Chapter 5

# Analysis and Reconstruction of the Algorithmic Composition of the Muqarnas of the Main Gate of the Ottoman Mosque Atik Valide (Istanbul, Turkey): Creation of new geometric forms

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## 1. Introduction

### 1.1 Purpose of the Study

This study targets muqarnas, which are a characteristic component of Islamic architecture, but the method of making them is not always clear. Based on measured data, the study uses computer modelling of muqarnas in historical architecture to decipher their construction methods and intentions. At the same time, by reorganizing the compositional methods based on new intentions, new forms that develop from the muqarnas are explored. The analysis and reconstruction of the algorithms that generate the forms of muqarnas are central to the discussion.

### 1.2 Formative Techniques of Muqarnas

#### 1.2.1 What is Muqarnas

Muqarnas is a structural as well as an ornamental element of architecture which has been especially developed in Islamic Architecture. It is often used in the corners of square rooms and domes. Schematically, a muqarnas is a row of small horizontally aligned surfaces stacked vertically in layers." (Fukami, 1998, p12) There are two types of roles for muqarnas: structural and decorative. It can be said that this is a finding from an analysis of existing historical buildings.

In the structure, it is a corbelling technique and the muqarnas takes the load as a frame. When it is used to place a circular flat dome on a square room, the corners of the room are made into a polygonal shape by stretching out the corbelling steps from the corners of the room, and the corners of the square are brought closer to the dome. That polygon supports the load of the dome to be placed on top.

In decoration, it takes the form of a group of small surfaces formed by a geometric order that approximates a curved surface. Muqarnas are often used in domes and vaults, where the large, curved surfaces of the dome and vault are approximated by horizontal rows of small surfaces that are parts of the muqarnas, stacked vertically. When the dome is placed on top of the muqarnas, it forms a transition between the two that continuously joins the curved surfaces of the dome and the walls of the cubic chamber, creating an integral interior space. When a dome or vault is built on a muqarnas, a complex ceiling is created by a group of small surfaces rising from the lowest level to the apex of the uppermost level.

#### 1.2.2 Formative techniques of Muqarnas

From the standpoint of developing muqarnas' formative techniques and creating new forms,

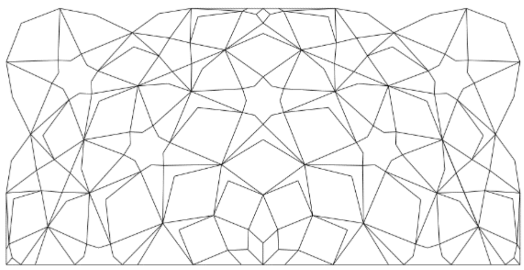

Figure 1 Planar projection plan of muqarnas, main gate of Atik Valide Mosque, Istanbul, Turkey. Author

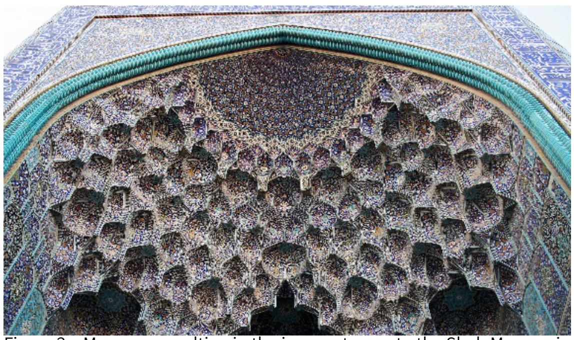

Figure 2 Muqarnas vaulting in the iwan entrance to the Shah Mosque in Isfahan, Iran. Wikimedia (2009)

| role | material | The idea behind construction methods |
|---|---|---|
| Structural and decorative | stone | Unitized materials, corresponding to a part or multiple parts, are piled up to form a curved surface. |
| | brick | Unitized materials are piled up to make parts, and this process is repeated to reach the curved surface shape. |
| Decorative only | tile | Unitized materials are combined and pasted to create parts, and this process is repeated to reach the curved surface shape. |
| | plaster | The surface is divided into layers, and the layers are then interlayered. The surface is then divided into smaller surfaces. |

Table 1 Ideas behind construction methods of muqarnas

knowledge about the process of constructing muqarnas is of interest.

It is known that muqarnas had a designer, who drew a two-dimensional drawing, i.e., a plane projection, and then followed the procedure of making it three-dimensional. Planar projection plans have centrality and share the characteristics of axial symmetry or symmetry.

Muqarnas was made of overlapping layers.

In contrast to this purely geometric design approach, when constructed as a real object, it depends on the physical properties of the materials and requires a corresponding method and approach.

Focusing on this phase of construction and understanding muqarnas's formative techniques as a combination of the construction method and the ideas behind it, it can be pointed out that different approaches exist, depending on the physical properties of the materials.

While both approaches have the expression of layered layers in common, there are two types of construction methods used by muqarnas. The differences in construction methods stem from the materials. The first is the stone and brick construction method. The parts are assembled horizontally, and then the resulting assembly of parts is stacked vertically. The second is the plaster construction method. A support frame is made of wood or plant stems on the back, to which polygonal wooden boards with uneven surfaces representing the cross-section of each layer are fixed. Plaster is applied to form small surfaces

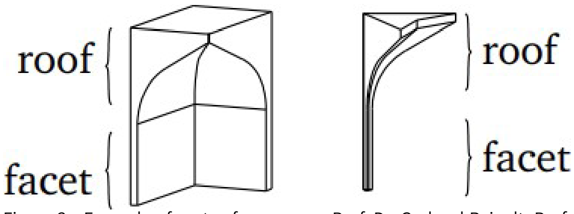


Figure 3 Example of parts of muqarnas. Prof. Dr. Gerhard Reinelt, Prof. Dr. Jan P. Hogendijk. (2006)

connecting the tops of the uneven surfaces of the upper and lower wooden boards.

It can be inferred that the thinking behind the different construction methods differed accordingly. The construction method using stones and tiles is based on the idea of piling up unitized building blocks to obtain a curved surface. On the other hand, the plaster construction method starts from a curved surface, divides it into layers, and then further divides the layers into smaller surfaces. The former is an inductive approach, while the latter is a deductive approach.

In this research the smallest unit of muqarnas, which form each micro surface, is defined as a muqarnas part. Parts have different shapes, and the main shape consists of a vertical part like a wall and a surface with curvature like a roof (Figure3). The classification of parts is explained in Section 2, 2.1.

## 1.3 Prior Studies and Precedent Work

In order to program the formative technique of muqarnas and to derive a new shape of muqarnas, previous studies on formative technique of muqarnas and studies on its reconstructing method with a computer program as a basis will be summarized in this research.

### 1.3.1 Historical Studies on the Perspective of Design

Fukami, N. (1998)., (2013)., (2017)., TAKAHASHI, S. (1970s)., Necipoğlu, G. (1995)., YVONNE, D. (1992)., Notkin, I.I. (1995)., Yasser, T. (2003)., and Harb, U. (1978). are the historical studies related to design of muqarnas.

However, the designers and craftsmen are gone, and there are only a limited number of examples that show how and with what philosophy the mukarnas were made. Many mysteries remain about the method of building muqarnas.

### 1.3.2 Studies of Composition and Modeling Based on Computer

YAMADA, G. (2020)., Asli Agirbas · Gulnur Yildiz. (2020)., Prof. Dr. Gerhard Reinelt, Prof. Dr. Jan P. Hogendijk. (2006)., Guéna, François. (2013)., and Mohammad Ali Yaghan. (1997). are studies of muqarnas through computer technology.

Since the 2000s, a new research project combining architectural history and computers has been conducted to solve the mystery of muqarnas's formative techniques.

This research focuses on the clarification of the existing muqarnas's formative principles from an interest in architectural history. Research that focuses on production and aims to elucidate the formative principles of muqarnas', and to develop the formative principles that decipher the production of a new muqarnas, has not been conducted.

### 1.3.3 Works Created Using the Formative Principle of Muqarnas

Some of the works that are said to have been produced by using formative principle of muqarnas include the following.

Muqarnas (2019-) Michael Hansmeyer: The appearance is flamboyant. However, it is not stated what aspects of Muqarnas's modeling principles were used. Rather than using the principles of muqarnas, it is thought that the work was inspired by muqarnas.

Folding Muqarnas, Ghazal Abbasy-Asbagh: A Case Study: It is a variant of the existing muqarnas. The shape of the centrality of the muqarnas is taken as a single unit, and the units are arranged successively. For each unit, a parallelogram deformation of the plane of the muqarnas is added, giving rise to a new form. However, the muqarnas is a given unit, and new forms are not derived by manipulating the components of the muqarnas and the way the elements are combined. It has a showy decorative effect.

StalacTile, Tessellated Manifolds(2010): This work transforms a part of muqarnas and adds transparent material and color to use it as a ceiling decoration. Although it is a new form, it is thought that the artist used a method of cutting it out of its entirety to attach it to the ceiling of a room.

Non-Modular Muqarnas(2017): This work maintains the symmetry of the joint relationship of the muqarnas and the drawings, and changes the horizontal and vertical length ratio of the small curved surfaces that make up the muqarnas, as well as the curvature of the curved surfaces, to reach a new form.

In the works created by using formative principle of muqarnas, a new modeling technique that focuses on parts has not yet been explored. This study explores the possibility that thinking in terms of the order of parts and their combinations can give rise to new forms that are free.

## 1.4 The Research Subject

In the 16th century, Islamic architecture was developed, and the Ottoman muqarnas is one of the representatives of this developed technology.

muqarnas can be found all over the world, and as the technique has spread to different regions and developed over time, the shapes and rules of muqarnas have also changed. However all existing muqarnas are drawings with solutions, in which a three-dimensional form is always generated given a planar projection. This study aims to create a new muqarnas with drawings, without just imagining the completed shape of muqarnas, by extracting the rules for generating three-dimensional forms from the planar projections of the conventional muqarnas examples, and to create a new muqarnas in a wide range of space and time. Among the cases, this study will focus on the muqarnas of the main gate of the Atik Valide Mosque, an example of an Ottoman muqarnas currently in Turkey. The reason for this is that the paper Asli Agirbas · Gulnur Yildiz. (2020)., which is based on the muqarnas of Atik Valide, refers the systematized way of drawing. Another reason is that Atik Valide's muqarnas has both star-shaped and pentagonal parts.

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## 2. Muqarnas Embodiment Process

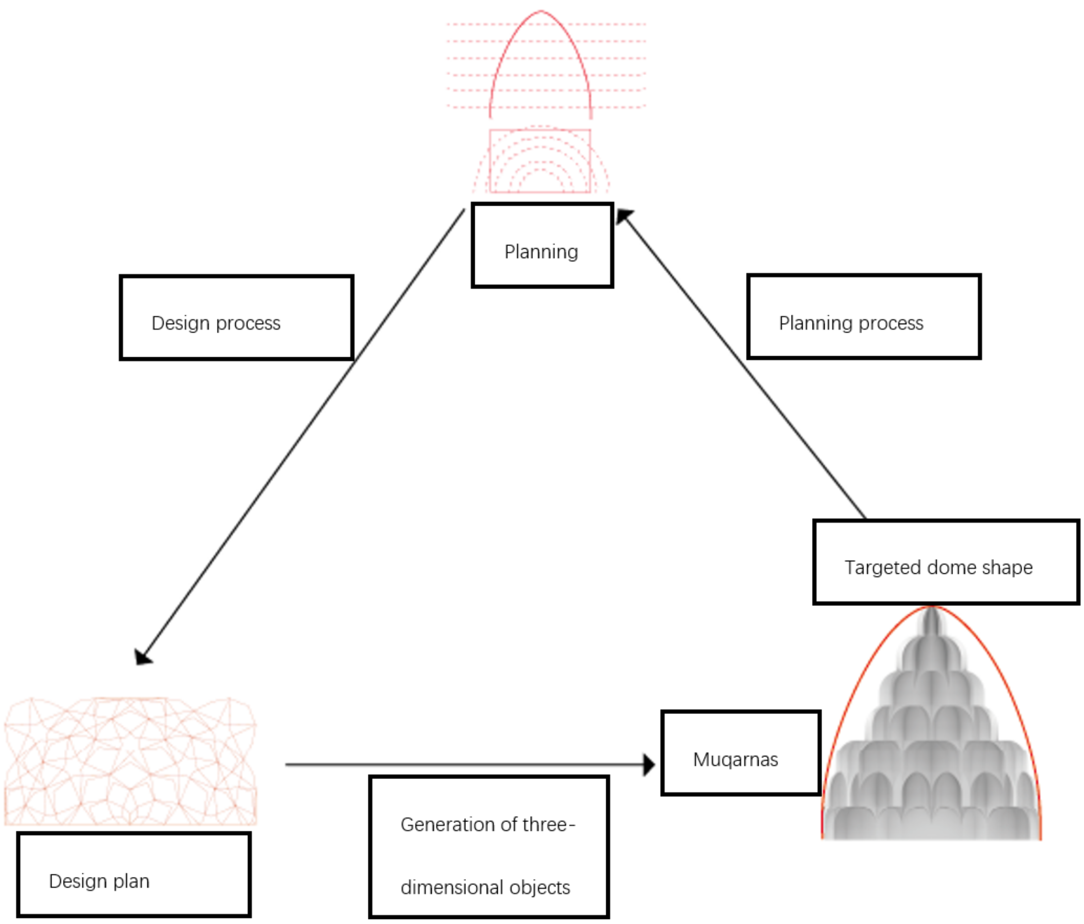



Figure 4 Image of muqarnas embodiment process. Author.

In this section, the parts have been organized as units of compositional elements of muqarnas and the compositional principle of muqarnas, referring to previous studies.

## 2.1 Muqarnas Components and Joining Rules

### 2.1.1 Parts of Muqarnas

Muqarnas are systematically created in a geometric order. The three-dimensional form of Muqarnas is composed of multiple parts that are continuously assembled horizontally and stacked vertically in layers. The parts of the same layer have the same height.

In this paper, the parts of muqarnas are classified into 5 major categories according to the position of the vertex. This is because they play different roles in terms of modeling.

The first is the cell. In terms of abstraction, a part is a cell, if one of its vertices is in a high vertical position (top) and the other vertices are in a low vertical position (bottom). From the plane, there are square cells and triangle cells. Cells connect the upper level, which has fewer vertices, to the lower level, which has more vertices, i.e., they spread the muqarnas from top to bottom. For "cell" parts, see "Dold-Samplonius, 1992, page 226. A cell is a shape that is curved (a roof) over two vertical surfaces (a facet) and is called a sail (a byat). There are also pyramidal parts without a cell or a curved surface. In this study, pyramidal parts are treated as the same as curved cells.

The second is the intermediate element. In terms of the abstracted form, a part with one vertex in a low vertical position (bottom) and the other vertices in a high vertical position (top) is an intermediate element. From the plane, there is a square intermediate element and a triangle intermediate element. The "intermediate element" part refers to "Harb, 1978, page34"; the one connecting the curved surfaces (roof) of two adjacent parts is an intermediate element.

Among the intermediate elements, there is a special part called a fletching part. The plane figure of the fletching part is a concave rectangle, and its plane figure and 3D shape are uniquely determined. This characteristic of the arrow vane part provides a hint to the constructor to identify the outline of the layers when reading the plane design drawings. For the designer, it is also an important part that separates the layers.

The third is a vertical plane. A part with one vertex in a lower vertical position (bottom) and two vertices in a higher vertical position (top) and a straight-line segment in the plane is a vertical plane. In other words, it is a part that has a solid body but no figure on the plane. The form of the vertical plane part is referred to FUKAMI, N. (1997). p27.

The fourth is a semi-tunnel vault. The number of vertices is four, and from the perspective of the abstracted form, a part with two of its vertices in a low vertical position (bottom) and the other two vertices in a high vertical position (top) is a half-tunnel vault. When viewed from the plane, only a rectangular half-tunnel vault is present. The shapes of the vertical plane parts are referred to FUKAMI, N. (1997). p27.

The fifth is the horizontal plane part. It is mainly polygonal and star-shaped and is used as a hanging ornament. The number of vertices is at least 4, and the vertices are all in the same vertical position. When viewed from the plane, they serve to increase the size of the layers, and when viewed from the vertical, they serve to increase the curvature of the large surface approximated by muqarnas. Horizontal surface parts are mainly represented as planes, umbrella-like shamse and hanging ornaments.

The 5 larger types can be further classified by eight types. Cells are triangles and two types of quadrilaterals, intermediate elements are triangles, three types of convex and concave quadrilaterals, vertical surfaces are one type, half-tunnel vaults are one type, and plane parts can also be grouped in one type. In all, there are 8 types. The concave rectangle part is called an arrow vane part.

The extant muqarnas can be described as a combination of 8 different parts of these five major categories.

### 2.1.2 Layers, Layer Lines and Joining of Muqarnas

Vertically up, layers are stacked on top of each other to form a three-dimensional muqarnas.

The joining of horizontal part combinations is that the parts are joined by sharing a ridge line with their neighbors. The joining of vertical upper layer stacking is the joining of two-layer lines next to each other, with the lower layer line of the upper layer sharing the upper layer line with the upper layer line of the lower layer. If the parts next to each other do not share a ridge line at the joint, there will be a chipped area with no solid at the joint, which is a break. The parts of muqarnas are joined without a tear.

The three-dimensional form of muqarnas was generated from a plan view. The only correspondence between the plan and the three-dimensional form is in the type of parts and the joining rules. In other words, when the plan is created, the shape of the three-dimensional object is automatically determined.

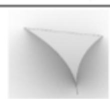
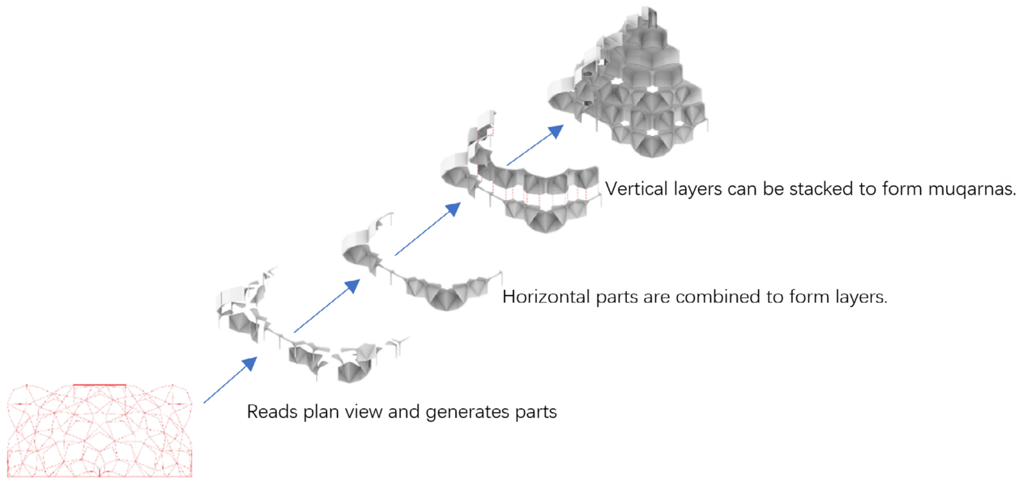
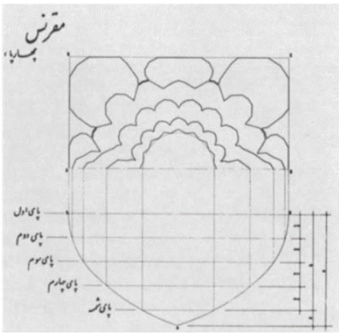
| Classification | type | planar figure | 3D shape |
|---|---|---|---|
| cell | triangle cell | triangle |  |
| | retangle cell | convex quadrangle |  |
| Intermediate Element | triangle intermeddiate element | triangle |  |
| | convex quadrangle intermediate element | convex quadrangle |  |
| | fletching part(concave rectangle intermediate element) | concave rectangle |  |
| vertical plane | vertical plane | straight line segment |  |
| Semi-tunnel Vault | Semi-tunnel Vault | convex quadrangle |  |
| Plane Parts | Plane Parts | polygon |  |

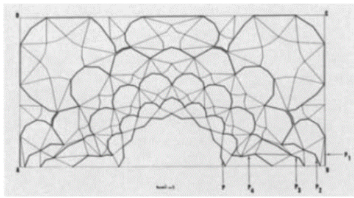
Table2 Types of parts of muqarnas. Author.




68. Asgar Shiʿrbaf, parallel projection of the muqarnas ground plan depicted in figure 67 to the vault of a portal, Iran, twentieth century, ink on paper. From Shiʿrbaf 1982–1983, 1: 94.



67. Asgar Shiʿrbaf, ground plan of a muqarnas portal hood, Iran, twentieth century, ink on paper. From Shiʿrbaf 1982–1983, 1: 91.



Top: Figure5 Generation process of muqarnas. Author.
Left: Figure6 Divide the dome into layers and draw outlines for each layer. Necipoğlu, G. (1995). P26.
Right: Figure7 Design parts for each layer. Necipoğlu, G. (1995). P26.

### 2.2 The Design Process of Muqarnas

The designer first draws a rectangle corresponding to the dimensions of the dome, sketches the outline of the front edge of each horizontal layer, and then imagines its three-dimensional image based on traditional rules of thumb, designing it in stages. (THE TOPKAPI SCROLL-GEOMETRY AND ORNAMENT IN ISLAMIC ARCHITECTURE page 46)

### 2.3 The Planning Process of Muqarnas

Muqarnas is used for domes and vaults. To reach the target dome shape, the dome is divided into several layers, and each layer is stacked to approximately a large, curved surface. The number of layers is considered to vary depending on the role, height, materials, construction method, technique, and design. Muqarnas that play a structural role tend to have larger parts and fewer layers than those that play only a decorative role.

## 3.Deciphering the Compositional Principle through the Reconstruction of Muqarnas.

In this section, we will construct a program to start up muqarnas by generating parts and stacking them in layers from the existing muqarnas and its planar projection plan. By analyzing the defects that occur in the process of building the program and their solutions one by one, the procedure and principles of construction of muqarnas are deciphered.

### 3.1 Drawings and Three-dimensional photos of the Research Subject

The actual muqarnas is a half-dome, but as some parts of the half-dome are cut in half, the plan view is compensated for the shape of the parts and a plan view of the whole dome is used.

### 3.2 Reference of the Foundation to Reconstruct Muqarnas

In this section's reconstruction of Atik Valide's

Figure8 Atik Valide main gate muqarnas. Asli Agirbas. (2020)

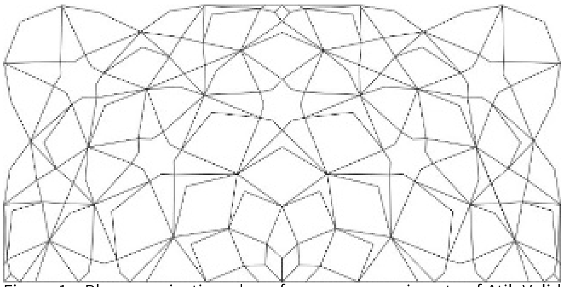


Figure 1 Planar projection plan of muqarnas, main gate of Atik Valide Mosque, Istanbul, Turkey. Author

Muqarnas and the creation of the plan projection plan is based on Asli Agirbas · Gulnur Yildiz. (2020). The development of the program is referenced in YAMADA. (2020).

In terms of planar projection plans, Asli Agirbas · Gulnur Yildiz. (2020) describes a method for creating planar projection plans by creating intersecting elliptical shapes with point symmetry and connecting the points of intersection.

For three-dimensional generation, the same mechanism is used as in YAMADA. (2020). The parts of the muqarnas are oriented. For example, although they are the same square in plan, the shape of this part changes throughout the muqarnas depending on which of the four corners it faces. Similarly, the shape of the type also changes, depending on whether it is a cell or an intermediate element, even if it is the same rectangle in plan. The paper referred to above uses a mechanism for generating a three-dimensional projection by classifying the parts and organizing their orientations to derive the correct three-dimensional shape from the planar projection. This section uses the same mechanism to reproduce Atik Valide's muqarnas.

### 3.3 Purpose of the Reconstruction of Muqarnas

In terms of analysis of the actual samples, a muqarnas is composed of several parts stacked on top of each other. From the point of view of the process of making a muqarnas, the three-dimensional form is derived from the comprehension of the planar projection drawing. Therefore, the comprehension of the planar projection drawing is the identification of the three-dimensional form of the parts by using the formative principle of the muqarnas to correspond the three-dimensional form of the parts to the two-dimensional figure of the parts depicted in the planar projection drawing.

However, unlike the process of creating a muqarnas from a planar projection, the reproduction of the muqarnas of the main gate of the Atik Valide mosque, described in this section, involves knowing the planar projection of the muqarnas and the three-dimensional shape in advance. The aim is to find out how to design a program that can be used to derive the known three-dimensional shape from the known planar projection. The aim is to decipher muqarnas' formative principles by reproducing them in this program.

### 3.4 Parts Used in the Muqarnas of the Main Gate of Atik Valide

Six types of parts used in the muqarnas of the main gate of Atik Valide. (Table3)

### 3.5 Hints to Determine Shapes of Parts

The 6 types of muqarnas' parts planographic figures are triangles, convex rectangles, concave rectangles, polygons with a number of vertices of more than 4(hereafter abbreviated as polygons).

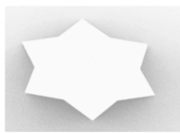
| Classification | type | planar figure | 3D shape |
|---|---|---|---|
| cell | triangle cell | triangle |  |
| | retangle cell | convex quadrangle |  |
| Intermediate Element | triangle intermeddiate element | triangle |  |
| | convex quadrangle intermediate element | convex quadrangle |  |
| | fletching part(concave rectangle intermediate element) | concave rectangle |  |
| Plane Parts | Plane Parts | polygon |  |

Table3 Parts used in muqarnas of main gate of Atik Valide. Author.

Of these, the triangular and convex rectangles are either two or more of the cell and intermediate elements. Also. The properties of each interior angle of triangles and convex rectangle are the same, and each vertex has no direction. Thereby, in terms of the direction in which the parts face, there are three possibilities in triangle parts and four possibilities in convex retangle parts. Therefore, there are six patterns of triangle parts and eight patterns of convex quadrilateral parts.

In contrast, the concave rectangles has only one corresponding part, the intermidiate element, and the concave retangle is a directional figure and is limited to one direction in which the part faces. Therefore, as a part, the concave rectangle in the plane projection diagram is uniquely identified from only a single figure. Concave rectangle parts are called flecthing shaped parts. Polygons in planar projection diagrams can also be uniquely identified as parts and from three-dimensional shapes. Therefore in this study, the flecthing parts and the polygon parts are specified, based on them, the program to operate the determination of shpaes of their suroundding parts are constructed

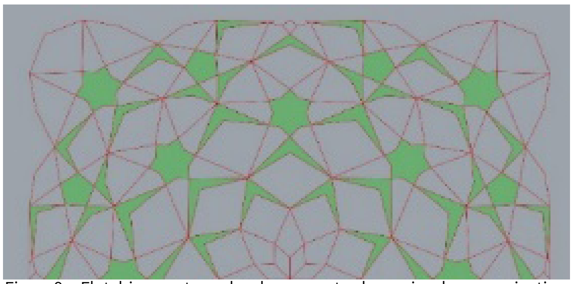


Figure9 Fletching parts and polygon parts shown in planar projection plan. Author.

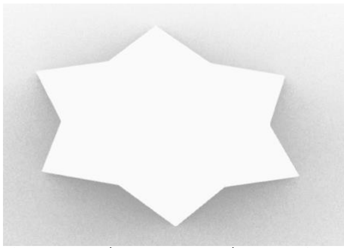


Figure10 Polygon parts. Author.

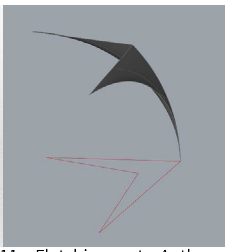


Figure11 Fletching parts. Author.

and the compositional principles of muqarnas are comprehenced.

### 3.6 Procedures of the Program of Reconstruction

Procedures of the program of reconstruciton of the main gate of Atik Valide Mosque are shown in figure12.

### 3.7 Discussion of the Reconstruction

In this section, the program with steps to resolve the troubles happened during the generation of shapes of parts, joining of part to

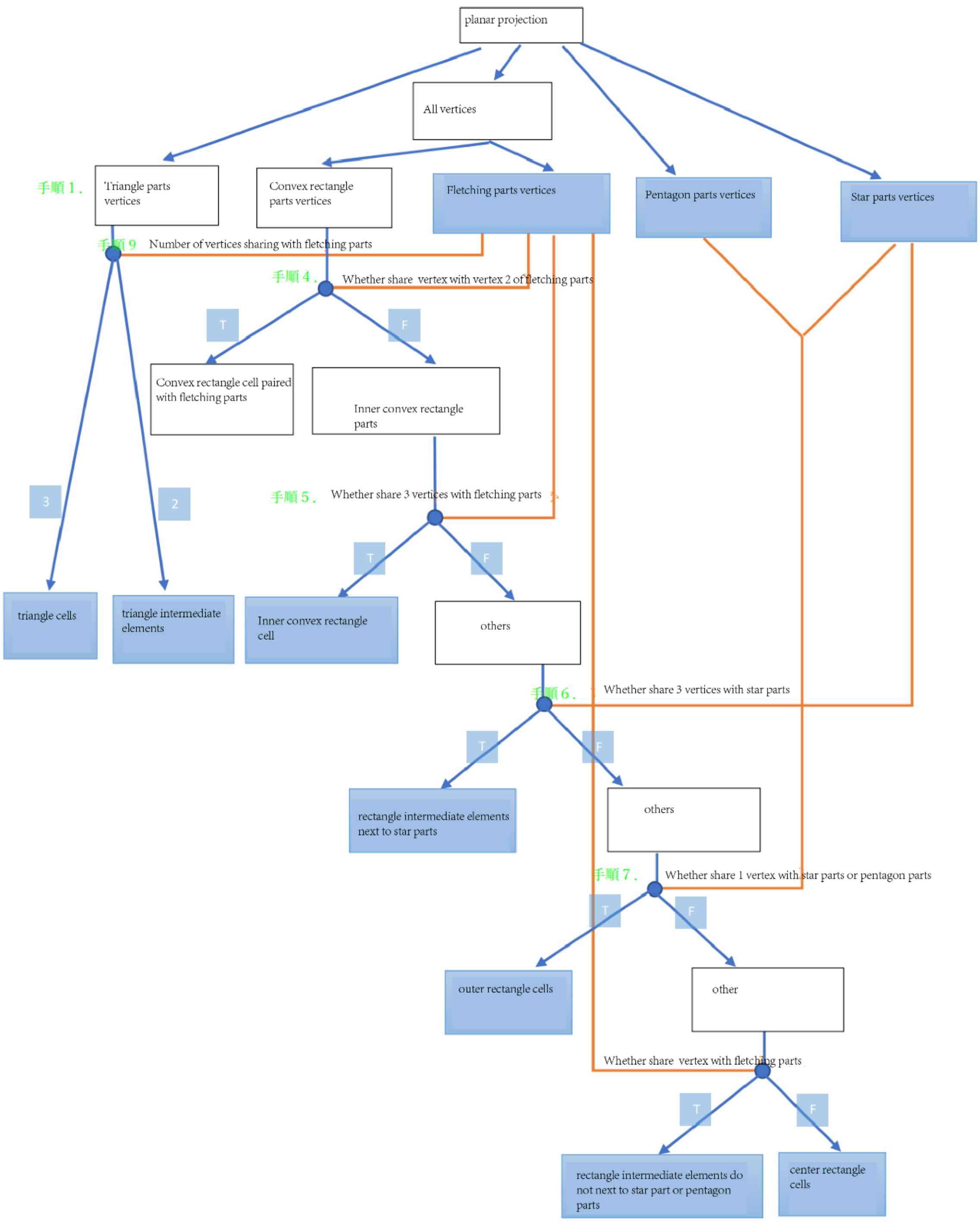



Figure12 Flow chart of program of reconstruction of muqarnas of the main gate of Atik Valide. Author.

Figure13 Model generated in reconstruction. Author.

part, the formation of layers and joining of layer to layer in the progress of construction of program will be explained.

In order to generate three-dimensional form of muqarnas through planar projection plan, there are 3 necessary steps, which are determination of the shape of each part, classification of which layers each part belongs to and assigning height to each layer.

For the first step, the shape of each part can be determined by two specific types of parts, which are fletching parts and polygon parts. Therefore, the following procedures are created. Firstly, parts whose shapes can be determined by joining relationships with fletching parts are identified. Secondly, parts whose shapes can be determined by joining relationships with polygon parts are identified. By treating center point of planar projection plan as infinitely small deformation of polygon parts, all parts of muqarnas can be identified through using the 2 specific parts, which are fletching parts and polygon parts.

For the second step, by focus on the bottom segment of cell parts, the layer-line of each layer

| types of parts \ jobs in each direction | horizontal (planar projection plan) | vertical(layer) |
|---|---|---|
| cell parts | × | ◯(lower layer line) |
| intermediate parts other than fletching parts | × | ◯(upper layer line) |
| fletching parts | ◯ | ◯(upper layer line) |
| polygon parts | ◯ | ◯(upper or lower layer line) |

Table4　Jobs of shape-determination in each direction. Author.

is determined. The lower layer line of each layer is formed by connecting outline of the bottom segment of each cell part and polygon parts. Parts enclosed by two layer lines are located in the same layer.

For the third steps, height of each layer is assigned. Within the several layer-lines which are determined in the second step, referring to the feature that the inner part is located in a higher layer, each layer composed by all parts are assigned with a height.

Among the types of shapes of the parts, there were two strong shapes, fletching parts and polygon parts, that served as the reference on determination of shapes. Their shapes can be generated automatically without additional information once the planar projection plan is drawn. By using these two types of parts, the shapes of their surrounding parts are determined, which ultimately construct the overall shape of muqarnas. It can be inferred that this is one of the methods of forming muqarnas. These two types of parts will be called "shape-determining parts" in this study. And all types of parts can be divided into two classifications, which are shape-determining parts with a higher hierarchy, and non-shape-determining parts, with a lower hierarchy, in terms of process of generation.

What is more, since upper layer line and lower layer line is form by the connection of either top segment or down segment of parts that belong to that layer, once the shapes of parts are determined, the shape of each layer is determined. Jobs of different types of parts are shown on the Table5.

The rules of types of parts and joining rules are

| Classification | type | planar figure | 3D shape |
|---|---|---|---|
| non-shape-determinig parts | retangle cell | | |
| | Semi-tunnel Vault | | |
| | rectangle intermediate element | | |
| | triangle cell | | |
| | triangle intermediate element | | |
| shape-determinig parts | fletching part(concave rectangle intermediate element) | | |
| | Polygon Parts | | |

Table5　Classification of types of parts based on traditional muqarnas

used to specify the overall shape of muqarnas. The rules of types of parts and joining rules of parts are formative principles of muqarnas. Based on this formative principle, once the planar projection is determined, the overall shape of the muqarnas will be determined automatically. It can be climbed that this program which automatically generates the overall shape by certain rules shows relationship to the order of God that can generate the whole world.

## 4.Generation of new Form by reorganizing -Referred Compositional Pinciples of Muqarnas

This section is about design of a program to generate new shapes of muqarnas on a free unsymmetrical planar projection plan whose shapes of parts cannot be identified automatically, based on understanding of the compositional principles of muqarnas and program developed in Section 3.

### 4.1 Key to Generate New Shape of Muqarnas

Based on Section 3, it is clear that among all types of parts, parts that serve as reference in planar and three-dimensional aspects, can be used to determine parts surrounding them, and finally construct the overall shape of muqarnas. Because of that, the method described above is attempted to try to generate a new shape of muqarnas with free planar projection plan. Thereupon, demands to determine the shape of a single part based on different shape-determining parts may be batting against each other. When that happens, the determination of shape of that part follows one of the demands, and a tear of joining parts will occur. These two shape-determining pars have the power to lead the directions of shape-generation of their surrounding parts. However, without understanding the full portrait of them, placing them in a free planar projection plan leads to the appearance of tears.

Thus, the key to construct the shape of muqarnas is shifted from using shape-determining parts to how to join parts to generate an overall shape of muqarnas with no tears. Base on it, Muqarnas can be constructed by repeating these steps: generating parts, joining the parts, forming layers, and joining the layers. The key point in this process is how the parts are determined.

Although the focus on shape-determination is shifted, the shape of different types of parts, and existence of layer remain. In Section 4, developing new programs to construct new shapes of muqarnas is still based on features of muqarnas.

### 4.2 General Program of Generation of Muqarnas

By putting the focus on joining parts with no tears, a new general program is designed. In terms of operation, muqarnas can be constructed by repeating these steps progressively: generating parts, joining the parts, forming layers, and joining the layers. To develop these steps to a more general level, forming layers can be treated as forming and assembling parts horizontally while joining layers can be treated as forming and assembling parts vertically. Because of this transition, the construction becomes forming and assembling in two directions with no hierarchy instead of forming basic units to complex assemblies progressively. In a word, the general program of constructing muqarnas is based on repetition of forming and assembling parts until shapes of all parts are created.

In order to start the generation of shape of parts, unlike the traditional muqarnas, which has clear frame serves as the starting and a center serve as the end point of formation in order to complete the overall shape, muqarnas with free planar projection plan needs a starting or ending given by manually.

In terms of parts, according to classification in Section 3, seven types of parts are classified with two hierarchy, which are rectangle cells, rectangle intermediate elements, triangle cells, triangle intermediate elements, a semi-tunnel vaults, fletching parts, and polygonal parts.

Like the operation, the classification of types of parts needs to be developed to a more general level, parts cannot be treated with different hierarchy. To satisfy this, the planar figures of rectangle parts and fletching parts are treated as quadrangle parts. Therefore, there are three types of planar figure of parts, which are quadrangle, triangle, and polygon. The three planar figures are corresponding to six types of shapes which are quadrangle cell, quadrangle intermediate element, semi-tunnel vault, triangle cell, triangle intermediate element, and polygon parts, shown in Table6.

Firstly, the top layer line of the highest layer manually is set by giving a 3d coordinate to each vertex of this layer line. Parts with vertices shared with the known layer line are specified, and among them, shapes of parts with enough

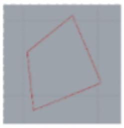
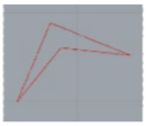
| planar figure | | type of shapes |
|---|---|---|
| quadrangle |   | quadrangle cell |
| | | Semi-tunnel Vault |
| | | quadrangle element |
| triangle |  | triangle cell |
| | | triangle intermediate element |
| polygon |  | Polygon Parts |

Table6 Classification of types of parts at a general level

| | General program of shape-determination |
|---|---|
| Basic | a. All parts are identified by their vertices. |
| | b. All parts share the same height. |
| | c. In terms of planar figures, parts is classified as quadrangle parts, triangle parts, and polygon parts. |
| | d. In terms of planar figures, parts is classified as quadrangle parts, triangle parts, fletching parts, and polygon parts. |
| | e. Because the corresponding of planar figure and 3D shape, polygon parts function as shape-determining parts. |
| | f. Because the corresponding of planar figure and 3D shape, fletching parts function as shape-determining parts. |
| | g. When the parts are joined together horizontally, planes of equal height are formed at their top and bottom, respectively, and all vertices are located in one of the two horizontal planes. In other words, layers are formed. |
| | h. When parts are joined vertically to each other, a new plane of equal height is formed at the top or bottom of each other. In other word, two layers joined and three layer lines with different height are formed. |
| Steps of shape-determination | 1. Set a enclosed polygon as the top layer line of the highest layer. |
| | 2. Using the method of determination of shape to determine the shapes of parts that satisfies the basic rules selected. |
| | 3. Check if shapes of all parts are determined. If so, program finishes. if not, go to step 4. |
| | 4. Check if there are any parts whose shape can be newly determined as a result of performing step 2. If so, the process returns to step 2., again. If not, go to step 5. |
| | 5. By introducing a new joining rule, the supporting joining rule① to determine the shapes of parts. Then go to step 6. |
| | 6. Check if there are parts newly determined in step 5. If so, go back to step. 2., if not, go to step 7. |
| | 7. By introducing a new joining rule, the supporting joining rule② to determine the shapes of parts. Then go to step 8. |
| | 8. Check if there are parts newly determined in step 7. If so, go back to step. 2., if not, go to step 9. |
| | 9. By introducing a new joining rule, the supporting joining rule③ to determine the shapes of parts. Then go to step 10. |
| | 10. Check if there are parts newly determined in step 9. If so, go back to step. 2., if not, go to step 11. |
| | ※Once no shape of part is newly determined by using step 2.-3, a step new joining rule will be added to continue the shape-determination. In this case, step 2n-1. ($n\geq3$) and step 2n. ($n\geq3$) are introduced as one set. Step 2n-1. ($n\geq3$) is determining new shapes of parts while step 2n. ($n\geq3$) is determining the direction of the flow of program. In this table, it is shown the case that three new supporting rules are added. |
| | … |

Table7 General program of shape-determination

shape-determining conditions are determined. Those parts are placed in layers they belong to. Once shapes of parts are determined newly, their coordinates of vertices become new information to fulfil the shape-determining conditions of other parts. By repeating these procedures, the general

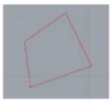
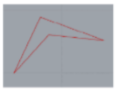
| Classification | planar figure | relationship with upper layer line | locations of vertices | type of parts | 3D shape |
|---|---|---|---|---|---|
| non-shape-determinig parts |  | share 1 vertex with upper layer line | 1 on upper layer line<br>3 on lower layer line | retangle cell |  |
| | | share 2 vertices with upper layer line | 2 on upper layer line<br>2 on lower layer line | Semi-tunnel Vault |  |
| | | share 3 vertices with upper layer line | 3 on upper layer line<br>1 on lower layer line | rectangle intermediate element |  |
| |  | share 1 vertex with upper layer line | 1 on upper layer line<br>2 on lower layer line | triangle cell |  |
| | | share 2 vertices with upper layer line | 2 on upper layer line<br>1 on lower layer line | triangle intermeddiate element |  |
| shape-determinig parts |  | | 3 on upper layer line<br>1 on lower layer line | fletching part(concave rectangle intermediate element) |  |
| |  | | all on either upper or lower layer line | Polygon Parts |  |

Table8　The method of determination of shape of Type 1

| | General program of shape-determination | Type 1 |
|---|---|---|
| Basic | a. All parts are identified by their vertices. | ○ |
| | b. All parts share the same height. | ○ |
| | c. In terms of planar figures, parts is classified as quadrangle parts, triangle parts, and polygon parts. | × |
| | d. In terms of planar figures, parts is classified as quadrangle parts, triangle parts, fletching parts, and polygon parts. | ○ |
| | e. Because the corresponding of planar figure and 3D shape, polygon parts function as shape-determining parts. | ○ |
| | f. Because the corresponding of planar figure and 3D shape, fletching parts function as shape-determining parts. | ○ |
| | g. When the parts are joined together horizontally, planes of equal height are formed at their top and bottom, respectively, and all vertices are located in one of the two horizontal planes. In other words, layers are formed. | ○ |
| | h. When parts are joined vertically to each other, a new plane of equal height is formed at the top or bottom of each other. In other word, two layers joined and three layer lines with different height are formed. | ○ |
| Steps of shape-determination | 1. Set a enclosed polygon as the top layer line of the highest layer. | ○ |
| | 2. Using the method of determination of shape to determine the shapes of parts that satisfies the basic rules selected. | ○ |
| | 3. Check if shapes of all parts are determined. If so, program finishes. if not, go to step 4. | ○ |
| | 4. Check if there are any parts whose shape can be newly determined as a result of performing step 2. If so, the process returns to step 2., again. If not, go to step 5. | ○ |
| | 5. By introducing a new joining rule, the supporting joining rule① to determine the shapes of parts. Then go to step 6. | × |
| | 6. Check if there are parts newly determined in step 5. If so, go back to step. 2., if not, go to step 7. | × |
| | 7. By introducing a new joining rule, the supporting joining rule② to determine the shapes of parts. Then go to step 8. | × |
| | 8. Check if there are parts newly determined in step 7. If so, go back to step. 2., if not, go to step 9. | × |
| | 9. By introducing a new joining rule, the supporting joining rule③ to determine the shapes of parts. Then go to step 10. | × |
| | 10. Check if there are parts newly determined in step 9. If so, go back to step. 2., if not, go to step 11. | × |
| | ※Once no shape of part is newly determined by using step 2.-3, a step new joining rule will be added to continue the shape-determination. In this case, step 2n-1. (n≥3) and step 2n. (n≥3) are introduced as one set. Step 2n-1. (n≥3) is determining new shapes of parts while step 2n. (n≥3) is determining the direction of the flow of program. In this table, it is shown the case that three new supporting rules are added. | × |
| | … | × |

Table9　Table9 The program of shape-determination of Type 1

program of shape-determination is design and shown in Table7.

By applying two different joining rules, two methods of shape-determination are created, which leads to two different types of forms of generation which are Type 1 and Type2.

### 4.3 Type 1

#### 4.3.1 Program of Type 1

Program of Type 1 puts relationship between parts and layer line as the fundamental rule of the shape-determination. Because of the sharing of layer line between the adjacent layers, it is guaranteed that an overall shape of muqarnas with no tears can be generated if each layer, which is each assembly of parts, shows no tears.

Since the relationship between parts and layer lines is the fundamental rule of Type 1, the hierarchy is brought back in the operation. Therefore, the classification of parts where seven types exist is used in Type 1. Generating parts, joining the parts, forming layers, and joining the layers happen progressively and repeatedly in program of Type 1.

Location of vertices of parts are either on the upper layer line or lower layer line of the same layer. Once locations of certain vertices of a part are clarified, the other vertices of this part will be located automatically on the layer line other than the vertices clarified were located on. By this way, enclosed layer lines of each layer are generated sequentially from the top to the bottom. This characteristic of shape that stacking layers one on the other are shared both in the reconstruction of Atik Valide, which is a traditional muqarnas and Type 1.

The method of determination shape of Type 1, based on relationships with upper layer line is shown in Table8.

By setting the top layer line of the highest layer, the generation of shapes starts. Table9 shows program of shape-determination of Type1.

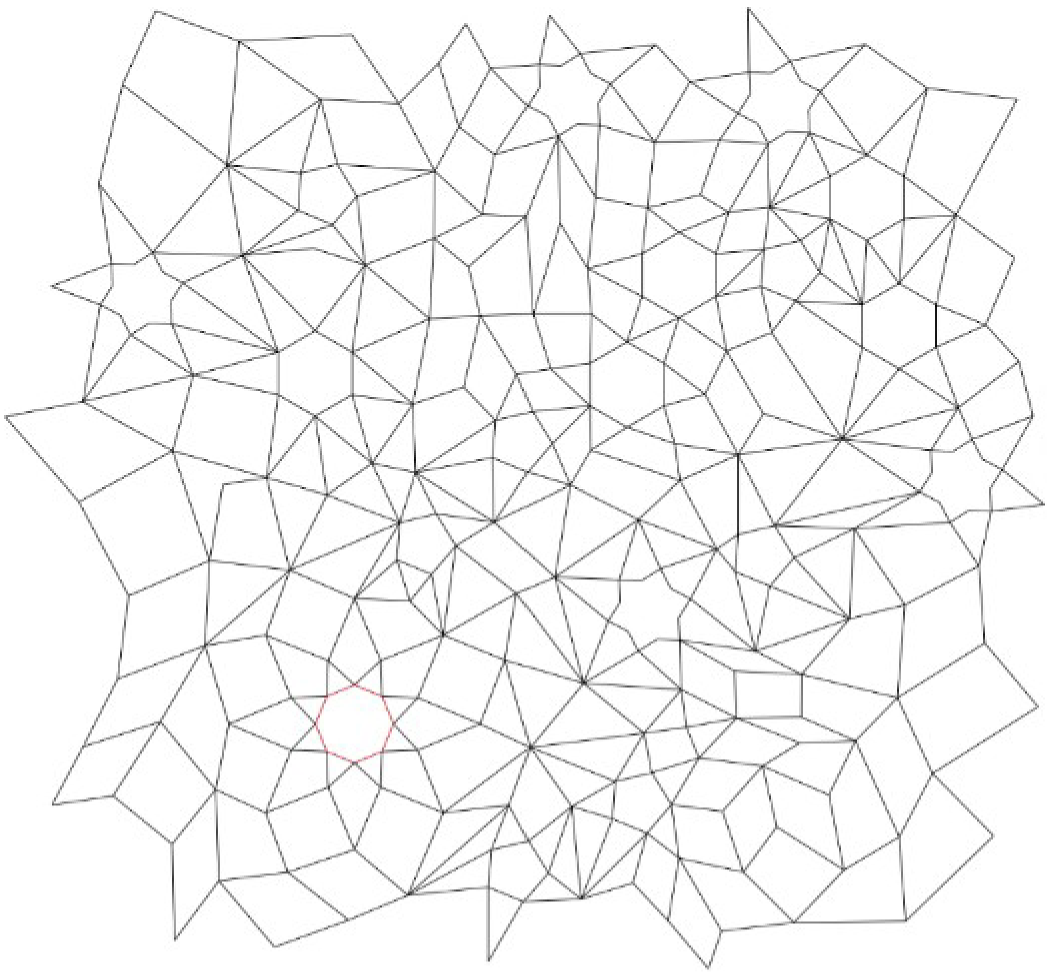


Figure14 Planar projection plan of Type 1

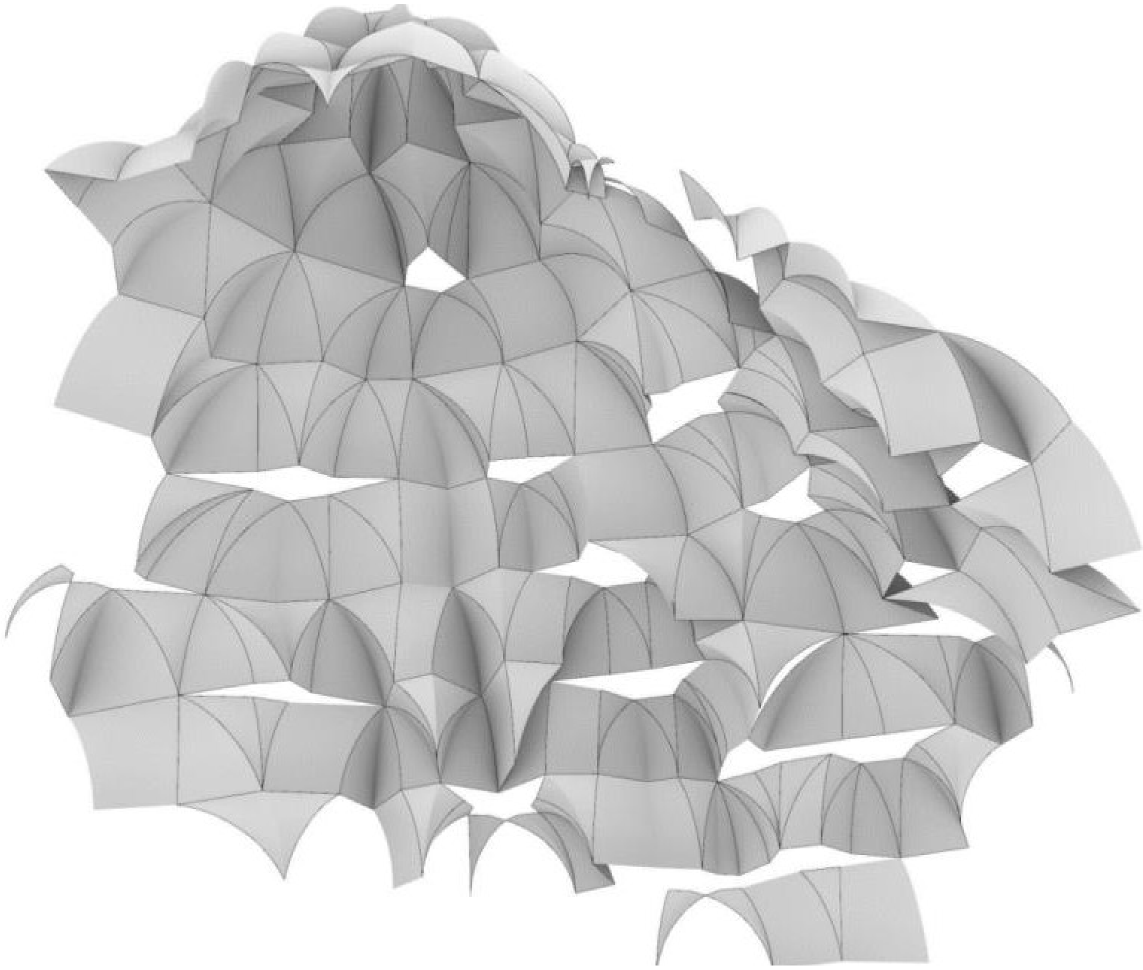


Figure15　The overall shape of Type1

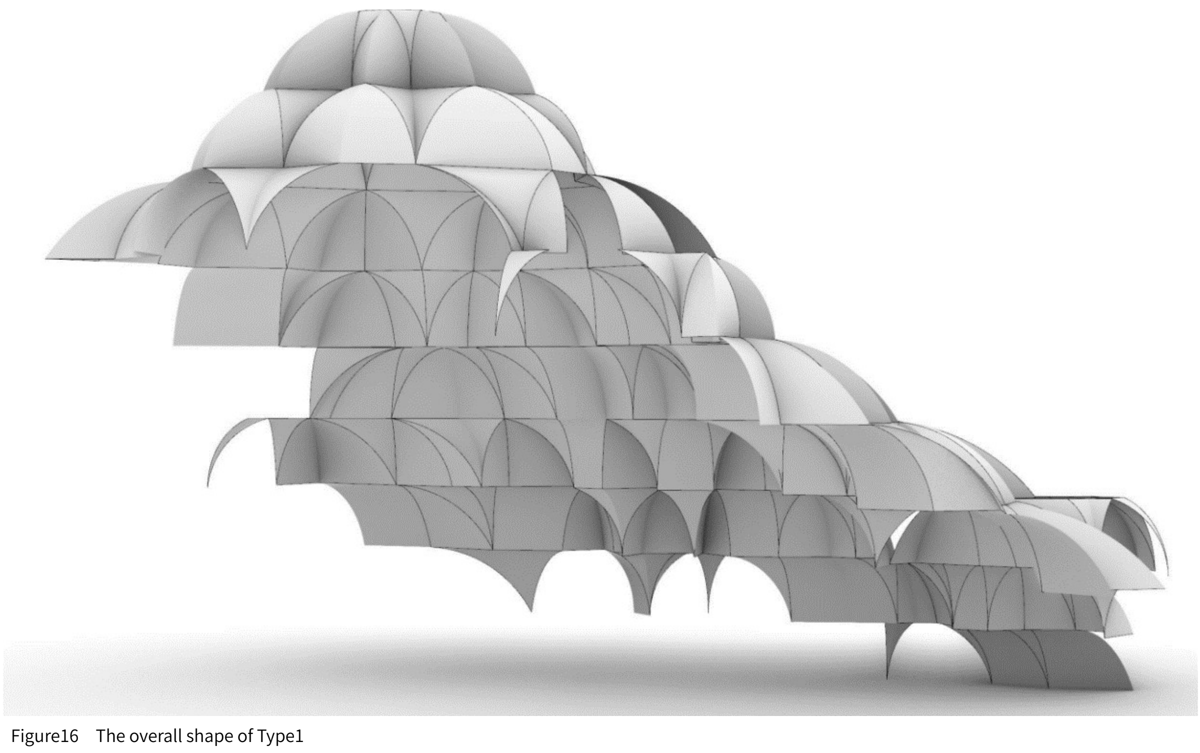


Figure16　The overall shape of Type1

#### 4.3.2 The Planar Projection Plan and Model of Type 1

The planar projection plan is created by the following steps. Firstly, planar figure of polygon are placed randomly. Secondly, by adding triangles or quadrangles around polygons, figures of star are created. Finally, by drawing quadrangles and triangles, all planar figures are connected in a plan. In this case, planar figures of feltching part are not used in order to aviod tears due to inconsistency between intention and placement of them. The paln is shown in Figure14.

The overall shape of Type 1 is shown in Figure15 and Figure16.

#### 4.3.3 Evaluation of Type 1

A similar shape compared to the traditional muqarnas, with a smooth and continuous surface is generated. Through the process of generation, the lack of freedom is observed in Type 1. After assigning the coordinates of the top layer line of the highest layer, the shape of each part is determined even with a single vertex shared with each layer line. Since that, the overall shape is determined automatically once the assignment of coordinates of the first layer line finished.

Through the reconstruction of muqarnas in Section 3 and the construction of Type 1, the automatic generation of the overall shape of muqarnas seems to correspond to the monotheistic Islamic ideology. The diverse worlds are formed in an order determined by God. The world emerges at the point of beginning, into the future, in a fundamental order determined by God. 'One and many', and 'fundamental order' are expressed in muqarnas. Complex forms of muqarnas are generated by simple types of parts and joining rules. Different blueprints and the same rules can generate a variety of forms. Also, a three-dimensional form is uniquely determined by a two-dimensional blueprint, the type of parts and the rules for joining them. The generative process seems to correspond to the generation of the world in the divine order.

### 4.4 Type 2

#### 4.4.1 Program of Type 2

Type 1 is based on the relationship between parts and layer line. That program depends on a regulation that layer line is an enclosed set of straight line segments. However, this regulation also leads to the lack of freedom of shapes in Type 1. In order to enlarge the freedom of shapes, the regulation about layer line is eliminated, which means that diverse shapes of layer lines are allowed. And the diversity of shapes of layer lines may lead to a larger possibility in shapes of parts.

The method of determination of shape of Type 2 based on relationships between a part and a part is shown in Table10.

By setting the top layer line of the highest layer, the generation of shapes starts. Table11 shows the program of shape-determination of Type 2.

#### 4.4.2 Problems of Generation and Supporting Joining Rules of Type 2

Decreasing the number of limitations lead to a lack of condition to determine the shapes of the parts, the generation stopped at a very early step. In order to continue the generation, supporting joining rules are added. There three types of additional supporting joining rules were identified. The first one is based on a shape-determining part, the fletching part. The second one is the additional conditions to form typical parts of muqarnas. The third one is a method to generate new types of parts that has not been seen in traditional muqarnas. They are shown in Table12.

The first and second additional joining rules are based on the traditional muqarnas, and these new conditions let the generation continue. However, due to absent of hierarchy of forming layers and joining layers, layers are formed neither progressively nor continuously. Because of that, some parts which belong to a lower layer were formed before the formation of the parts that belong to a higher layer. What is more, parts with a span over several layers are formed, and it led

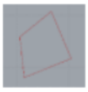
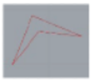
| planar figure | | vertices determined and their location | type of parts | 3D shape |
|---|---|---|---|---|
| quadrangle |  | 3 vertices determined; 1 on upper layer line, 2 on lower layer line<br>4 vertices determined; 1 on upper layer line, 3 on lower layer line | quadrangle cells |  |
| |  | 4 vertices determined; 2 on upper layer line, 2 on lower layer line | Semi-tunnel Vault |  |
| | | 3 vertices determined; 2 on upper layer line, 1 on lower layer line<br>4 vertices determined; 3 on upper layer line, 1 on lower layer line | quadrangle intermediate element |  |
| triangle |  | 3 vertices determined; 1 on upper layer line, 2 on lower layer line | triangle cells |  |
| | | 2 vertices determined; 2 on the same layer line<br>3 vertices determined; 2 on upper layer line, 1 on lower layer line | triangle intermediate element |  |
| polygon |  | even only 1 vertex is determined, the shape is generated | polygon parts |  |

Table10 The method of determination of shape of Type 2

| | General program of shape-determination | Type 1 | Type 2 |
|---|---|---|---|
| Basic | a. All parts are identified by their vertices. | ○ | ○ |
| | b. All parts share the same height. | ○ | × |
| | c. In terms of planar figures, parts is classified as quadrangle parts, triangle parts, and polygon parts. | × | ○ |
| | d. In terms of planar figures, parts is classified as quadrangle parts, triangle parts, fletching parts, and polygon parts. | ○ | × |
| | e. Because the corresponding of planar figure and 3D shape, polygon parts function as shape-determining parts. | ○ | ○ |
| | f. Because the corresponding of planar figure and 3D shape, fletching parts function as shape-determining parts. | ○ | × |
| | g. When the parts are joined together horizontally, planes of equal height are formed at their top and bottom, respectively, and all vertices are located in one of the two horizontal planes. In other words, layers are formed. | ○ | × |
| | h. When parts are joined vertically to each other, a new plane of equal height is formed at the top or bottom of each other. In other word, two layers joined and three layer lines with different height are formed. | ○ | × |
| Steps of shape-determination | 1. Set a enclosed polygon as the top layer line of the highest layer. | ○ | ○ |
| | 2. Using the method of determination of shape to determine the shapes of parts that satisfies the basic rules selected. | ○ | ○ |
| | 3. Check if shapes of all parts are determined. If so, program finishes. if not, go to step 4. | ○ | ○ |
| | 4. Check if there are any parts whose shape can be newly determined as a result of performing step 2. If so, the process returns to step 2., again. If not, go to step 5. | ○ | ○ |
| | 5. By introducing a new joining rule, the supporting joining rule① to determine the shapes of parts. Then go to step 6. | × | ○ |
| | 6. Check if there are parts newly determined in step 5. If so, go back to step. 2., if not, go to step 7. | × | ○ |
| | 7. By introducing a new joining rule, the supporting joining rule② to determine the shapes of parts. Then go to step 8. | × | ○ |
| | 8. Check if there are parts newly determined in step 7. If so, go back to step. 2., if not, go to step 9. | × | ○ |
| | 9. By introducing a new joining rule, the supporting joining rule③ to determine the shapes of parts. Then go to step 10. | × | ○ |
| | 10. Check if there are parts newly determined in step 9. If so, go back to step. 2., if not, go to step 11. | × | ○ |
| | ※Once no shape of part is newly determined by using step 2.-3, a step new joining rule will be added to continue the shape-determination. In this case, step 2n-1. (n≥3) and step 2n. (n≥3) are introduced as one set. Step 2n-1. (n≥3) is determining new shapes of parts while step 2n. (n≥3) is determining the direction of the flow of program. In this table, it is shown the case that three new supporting rules are added. | × | ○ |
| | … | × | ○ |

Table11 The program of shape-determination of Type 2

to the appearance of the space that cannot be filled by parts used so far. Therefore, the third supporting joining rule is introduced. As a result, broken layer lines are formed.

### 4.4.3 The Planar Projection Plan and Models of Type 2

By adding planar figures of fletching parts, the planar projection plan of Type 2 is created. The plan is shown in Figure18.

The overall shape of Type 2 is generated by the program of shape-determination of Type1, it is shown in Figure19 and Figure20.

### 4.4.4 Evaluation of Type 2

The process of generation, which is shown in Figure21, 22, and 23, is not a concentric diffusion process, but a process in which certain areas are generated first, follows by generation in three directions, and finally the spaces in between are gradually filled in.

During the progress of generation, the possibility of shapes of parts that are not determined in the early stage is accumulated step by step. Parts formed in a lower layer leads to the allowance of generation of a multi-layer-height part which

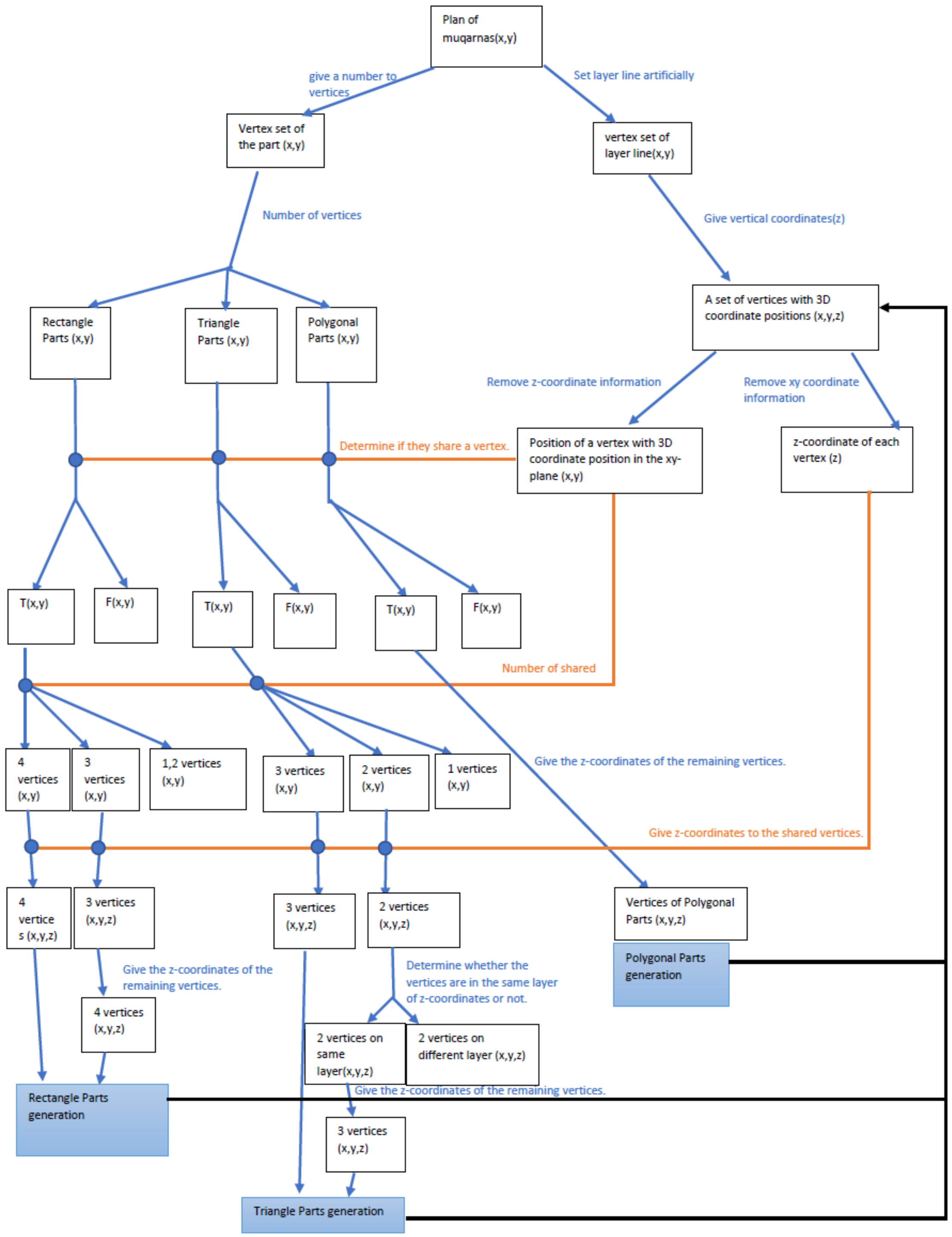



Figure17 The flow chart of step 1.-3. of program of shape-determination of Type 2

belongs to a higher layer. What is more, the multi-layer-height parts also lead to a generation of parts with shapes that do not exist in a traditional muqarnas.

From a two-dimensional perspective, a program was created to generate new muqarnas formations with free planar projection diagrams. Conventional muqarnas are a part of architecture, such as corners, domes, and vaults, and most of the planes are symmetrical, point-symmetrical, or other forms of symmetry. It is thought that new forms of muqarnas, or forms with the nature of muqarnas, can respond to more flexible conditions.

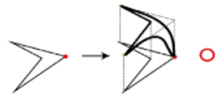
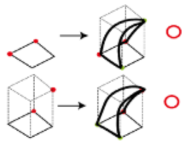
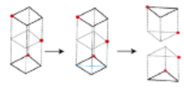
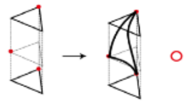
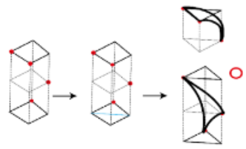
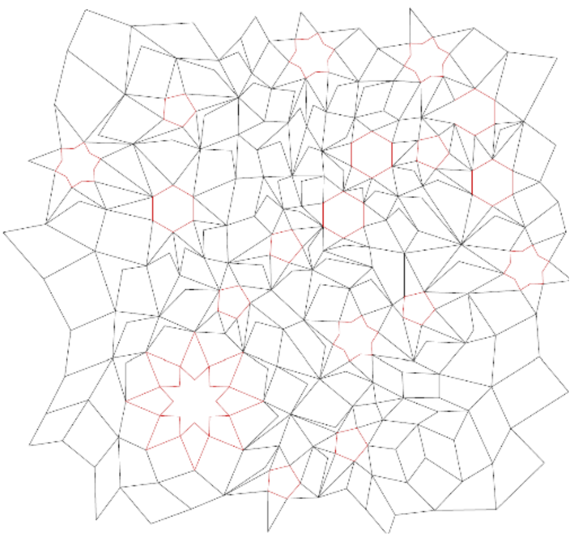
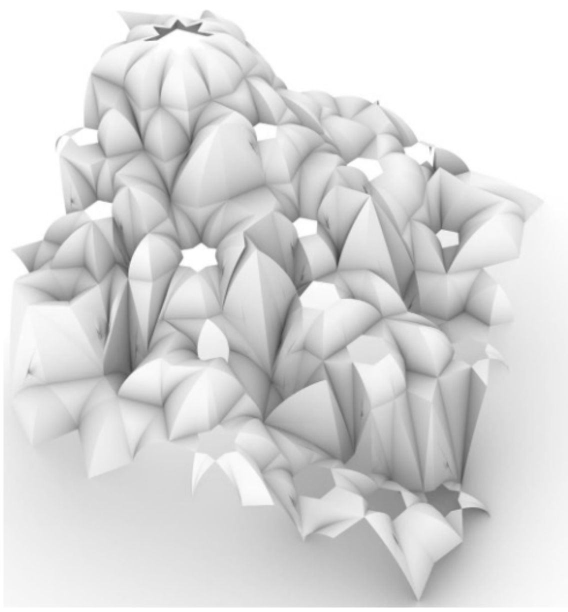
| | | |
|---|---|---|
| Supporting joining rule 1 | a. For a fletching part, the solid form is identified when one of the four vertices is determined; when one of the four vertices is determined, the solid form is determined by determining whether it is consistent with the solid form of the arrow-feather part, and if not, the solid form is identified. |  |
| Supporting joining rule 2 | a. For a rectangle part, If two vertices are in the same vertical position, the remaining two vertices are positioned at the top and bottom of the part, respectively, to form a cell; if two vertices are in different vertical positions, the remaining two vertices are positioned at the bottom of the part to form a cell. The following table shows the number of the elements in the table. |  |
| | b. For a rectangle part, if three of the four vertices of a rectangle part have been determined belong to three different planes, draw a diagonal line, divide the rectangle part into two triangle parts, and determine the shape as a triangle part using method of dertermination of shape of Type 2. The triangle parts that have been divided and whose shapes have not yet been determined are placed in the triangle parts set. |  |
| Supporting joining rule 3 | a. For both retangle parts and triangle parts, planar shape is allowed |  |
| | b. For a triangle part, if three of the vertices are determined, but each in a different vertical position, a form without a conventional muqarnas part is allowed. |  |
| | c. For a rectangle part, if 4 of the 4 vertices are determined, but they belong to tree different planes, draw a diagonal line and divide the rectangle part into two triangle parts, and operate method of dertermination of shape of Type 2 andd supporting joining rule 3b. Triangle parts that have been divided and whose shapes have not yet been determined are placed in the triangle parts set. |  |

Table12　The program of shape-determination of Type 2





Left:　Figure18　The planar projection plan of Type 2
Right:　Fighure19　The overall shape of Type 2

## 4.5　Conclusion of Generation of New Form

The geometric form generated in this paper can be described as a development of muqarnas, or another form developed from muqarnas. By analyzing and examining algorithms through computer, it become possible to generate new geometric forms that do not directly duplicate the forms of historical buildings but are rather developed from regularity of historical buildings and extended to today's architecture.

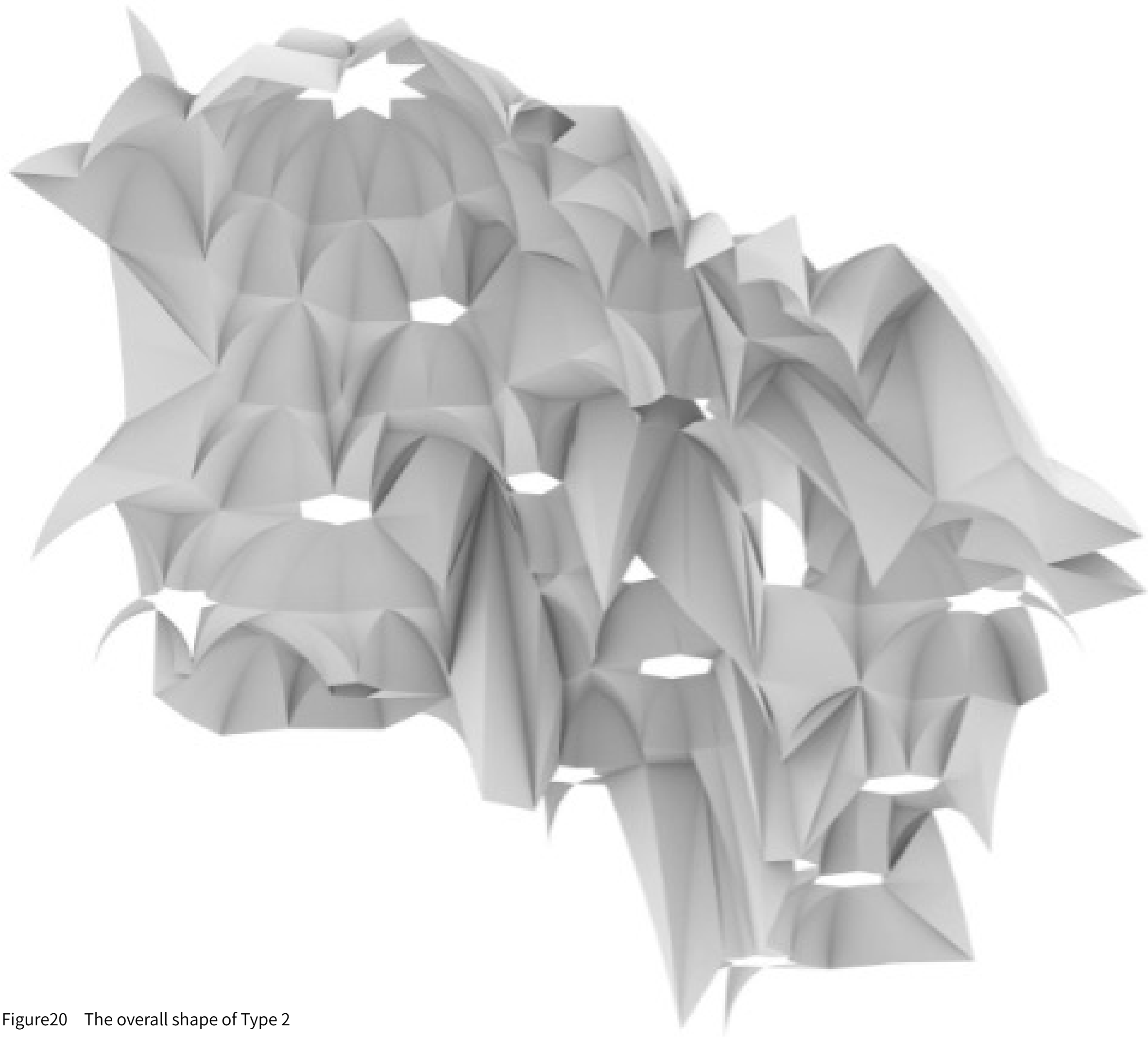


Figure20　The overall shape of Type 2

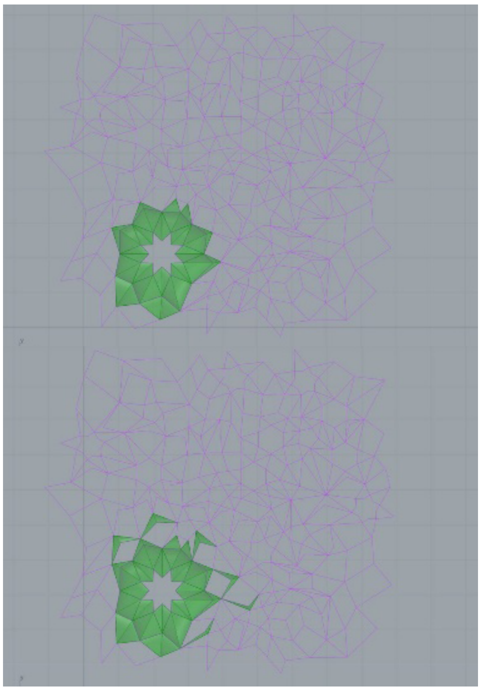




Figure21　Starting stages of the process of generation of Type 2, types generated around the layer line set manually

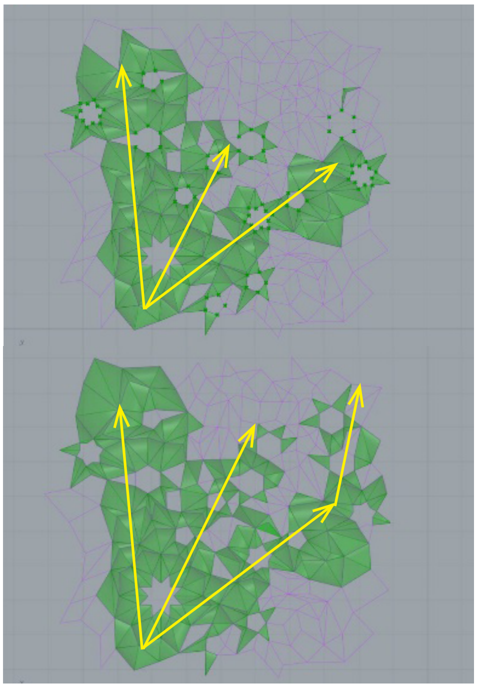




Figure22　The process of generation of Type 2, showing three directions of generation

## 5.An Architectural Work Based on the Algorithm Developed

"Muqarnas Pavilion: Amakubo boat house"(Figure24, 25), which is author's graduation design, is a work based on studies of this paper. By using the program of shape-determination of Type2, Author succeeded in generating the form of "Muqarnas Pavilion" from a planar projection plan with design intention.

Modeling created virtually can made in real space through digital fabrication technology using 3d printers.

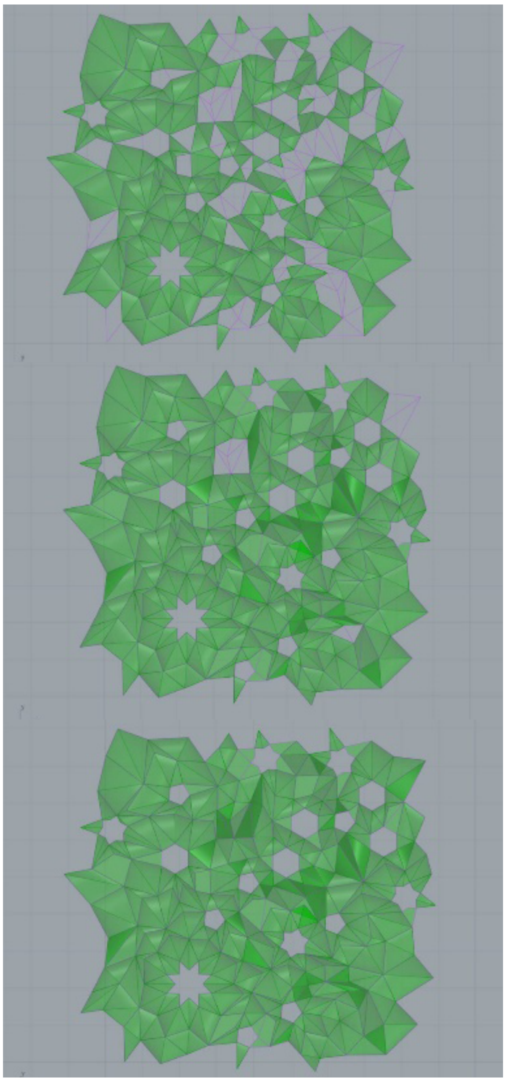

5
6
7

Figure23 The final stage of process of generation of Type 2

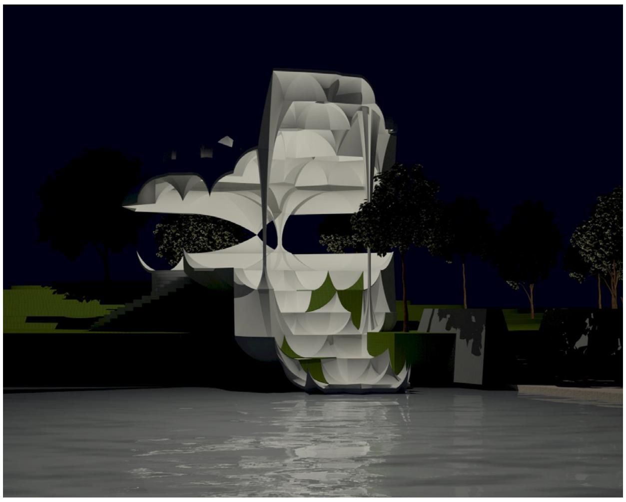


Figure24 A night view

Figure25 A daytime view

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