Decentralized navigation in 3D space of a robotic swarm with heterogeneous abilities

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Abstract. This paper proposes a decentralized method for navigation of multiple robots, each of which has different abilities, by a single leader robot in 3D space, especially focusing on the connectivity maintenance. We assume a swarm of robots whose sensing ranges, maximum speeds and maximum accelerations are different. For such robots, we propose a control method for maintaining the whole connectivity by each agent's keeping local connectivity in a decentralized way. We also mathematically prove that the proposed method can enable multiple robots to navigate in 3D space while keeping the connectivity. Finally, numerical simulation results are presented to confirm the effectiveness of the proposed method.

Keywords: swarm control, navigation control, heterogeneous agents

1 Introduction

The robotic swarm consists of multiple robots and requires their cooperation to achieve specific tasks. It is pointed out that swarm robots are useful due to their "robustness", "flexibility" and "scalability" [1, 2]. "Robustness" is a property that prevents some agents' failure from depriving the original function of the whole swarm, and it is important when using the robotic swarm in a harsh environment. "Flexibility" is a property that allows a robotic swarm to adapt to various situations and tasks. For example, a large object that is too large for a single robot to carry could be transported by a multiple robots' cooperation. "Scalability" is a property that the system still works well even if the number of agents increases, when each agent determines their action by only using local information (called "decentralized control"). In decentralized control, even though each agent acts according to their simple control law, the whole swarm exhibits good performance due to each agent's interaction. For these reasons, robotic swarms are expected to be applied to various situations.

Robotic swarms are used in such tasks as exploration [3], transportation [4], and surveillance [5]. They have a basic task in common: "moving to the destination while maintaining the swarm structure," and to realize this task by decentralized control, each agent needs to get information about the surroundings by sensing or communication and to determine their action based on it. Considering that agents have a physical limit to the sensing/communication distance, the distance between agents should be kept to such an extent that each agent can obtain the necessary information.

There have been some researches on connectivity maintenance control, but many of them have an assumption that agents have the same capabilities [6, 7]. On the other hand, the robotic swarm consisting of heterogeneous agents is expected to adapt to a wider variety of tasks as each of them makes use of their abilities and cooperate with each other. In [8], the authors state that we have to incorporate heterogeneity into swarm robot systems in order to apply them to the actual environment. They proposed a swarm system composed of hand-bots that can climb up or down walls and work as a manipulator, foot-bots that navigate and carry the hand-bots, and eye-bot to scan the environment and send information. In [9], a task example for heterogeneous robotic swarms are shown, in which drones with different abilities monitor several targets which moves differently.

There are some researches on "heterogeneous" (meaning "agents have different capabilities" in this context) swarms [10]-[12]. In [10], a decentralized control method is proposed with which agents with different sensing ranges estimate connectivity from the graph Laplacian of the connected graph only using local information, and move in such a way as to keep connectivity. However, the research has an assumption that each agent's sensing range could be enlarged if needed, and does not consider agents' physical constraints such as the maximum speed. On the other hand, [11] proposed a control method for mobile robots with different sensing range, maximum speed and maximum acceleration. In this method, a single leader navigates the other agents while maintaining connectivity, without agents' communicating or getting information about their surroundings, and each agent moves based only on their relative position to neighboring agents. [12] is the expansion of this research, to which the authors have added the constraint on agents' viewing angle, and the stability analysis is proved in [13]. However, [11, 12] handles moving robots in a 2D plane, and navigations method for heterogeneous robots in 3D space have not been proposed.

When we use the method [11] to 3D space, we have two main problems. The first problem is in the vector resolution of the control input. As [11], where the input vector is decomposed using circular coordinates, let us consider the vector resolution of the control input using the spherical coordinates. The singular point in circular coordinates is the point where an agent and its target are in the same position. Here, this situation cannot physically occur. However, in spherical coordinates, the singular points are points where an agent comes right above or below its target, and this situation is likely enough to occur. At this singularity, the basis vectors are no longer determined uniquely, which is a big obstacle to maintaining the continuity of the agent's input vector.

The second problem is about decentralized control. In [11], the leader's speed constraint is a constant and can be uniquely determined from each follower's ability, which is known in advance by the leader. However, when trying to apply the same method to our study, the leader's constraint will include the variables that only followers could obtain. Specifically, the leader would have to know the angle $\theta_i(t)$ (introduced in Chapter 3.3), which the leader cannot obtain without assuming the unlimited sensing range. We will have to use a centralized control system where the leader can obtain each follower's measured value with communication. Moreover, another problem also occurs that the leader's speed constraint is no longer determined uniquely and depends on time.

In this paper, based on the study [11], we propose a control method that can solve the problems above. In other words, we propose a decentralized control method in which a single leader robot navigates the other robots, each of which has different capabilities, while maintaining connectivity in 3D space. We assume that each agent has a different sensing range, maximum speed, and maximum acceleration and does not use the communication with other agents.

2 Problem Setting

2.1 Modeling of Agents

We consider a leader-follower navigation problem in 3D space $D \in \mathbb{R}^3$ with no obstacles. The robotic swarm consists of a single leader and n followers, and followers are numbered $1, 2, \ldots, n$ while the leader is numbered $n+1$. These numbers are set only for the convenience of description, and agents do not recognize them in the actual situation. We define sets of indices of all agents as $\mathcal{A} = \{1, 2, \ldots, n+1\}$, and of all followers as $\mathcal{F} = \{1, 2, \ldots, n\}$.

We define a position vector and control input for agent $i \in \mathcal{A}$ at time t as $\boldsymbol{x}_i(t)$ and $\boldsymbol{u}_i(t)$ respectively, and its equation of motion is described as $\dot{\boldsymbol{x}}_i(t) = \boldsymbol{u}_i(t)$.

We assume that each follower $i \in \mathcal{F}$ has the following physical constraints about their input vector and its time derivative:

$$\|\boldsymbol{u}_i(t)\| \leq U_i,\ \|\dot{\boldsymbol{u}}_i(t)\| \leq A_i,\ \boldsymbol{u}_i(t)\text{: Continuous.} \tag{1}$$

Here, we define $\dot{\boldsymbol{u}}_i(t)$ as the larger of left and right derivative of $\boldsymbol{u}_i(t)$ at time t, so we do not need to think of acceleration constraints. If we consider the flying robot as the agent, the actual flying robot has a limit to speed and acceleration, so we assume that they cannot do such movements that exceed the limit. In addition to that, the input vector should be continuous for controlling the actual robots. The leader agent does not have those constraints above, other than the next section's speed constraint.

We assume that each follower $i \in \mathcal{F}$ can measure with an on-board sensor its relative displacement $\boldsymbol{x}_{ij}(t)$ to any agent $j \in \mathcal{A}$ in the neighborhood area $S_i(t)$, where $\boldsymbol{x}_{ij}(t) \coloneqq \boldsymbol{x}_j(t) - \boldsymbol{x}_i(t)$ and $S_i(t) = \{\boldsymbol{X}(t) \in D \mid \|\boldsymbol{x}_i(t) - \boldsymbol{X}(t)\| \leq \rho_i,\ \rho_i > 0\}$. Here, ρ_i is the sensing range of agent i and is not necessarily uniform for all agents. We also assume that followers can always get the vertically upward direction $\boldsymbol{e}_z$ on the inertial frame. In addition, we assume that as long as an agent in $S_i(t)$ is staying there, each follower can distinguish it from the other agents inside $S_i(t)$.

2.2 Connectivity of Agents

We use graph theory to describe connectivity between agents [11].

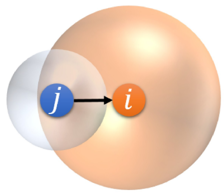


Fig. 1. ij semi-connection.

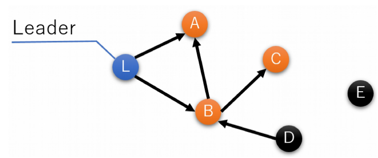


Fig. 2. Connectivity expression as a directed graph.

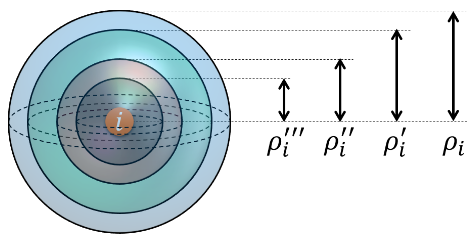


Fig. 3. Sensing radius ρ_i and design parameters ρ_i', ρ_i'', ρ_i''' for agent i.

We define a group of nodes as $\mathcal{N} := \{N_1, N_2, \ldots, N_{n+1}\}$, and a group of directed edges between nodes at time t as $\mathcal{E}(t)$. A directed graph can be described $\mathcal{G}(t) = \mathcal{N} \times \mathcal{E}(t)$, and we describe a directional branch from N_j to N_i as $E_{ji} \in \mathcal{E}(t)$. If E_{ji} exists, we call N_j a parent of N_i, and call N_i a child of N_j. If a path can be tracked along directional branches (like E_{kj}, E_{ji}), it is called a directed path. If N_{n+1} has no parent and every node in $\mathcal{N}$ other than N_{n+1} has only one parent, $\mathcal{G}(t)$ is called a spanning tree with a root N_{n+1} We define the following leader semi-connectivity [11].

Definition 1 (Leader Semi-Connectivity). *If E_{ji} exists, $i \in \mathcal{A}$ and $j \in \mathcal{A}$ are ij semi-connected (Fig. 1). If there is a directed path from N_{n+1} to N_i, follower $i \in \mathcal{F}$ is leader semi-connected. A swarm of agents is leader semi-connected if all followers are leader semi-connected. In other words, the directed graph $\mathcal{G}(t)$ is leader semi-connected if $\mathcal{G}(t)$ has a spanning tree with a root N_{n+1}.*

Fig. 2 is an example of directed paths for a robot swarm with five followers. Here, followers A, B, C are leader semi-connected while D, E are not.

Next, we add an assumption about the initial position at time $t = 0$. Assumption 1 is an assumption for the initial position of each followers.

Assumption 1 (Initial Placement) *We define a positive constants ρ_i', ρ_i'', ρ_i''' satisfying $0 < \rho_i''' < \rho_i'' < \rho_i' < \rho_i$, where ρ_i' and ρ_i'' are criteria for switching the control input and ρ_i''' is equivalent to the lower limit for the distance between agent i and the agent which agent i is following (Fig. 3). For follower $i \in \mathcal{F}$, we set regions $S_i'(t)$ and $S_i''(t)$ as follows:*

$$S_i'(t) := \{\boldsymbol{X}(t) \in D \mid \|\boldsymbol{x}_i(t) - \boldsymbol{X}(t)\| \le \rho_i'\},$$
$$S_i''(t) := \{\boldsymbol{X}(t) \in D \mid \|\boldsymbol{x}_i(t) - \boldsymbol{X}(t)\| \le \rho_i''\}.$$

and define their boundary as $\partial S_i'(t), \partial S_i''(t)$, respectively.

We set $B_i(t) := S_i'(t) \setminus \{\partial S_i'(t) \cup S_i''(t)\}$ and assume that all followers have at least one other agent in $B_i(t)$, do not have any other agents in $S_i''(t)$, and are static ($\boldsymbol{u}_i(0) = \mathbf{0}$), at the time $t = 0$. We also assume that at least one follower includes the leader in $B_i(t)$ at $t = 0$.

2.3 Control Objective

In this paper, we propose a decentralized control method and leader's constraint to keep the state where, for any $t \geq 0$, each agent satisfies the physical constraints (1), and every follower is leader semi-connected. The leader is assumed to obtain the followers' properties (ρ_i, ρ_i', ρ_i'', ρ_i''', U_i, A_i) off-line in advance, and to move within the speed constraint shown below. Each follower can only use information about itself and its relative position to other agents within its sensing range.

3 Proposed Method

We show a method for creating and maintaining a spanning tree structure of $\mathcal{G}(t)$. In this method, each follower chooses one other agent as a target, and makes efforts to keep semi-connectivity with it, which leads to leader semi-connectivity of all agents. We also give them some degrees of freedom for changing the swarm shape. The proposed method consists of three parts: leader's speed constraint, selection of a target agent, and control inputs for maintaining connectivity.

3.1 Leader's Speed Constraint

In order to maintain leader semi-connectivity, we constrain the leader's speed $\|\boldsymbol{u}_{n+1}(t)\|$ as $\|\boldsymbol{u}_{n+1}(t)\| \leq U_{n+1}$, where U_{n+1} is determined by:

$$U_{n+1} \leq \min_{i \in \mathcal{F}} U_i, \quad U_{n+1} \leq \min_{i \in \mathcal{F}} \sqrt{A_i h_i / \{\sqrt{5}(2 + \sqrt{3})\}} \tag{2}$$

where $h_i := \min\{\rho_i - \rho_i', \rho_i'' - \rho_i'''\}$. The first constraint is for obeying followers' speed constraints and is derived from the proof of Theorem 2. The second one is for acceleration constraints and arises from the proof of Theorem 4.

3.2 Target Selection Process

Each follower $i \in \mathcal{F}$ chooses as its target the first agent which has passed $\partial S_i'(t)$ or $\partial S_i''(t)$ at $t > 0$. If multiple agents become the target candidates simultaneously, the target will be selected from them randomly. From Assumption 1, no followers start moving earlier than the leader, so the spanning tree of $\mathcal{G}(t)$ is created through this process. We define t_i as the time when agent i has determined its target, and the target does not change after that.

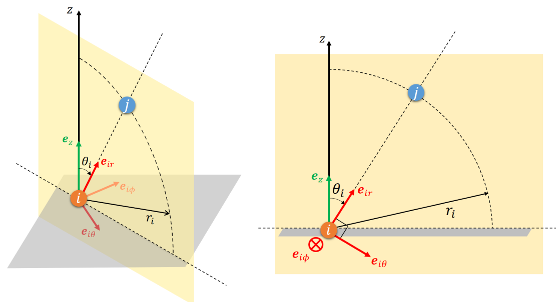


Fig. 4. Local basis vectors $\boldsymbol{e}_{ir}(t), \boldsymbol{e}_{i\theta}(t), \boldsymbol{e}_{i\phi}(t)$ of agent i.

3.3 Control Input

We express the control input $\boldsymbol{u}_i(t)$ to follower $i \in \mathcal{F}$ as follows (Fig. 4):

$$\boldsymbol{u}_i(t) = u_{ir}(t)\boldsymbol{e}_{ir}(t) + u_{i\theta}(t)\boldsymbol{e}_{i\theta}(t) + u_{i\phi}(t)\boldsymbol{e}_{i\phi}(t). \tag{3}$$

Here, $\boldsymbol{e}_{ir}(t)$ is defined as $\boldsymbol{e}_{ir}(t) := \boldsymbol{x}_{ij}(t)/r_i(t)$ for $r_i(t) > 0$, where $r_i(t) := \|\boldsymbol{x}_{ij}(t)\|$ and agent j is the target of agent i. We also define $\theta_i(t)$ as the angle measured from $\boldsymbol{e}_z$ to $\boldsymbol{e}_{ir}(t)$, which ranges from 0 to π. In addition, $\boldsymbol{e}_{i\theta}(t)$ is an unit vector which is on the same plane as $\boldsymbol{e}_{ir}(t)$ and $\boldsymbol{e}_z$ and is orthogonal to $\boldsymbol{e}_{ir}(t)$. Also, $\boldsymbol{e}_{i\theta}(t)$ faces toward the direction in which $\theta_i(t)$ increases. Further, $\boldsymbol{e}_{i\phi}(t)$ is an unit vector which faces such direction that $\boldsymbol{e}_{ir}(t)$, $\boldsymbol{e}_{i\theta}(t)$, $\boldsymbol{e}_{i\phi}(t)$ make a right-handed coordinate system. Note that $\boldsymbol{e}_{i\theta}(t)$ and $\boldsymbol{e}_{i\phi}(t)$ cannot be uniquely determined when $\sin\theta_i(t) = 0$. If $r_i(t) = 0$, we set $\boldsymbol{e}_{ir}(t) = \boldsymbol{e}_{i\theta}(t) = \boldsymbol{e}_{i\phi}(t) = \boldsymbol{0}$.

We set $\boldsymbol{u}_i(t) = \boldsymbol{0}$ for $t < t_i$, and design the control input for $t \geq t_i$ as follows:

1. The case of $\sin\theta_i(t) \neq 0$
   (a) $\rho_i''' \leq r_i(t) \leq \rho_i''$:

$$u_{ir}(t) = a_i'(r_i(t) - \rho_i''), \quad u_{i\theta}(t) = u_{i\phi}(t) = 0 \tag{4}$$

   (b) $\rho_i'' < r_i(t) < (\rho_i'' + \rho_i')/2$:

$$\begin{aligned} &u_{ir}(t) = 0, \quad u_{i\theta}(t) = 2k_i l_i \sigma_i(t) U_i'(t)(r_i(t) - \rho_i'')\sin\theta_i(t)/(\rho_i' - \rho_i'') \\ &u_{i\phi}(t) = 2\sqrt{1 - k_i^2} l_i \sigma_i(t) U_i'(t)(r_i(t) - \rho_i'')\sin\theta_i(t)/(\rho_i' - \rho_i'') \end{aligned} \tag{5}$$

   (c) $(\rho_i'' + \rho_i')/2 \leq r_i(t) < \rho_i'$:

$$\begin{aligned} &u_{ir}(t) = 0, \quad u_{i\theta}(t) = 2k_i l_i \sigma_i(t) U_i'(t)(\rho_i' - r_i(t))\sin\theta_i(t)/(\rho_i' - \rho_i'') \\ &u_{i\phi}(t) = 2\sqrt{1 - k_i^2} l_i \sigma_i(t) U_i'(t)(\rho_i' - r_i(t))\sin\theta_i(t)/(\rho_i' - \rho_i'') \end{aligned} \tag{6}$$

   (d) $\rho_i' \leq r_i(t) < \rho_i' + U_i'(t)/(2a_i)$:

$$\begin{aligned} &u_{ir}(t) = a_i(r_i(t) - \rho_i'), \quad u_{i\theta}(t) = k_i \sigma_i(t)\sin\theta_i(t) u_{ir}(t) \\ &u_{i\phi}(t) = \sqrt{1 - k_i^2}\sigma_i(t)\sin\theta_i(t) u_{ir}(t) \end{aligned} \tag{7}$$

(e) $\rho_i' + U_i'(t)/(2a_i) \le r_i \le \rho_i' + U_i'(t)/a_i$:

$$\begin{aligned} u_{ir}(t) &= a_i(r_i(t) - \rho_i'), \quad u_{i\theta}(t) = k_i \sigma_i(t) \sin\theta_i(t)(U_i'(t) - u_{ir}(t)) \\ u_{i\phi}(t) &= \sqrt{1 - k_i^2}\sigma_i(t) \sin\theta_i(t)(U_i'(t) - u_{ir}(t)) \end{aligned} \tag{8}$$

2. The case of $\sin\theta_i(t) = 0$
Though $\boldsymbol{e}_{i\theta}(t)$ and $\boldsymbol{e}_{i\phi}(t)$ cannot be uniquely determined, we do not have to consider the direction of these basis vectors when we set $u_{i\theta}(t) = u_{i\phi}(t) = 0$. Then (3) gives $\boldsymbol{u}_i(t) = u_{ir}(t)\boldsymbol{e}_{ir}(t)$, and we set $u_{ir}(t)$ the same as (4)–(8).

Here, $a_i := V_i/(\rho_i - \rho_i')$, $a_i' := V_i/(\rho_i'' - \rho_i''')$, $k_i, \sigma_i(t) \in [-1, 1]$, $U_i'(t) := \max_{0\le\tau\le t} |u_{ir}(\tau)|$. Further, l_i and V_i are defined as follows:

$$l_i := \min\left\{1, \frac{(\rho_i' - \rho_i'')A_i}{2\sqrt{9 + 2\sqrt{6}}V_i^2}\right\}, \quad V_i := \min\left\{U_i, \sqrt{\frac{A_i h_i}{\sqrt{5}(2 + \sqrt{3})}}\right\}. \tag{9}$$

In (9), V_i corresponds to the more stringent speed constraint imposed on the leader. $k_i \in [-1, 1]$ is an arbitrary constant which can be set for each agent, and determines the length of the input components in the direction of $\boldsymbol{e}_{i\theta}$ and $\boldsymbol{e}_{i\phi}$.

From the definition of $u_{ir}(t)$ when $r_i(t) \ge \rho_i'$, we obtain $r_i(t) = \rho_i' + u_{ir}(t)/a_i$. Since $a_i > 0$ and $u_{ir}(t) \le U_i'(t)$, $r_i(t) \le \rho_i' + U_i'(t)/a_i$ holds as long as ij semi-connectivity is kept. Then, since $r_i(t) > \rho_i'''$ also holds, we just need to consider the input for $\rho_i''' \le r_i(t) \le \rho_i' + U_i'(t)/a_i$. The parameter $\sigma_i(t) \in [-1, 1]$ represents how wide agents spread. The larger $|\sigma_i(t)|$ is, the wider they spread.

3.4 What is Guaranteed by the Proposed Method

Under Assumption 1, when each agents moves according to the mentioned method, all followers are leader semi-connected for all $t \ge 0$, satisfy the physical constraints (1) for all $t \ge t_i$, and keep semi-connectivity to their target.

4 Mathematical Proof

Now, we prove that each follower keeps semi-connectivity with its target and satisfies its physical constraints (1) under the control method in section 3. We assume that agent i's target is j and agent j's velocity vector is expressed as $\boldsymbol{u}_j(t) = u_{jr}(t)\boldsymbol{e}_{ir}(t) + u_{j\theta}(t)\boldsymbol{e}_{i\theta}(t) + u_{j\phi}(t)\boldsymbol{e}_{i\phi}(t)$ using agent i's basis vectors.

Theorem 1 (Semi-connectivity with the target). *Suppose Assumption 1 holds and $\boldsymbol{u}_i(t)$ is set according to* (3)–(8). *If $\|\boldsymbol{u}_j(t)\| \le U_{n+1}$, then $\rho_i'' - U_{n+1}/a_i' \le r_i(t) \le \rho_i' + U_{n+1}/a_i$.*

Proof: This can be proved in the same way as Theorem 1 in [12]. □

Theorem 2 (Maximum speed). *Suppose $\boldsymbol{u}_i(t)$ is set according to* (3)–(8). *If $\|\boldsymbol{u}_j(t)\| \le U_{n+1}$, then $\|\boldsymbol{u}_i(t)\| < U_{n+1}$.*

Proof: If $t < t_i$, the theorem clearly holds since $\boldsymbol{u}_i(t) = \boldsymbol{0}$. We discuss the case $t \geq t_i$. From Theorem 1, $\rho'' - U_{n+1}/a' \leq r \leq \rho' + U_{n+1}/a$ holds and, combining it with the control inputs (4)–(8), we can say that the velocity in the target direction u_{ir} satisfies $|u_{ir}| \leq U_{n+1}$.

1. The case of $\sin\theta = 0$ or $\rho'' - U_{n+1}/a' < r \leq \rho''$:
   Since $u_{i\theta} = u_{i\phi} = 0$, $\|\boldsymbol{u}_i\| = |u_{ir}| < U_{n+1}$ holds.
2. The case of $\sin\theta \neq 0$ and $\rho'' < r < \rho' + U'/a$:
   (a) $\rho'' < r < (\rho'' + \rho')/2$: From $0 < (r - \rho'')/(\rho' - \rho'') < 1/2$, $|l| \leq 1$, and $|\sigma| \leq 1$, the following holds:

   $$\|\boldsymbol{u}_i\|^2 = u_{i\theta}^2 + u_{i\phi}^2 = 4l^2\sigma^2U'^2\left(\frac{r - \rho''}{\rho' - \rho''}\right)^2 \sin^2\theta < U'^2.$$

   Because of $|u_{ir}| \leq U_{n+1}$ and $U_i'(t) := \max_{0\leq\tau\leq t}|u_{ir}(\tau)|$, we obtain $U_i'(t) \leq U_{n+1}$. Therefore, $\|\boldsymbol{u}_i\| < U_{n+1}$ holds.

   (b) $(\rho'' + \rho')/2 \leq r < \rho'$: From $0 < (\rho' - r)(\rho' - \rho'') \leq 1/2$, $|l| \leq 1$, and $|\sigma| \leq 1$, the following holds:

   $$\|\boldsymbol{u}_i\|^2 = u_{i\theta}^2 + u_{i\phi}^2 = 4l^2\sigma^2U'^2\left(\frac{\rho' - r}{\rho' - \rho''}\right)^2 \sin^2\theta \leq U'^2.$$

   Using $U' < U_{n+1}$, we obtain $\|\boldsymbol{u}_i\| < U_{n+1}$.

   (c) $\rho' \leq r < \rho' + U'/(2a)$: From (7), $|u_{ir}| < U'/2$ holds. Using $U' < U_{n+1}$, we obtain $|u_{ir}| < U_{n+1}/2$. From $|\sigma| \leq 1$, the following holds:

   $$\|\boldsymbol{u}_i\|^2 = u_{ir}^2 + u_{i\theta}^2 + u_{i\phi}^2 = (1 + \sigma^2\sin^2\theta)u_{ir}^2 \leq 2u_{ir}^2 < U_{n+1}^2/2$$

   Therefore, $\|\boldsymbol{u}_i\| < U_{n+1}$ holds.

   (d) $\rho' + U'/(2a) \leq r < \rho' + U'/a$: From (8), $U'/2 \leq u_{ir} < U'$ holds. From $|\sigma| \leq 1$, the following holds:

   $$\begin{aligned}\|\boldsymbol{u}_i\|^2 = u_{ir}^2 + \sigma^2\sin^2\theta(U' - u_{ir})^2 &\leq u_{ir}^2 + (U' - u_{ir})^2 \\ &= 2\left(u_{ir} - \frac{U'}{2}\right)^2 + \frac{U'^2}{2} < U'^2\end{aligned}$$

   Combining it with $U' < U_{n+1}$, $\|\boldsymbol{u}_i\| < U_{n+1}$ holds.

Therefore, it is proved that if $\|\boldsymbol{u}_j(t)\| \leq U_{n+1}$, $||\boldsymbol{u}_i(t)|| < U_{n+1}$ holds. □

Theorem 3 (Continuity of the control input).
Suppose Assumption 1 holds and $\boldsymbol{u}_i(t)$ is set according to (3)–(8). If $\|\boldsymbol{u}_j(t)\| \leq U_{n+1}$, then $\boldsymbol{u}_i(t)$ is continuous for any t.

Proof: If $t < t_i$, $\boldsymbol{u}_i$ is always zero and the theorem clearly holds. We discuss the case $t \geq t_i$.

1. The case of $\sin\theta \neq 0$:
   Since $\|\boldsymbol{u}_j\| \leq U_{n+1}$, $|u_{jr}| \leq U_{n+1}$ and $|u_{ir}| \leq U_{n+1}$ holds. Hence $\dot{r} = u_{jr} - u_{ir}$ is bounded and then r is continuous for any t. Because $u_{ir}, u_{i\theta}, u_{i\phi}$ are continuous for r, these are also continuous for t.
   The basis vectors $\boldsymbol{e}_{ir}, \boldsymbol{e}_{i\theta}, \boldsymbol{e}_{i\phi}$ change according to $\dot{\theta}_i$ and $\dot{\phi}_i$, which are the relative velocity between agents i and j in the directions of $\boldsymbol{e}_{i\theta}$ and $\boldsymbol{e}_{i\phi}$ respectively. Here, $\dot{\phi}_i$ corresponds to the angular velocity around the Z-axis (See Fig. 4), and define the counterclockwise direction (viewed from $+Z$ direction) as positive.
   Using $r > 0$ and $\sin\theta \neq 0$, $\dot{\theta}$ and $\dot{\phi}$ is expressed as $\dot{\theta} = (u_{j\theta} - u_{i\theta})/r$, $\dot{\phi} = (u_{j\phi} - u_{i\phi})/(r\sin\theta)$. Since $\|\boldsymbol{u}_j\| \leq U_{n+1}$ and $\|\boldsymbol{u}_i\| \leq U_{n+1}$, they can be defined as bounded values. Therefore, the time derivative of basis vectors $\dot{\boldsymbol{e}}_{ir} = \dot{\theta}\boldsymbol{e}_{i\theta} + \dot{\phi}\sin\theta\boldsymbol{e}_{i\phi}$, $\dot{\boldsymbol{e}}_{i\theta} = -\dot{\theta}\boldsymbol{e}_{ir} + \dot{\phi}\cos\theta\boldsymbol{e}_{i\phi}$, $\dot{\boldsymbol{e}}_{i\phi} = -\dot{\phi}(\sin\theta\boldsymbol{e}_{ir} + \cos\theta\boldsymbol{e}_{i\theta})$ are also bounded values, so $\boldsymbol{e}_{ir}$, $\boldsymbol{e}_{i\theta}$, $\boldsymbol{e}_{i\theta}$ is continuous for any t.
   From the discussion above, $\dot{\boldsymbol{u}}_i = \dot{u}_{ir}\boldsymbol{e}_{ir} + u_{ir}\dot{\boldsymbol{e}}_{ir} + \dot{u}_{i\theta}\boldsymbol{e}_{i\theta} + u_{i\theta}\dot{\boldsymbol{e}}_{i\theta} + \dot{u}_{i\phi}\boldsymbol{e}_{i\phi} + u_{i\phi}\dot{\boldsymbol{e}}_{i\phi}$ can be defined as bounded values and $\boldsymbol{u}_i(t)$ is continuous for any t.
2. The case of $\sin\theta = 0$:
   Since $u_{i\theta} = 0$ and $u_{i\phi} = 0$, $\dot{\boldsymbol{u}}_i$ is expressed as $\dot{\boldsymbol{u}}_i = \dot{u}_{ir}\boldsymbol{e}_{ir} + u_{ir}\dot{\boldsymbol{e}}_{ir} + \dot{u}_{i\theta}\boldsymbol{e}_{i\theta} + \dot{u}_{i\phi}\boldsymbol{e}_{i\phi}$, where $\dot{u}_{i\theta}$ and $\dot{u}_{i\phi}$ derives from (4)–(8).
   Using $u_{i\theta} = 0$ and $\sin\theta = 0$, $\dot{\boldsymbol{e}}_{ir}$ is expressed as $\dot{\boldsymbol{e}}_{ir} = (u_{j\theta}/r)\boldsymbol{e}_{i\theta}$, hence we obtain $\dot{\boldsymbol{u}}_i = \dot{u}_{ir}\boldsymbol{e}_{ir} + \{(u_{ir}u_{j\theta})/r + \dot{u}_{i\theta}\}\,\boldsymbol{e}_{i\theta} + \dot{u}_{i\phi}\boldsymbol{e}_{i\phi}$.
   Here, u_{ir}, $u_{i\theta}$, and $u_{i\phi}$ are continuous and bounded for any t regardless of whether $\sin\theta$ goes to zero. Since $|u_{j\theta}| \leq U_{n+1}$, $\dot{\boldsymbol{u}}_i(t)$ can be defined as a bounded value regardless of the directions of $\boldsymbol{e}_{i\theta}$ and $\boldsymbol{e}_{i\phi}$, therefore $\boldsymbol{u}_i(t)$ is continuous for any t.

In conclusion, it is proved that $\boldsymbol{u}_i(t)$ is continuous for any t if $\|\boldsymbol{u}_j(t)\| \leq U_{n+1}$. □

Theorem 4 (Maximum Acceleration). *Suppose Assumption 1 holds,* $\boldsymbol{u}_i(t)$ *is set according to* (3)–(8)*, and* $\sigma(t)$ *is constant. If* $\|\boldsymbol{u}_j(t)\| \leq U_{n+1}$*, then* $\|\dot{\boldsymbol{u}}_i(t)\| < A_i$*.*

Proof: If $t < t_i$, $\boldsymbol{u}_i$ is always zero and the theorem clearly holds. We discuss the case $t \geq t_i$. When $\sin\theta \neq 0$, the acceleration vector $\dot{\boldsymbol{u}}_i$ is expressed as $\dot{\boldsymbol{u}}_i = (\dot{u}_{ir} - \dot{\theta}u_{i\theta} - \dot{\phi}\sin\theta u_{i\phi})\boldsymbol{e}_{ir} + (\dot{\theta}u_{ir} + \dot{u}_{i\theta} - \dot{\phi}\cos\theta u_{i\phi})\boldsymbol{e}_{i\theta} + (\dot{\phi}\sin\theta u_{ir} + \dot{\phi}\cos\theta u_{i\theta} + \dot{u}_{i\phi})\boldsymbol{e}_{i\phi}$.

1. The case of $\sin\theta \neq 0$ and $\rho'' - U_{n+1}/a' < r(t) \leq \rho''$:

$$\begin{aligned}\|\dot{\boldsymbol{u}}_i\|^2 &= \dot{u}_{ir}^2 + (\dot{\theta}^2 + \dot{\phi}^2\sin^2\theta)u_{ir}^2 = a'^2\dot{r}^2 + \frac{u_{j\theta}^2 + u_{j\phi}^2}{r^2}a'^2(r-\rho'')^2 \\ &\leq a'^2\{(u_{jr} - u_{ir})^2 + \frac{(r-\rho'')^2}{r^2}(U_{n+1}^2 - |u_{jr}|^2)\}.\end{aligned}$$

   Since $(u_{jr} - u_{ir})^2 \leq (|u_{jr}| + U_{n+1})^2$ and $(r-\rho'')^2/r^2 \leq (\rho''/\rho''' - 1)^2 \coloneqq c(>0)$, we get the following:

$$\begin{aligned}\|\dot{\boldsymbol{u}}_i\|^2 &\leq a'^2\left\{(|u_{jr}| + U_{n+1})^2 + c(U_{n+1}^2 - |u_{jr}|^2)\right\} \\ &= a'^2\left\{(1-c)|u_{jr}|^2 + 2U_{n+1}|u_{jr}| + (1+c)U_{n+1}^2\right\}.\end{aligned}$$

For $0 \le |u_{jr}| \le U_{n+1}$, $\|\dot{\boldsymbol{u}}_i\|^2$ gets the maximum value $4a'^2U_{n+1}^2$ at $|u_{jr}| = U_{n+1}$. Since $V_i < \sqrt{Ah/2}$, $U_{n+1} \le V_i$, and $h \le \rho'' - \rho'''$, it is proved that $\|\dot{\boldsymbol{u}}_i\|^2 \le 4a'^2U_{n+1}^2 < A^2$.

2. The case of $\sin\theta \ne 0$ and $\rho'' < r(t) < (\rho'' + \rho')/2$:

$$\dot{u}_{i\theta} = kl\sigma U' \frac{2}{\rho' - \rho''} \left\{ u_{jr}\sin\theta + \frac{r - \rho''}{r}(u_{j\theta} - u_{i\theta})\cos\theta \right\}$$
$$\dot{u}_{i\phi} = \sqrt{1 - k^2}\, l\sigma U' \frac{2}{\rho' - \rho''} \left\{ u_{jr}\sin\theta + \frac{r - \rho''}{r}(u_{j\theta} - u_{i\theta})\cos\theta \right\}.$$

Calculating $\|\dot{\boldsymbol{u}}_i\|^2$ with the formulae above and evaluating it with $(r - \rho'')^2/r^2 < 1$, $|k| \le 1$, and $|\sigma| \le 1$, we get the following:

$$\begin{aligned} \|\dot{\boldsymbol{u}}_i\|^2 &< \left(\frac{2lU'}{\rho' - \rho''}\right)^2 \left\{ u_{jr}^2 + (u_{j\theta} - u_{i\theta})^2 (|u_{j\theta} - u_{i\theta}| + |u_{j\phi} - u_{i\phi}|)^2 \right\} \\ &\le \left(\frac{2lU'}{\rho' - \rho''}\right)^2 \left\{ (u_{jr}^2 + u_{j\theta}^2) + u_{i\theta}^2 \right. \\ &\qquad \left. + 2|u_{j\theta}||u_{i\theta}| + (|u_{j\theta}| + |u_{j\phi}| + |u_{i\theta}| + |u_{i\phi}|)^2 \right\}. \end{aligned}$$

Since $\|\boldsymbol{u}_j\| \le U_{n+1}$ and $\|\boldsymbol{u}_i\| \le U_{n+1}$, we get $|u_{jr}| + |u_{j\theta}| + |u_{j\phi}| \le \sqrt{3}U_{n+1}$ and $|u_{i\theta}| + |u_{i\phi}| < \sqrt{2}U_{n+1}$. Using them, we can further evaluate $\|\dot{\boldsymbol{u}}_i\|^2$ as $\|\dot{\boldsymbol{u}}_i\|^2 < (2lU'/(\rho' - \rho''))^2\{4 + (\sqrt{3} + \sqrt{2})^2\}U_{n+1}^2 = (9 + 2\sqrt{6})(2lU'U_{n+1}/(\rho' - \rho''))^2$. Using (9) and $U' < U_{n+1} \le V_i$, $\|\dot{\boldsymbol{u}}_i\|^2 < A^2$ holds.

For the other cases (including the case $\sin\theta = 0$), we can also prove $\|\dot{\boldsymbol{u}}_i\|^2 < A^2$ in the same way of evaluation as the two cases above. □

From Theorem 1–4, it is shown that the followers can keep leader semi-connected and satisfy all of their physical constraints (1).

5 Simulation

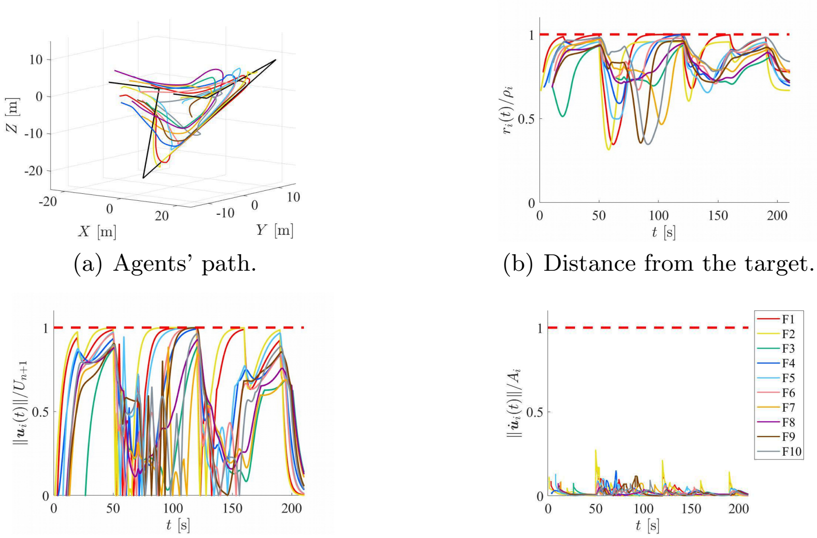
We show the simulation results for the proposed navigation method. We set the number of followers as $n = 10$, sampling period as 0.001 s, and $\sigma_i(t)$ as constants. We set the follower parameters as shown in Table 1. Here, the leader's speed constraints derived from (1) give $U_{n+1} = 0.6000$ m/s.

In this simulation, we consider a situation where a leader's trajectory includes drastic turns, and confirm that followers satisfy their physical constraints. The leader moves at its max speed U_{n+1} [m/s]. The leader starts from $X = Y = Z = 0$ m, and change its direction at 20 s, 50 s, 120 s, 160 s, and stops at 190 s.

Each agent's trajectory is shown in Fig. 5(a). The black line corresponds to the leader's trajectory. Fig. 5(b), 5(c), and 5(d) show the results of the simulation, each of which corresponds to the values of distance indicators $r_i(t)/\rho_i$, speed indicators $\|\boldsymbol{u}_i(t)\|/U_{n+1}$, acceleration indicators $\|\dot{\boldsymbol{u}}_i(t)\|/A_i$, and θ_i for each follower, respectively. From Fig. 5(b) we can say that all followers successfully keep

Table 1. Follower parameters

i	1	2	3	4	5	6	7	8	9	10
ρ_i [m]	9.0	6.0	8.0	8.0	7.0	7.0	8.0	8.0	7.0	9.0
ρ_i' [m]	7.0	4.0	5.5	5.5	5.0	5.5	6.0	5.5	5.0	7.5
ρ_i'' [m]	4.0	3.0	4.0	4.5	3.0	3.0	4.0	3.5	3.0	4.0
ρ_i''' [m]	2.0	1.0	1.5	2.0	1.0	1.5	2.0	1.0	1.0	2.5
U_i [m/s]	0.6	0.7	0.8	0.6	0.7	0.8	0.6	0.7	0.8	0.6
A_i [m/s^2]	2.0	2.0	1.5	1.5	2.0	2.5	2.0	1.5	2.0	2.5
k_i	0.7	-0.9	0.5	-0.3	0.7	0.1	-0.5	-0.9	0.3	-0.7
σ_i	1	-1	-1	1	-1	-1	-1	1	1	-1



(a) Agents' path.

(b) Distance from the target.

(c) Followers' speed.

(d) Followers' acceleration.

Fig. 5. Simulation results.

semi-connectivity with their target. In Fig. 5(d), some followers' acceleration values get larger values when the leader changes direction (especially at 50 s and 120 s) or stops moving, but Fig. 5(c) and 5(d) show that all followers still satisfy their physical constraints.

6 Conclusion

This paper proposed a control method for navigating robots with heterogeneous capabilities by a single leader in 3D space. In the proposed method, agents create a spanning tree by choosing the target agent to keep connectivity in a distributed way, and maintain the tree by keeping local connectivity with their target. This control method is decentralized, as each follower determines its action only using the local information. We have mathematically proved that leader semi-connectivity of the whole swarm is guaranteed for any leader's motion (under the leader's constraint) in 3D space with no obstacles, and confirmed the effectiveness of the proposed method by numerical simulation. For future work, we will try to show the proposed method can be applied to the actual environment by experiment. In addition, we hope that we could improve the control method for obstacle avoidance or inter-agent collision avoidance.

Acknowledgments

This work was supported in part by JST SICORP Grant Number JPMJSC18E4, Japan.

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