Various Phenomena of Vibrated Non-Uniform Granular Particles

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Identical granular particles in multiple compartments on a vertically shaking table may show an aggregation phenomenon termed *granular Maxwell's demon* for a suitable choice of parameters. Vertically vibrated binary granular particles may yield *granular Maxwell's demon* or the *granular clock*. Horizontally vibrated binary granular particles may form stripe patterns perpendicular or parallel to the vibratory direction for a suitable choice of parameters. Influences on the above-mentioned phenomena were analyzed, when the diameter or the mass of granular particles was distributed.

Key words: Granular Maxwell's demon, Granular clock, Aggregation phenomenon, Brazil nut effect

1. Introduction

Granular particles under the effect of periodic forcing show various collective motions. For example, there are granular particles of two sizes that are horizontally forced form a stripe pattern parallel or perpendicular to the external forcing. A stripe consists of large particles. The stripes on both sides of the above stripe consist of small particles (Fujii *et al.*, 2012).

An aggregation, termed granular Maxwell's demon (Eggers, 1999), is observed, when granular particles of the same size are in a compartmentalized container under the effect of vertical shaking. Both an aggregation and an oscillation, termed the granular clock (Lambiotte *et al.*, 2005), are also observed in the case of granular particles of the two sizes.

It is recommended that lay experts perform appropriately selected numerical analyses to obtain various granular collective motions, such as the discrete element method (Cundall-Strack, 1979) and the event-driven method (Isobe, 1999), according to the analysis objective. The former and latter are suited for aggregation and gas-like phenomena, respectively.

We analyzed influences on the above-mentioned phenomena, when the diameter or the mass of the granular particles was distributed. Some observations obtained from numerical simulations are mainly reported in this paper. In the second section, numerical results of a netlike appearance of non-uniform particles in the case of horizontal shaking are presented. The third section shows numerical results of a shift of the critical point of granular Maxwell's demon of non-uniform particles in the case of vertical shaking. The final section is devoted to concluding remarks.

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2. A Netlike Appearance of Non-Uniform Particles in the Case of Horizontal Shaking

Our starting point is a stripe pattern described in a previous paper (Fujii *et al.*, 2012). Parameters were mostly selected as listed in TABLE I of the paper (Fujii *et al.*, 2012). The area fractions were of large and small particles, ρ_L and ρ_S in units of cm^{-1}, respectively, and the viscous friction coefficient between the particle and tray μ was in units of $\mathrm{g{\cdot}s^{-1}}$. We set $(\rho_L, \rho_S) = (0.25, 0.3)$ and $\mu = 0.17$ (0.14) for large (small) particles. The elastic coefficient, amplitude of oscillation of the tray, and its frequency were $1.0\times10^4\mathrm{g{\cdot}cm^2\ {\cdot}s^{-2}}$, 7.5 cm, and 2 Hz, respectively. Masses of large and small particles were 1 and 0.3 g, respectively. Diameters of large and small particles were 0.5 and 0.25 cm, respectively. Coefficients of restitution between large particles, between small particles, and between large and small particles were 0.2, 0.9, and 0.5, respectively.

Next, we introduced uniform distributions defined in the interval $[0.5-0.0325, 0.5+0.0325]$ ($[0.25-0.0325, 0.25+0.0325]$) around the mean diameter of the large (small) particle. According to these distributions, we prepared granular particles and performed numerical simulations based on the discrete element method (Cundall-Strack, 1979). Thus, we did not obtain stripe patterns, but a fishnet structure, as shown in Fig. 1.

3. A Shift of the Critical Point of Granular Maxwell's Demon of Non-Uniform Particles on Vertical Shaking

Jens Eggers termed a dilute gas of granular material inside box *Sand as Maxwell's demon*, in which a wall separates the box into two identical compartments except for a small hole at some finite height, kept in a stationary state by vertical vibrations, since the particles preferentially occupy one side of the box at a suitable value of vibration intensity. He analyzed this clustering phenomenon based on a thermodynamical approach to granular material, and constructed a

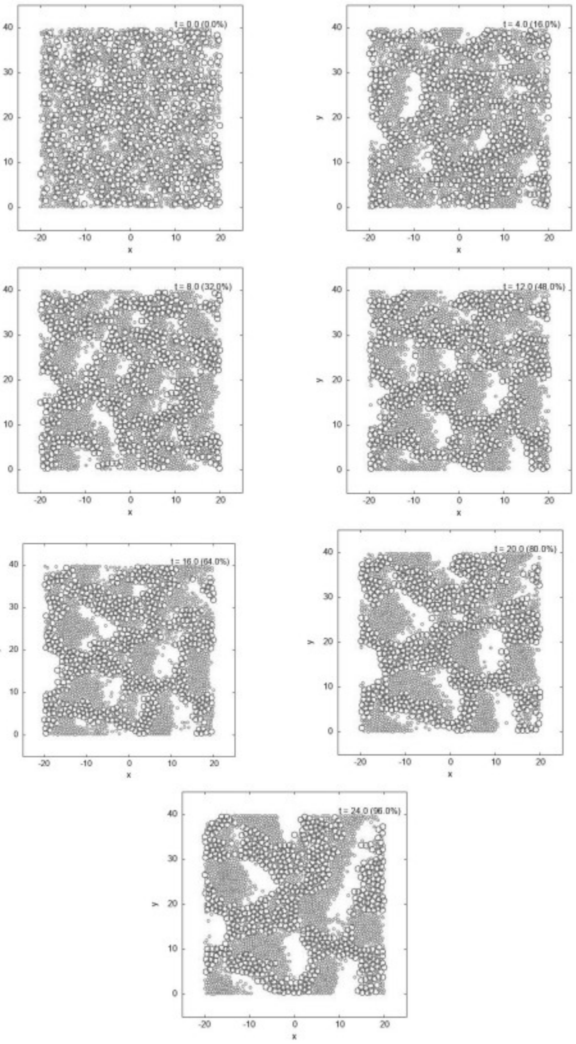

Fig. 1. A fishnet-like pattern gradually formed over the course of time. These snapshots started from random initial conditions at $t = 0.0$, followed by $t = 4.0$, 8.0, 12.0, 16.0, 20.0, and 24.0. Read from left to right and top down.

Table 1. Parameters used for the simulation in Fig. 2.

N	ρ	$\langle r\rangle$	Δ	e
720	$10^4/\pi$	0.01	0.15	0.95
g	W	U	l	
1.0	3.2	0.149	0.025	

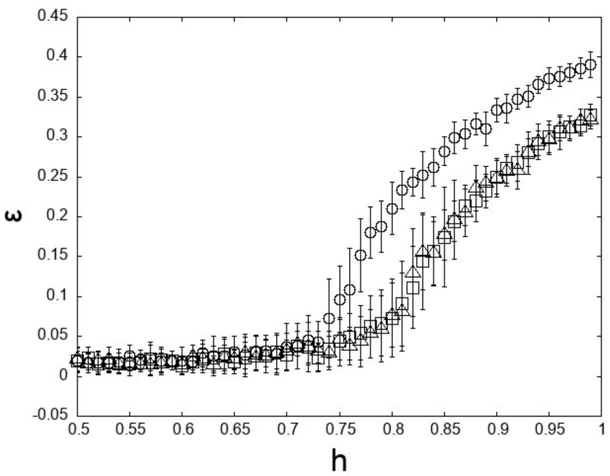

Fig. 2. The bias $\epsilon = \dfrac{2N_l - N}{2N}$ at equilibrium versus the hole height h. The three cases of radii are indicated by the symbols □ (uniform distribution), △ (two-valued distribution), and ∘ (single-valued case). The variance of the 20 samples is represented by error bars.

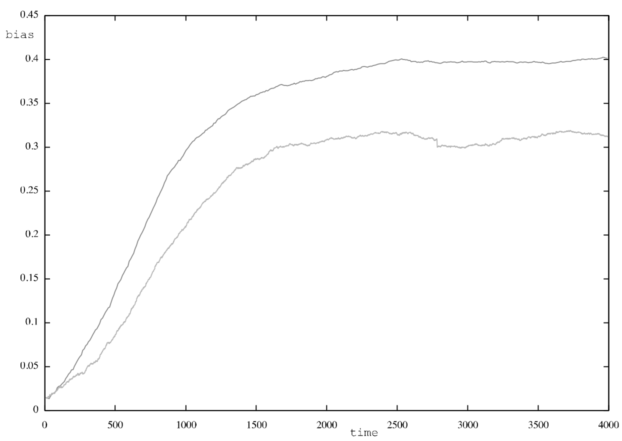

Fig. 3. The bias $\epsilon = \dfrac{2N_l - N}{2N}$ at equilibrium versus time t for the hole height $h = 1.0$. The upper and lower lines correspond to the single-valued case and uniformly distributed case of the radii, respectively.

mean-field model (Eggers, 1999). Schlichting and Nordmeier described the first experimental implementation in a paper entitled: *Strukturen im Sand*, a literal translation of the German title: Structures in the sand, in which one of the section titles, *Experimente mit Mitteln der Schulphysik* (experiments as educational tools for school physics), suggested an educational purpose (Schlichting and Nordmeier, 1996). A generalization from a single dispersion to a double dispersion in radius was analyzed to yield a granular clock (Lambiotte *et al.*, 2005). It is notable that not only the granular clock but also granular Maxwell's demon occurs in the case of a double dispersion radius. One of the authors (S.M.) advised high-school students studying the granular clock, and the students received *yuushuushoo* (award of excellence) in the Jr. session of the annual meeting of the Physical Society of Japan in 2015 (Inomata *et al.*, 2015).

We first used a system consisting of a box with width W and N granular particles. The particles with radius r and mass $m = \rho \cdot \pi r^2$ collide with a coefficient of normal restitution e, where ρ is the mass density. The following three cases are considered:

Uniform distribution The radii obey the uniform distribution defined in the interval $[\langle r\rangle(1 - \sqrt{3}\Delta), \langle r\rangle(1 + \sqrt{3}\Delta)]$ around the mean diameter $\langle r\rangle$ with $\langle r\rangle = 0.01$ and $\Delta = 0.15$.

Two-valued distribution The two-valued distribution $\frac{1}{2}[\delta(r - \langle r\rangle(1 - \Delta)) + \delta(r - \langle r\rangle(1 + \Delta))]$ has the same average $\langle r\rangle$ and variance $\Delta^2\langle r\rangle^2$ as the above uniform distribution.

Single-valued case Uniform particles with radius $r = \langle r\rangle$

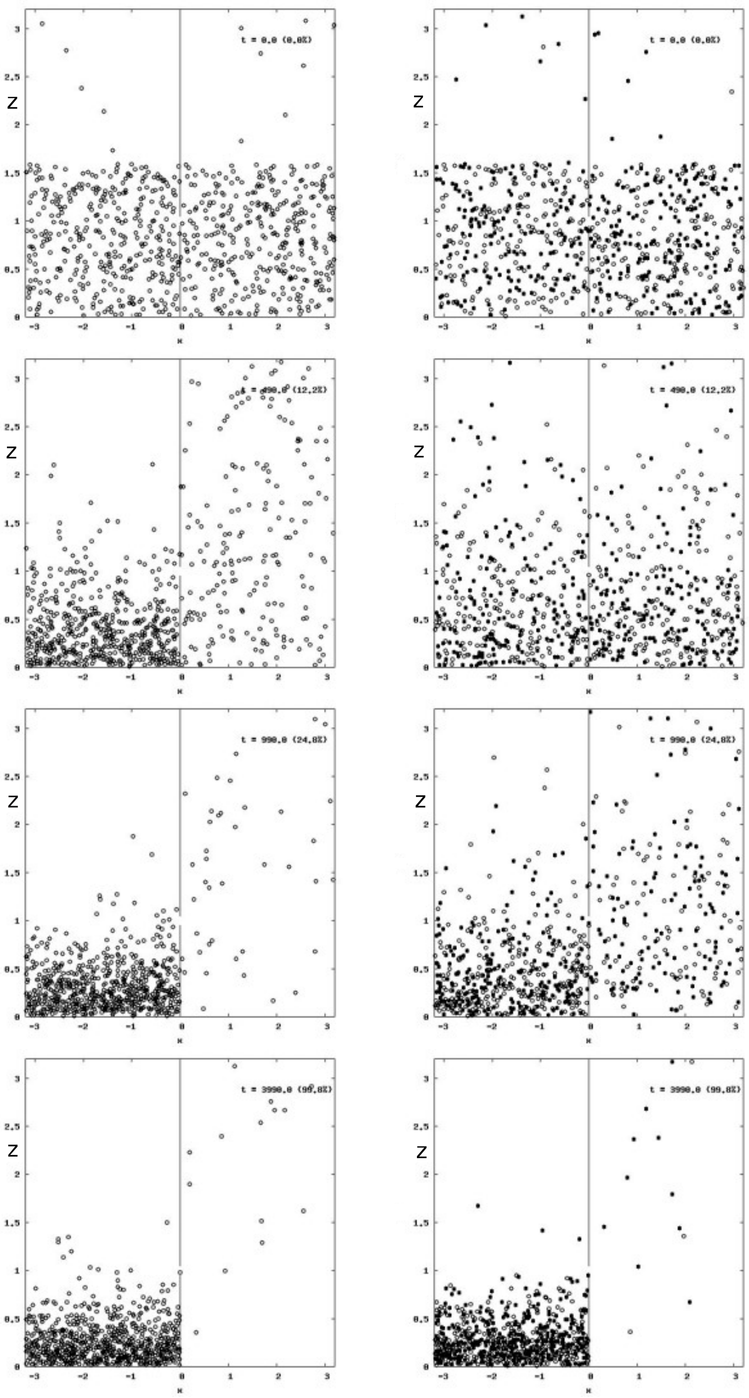

Fig. 4. Snapshots of particles in the container at time $t = 0.0$, 490.0, 990.0, and 3990.0 corresponding to the first, second, third, and fourth lines, respectively. The right and left columns correspond to the large ($\bullet$) and small ($\circ$) particles and uniform particles, respectively. The hole of the dividing wall is located at $z = 1$.

are considered in comparison with the above two cases.

The container consists of two identical compartments and a dividing wall. The latter has a small hole at height h through which particles can pass to the other side. The box is mounted on a shaker with a bottom vibration pattern and saw-tooth amplitude function; therefore, the velocity of the bottom has a constant positive value, U. Particles accelerated by gravity g bounce back on the floor and gain energy.

According to the preceding study of Eggers (Eggers 1999), we defined the bias as $\epsilon = \dfrac{2N_l - N}{2N}$, and we calculated it at equilibrium as a function of h based on the Extended Exclusive Particle Grid Method (EEPGM) (Isobe, 1999), with the results are shown in Fig. 2, where N_l is the particle number in the left compartment. The three cases are indicated by the symbols $\square$ (uniform distribution), $\triangle$ (two-valued distribution), and $\circ$ (single-valued case). The parameters shown in Table 1 were used for the simulation, and 20 samples were taken for each value of hole height h, and the bias ϵ was measured at $t = 6000.0$, which was considered to be larger than the relaxation time. The variance of the 20 samples is represented by error bars in Fig. 2. The width of the hole was $2l = 0.05$.

As shown in Fig. 2, The case of uniform distribution is fairly well-approximated by the case of two-valued distribution. For the fixed value of the hole height, $h = 1.0$, the bias ϵ of the case of uniform distribution has a smaller value than the single-valued case, suggesting that uniform distribution causes less condensation in a compartment than the single-valued case. For a fixed hole height, we plot the bias ϵ against time t in cases of single (upper line) and uniformly distributed (lower line) values of the radii in Fig. 3. The equilibrium value and gradient of the curve, which is identical to the speed to the equilibrium value, of single-valued radii are larger than those of uniformly distributed radii. The variance of the radii suppresses the condensation. Snapshots of the particles in the container at time $t = 0.0$, 490.0, 990.0, and 3990.0 are shown in Fig. 4, in which the right and left columns correspond to the uniform particles and the large ($\bullet$) and small ($\circ$) particles, respectively.

4. Concluding Remarks

We demonstrated the numerical results with the limited number of pages available. The fishnet or stripe patterns discussed in the second section should be quantitatively analyzed by two-dimensional spatial spectrum, with the intensity of the spatial Fourier coefficients as a function of the two-dimensional wave-number vector. The phase transition of the single-valued radius discussed in the third section was theoretically studied (Eggers, 1999). Theoretical treatments of the case of uniform distribution of the radius may be difficult. The two-valued distribution is promising as an approximation, and the approach taken by Eggers may be extended to the two-valued case. Vertical non-uniformity of periodically forced granular particles, also known as the Brazil nut effect (Rosato *et al.*, 1987), is essential to yield a granular clock. Phenomenologically derived equations of motion for the case of two-valued radii (Lambiotte *et al.*, 2005) can be used not only for the granular clock but also for condensation in a single compartment (granular Maxwell's demon). These will be reported in a separate paper.

References

Cundall, P. A. and Strack, O. D. L. (1979) A discrete numerical model for granular assembles, *Geotechnique*, **29**, 47–65.

Eggers, J. (1999) Sand as Maxwell's Demon, *Phys. Rev. Lett.*, **83**, 5322–5325.

Fujii, M., Awazu, A. and Nishimori, H. (2012) Segregation-pattern reorientation of a granular mixture on a horizontally oscillating tray, *Phys. Rev. E*, **85**, 041304.

Inomata, M. *et al.* (2015) The Nihon Butsuri Gakkaishi (Butsuri), **70**, 154 [in Japanese].

Isobe, M. (1999) Simple And Efficient Algorithm For Large Scale Molecular Dynamics Simulation In Hard Disk System, *International Journal of Modern Physics C*, **10**, 1281–1293.

Lambiotte, R., Salazar, J. M. and Brenig, L. (2005) From particle segregation to the granular clock, *Physics Letters A*, **343**, 224–230.

Rosato, A., Strandburg, K. J., Prinz, F. and Swendsen, R. H. (1987) Why the Brazil nuts are on top: Size segregation of particulate matter by shaking, *Phys. Rev. Lett.*, **58**, 1038–1040.

Schlichting, H. J. and Nordmeier, V. (1996) *Math. Naturwiss. Unterr.*, **49**, 323–332 [in German].