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Ambient vibration based modal analysis and cable tension estimation for a cable-stayed bridge with Bayesian approaches

W.J. Jiang[1], C.W. Kim[1,*], X. Zhou[1] and Y. Goi[1]

[1] Department of Civil & Earth Resources Engineering, Graduate School of Engineering, Kyoto University, Japan.
*E-mail: kim.chulwoo.5u@kyoto-u.ac.jp

ABSTRACT:

This study aims to investigate ambient vibration based modal analysis and cable tension estimation with Bayesian approaches, associating with an ambient vibration testing in a cable-stayed bridge. Firstly, a Bayesian fast Fourier transform (FFT) is introduced to the target bridge for operational modal analysis using ambient vibration data. Considering a mixing effect of local modes and global modes in the vibration measurements of cables, the identification is implemented in three scenarios: global modal analysis without data of cables, global modal analysis with data of cables, local modal analysis of each cable. The effects of involving measurements of cables into the global modal analysis of the whole bridge are discussed, and the identification uncertainty of modal frequencies in each scenario is investigated. Then, considering the identified modal frequencies of multiple modes in each cable, a Bayesian cable tension estimation framework including a Bayesian linear regression (BLR) is proposed, with the ability of simultaneously estimating the cable tension and flexural rigidity in a probabilistic way. The estimation is conducted with the identification results by both ambient vibration and hammer test. The results showed different accuracy and uncertainty in the estimation of cable tension and flexural rigidity, as well as among different cable conditions.

Keywords: *Ambient vibration, cable-stayed bridge, modal analysis, Bayesian FFT, Bayesian cable tension estimation.*

1. INTRODUCTION

With excellent performance to realize long-span crossing, cable-stayed bridge is widely constructed around the world in the past few years. The stayed cable, as a crucial component connecting bridge deck with tower in cable-stayed bridge, is always faced with a long-term deterioration caused by traffic, ambient vibration, corrosion, fatigue, etc. Therefore, it is well-acknowledged that the monitoring of cable is of great meaning for the safety and maintenance of cable-stayed bridge. There have been many researches on the structural health monitoring (SHM) of cable-stayed bridge [1,2]. Among those studies, the dynamic characteristics and cable tension are generally valued as informative features reflecting the condition of cable and whole bridge.

To get the dynamic characteristics of cable and bridge, field test is required. From the view of accuracy in structural identification, a hammer test or a weight-drop-off test may be a choice (despite in some large bridge the artificial excitation may be deficient). However, from the view of convenience in conducting long-term SHM, ambient vibration based modal analysis played a more important role due to its release of artificial excitation and continuity. Various methods have been proposed for ambient vibration based modal analysis, like frequency domain decomposition (FDD), stochastic subspace identification (SSI), a series of Bayesian operational modal analysis (BAYOMA) methods, etc., which make the output-only system identification (general case in ambient vibration) efficient and flexible. Compare to cable dynamics, cable tension is a more intuitional mechanical feature for assessing the cable and bridge state. Thus, based on the relation between modal frequency and cable tension, there are also some studies on the vibration-based cable tension estimation [3,4].

However, in the entire process as above, some issues remained and heavily affected the reliability of SHM using ambient vibration. Firstly, the cable-stayed bridge is a complex structure incorporating both global modes of bridge and local modes of cable under ambient vibration. The interactive influence between the global modes and local modes will augment the uncertainty in the structural identification, especially in the cable. Secondly, due to lack of input information, low signal-to-noise ratio (SNR), model assumptions, etc., the identification uncertainty may increase in the operational modal analysis and pollute the damage effect. Then, even though the previous study [5] well demonstrated the feasibility of vibration based cable tension estimation using hammer test, the increased uncertainty in ambient vibration based modal analysis may seriously reduce the reliability of cable tension estimation since the estimate tends to be sensitive to the identification uncertainty.

Therefore, with a concern on these issues of the uncertainty and accuracy in particular, this paper presents a study on the modal analysis and cable tension estimation in a cable-stayed bridge under ambient vibration and Bayesian framework.

2. THEORETICAL BASIS

2.1. Fast Bayesian FFT

As one of the Bayesian operational modal analysis methods possessing the ability of uncertainty measurement, a fast Bayesian FFT approach [6,7] is introduced here. By associating Bayesian inference with the FFTs of vibration response, Bayesian FFT gives a basic form as

$$p\left(\theta\middle|\left\{\hat{F}_k\right\}\right)=\frac{p\left(\left\{\hat{F}_k\right\}\middle|\theta\right)p(\theta)}{p\left(\left\{\hat{F}_k\right\}\right)} \tag{1}$$

where θ denotes the system parameters of the structure to be identified, and $\hat{F}_k$ are the estimated FFTs data at different frequencies f_k.

Then, given that the FFTs are Gaussian and independent at different frequencies, it can be written as

$$p\left(\theta\middle|\left\{\hat{F}_k\right\}\right)\propto p\left(\left\{\hat{F}_k\right\}\middle|\theta\right)=\frac{\pi^{-nN_f}}{\prod_k\left|E_k(\theta)\right|}\exp\left[-\sum_k \hat{F}_k^* E_k(\theta)^{-1}\hat{F}_k\right]=e^{-L(\theta)} \tag{2}$$

where

$$E_k(\theta)=E\left[F_kF_k^*\middle|\theta\right]+E\left[\varepsilon_k\varepsilon_k^*\middle|\theta\right]=\sum_{i=1}^{m}\sum_{j=1}^{m}h_{ik}h_{jk}^*S_{ij}\varphi_i\varphi_j^T+S_eI_n \tag{3}$$

is the theoretical power spectral density (PSD) matrix of data at the kth FFT for given θ. And

$$L(\theta)=nN_f\ln\pi+\sum_k\ln\left|E_k(\theta)\right|+\sum_k\hat{F}_k^*E_k(\theta)^{-1}\hat{F}_k \tag{4}$$

is the 'negative log-likelihood function' (NLLF), and the most probable value (MPV) $\hat{\theta}=arg\,min_{\theta}L(\theta)$.

Here, the system parameter θ comprises modal frequencies $f_{i}{}_{i=1}^{k}$ and modal damping ratios $\zeta_{i}{}_{i=1}^{k}$ denoted in transfer functions $h_{ik}{}_{i=1}^{k}$ corresponding to each mode, partial mode shapes $\varphi_{i}{}_{i=1}^{k}$, PSD matrix of modal forces $S=\left[S_{ij}\right]_{k\times k}$, and the PSD matrix of prediction errors S_eI_n. In addition, m represents the number of dominant modes in a specified frequency band where the estimation is conducted. n is the number of sensors to collect the ambient vibration response. N_f is the number of FFT points in the specified frequency band.

It has been demonstrated that the posterior PDF can be well approximated as a Gaussian form for typical data size. Au [7] showed that a second-order Taylor approximation of the NLLF at the MPV engenders a Gaussian approximation of the posterior PDF as

$$p\left(\theta\middle|\left\{\hat{F}_k\right\}\right)\approx(2\pi)^{-n_\theta/2}\left|\hat{C}\right|^{-1/2}\exp\left[-\frac{1}{2}\left(\theta-\hat{\theta}\right)^T\hat{C}^{-1}\left(\theta-\hat{\theta}\right)\right] \tag{5}$$

where $\hat{\theta}$ is the MPV, and $\hat{C}$ denotes the posterior covariance matrix which is the inverse of the Hessian of NLLF at the MPV. Also, n_θ represents the number of parameters in θ. The posterior covariance matrix $\hat{C}$ can provide useful information reflecting the identification uncertainty.

2.2. Modal frequency versus cable tension

Given some assumptions, the relation between modal frequency of cable and cable tension can be derived from the free vibration differential equation of cable as below.

$$m\frac{\partial^2 v(x,t)}{\partial t^2}+EI\frac{\partial^4 v(x,t)}{\partial x^4}-T\frac{\partial^2 v(x,t)}{\partial x^2}-h(t)\frac{\partial^2 v(x,t)}{\partial x^2}=0 \tag{6}$$

where $v(x,t)$ denotes the vertical vibration deflection, x is the cable longitudinal coordinate and t denotes time. The symbol m is the mass of cable per unit length, EI denotes the flexural rigidity of cable and T is the cable tension force. The notation $h(t)$ is the derivative cable tension force caused by vibration.

According to [3], the influence of vibration-induced derivative cable tension $h(t)$ and the cable sag is generally small and ignorable for simplicity. Then, given that the boundary condition is hinged support, the solution of Eq. (6) can be presented as follows.

$$\left(\frac{f_i}{i}\right)^2 = \frac{\pi^2 i^2}{4ml^4} EI + \frac{1}{4ml^2} T \tag{7}$$

where i is mode order and f_i denotes the i^{th} modal frequency of cable; l is the length of cable.

Then, when the cable vibration is more similar to a string (the contribution of EI on modal frequency is rather small), the equation can be further simplified as

$$\left(\frac{f_i}{i}\right)^2 = \frac{1}{4ml^2} T \tag{8}$$

Generally, Eq. (7) and Eq. (8) can be regarded as hinged beam model and string model, respectively, depending on the feature of cable vibration. To see whether the cable performs like a beam or a string, a parameter can be referred as below.

$$\xi = \sqrt{\frac{T}{EI}} \cdot l \tag{9}$$

where, the larger the value of ξ is, the more similar to a string the cable performs.

2.3. Bayesian cable tension estimation

With multiple modal frequencies of cable identified, the estimation of cable tension from Eq. (7) and Eq. (8) can be treated as a regression issue.

Given that a basic form of Bayesian linear regression (BLR) model can be denoted as below.

$$y = X\beta + \varepsilon,\ \varepsilon \sim N\left(0,\sigma^2\right) \tag{10}$$

where y is a $n\times 1$ vector of response variable; X is a $n\times d$ matrix of predictor variables; β is a $d\times 1$ vector of coefficients; ε denotes the *iid* error term which obeys a normal distribution with zero mean and variance σ^2 for each observation; n is the number of observations, and d is the number of predictor variables.

Then, the posterior distribution of $\left(\beta,\sigma^2\right)$ can be obtained by Bayesian reference like

$$p\left(\beta,\sigma^2 \middle| y,X\right) = \frac{p\left(y \middle| X,\beta,\sigma^2\right)\cdot p\left(\beta,\sigma^2\right)}{p\left(y \middle| X\right)} \tag{11}$$

Further, the marginal posterior of β can be given as

$$p\left(\beta \middle| y,X\right) = \int p\left(\beta,\sigma^2 \middle| y,X\right) d\sigma^2 \tag{12}$$

Given a Jeffreys non-informative prior

$$p\left(\beta,\sigma^2\right) \propto \frac{1}{\sigma^2} \tag{13}$$

the marginal posterior of β is analytically tractable and follows a d dimensional *t-location-scale* distribution as

$$\beta \middle| y,X \sim t_d\left(\hat{\beta}, \frac{\left(y-X\hat{\beta}\right)'\left(y-X\hat{\beta}\right)}{n-d}\left(X'X\right)^{-1}, n-d\right) \tag{14}$$

where the three parts in the right hand are the location parameter, scale parameter, degree of freedom, in sequence. The notations (y, X, β, n, d) are the same as before, while $\hat{\beta}$ is the least-squares estimate of β with a form as follows.

$$\hat{\beta}=\left(X'X\right)^{-1}X'y \tag{15}$$

Without loss of generality, taking Eq. (7) into the form as Eq. (10), a Bayesian cable tension estimation framework can be established with

$$y=\left\{\left(\frac{f_i}{i}\right)^2\right\}_{n\times 1},\ X=\begin{bmatrix}\frac{\pi^2 i^2}{4ml^4} & \frac{1}{4ml^2}\end{bmatrix}_{n\times 2},\ \beta=\begin{Bmatrix}EI\\ T\end{Bmatrix}_{2\times 1} \tag{16}$$

and the posterior distribution of β contributes to a simultaneous estimation of cable tension and flexural rigidity, along with a depiction of the estimation uncertainty.

3. VIBRATION TEST ON CABLE-STAYED BRIDGE

3.1. Field experiment

The target bridge is a single-tower cable-stayed bridge shown in Fig. The span length of the bridge is about 124 m and tower height is about 48 m (see Fig. 2). A short-term field test including forced excitation test and ambient vibration test was carried out on November, 2020. The corresponding sensor setup and structural layout are shown in Fig. 2. Vibration signals from cables at anchor, cables at bridge deck, bridge deck nodes and a tower node were collected during the test. The forced excitation test was conducted just on the cables, while the ambient vibration test was implemented on the whole bridge involving cables, bridge deck and tower.

Figure 1. A side view of investigated bridge

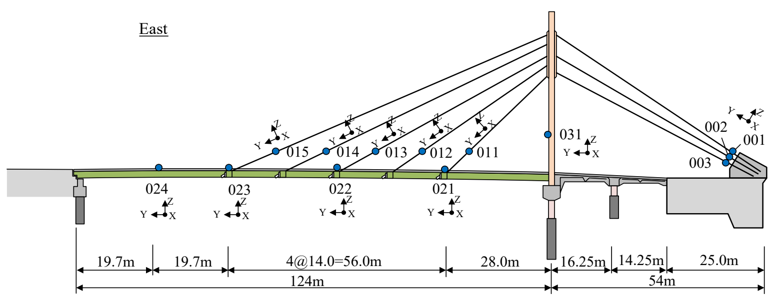


Figure 2. Bridge layout and sensor deploying map.

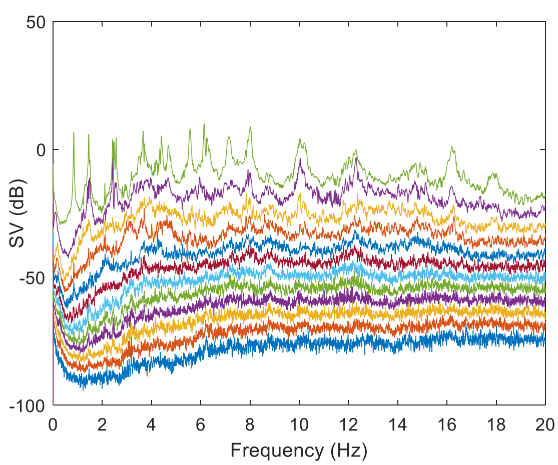


Figure 3. Singular value spectrum of signal set from bridge deck nodes.

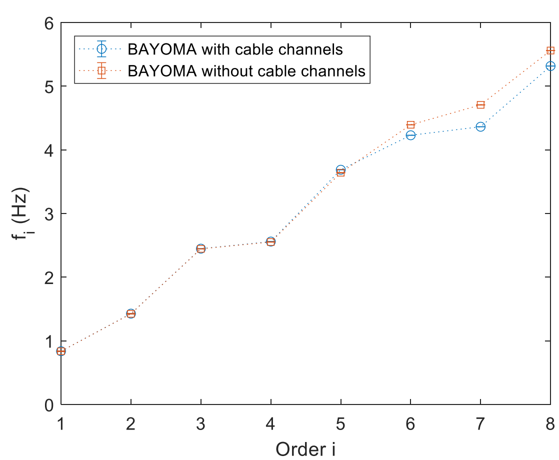
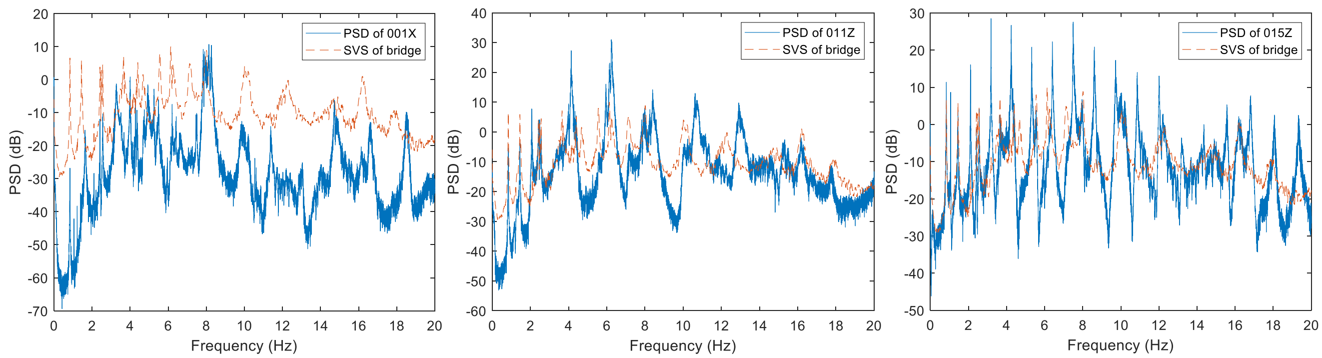


Figure 4. Comparison of OMA results for bridge with and without signal from cables.



(a) Cable 001 at anchor *(b) Cable 011 at bridge deck* *(c) Cable 015 at bridge deck*

Figure 5. PSDs of three cables under ambient vibration.

3.2. Operational modal analysis of bridge

As discussed before, there is an interactive influence between global modes of bridge and local modes of cables. Thus, operational modal analysis (OMA) of the bridge is conducted in two signal scenarios: bridge with cable channels, and bridge without cable channels. To specify a prior set including initial value and frequency band for the optimization algorithm of the fast Bayesian FFT, a singular value spectrum (SVS) as shown in Fig. 3 is used. A peak in the highest spectral line of SVS indicates a possible modal frequency. More information about use of SVS in the fast Bayesian FFT can be found in [7].

Figure 4 showed a comparison of the identified modal frequencies of the whole bridge with and without signals from cable channels. It can be noted that the identification uncertainty is low, and the error between two signal scenarios is also small in lower modes while increasing in some higher modes. This phenomenon indicated that the OMA of the whole bridge under ambient vibration is of efficacy, and the interference from the local modes of cables only makes a clear difference in some higher global modes of bridge under ambient vibration.

3.3. Modal analysis of cables

Three cables with sensors are considered in this paper to simplify the discussion even though vibrations of all the cables were examined. A cable at the anchor, the shortest cable at bridge deck and the longest cable at bridge deck are selected, and each cable is named as cable 001, cable 011, cable 015 respectively as shown in Fig. 2.

To specify a prior set about the initial value and frequency band for the fast Bayesian FFT, a power spectral density (PSD) plot along with the highest spectral line in SVS of bridge to help eliminate some fake peaks caused by the global modes of bridge is given as shown in Fig. 5. Comparing with the SVS, it can be noted that some global modes in SVS also appear in the PSD of cables, which will affect the identification about local modal frequency of cables and further affect the accuracy of vibration-based cable tension estimation. In addition, observations on the PSDs of three cables show different SNRs in three groups of cables, and further different accuracy of OMA. The PSD curve of the cable at anchor indicated a low SNR due to weak ambient excitation comparing to the cables at the bridge deck. The short cable at bridge deck performs better since the higher ambient excitation comparing to the cable at the anchor. The long cable at bridge deck shows clear PSD peaks as a result of both higher ambient excitation.

As mentioned previously, the forced excitation test was also conducted in the cables. Generally, the forced excitation test is believed to offer a modal identification with higher accuracy and lower uncertainty compared to the ambient vibration test, since the strength and frequency domain feature of the forced excitation is more ideal than the ambient vibration test. Another reason is that, the forced excitation test just excites local vibration of cable, while ambient vibration of cable is mixed with interference from global vibration for the bridge. Therefore, taking the forced excitation test results as a reference, the fast Bayesian FFT results of cables under ambient vibration are summarized in Fig. 6.

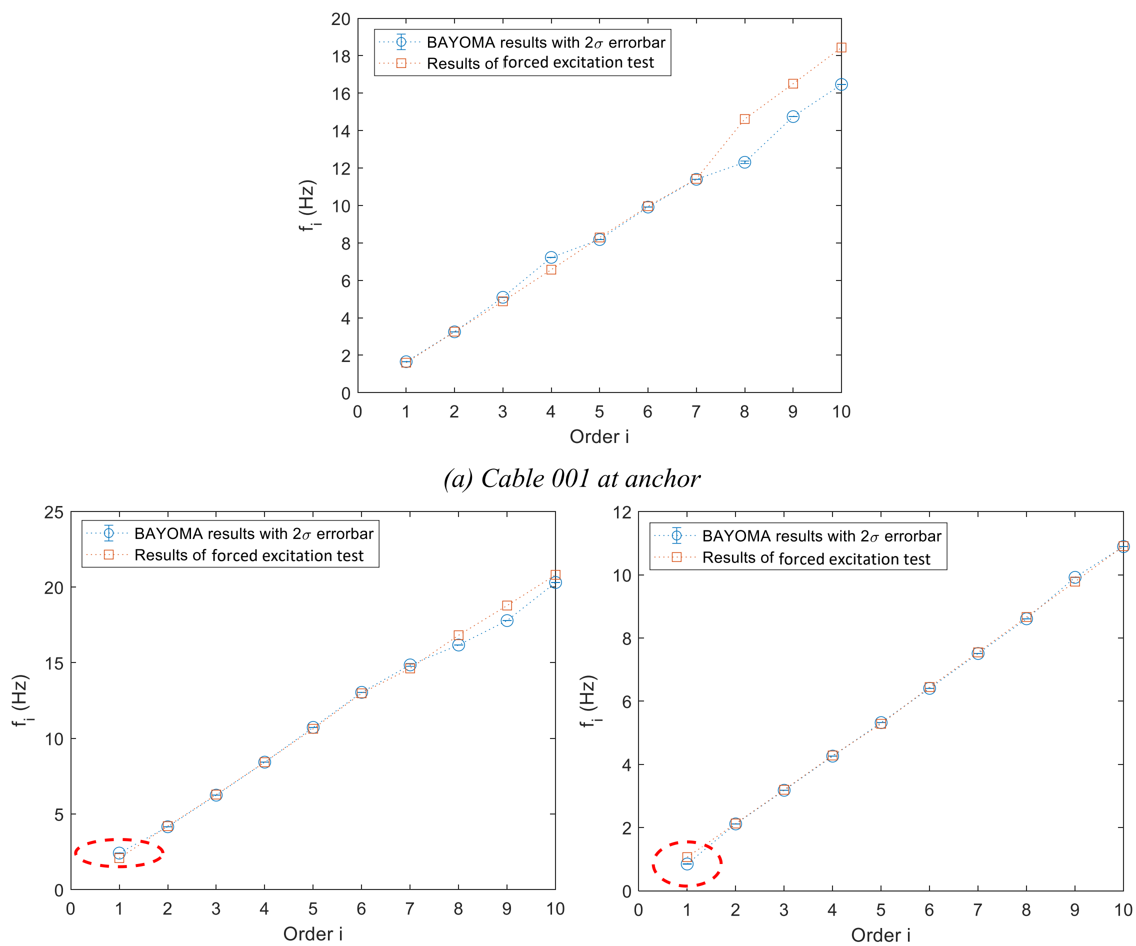
As shown in Fig. 6, the identification performances of each cable by means of the fast Bayesian FFT were different in comparison to the forced excitation test results. Generally, the identification uncertainty from the ambient vibration was rather small, but clear difference between the fast Bayesian FFT (denoted as BAYOMA in Fig. 6) and the forced excitation test was in some higher modes: 8th, 9th and 10th modes of Cable 001 at the anchor and Cable 011 at the bridge deck. Good consistency was observed in lower modes of the cables at bridge deck, except for the identified 1st modal frequency by the fast Bayesian FFT which is close to the global modal frequency of the bridge deck. This phenomenon indicated the 1st modal frequency in the cable at the bridge deck is apt to be polluted by the global vibration of the bridge under ambient vibration. In further research of cable tension estimation, these polluted results need to be removed. Hence, modal frequencies from 2nd to the 6th modes are considered in the cable tension identification. On the other hand, it is noted that while the interference of global vibration of the bridge deck is small in the cable at the anchor, the inconsistency in lower modes indicated a lower accuracy of identification in this cable.

3.4. Estimation of cable tension

With the identified modal frequencies of cables, Bayesian cable tension estimation is carried out. Because there is no direct and real-time cable tension measurement of the target bridge, the original design value of cable tension and calculated flexural rigidity with approximate cross-section are taken as a reference value. In addition, since the good performance of the forced excitation test in the modal analysis, the estimation from data of the forced excitation test is taken as another reference values. Those two reference values from design and the forced vibration test are named as “R1” and “R2” respectively.

According to Eq. (7), there is a linear relation between $(f_i/i)^2$ and i^2 for the hinged beam model. Then, with the BLR model, cable tension and flexural rigidity can be simultaneously estimated. Figure 7 showed the observations and fitted line by BLR in three cables. It can be noted that generally a linear relation can be observed in both the Bayesian FFT results and the forced excitation test results, except for the Bayesian FFT results in the cable at the anchor which indicated a high identification uncertainty.

The posterior estimate of flexural rigidity *EI* and cable tension *T* is summarized in Table 1 and Table 2, respectively. It can be noted that generally the MPV of posterior estimate of *EI* showed a large deviation to the value of both R1 and R2, while the posterior estimate of *T* performed much better. This observation indicated that the estimate of *EI* is more sensitive to the uncertainty in the ambient vibration based modal analysis than *T*, which indicates the cable vibration of the bridge is more similar to a string model than to the hinged beam model when we consider the frequencies up to 10th modes.

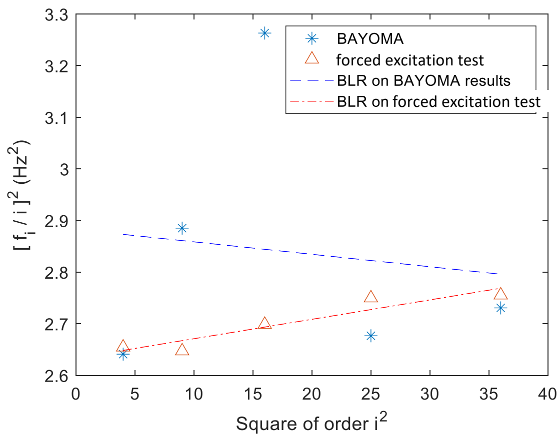


(a) Cable 001 at anchor

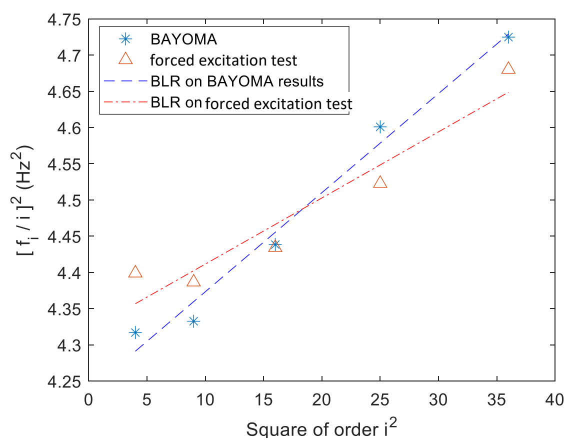
(b) Cable 011 at bridge deck

(c) Cable 015 at bridge deck

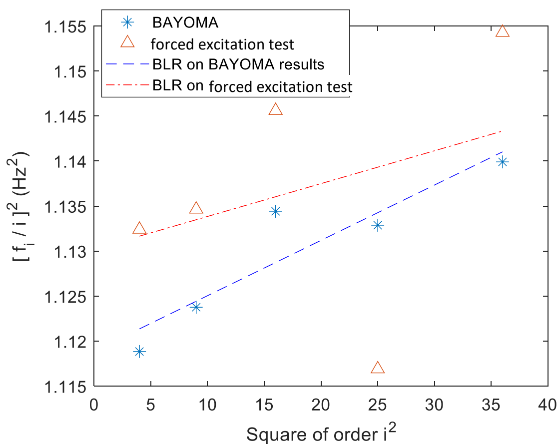
Figure 6. Identified modal frequency w.r.t. mode order for three cables.



(a) Cable 001 at anchor.



(b) Cable 011 at bridge deck



(c) Cable 015 at bridge deck

Figure 7. Observations and BLR fitted line with a hinged beam model in three cables.

Table 1. Posterior estimate of flexural rigidity in three cables.

Cable	**BAYOMA**		**Forced excitation test (R2)**		**Design (R1)**	**Relative error R2 (%)**	**Relative error R1 (%)**
	EI (kN·m^2)	std (kN·m^2)	EI (kN·m^2)	std (kN·m^2)	EI (kN·m^2)		
001	-1702	13947	2655	933	10048	-164.1	-116.9
011	520	70	347	102	2337	49.9	-77.8
015	1174	486	697	1999	6273	68.5	-81.3

Table 2. Posterior estimate of cable tension in three cables.

Cable	**BAYOMA**		**Forced excitation test (R2)**		**Design (R1)**	**Relative error R2 (%)**	**Relative error R1 (%)**
	T (kN)	std (kN)	T (kN)	std (kN)	T (kN)		
001	4605	673	4206	45	4312	9.5	6.8
011	978	9	997	13	1332	-1.9	-26.5
015	2465	12	2490	49	3057	-1.0	-19.4

The MPV of posterior estimate of T of the cables at the bridge deck resulted in differences of 20% to 26% comparing to the design values (R1), and that of the cable at the anchor showed 7% difference comparing to the design value (R1). However, those estimated tensions were highly comparable to the estimate from vibrations of the forced excitation test (R2). Meanwhile, a difference between the cable tensions at the anchor and at the bridge deck was observed. For the Cable 011 and Cable 015 at bridge deck, the estimated T showed low posterior uncertainty and was very consistent to those from vibrations of the forced excitation test (R2). But on the other hand, for Cable 001 at the anchor, the estimated of both EI and T showed large posterior uncertainty. Therefore, for the target bridge, it can be concluded that the ambient vibration-based cable tension estimation in the cable at the bridge deck may be more feasible due to higher SNR and accuracy in OMA.

4. CONCLUSIONS

This paper investigated the performance and uncertainty quantification of ambient vibration based modal analysis and cable tension estimation in a cable-stayed bridge under a Bayesian framework. The uncertainty sources and propagation in this process from original vibration response to operational modal analysis, and further to vibration-based cable tension estimation was discussed. Based on the above investigation, it can be concluded as follows.

(1) An interactive interference exists between the global modal analysis of bridge and the local modal analysis of cable under ambient vibration, which should be considered in the long-term SHM study using ambient vibration. In particular, the global vibration of bridge affects the accuracy of OMA in 1st modal frequency of cables at the bridge deck.

(2) Under ambient vibration, the performance of OMA and further cable tension estimation also differs with the cable conditions in cable-stayed bridge. Generally, cable at the bridge deck shows higher accuracy and lower uncertainty in OMA and cable tension estimation than cable at anchor except for a situation that cable at the bridge deck is easy to be perturbed by the global vibration of bridge.

(3) Under ambient vibrations, the proposed Bayesian cable tension estimation approach shows different accuracy and uncertainty between the posterior estimate of flexural rigidity and cable tension. Generally, the posterior estimate of flexural rigidity presents high uncertainty and low accuracy, while the estimate of cable tension performs better and almost consistent with the estimate by the forced excited vibration. It indicates that the cable vibration of the bridge is more similar to a string model than to the hinged beam model. It is noted that when the string model was applied, the difference between design and estimate values were less than 20% for cables at the bridge deck and less than 5% for cables at the anchor.

To sum up, ambient vibration-based SHM in cable-stayed bridge is a complicated issue encountered with various sources affecting the accuracy and uncertainty such as low SNR, deviation with the idealized frequency domain feature of ambient excitation in the fast Bayesian FFT, interference from global vibration of bridge, simplification in the model of cable tension estimation, and so on. It is noted that the environmental variation should be considered as a source of uncertainty in the long-term SHM campaign, which will be discussed in further study.

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