, which is, again, verified by comparing the firm's first-order condition with Eq. (1). Again, the marginal change in deadweight loss and the incidence are obtained by substituting these expressions into those of Propositions 3 and 4.

Proposition B.2.2. *Under symmetric oligopoly with quantity competition and with a possibly non-constant marginal cost, the unit tax pass-through and the ad valorem tax pass-through are characterized by*

$$\rho_t = \frac{1}{1-v}\cdot\frac{1}{1+\dfrac{\eta_{own}}{\eta}-\sigma+\left(\dfrac{1-\tau}{1-v}-\eta_{own}\right)\dfrac{\chi}{\eta}}$$

and

$$\rho_v = \frac{1}{1-v}\cdot\frac{(1-\eta_F)}{1+\dfrac{\eta_{own}}{\eta}-\sigma+\left(\dfrac{1-\tau}{1-v}-\eta_{own}\right)\dfrac{\chi}{\eta}},$$

respectively.

Proof. In the case of quantity setting, $\theta = \eta_{own}/\eta$, so $(\eta+\chi)\theta = (1+\chi/\eta)\eta_{own}$ and $(\theta\eta)'\epsilon q = q(\eta_{own})'/\eta = (1+\eta-\sigma\eta/\eta_{own})\eta_{own}/\eta$, where in the last equality we utilize the expression for the elasticity of $\eta_{own}(q)$ and $\sigma_{own}+\sigma_{cross} = \sigma\eta/\eta_{own}$ from Subsubsection B.1.2 above. Substituting these into the expression for ρ_t in Proposition 2 gives

$$\rho_t = \frac{1}{1-v}\cdot\frac{1}{1-\left(1+\frac{\chi}{\eta}\right)\eta_{own}+\frac{1}{\eta}\left(1+\eta-\frac{\sigma\eta}{\eta_{own}}\right)\eta_{own}+\frac{1-\tau}{1-v}\frac{1}{\eta}\chi},$$

which is equivalent to the expression for ρ_t in the proposition. Since $\theta = \eta_{own}/\eta$, Proposition 1 implies $\rho_v = (\epsilon-\theta)\rho_t/\epsilon = (1/\eta-\eta_{own}/\eta)\rho_t\eta = (1-\eta_{own})\rho_t$, which can be used to verify the expression for ρ_v. □

This proposition is similar to Proposition B.2.1 above. Recall again that

$$\rho_t = \frac{1}{1-v}\cdot\frac{1}{\underbrace{[(1-\theta\eta)+(\theta\eta)'\epsilon q]}_{\text{revenue increase}}+\underbrace{\left[\dfrac{1-\tau}{1-v}\epsilon-\theta\right]\chi}_{\text{cost savings}}}.$$

Then, $\theta = \eta_{own}/\eta$ implies $(1/\epsilon_S-\eta)\theta = [(1/\epsilon_S\eta)-1]\eta_{own}$ and $(\theta\eta)'(q/\eta) = q(\eta_{own})'/\eta = (1+\eta-\sigma_{own}-\sigma_{cros})(\eta_{own}/\eta)$. Thus, the equality above is rewritten as

$$\rho_t = \frac{1}{1-v}\cdot\frac{1}{\underbrace{\left[(1-\eta_{own})+\dfrac{1+\eta-\sigma\eta/\eta_{own}}{\eta}\eta_{own}\right]}_{\text{revenue increase}}+\underbrace{\left[\dfrac{1-\tau}{1-v}\cdot\dfrac{1}{\epsilon_S\eta}-\dfrac{\eta_{own}}{\epsilon_S\eta}\right]}_{\text{cost savings}}}$$

$$= \frac{1}{1-v}\cdot\frac{1}{\underbrace{\left[1+\dfrac{\eta_{own}-\sigma\eta}{\eta}\right]}_{\text{revenue increase}}+\underbrace{\left[\dfrac{1-\tau}{1-v}-\eta_F\right]\dfrac{1}{\epsilon_S\eta}}_{\text{cost savings}}}.$$

To further facilitate the understanding of the connection of this result for to Proposition 2, consider the case of zero initial taxes ($t = v = \tau = 0$) again. Then, Proposition B.2.2 shows that

$$\rho_t = \frac{1}{1+\epsilon\chi-\theta\chi+\left[-\eta\cdot\frac{\eta_{own}}{\eta}+\left(1+\frac{1}{\eta}-\frac{\sigma}{\eta_{own}}\right)\eta_{own}\right]}$$
$$= \frac{1}{1+\epsilon\chi-\theta\chi+\left(1-\frac{\sigma}{\theta}\right)\theta}$$

because $\theta = \eta_{own}/\eta$. Here, the term $(1-\sigma/\theta)\theta$ demonstrates the *effects of the industry's inverse demand curvature, σ, on the pass-through*: as the inverse demand curvature becomes larger (i.e., as the industry's inverse demand becomes more convex), the pass-through becomes higher. Interestingly, in contrast to the case of price competition, this effect is not mitigated by the intensity of competition, θ.

Appendix C. Multi-dimensional pass-through framework under firm heterogeneity

As shown below, it turns out that it is useful to consider a general version of multi-dimensional interventions because specific and ad valorem taxation can be deemed as a special case of a two-dimensional intervention. A key concept is *multi-dimensional pass-through*, which is defined as the impact of infinitesimal changes in interventions $\mathbf{T} \equiv (T_1,\ldots,T_d)$— a d-dimensional vector of tax instruments—on the equilibrium price p_i for firm $i = 1,\ldots,n$. Multi-dimensional pass-through corresponds to a matrix in the case of heterogeneous firms, which can be simplified as a vector under symmetric oligopoly. We argue that multi-dimensional pass-through is an important determinant of the welfare effects of various kinds of government intervention and external changes, not limited to the two-dimensional taxation.

C.1. Price sensitivity and quantity sensitivity of taxes

Consider a tax structure under which firm i's tax payment is expressed as $\phi_i(p_i,q_i,\mathbf{T})$, so that the firm's profit is written as $\pi_i = p_iq_i - c_i(q_i) - \phi_i(p_i,q_i,\mathbf{T})$.[47] Note that the production cost, and hence, the marginal cost $mc_i(q_i)$ of firm i is also allowed to depend on the identity of the firm, and we denote its elasticity by $\chi_i(q_i) \equiv mc_i'(q_i)q_i/mc_i(q_i)$. In the special case of a unit tax t and an ad valorem tax v, $\phi_i(p_i,q_i,\mathbf{T}) = tq_i + vp_iq_i$, where $\mathbf{T} = (t,v)$. Below, we argue how to generalize our previous framework with two policy instruments by defining analogs of t and v even for general interventions that may include multiple instruments, not just two.

We aim to express a decomposition of $\phi_i(p_i,q_i,\mathbf{T})$ analogous to $\phi_i(p_i,q_i,\mathbf{T}) = tq_i + vp_iq_i$. Specifically, we argue that it is possible to write $\phi_i(p_i,q_i,\mathbf{T}) = \bar{t}q_i + \bar{v}p_iq_i$, where $\bar{t}$ and $\bar{v}$ are the averages of appropriately defined functions t and v over the ranges $(0,q_i)$ and $(0,p_iq_i)$. In the special case of specific and ad valorem taxes, these functions should reduce to constants t and v. We verify this property by decomposing $\phi_i(p_i,q_i,\mathbf{T})$ into infinitesimal contributions, each of which resembles specific and ad valorem taxes, respectively. If we set the tax burden at zero quantities and prices: $\phi_i(0,0,\mathbf{T}) = 0$, we can write the desired relationship $\phi_i(p_i,q_i,\mathbf{T}) = \bar{t}q_i + \bar{v}p_iq_i$ as $\phi_i(p_i,q_i,\mathbf{T}) = \int_0^{q_i} t(\tilde{p},\tilde{q},\mathbf{T})\,d\tilde{q} + \int_0^{p_iq_i} v(\tilde{p},\tilde{q},\mathbf{T})\,d(\tilde{p}\tilde{q})$, or alternatively as

[47] To be precise, $\phi(p,q,\mathbf{T})$ represents a simplified notation for a function $\phi(p,q,T_1,\ldots,T_d)$ with $d+2$ arguments.

$$\phi_i(p_i, q_i, \mathbf{T}) = \int_0^1 \left[\left(\frac{t(\tilde{p}_i(s), \tilde{q}_i(s), \mathbf{T})}{\tilde{p}_i} + v(\tilde{p}_i(s), \tilde{q}_i(s), \mathbf{T}) \right) \tilde{p}_i \frac{d\tilde{q}_i}{ds} + v(\tilde{p}_i(s), \tilde{q}_i(s), \mathbf{T}) \tilde{q}_i \frac{d\tilde{p}_i}{ds} \right] ds,$$

where the integration is over an auxiliary parameter s that parameterizes a path $(\tilde{p}_i(s), \tilde{q}_i(s))$ in the price-quantity plane such that $(\tilde{p}_i(0), \tilde{q}_i(0)) = (0, 0)$ and $(\tilde{p}_i(1), \tilde{q}_i(1)) = (p_i, q_i)$.

At the same time, $\phi_i(p_i, q_i, \mathbf{T})$ can be expressed by an integral of its total differential:

$$\phi(p_i, q_i, \mathbf{T}) = \int_0^1 \left[\phi_{\tilde{q}_i}(\tilde{p}_i(s), \tilde{q}_i(s), \mathbf{T}) \frac{d\tilde{q}_i}{ds} + \phi_{\tilde{p}_i}(\tilde{p}_i(s), \tilde{q}_i(s), \mathbf{T}) \frac{d\tilde{p}_i}{ds} \right] ds,$$

where a subscript notation is used for partial derivatives. We observe that if we identify

$$\begin{cases} \left(\frac{t\left(\tilde{p}_i(s), \tilde{q}_i(s), \mathbf{T}\right)}{\tilde{p}_i} + v\left(\tilde{p}_i(s), \tilde{q}_i(s), \mathbf{T}\right) \right) \tilde{p}_i = \phi_{\tilde{q}_i}\left(\tilde{p}_i(s), \tilde{q}_i(s), \mathbf{T}\right) \\ v\left(\tilde{p}_i(s), \tilde{q}_i(s), \mathbf{T}\right) \tilde{q}_i = \phi_{\tilde{p}_i}\left(\tilde{p}_i(s), \tilde{q}_i(s), \mathbf{T}\right), \end{cases}$$

then the desired relationship $\phi_i(p_i, q_i, \mathbf{T}) = \bar{t}q_i + \bar{v}p_i q_i$ is satisfied.

Now, we define the (*first-order*) *price sensitivity* of the (per-firm) tax revenue by

$$v_i(p_i, q_i, \mathbf{T}) \equiv \frac{1}{q_i} \frac{\partial}{\partial p_i} \phi_i(p_i, q_i, \mathbf{T}),$$

and the (*first-order*) *quantity sensitivity* by

$$\tau_i(p_i, q_i, \mathbf{T}) \equiv \frac{1}{p_i} \frac{\partial}{\partial q_i} \phi_i(p_i, q_i, \mathbf{T})$$

so that $t_i(p_i, q_i, \mathbf{T}) = \tau_i(p_i, q_i, \mathbf{T}) p_i + v_i(p_i, q_i, \mathbf{T})$. Note that both the first-order and second-order sensitivities are dimensionless.

C.2. Pricing strength index

We now introduce the *pricing strength index* $\psi_i(\mathbf{q})$ of firm i as a function of $\mathbf{q}$—but independent of the cost side—such that the first-order condition for firm i is:

$$\{1 - \tau_i(p_i(\mathbf{q}), q_i, \mathbf{T}) - \psi_i(\mathbf{q})[1 - v_i(p_i(\mathbf{q}), q_i, \mathbf{T})]\} p_i(\mathbf{q}) = mc_i(q_i). \quad (12)$$

In the special case of symmetric firms, this pricing strength index is expressed by $\psi_i = \eta \cdot \theta$ for all i. Because of this simplicity, analyzing oligopoly in terms of the pricing strength index does not differ from analyzing it in terms of the conduct index. However, these two approaches would differ for heterogeneous firms. An innovation of this paper is to provide an oligopoly analysis in terms of the pricing strength index.

Note here that in the case of specific and ad valorem taxation, it is verified that

$$\tau_i(p_i, q_i, \mathbf{T}) \equiv \frac{1}{p_i} \frac{\partial \phi_i}{\partial q_i}(p_i, q_i, \mathbf{T}) = \frac{t}{p_i} + v$$

and

$$v_i(p_i, q_i, \mathbf{T}) \equiv \frac{1}{q_i} \frac{\partial \phi_i}{\partial p_i}(p_i, q_i, \mathbf{T}) = v$$

so that Eq. (12) becomes

$$\left[\left(1 - \frac{t}{p_i(\mathbf{q})} - v \right) - \psi_i(\mathbf{q})(1 - v) \right] p_i(\mathbf{q}) = mc_i(q_i),$$

as appeared in the main text.

C.3. Pass-through

We express the pass-through rate matrix in terms of these pricing strength indices. Specifically, the pass-through rate is an $n \times d$ matrix $\tilde{\boldsymbol{\rho}}$ whose (i, T_ℓ) element is $\tilde{\rho}_{iT_\ell} = \partial p_i / \partial T_\ell$.
First, we define the following functions:

$$\kappa_i(p_i, q_i, \mathbf{T}) \equiv \frac{\partial^2 \phi_i(p_i, q_i, \mathbf{T})}{\partial p_i \partial q_i},$$

$$v_{(2),i}(p_i, q_i, \mathbf{T}) \equiv \frac{p_i}{q_i} \frac{\partial^2 \phi_i(p_i, q_i, \mathbf{T})}{\partial p_i^2},$$

$$\tau_{(2),i}(p_i, q_i, \mathbf{T}) \equiv \frac{q_i}{p_i} \frac{\partial^2 \phi_i(p_i, q_i, \mathbf{T})}{\partial q_i^2},$$

$$\epsilon_{ij} \equiv -\frac{p_i}{q_i} \frac{\partial q_i(\mathbf{p})}{\partial p_j},$$

and

$$\Psi_{ij} \equiv \frac{p_i}{\psi_i} \frac{\partial \psi_i[\mathbf{q}(\mathbf{p})]}{\partial p_j}.$$

Then, the following proposition is obtained.

Proposition C.3. *The pass-through rate equals*

$$\underbrace{\tilde{\boldsymbol{\rho}}_{T_\ell}}_{n\times 1} = \underbrace{\mathbf{b}^{-1}}_{n\times n} \underbrace{\iota_{T_\ell}}_{n\times 1}, \quad (13)$$

where $\mathbf{b}$ *is an* $n \times n$ *matrix, independent of the choice of* T_ℓ*, with the* (i,j) *element being:*

$$\begin{aligned} b_{ij} \quad = \quad & [1 - \kappa_i - (1 - v_i - v_{(2)i})\psi_i]\delta_{ij} - (1 - v_i)\psi_i \Psi_{ij} \\ & + \{\tau_{(2)i} + (v_i - \kappa_i)\psi_i + [1 - \tau_i - (1 - v_i)\psi_i]\chi_i\}\epsilon_{ij}, \end{aligned}$$

where δ_{ij} *is the Kronecker delta, and for each tax* T_ℓ*,* ι_{T_ℓ} *is an n-dimensional vector with i-th element being:*

$$\iota_{iT_\ell} \equiv p_i \cdot \left(\frac{\partial \tau_i(p_i, q_i, \mathbf{T})}{\partial T_\ell} - \psi_i \frac{\partial v_i(p_i, q_i, \mathbf{T})}{\partial T_\ell} \right).$$

Proof. Eq. (12) indicates that

$$\begin{aligned} & \left[p_i \cdot \left(\frac{\partial \tau_i}{\partial p_i} - \psi_i \cdot \frac{\partial v_i}{\partial p_i} \right) + (1 - v_i)\psi_i - (1 - \tau_i) \right] dp_i \\ & + \left[p_i \cdot \left(\frac{\partial \tau_i}{\partial q_i} - \psi_i \cdot \frac{\partial v_i}{\partial q_i} \right) + mc_i' \right] dq_i + p_i \cdot \left(\frac{\partial \tau_i}{\partial T} - \psi_i \cdot \frac{\partial v_i}{\partial T} \right) dT \\ & + p_i(1 - v_i) d\psi_i = 0, \end{aligned}$$

implying that

$$\begin{aligned} \iota_{iT_\ell} dT_\ell = & [1 - \kappa_i - (1 - v_i - v_{(2),i})\psi_i] dp_i - (1 - v_i)\psi_i \left(\sum_{j=1}^n \Psi_{ij} dp_j \right) \\ & + \{\tau_{(2),i} + (v_i - \kappa_i)\psi_i + [1 - \tau_i - (1 - v_i)\psi_i]\chi_i\} \left(\sum_{j=1}^n \epsilon_{i,j} dp_j \right), \end{aligned}$$

where $dq_i = -(q_i/p_i)\sum_{j=1}^n \epsilon_{i,j} dp_j$, $d\psi_i = (\psi_i/p_i)\sum_{j=1}^n \Psi_{ij} dp_j$ and $mc_i' = \chi_i mc_i / q_i = (p_i/q_i)[1 - \tau_i - (1 - v_i)\psi_i]\chi_i$ are used.[48] Hence

[48] Note that $\partial v_i/\partial p_i = v_{(2),i}/p_i$, $\partial v_i/\partial q_i = (\kappa_i - v_i)/q_i$, $\partial \tau_i/\partial p_i = (\kappa_i - \tau_i)/p_i$ and $\partial \tau_i/\partial q_i = \tau_{(2),i}/q_i$ are also used.

$$\underbrace{\iota_{T_\ell}}_{n\times 1} = \underbrace{\begin{pmatrix} 1-\kappa_1-(1-v_1-v_{(2),1})\psi_1 \\ \vdots \\ 1-\kappa_j-(1-v_j-v_{(2),j})\psi_j \\ \vdots \\ 1-\kappa_n-(1-v_n-v_{(2),n})\psi_n \end{pmatrix}}_{n\times 1} \circ \underbrace{\tilde{\boldsymbol{\rho}}_{T_\ell}}_{n\times 1} - \underbrace{\begin{pmatrix} \ddots & & \\ & (1-v_i)\psi_i\Psi_{ij} & \\ & & \ddots \end{pmatrix}}_{n\times n} \cdot \boldsymbol{\rho}^{\sim}{}_{T_\ell}$$

$$+ \underbrace{\begin{pmatrix} \ddots & & \\ & \{\tau_{(2),i}+(v_i-\kappa_i)\psi_i+[1-\tau_i-(1-v_i)\psi_i]\chi_i\}\epsilon_{i,j} & \\ & & \ddots \end{pmatrix}}_{n\times n} \cdot \boldsymbol{\rho}^{\sim}{}_{T_\ell},$$

and thus, assuming that $\mathbf{b}$ is invertible, Eq. (13) holds. □

Under the two-dimensional taxation, it is verified that $v_{(2)i}(p,q,\mathbf{T})=0$, $\tau_{(2)i}(p,q,\mathbf{T})=0$, and $\kappa_i(p,q,\mathbf{T})=v$. In addition, $\iota_{i\,t}=1$ and $\iota_{i\,v}=p_i\cdot(1-\psi_i)$ because $\partial\tau_i/\partial t=1/p_i, \partial v_i/\partial t=0$, $\partial\tau_i/\partial v=1$, and $\partial v_i/\partial v=1$.

C.4. Welfare changes

So far, we have introduced $\phi_i(p_i,q_i,\mathbf{T})$ as an additional cost in the firm's profit function: $\pi_i=p_iq_i-c_i(q_i)-\phi_i(p_i,q_i,\mathbf{T})$. Here $\mathbf{T}$ is a vector of interventions (in governmental and other external circumstances), which may or may not include traditional taxes. To evaluate welfare changes, we also need to know what part of this cost is collected by the government in the form of taxes. We now introduce the notation $\widehat{\phi}_i(p_i,q_i,\mathbf{T})$ for the tax payment of the firm: in the main text, this corresponds to R_i. The difference $\phi_i(p_i,q_i,\mathbf{T})-\widehat{\phi}_i(p_i,q_i,\mathbf{T})$ corresponds to additional non-tax costs the firm faces. In the case of pure taxation, $\widehat{\phi}_i(p_i,q_i,\mathbf{T})=\phi_i(p_i,q_i,\mathbf{T})$.[49] Then, for each firm i, we define

$$\widehat{v}_i(p_i,q_i,\mathbf{T}) \equiv \frac{1}{q_i}\frac{\partial}{\partial p_i}\widehat{\phi}_i(p_i,q_i,\mathbf{T}),$$

and

$$\widehat{\tau}_i(p_i,q_i,\mathbf{T}) \equiv \frac{1}{p_i}\frac{\partial}{\partial q_i}\widehat{\phi}_i(p_i,q_i,\mathbf{T}).$$

We also write

$$\mathbf{f}_i \equiv \frac{1}{q_i}\nabla\phi_i(p_i,q_i,\mathbf{T}),$$

where $\nabla\phi_i$'s components are

$$\phi_{i\,T_\ell}(p,q,\mathbf{T}) \equiv \partial\phi_i(p_i,q_i,\mathbf{T})/\partial T_\ell,$$

and

$$\widehat{\mathbf{f}}_i \equiv \frac{1}{q_i}\nabla\widehat{\phi}_i(p_i,q_i,\mathbf{T})$$

is also defined analogously.[50]

Let $\boldsymbol{\epsilon}_i$ be an n-dimensional row vector with its j-th component equal to ϵ_{ij} for each $i:\boldsymbol{\epsilon}_i=(\epsilon_{i1},\ldots,\epsilon_{ij},\ldots\epsilon_{in})$. For convenience, we also define $\mathbf{e}_i$ to be an n-dimensional indicator vector with the i-th component equal to 1 and other components zero: $\mathbf{e}_i=(0,\ldots,\underbrace{1}_{\mathrm{i}-th},\ldots,0)$. Then, the following proposition is obtained.

[49] If all of the additional cost to the firm comes from the production side, we have $\widehat{\phi}_i(p_i,q_i,\mathbf{T})=0$ (i.e., the tax payment is zero).

[50] For the two-dimensional taxation, it is verified that
$\mathbf{f}_i \equiv (1/q_i)\nabla\phi_i(p_i,q_i,\mathbf{T}) = (1/q_i)\begin{pmatrix}\partial\phi_i(p_i,q_i,\mathbf{T})/\partial t \\ \partial\phi_i(p_i,q_i,\mathbf{T})/\partial v\end{pmatrix} = \begin{pmatrix}1\\ p_i\end{pmatrix}$
because $\begin{cases}\phi_{i,t}(p_i,q_i,\mathbf{T}) \equiv \frac{\partial\phi_i(p_i,q_i,\mathbf{T})}{\partial t} = q_i \\ \phi_{i,v}(p_i,q_i,\mathbf{T}) \equiv \frac{\partial\phi_i(p_i,q_i,\mathbf{T})}{\partial v} = p_iq_i.\end{cases}$

Proposition C.4.1. *The intervention gradients of consumer surplus, producer surplus, tax revenue, and social welfare with respect to the taxes are*

$$\frac{1}{q_i}\nabla CS_i = -\mathbf{e}_i\tilde{\boldsymbol{\rho}},$$

$$\frac{1}{q_i}\nabla PS_i = (1-v_i)(\mathbf{e}_i-\psi_i\boldsymbol{\epsilon}_i)\tilde{\boldsymbol{\rho}}-\mathbf{f}_i,$$

$$\frac{1}{q_i}\nabla R_i = \left(\widehat{v}_i\mathbf{e}_i-\widehat{\tau}_i\boldsymbol{\epsilon}_i\right)\tilde{\boldsymbol{\rho}}+\widehat{\mathbf{f}}_i,$$

$$\frac{1}{q_i}\nabla W_i = -\left[\widehat{\tau}_i+\psi_i(1-v_i)\right]\boldsymbol{\epsilon}_i\tilde{\boldsymbol{\rho}}+\left(\widehat{v}_i-v_i\right)\mathbf{e}_i\tilde{\boldsymbol{\rho}}+\widehat{\mathbf{f}}_i-\mathbf{f}_i,$$

respectively.

Proof. The result for $(1/q_i)\nabla CS_i$ is straightforward. It suffices to provide expressions for $(1/q_i)\nabla PS_i$ and $(1/q_i)\nabla R_i$ since $(1/q_i)\nabla W_i$ equals the sum of the other three expressions. Note first that in response to a change $T_\ell\to T_\ell+dT_\ell$, we have $dPS_i=d(p_iq_i-c_i(q_i)-\phi_i(p_i,q_i,\mathbf{T}))$ and $d\widehat{\phi}_i(p_i,q_i,\mathbf{T})=p_i\widehat{\tau}_i(p_i,q_i,\mathbf{T})dq_i+q_i\widehat{v}_i(p_i,q_i,\mathbf{T})dp_i+\left(\partial\widehat{\phi}_i/\partial T_\ell\right)dT_\ell$. Then by using Eq. (12), one can rewrite:

$$\begin{aligned} dPS_i &= [-(v_i\psi_i-\tau_i-\psi_i+1)p_i-p_i\tau_i+p_i]dq_i+(q_i-v_iq_i)dp_i-\frac{\partial\phi_i}{\partial T_\ell}dT_\ell \\ &= (1-v_i)\left[p_i\psi_i\cdot\left(\sum_{j=1}^n\frac{\partial q_i}{\partial p_j}dp_j\right)+q_idp_i\right]-\frac{\partial\phi_i}{\partial T_\ell}dT_\ell \\ &= (1-v_i)\left[p_i\psi_i\cdot\left(\sum_{j=1}^n\frac{\partial q_i}{\partial p_j}\tilde{\rho}_{jT_\ell}\right)+q_i\tilde{\rho}_{iT_\ell}\right]dT_\ell-\frac{\partial\phi_i}{\partial T_\ell}dT_\ell \\ &= (1-v_i)q_i\left[\psi_i\cdot\left(\sum_{j=1}^n\frac{p_i\partial q_i}{q_j\partial p_j}\tilde{\rho}_{jT_\ell}\right)+\tilde{\rho}_{iT_\ell}\right]dT_\ell-\frac{\partial\phi_i}{\partial T_\ell}dT_\ell, \end{aligned}$$

which indicates that

$$\begin{aligned}(1/q_i)\nabla PS_i &= (1-v_i)\left[\tilde{\rho}_{iT_\ell}-\psi_i\cdot\left(\sum_{j=1}^n\epsilon_{ij}\tilde{\rho}_{jT_\ell}\right)\right]-\mathbf{f}_i \\ &= (1-v_i)(\mathbf{e}_i-\psi_i\boldsymbol{\epsilon}_i)\,\tilde{\rho}-\mathbf{f}_i.\end{aligned}$$

Next, note first that $dR_i=\widehat{v}_iq_idp_i+\widehat{\tau}_ip_idq_i+\left(\partial\widehat{\phi}_i/\partial T_\ell\right)dT_\ell$, where $\widehat{v}_i=(1/q_i)\widehat{\phi}_{i,p_i}$ and $\widehat{\tau}_i=(1/p_i)\widehat{\phi}_{i,q_i}$ are used. Then, by using $dq_i=\sum_{j=1}^n\partial q_i/\partial p_jdp_j$ and $\tilde{\rho}_{iT_\ell}=\partial p_i/\partial T_\ell$, one can further proceed:

$$\begin{aligned}\frac{dR_i}{dT_\ell} &= \widehat{v}_iq_i\tilde{\rho}_{iT_\ell}+\widehat{\tau}_ip_i\cdot\left(\sum_{j=1}^n\frac{\partial q_i}{\partial p_j}\tilde{\rho}_{jT_\ell}\right)+\frac{\partial\widehat{\phi}_i}{\partial T_\ell} \\ &= \widehat{v}_iq_i\tilde{\rho}_{iT_\ell}-\widehat{\tau}_ip_i\cdot\left(-\sum_{j=1}^n\frac{q_i}{p_i}\epsilon_{ij}\tilde{\rho}_{jT_\ell}\right)+\frac{\partial\widehat{\phi}_i}{\partial T_\ell} \\ &= q_i\widehat{v}_i\tilde{\rho}_{iT_\ell}-q_i\widehat{\tau}_i\cdot\left(\sum_{j=1}^n\epsilon_{ij}\tilde{\rho}_{jT_\ell}\right)+\frac{\partial\widehat{\phi}_i}{\partial T_\ell},\end{aligned}$$

which indicates that $(1/q_i)\nabla R_i=\left(\widehat{v}_i\mathbf{e}_i-\widehat{\tau}_i\boldsymbol{\epsilon}_i\right)\tilde{\boldsymbol{\rho}}+\widehat{\mathbf{f}}_i$, completing the proof. □

Now, we define the *pass-through quasi-elasticity matrix* $\boldsymbol{\rho}$ as an $n\times d$ matrix with elements: $\rho_{iT_\ell}=\tilde{\rho}_{iT_\ell}/f_{iT_\ell}(p_i,q_i,\mathbf{T})$, and with rows denoted $\boldsymbol{\rho}_{T_\ell}$.[51] We also define, for each firm $i, g_{iT_\ell}\equiv\widehat{f}_{iT_\ell}/f_{iT_\ell}$. Then, for the firm-specific welfare change ratios, we obtain the following

[51] For the two-dimensional taxation, it is easily verified that $\rho_{it}=1/f_{it}\,\tilde{\rho}_{it}=\tilde{\rho}_{it}$ and $\rho_{iv}=1/f_{iv}\,\tilde{\rho}_{iv}=\tilde{\rho}_{iv}/p_i$.

proposition by using the results of Proposition C.4.1.

Proposition C.4.2. *Let* $\epsilon^{\rho}_{iT_\ell} \equiv \boldsymbol{\epsilon}_i \tilde{\boldsymbol{\rho}}_{T_\ell} / \tilde{\rho}_{iT_\ell} = \boldsymbol{\epsilon}_i \boldsymbol{\rho}_{T_\ell} / \rho_{iT_\ell}$. *Then, the marginal value of public funds associated with intervention* T_ℓ, $MVPF_{iT_\ell} = [(\nabla CS_i)_{T_\ell} + (\nabla PS_i)_{T_\ell}]/(-\nabla R_i)_{T_\ell}$, *is characterized by:*

$$MVPF_{iT_\ell} = \frac{\dfrac{1/\rho_{iT_\ell} + v_i}{\epsilon^{\rho}_{iT_\ell}} + (1 - v_i)\,\dfrac{\psi_i\,\epsilon^{\rho}_{iT_\ell}}{\epsilon^{\rho}_{iT_\ell}}}{\dfrac{g_{iT_\ell}/\rho_{iT_\ell} + \widehat{v}_i}{\epsilon^{\rho}_{iT_\ell}} - \widehat{\tau}_i},$$

and the incidence of this intervention, $I_{iT_\ell} = (\nabla CS_i)_{T_\ell}/(\nabla PS_i)_{T_\ell}$, *is characterized by:*

$$I_{iT_\ell} = \frac{1}{\dfrac{1}{\rho_{iT_\ell}} - (1 - v_i)\left(1 - \psi_i\,\epsilon^{\rho}_{iT_\ell}\right)}.$$

Appendix D. Conduct index and welfare changes

For heterogeneous firms, we can also consider the conduct index of firm i, instead of the pricing strength index, so that

$$\theta_i = -\frac{\sum_{j=1}^{n}\{p_j[1 - \tau_j(p_j, q_j, \mathbf{T})] - mc_j(q_j)\}\left(\frac{dq_j}{d\sigma_i}\right)}{\sum_{j=1}^{n}[1 - v_j(p_j, q_j, \mathbf{T})]\,q_j\left(\frac{dp_j}{d\sigma_i}\right)}$$

holds. In the special case of only unit taxation being present, this definition reduces to Weyl and Fabinger's (2013, p. 552) Eq. (4). In the special case of symmetric firms, the definition reduces to $[1 - \tau - (1 - v)\eta\,\theta]p = mc$ with $\theta_i = \theta$.

The conduct index θ_i is closely connected to the marker power index ψ_i, but not as closely as it would be in the case of symmetric oligopoly. Using the definitions of the indices, it is shown that

$$\theta_i = -\frac{\sum_{j=1}^{n}(1 - v_j)\psi_j\,p_j\left(\frac{dq_j}{d\sigma_i}\right)}{\sum_{j=1}^{n}(1 - v_j)\,q_j\left(\frac{dp_j}{d\sigma_i}\right)}.$$

For symmetric oligopoly, this equation reduces simply to $\theta = \epsilon\psi$.

The conduct index is used to express welfare component changes in response to infinitesimal changes in taxes. The relationships are a bit more complicated than when the pricing strength index is alternatively used. To see this, we define the price response to an infinitesimal change in the strategic variable σ_j of firm j by $\zeta_{ij} \equiv dp_i/d\sigma_j$. Since the vectors $\zeta_{i1}, \zeta_{i2}, \ldots, \zeta_{in}$ form a basis in the n-dimensional vector space to which $\tilde{\rho}_{iT_\ell}$ for a given ℓ belongs, we can write $\tilde{\rho}_{iT_\ell}$ as a linear combination of them for some coefficients λ_{iT_ℓ}: $\tilde{\rho}_{iT_\ell} = \sum_{j=1}^{n}\lambda_{jT_\ell}\zeta_{ij}$. For changes in consumer and producer surplus, we obtain:

$$\frac{dCS}{dT_\ell} = -\sum_{i=1}^{n} q_i \tilde{\rho}_{iT_\ell} = -\sum_{j=1}^{n}\left(\sum_{i=1}^{n} q_i \zeta_{ij}\right)\lambda_{jT_\ell},$$

$$\frac{dPS}{dT_\ell} = -\sum_{i=1}^{n} f_{iT_\ell}(p_i, q_i, \mathbf{T}) - \sum_{j=1}^{n}\hat{\zeta}_j\,(1 - \theta_j)\,\lambda_{jT_\ell},$$

where we use the notation $\hat{\zeta}_j \equiv \sum_{i=1}^{n}[1 - v_i(p_i, q_i, \mathbf{T})]q_i\,\zeta_{ij}$.

These surplus change expressions represent a generalization of the surplus expressions in Weyl and Fabinger's (2013) Section 5. Note, however, that the results in the previous subsections are significantly more straightforward and applicable than the ones in this subsection.

Appendix E. Supplementary material

Supplementary data associated with this article can be found, in the online version, at https://doi.org/10.1016/j.jpubeco.2021.104589.

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