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Type synthesis of 6-DOF mobile parallel link mechanisms based on screw theory

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Abstract
Mobile parallel mechanisms (MPMs), which are parallel mechanisms with moveable bases, have previously been proposed to resolve the limited workspace of conventional parallel mechanisms. However, most previous studies on the subject focused on the kinematic analysis of some specific MPMs and did not discuss a type synthesis method for MPMs. With this in mind, we propose a screw theory-based type synthesis method to find out possible 6-degrees-of-freedom (DOF) MPM structures. In our proposed method, the 6-DOF mobility is divided into 3-DOF planar motion and 3-DOF spatial motion, both of which are realized by the transmitted planar motions of the driving units. Separately, the type synthesis of the entire MPM is divided into that of the driving unit and connecting chain. To realize 3-DOF spatial motion, two methods, applying singularity configuration and adding an additional chain, are proposed as ways to restrict undesired motions for the synthesis of the connecting chain. The driving unit is synthesized via the same type-synthesis method as the connecting chain by considering the driving unit as a planar mechanism. The method used to integrate the driving unit and the connecting chain was constructed based on whether the end pair of the connecting chain should be connected with the driving unit directly or driven by it through an actuating mechanism. As a result, 284 possible types of MPM structure are suggested and four examples of MPMs with six DOFs were synthesized to verify the feasibility of the proposed method.

Keywords: Mobile parallel mechanism, Screw theory, Type synthesis, Driving unit, Constraint

1. Introduction

Comparing with serial robots, parallel mechanisms have the advantages of higher accuracy, rigidity, velocity, and payload-to-weight ratio (Furqan et al., 2017), so that they are widely used in a wide variety of fields, including medicine, machining, and factory automation. However, one of the main drawbacks of parallel mechanisms is their limited workspace.

To solve this problem, various researchers have tried to maximize their available workspace by addressing the nonlinear and complex input-output relationships of those mechanisms. Carretero et al. (2000) applied the quasi-Newton with Hessian update method to find the optimal structural parameters for maximizing workspace volume. Lou et al. (2005) employed a controlled random search method to maximize the effective regular workspace. Hosseini et al. (2011) divided Jacobian entries by units of length to produce a homogeneous twist array for the convenience of workspace optimization. Herrero et al. (2005) proposed a combined geometrical and discretization method for obtaining the most useful workspace containing the largest geometric volume. Che et al. (2020) employed the differential evolution algorithm to solve design problems related to maximizing the effective volume of transmission positional workspace. Shin et al. (2011) applied the Taguchi method to choose the most influential parameters in the optimization of workspace.

However, from a macroscopic perspective, the fixed base of a parallel mechanism still physically limits its available workspace. With this issue in mind, several researchers suggested changing the fixed base into moveable bases in order to achieve unlimited workspace, and two types of moveable parallel mechanism types have been proposed. One is a parallel mechanism fixed on a wheeled mobile robot, as shown in Fig. 1(a) (Chong et al., 2020; Fujita and Sugawara, 2014; Moosavian et al., 2009; Yamamoto and Yun, 1996). This type achieves unlimited workspace because the position and orientation of the base on the ground can be changed by moving the mobile robot. In this type, the output platform

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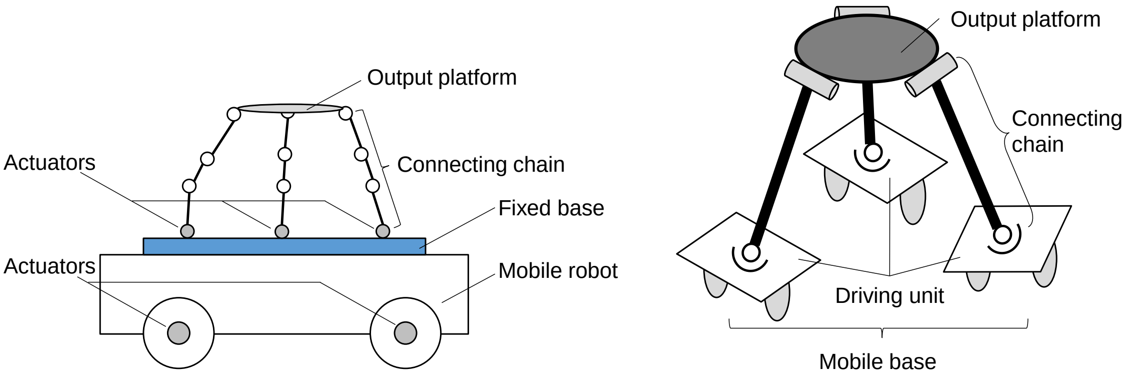
is controlled by different motors from those of the mobile robot.

The other type, which is called mobile parallel mechanism (MPM), is composed of an output platform, connecting chains, and a mobile base that has several independent driving units, as shown in Fig. 1(b) (Ben-Horin et al., 1998; Horin et al., 2006; Hu et al., 2011; Shoval and Shoham, 2003). In this type, each driving unit is connected to a connecting chain. As a result, when the mobile base is powered by the driving units and moves on the ground, the entire MPM moves. When one of the driving units moves relative to the other driving units, the motion is transmitted via the connecting chain and the output platform performs spatial motions, namely the change in height or tilt. Therefore, both the movements of the entire MPM on the ground and the spatial motion of the output platform are powered by the same driving units. Since the mobile base and the output platform share the same driving system in this type of MPM, it is possible to reduce the number of motors compared to previous ones for the same degrees of freedom (DOFs) mobility of the output platform. It is also notable that the stiffness of this MPM has proved to be higher than the conventional Stewart platform under some conditions (Tahmasebi and Tsai, 1995).



(a) Parallel mechanism on a mobile robot (b) Mobile parallel mechanism (MPM) [Horin et al., 2006]

Fig. 1 Schematic description of movable parallel manipulators.

Type synthesis is one of the most important stages in the design of a parallel mechanism. However, previous studies about MPMs have been a bit less and focused primarily on the kinematics and property analyses of the specified MPMs, while the type synthesis of MPMs was rarely reported. Therefore, it is possible that design solutions other than those used in existing MPMs could be developed. Many type synthesis methods have been proposed for conventional parallel mechanisms, such as the Lie subgroup synthesis method or the constraint synthesis method based on screw theory (Huang and Li, 2002a; Li et al., 2004; Kong and Gosselin, 2006; Meng et al., 2014). However, unlike conventional parallel mechanisms and those with fixed bases, an MPM is powered by several movable driving units. As a result, even though the abovementioned method can be applied to the type synthesis of a connecting chain, it cannot be used in the type synthesis of an entire MPM structure, which means it is necessary to establish an alternative type synthesis method for MPMs.

With this in mind, we propose an MPM type synthesis method by using the screw theory based on the integration of a connecting chain and mobile base. To that end, this paper explores possible structures for 6-DOF MPMs by first dividing those motions into 3-DOF planar motions (realized by the planar motion of the mobile base) and 3-DOF spatial motions (realized by the relative motions of the driving units through the connecting chains). As a result, the structures of the connecting chain for 3-DOF spatial motion, and the mobile base for 3-DOF planar motion, are enumerated separately based on their motion requirements. We then discuss the connecting chain structure by considering two possible solutions: applying a singularity configuration and adding another constraining chain. The mobile base is synthesized by regarding it as a planar parallel mechanism objected to the planar motion. Next, the method of integrating the connecting chain with the mobile base is discussed. Finally, feasible MPM structures are obtained based on the proposed type synthesis method, and four examples are provided to confirm its feasibility.

2. Type synthesis method for mobile parallel mechanism

The conditions and overview of the proposed MPM synthesis method are discussed in this section.

2.1 Conditions of motion and structure

As stated above, this study aims to find out the possible 6-DOF MPM structures. Based on the idea that a symmetrical structure makes the design and control of the MPM easier in practical applications (Huang and Li, 2003b), we set an input-symmetrical MPM as the object of type synthesis in this study. In this method, the number of the connecting chains is set to three, and each chain uses the same driving unit and connecting chain, as shown in Fig. 2.

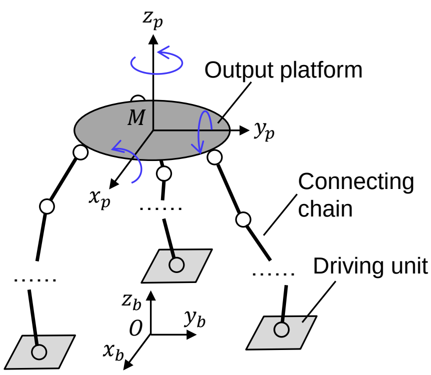


Fig. 2 Schematic description of an MPM with three connecting chains.

The MPM workflow proceeds as follows. First, the MPM moves to a specified location and then starts the required work. After finishing that job, the MPM moves to another location and continues. In such operations, the 6-DOF mobility of the MPM is divided into the planar motion of the mobile base and the spatial motion of the output platform. Additionally, to achieve unlimited workspace, the mobile base must be able to move in any direction on the ground. Therefore, 3-DOF planar motion, including the 2-DOF translation along the x_b- and y_b-axes and 1-DOF rotation mobility around the z_b-axis, are necessary to permit the mobile base to change the position and orientation of the MPM. The 3-DOF spatial motion of the output platform refer to translations along the z_p-axis and rotation around the x_p- and y_p-axes, which permit the end effector to access the target position and posture. Note that the coordinate systems M-$x_p y_p z_p$ and O-$x_b y_b z_b$ are fixed at the output platform center and on the ground, respectively, while the z_p- and z_b-axes are always perpendicular to the ground. The combination of the 3-DOF planar motion and 3-DOF spatial motion provides the MPM with 6-DOF mobility.

Since all of the pairs included in the connecting chains are passive, the motions of the mobile base are applied as a unique driving source to achieve the motions of the output platform. The planar motions of the mobile base are considered to be directly equivalent to those of the output platform. The relative movements of the driving units are transmitted to the output platform through the connecting chain in order to vary its orientation and/or position. More specifically, the motions of the driving units are transmitted to achieve the planar or spatial motion of the output platform.

In terms of accuracy, it is preferable that the relative motions of the driving units affect the spatial motions of the output platform but do not change the position of the whole MPM when the MPM is located at a specified position. To meet this requirement, this study considers two solutions. The first uses the singularity configuration. Here, if the MPM chains are designed appropriately, the singularity configuration results when the driving units travel along a specified direction, thus reducing the unrestricted 6-DOF motion to the desired 3-DOF spatial motion of the output platform. The other solution is achieved by adding an additional connecting chain. When the additional chain imposes 3-DOF constraints on the output platform, undesired motions coupled in the 6-DOF mobility can be limited. The former and latter methods are discussed in Sections 3.1 and 3.2, respectively.

2.2 Synthesis method and process

The screw theory-based constraint synthesis method is applied to the type synthesis of the MPM. The definitions and rules of the screw theory used in this paper can be found in Appendix A (Hunt, 1978). In brief, the motion of a rigid body can be represented by a screw called a twist. For example, the motions caused by the revolute pair and prismatic pair can be represented by zero-pitch and infinite-pitch screws, respectively. Similarly, the constraints imposed on a rigid body can be denoted by a screw called a wrench. Note here that zero-pitch and infinite-pitch wrenches are also named force and couple, respectively. The twist and wrench screws imposed on a rigid body are reciprocal to each other. A twist system composed of n linearly independent screws is called an n-system twist, and the corresponding wrench forms a

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$(6-n)$-system. Although both mathematical and geometrical methods have been proposed to calculate reciprocal screws (Zhao et al., 2009), a geometrical method is applied in this study due to its simplicity.

All kinds of pairs can be seen as equivalent to the combination of one or more revolute and prismatic pairs. Additionally, the mobile base of an MPM can be considered equivalent to a planar parallel mechanism when its wheels are regarded as the combination of lower pairs in the chain. Therefore, the structure of the connecting chain and mobile base in an MPM can be obtained by a series of reciprocal product calculations. As discussed in Section 2.1, after dividing the mobility of the MPM into the mobility of the output platform and that of the mobile base, the type synthesis of the connecting chain and the mobile base is performed separately. The required mobility of the output platform or mobile base can be represented by twists. Through the reciprocal product calculation of these twists, the wrench imposed on the output platform or mobile base is obtained. This result is the basis of the wrenches provided by all connecting chains or driving units. The wrench provided by each connecting chain or driving unit can be obtained easily by considering the structural symmetricity. The twist representing the connecting chain pairs or driving unit can be given through the reciprocal product of the abovementioned wrench. Meanwhile, the geometric relationship between pairs can be obtained via the reciprocal screw rules. For example, to form a zero-pitch wrench on the output platform that is perpendicular to the ground, all of the passive prismatic pair axes should be parallel to the ground, while the revolute pair axes should be coplanar with the zero-pitch wrench. By considering these conditions, the pair composition for the connecting chain or driving unit structure can be obtained. Finally, the MPM structure is obtained by integrating the type synthesis results of the connecting chain with those of the mobile base. Summarizing the above, our proposed MPM type synthesis process is as shown in Fig. 3.

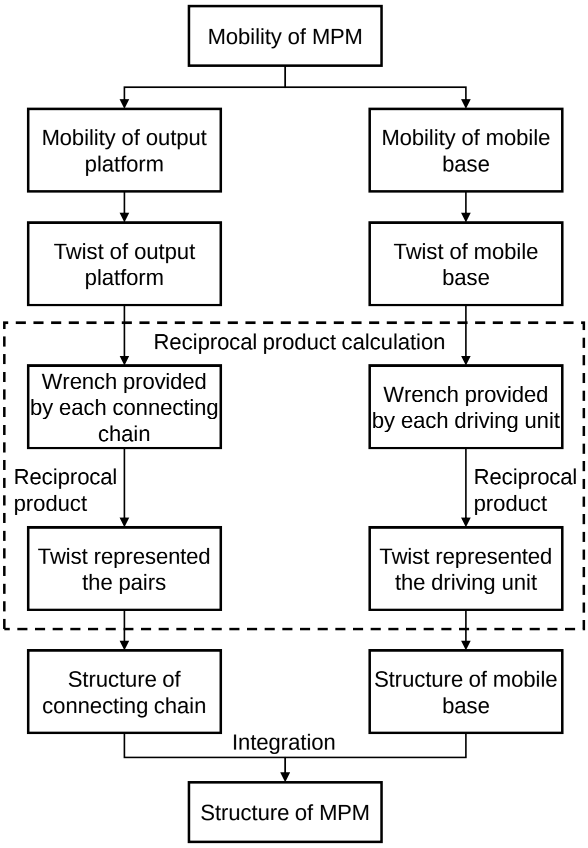

Fig. 3 MPM type synthesis flowchart.

3. Type synthesis of connecting chain

As discussed in Section 2, the connecting chains must facilitate the mobility of rotation around the x_p- or y_p-axis and translation along z_p-axis (RxRyTz) to the output platform when the driving units move in appropriate directions. Two solutions are considered to realize such spatial motions of the output platform. One is realizing motions under a singularity configuration, where motions other than the RxRyTz motion are restricted when the driving unit moves or rotates. The other is limiting the undesired motion by using an additional connecting chain. Both solutions will be discussed separately in this section.

3.1 Applying singularity configuration

Under the singularity configuration discussed here, the 6-DOF mobility of the MPM is limited to the RxRyTz motion. The RxRyTz mobility of the output platform can be represented by a 3-system twist composed of an infinite-pitch screw and two zero-pitch screws, in which the axis of the infinite-pitch screw is perpendicular to that of the zero-pitch screws. In this configuration, the possible wrench system provided by each connecting chain can be a 1-, 2- or 3-system, as shown in Fig. 4. When the wrench system is a 1-system, it contains only one zero-pitch wrench. When the wrench system is a 2-system, it is composed of pure zero-pitch wrenches or one zero-pitch wrench and one infinite-pitch wrench. When the wrench system is a 3-system, it is formed by one infinite-pitch wrench and two zero-pitch wrenches. Regardless of which of the above wrench systems is used, the axis of any infinite-pitch wrench must be perpendicular to the axes of zero-pitch wrenches provided by the same connecting chain. At least one zero-pitch wrench of each connecting chain must be in parallel with the ground to ensure that the wrench is also reciprocal to the equivalent pair representing the motion of the driving unit. Based on these conditions, the possible connecting chain structures for the RxRyTz mobility are enumerated and discussed below.

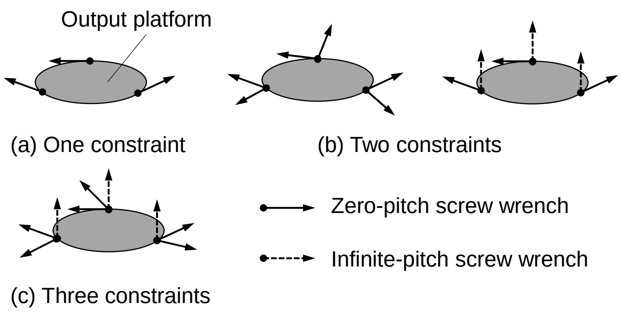

Fig. 4 Possible constraints imposed on the output platform.

3.1.1 Enumeration of connecting chains

3.1.1.1. Chain with one constraint

When each chain provides just one zero-pitch wrench to the output platform, as shown in Fig. 4(a), the screws of each chain form a 5-system twist, which means the maximum number of the independent 1-DOF pair screws is five. Additionally, based on Rule A3 of Appendix A, the number of revolute pairs in each chain is more than three. Revolute pair (R) and prismatic pair (P) are applied as the fundamental pairs because all other pairs can be equivalent to their combinations. At this point, the composition possibilities of each chain are 5R, 4R1P, or 3R2P. Because there is no infinite-pitch screw in the chain wrench system, the axes of more than three revolute pairs in each chain are non-coplanar. It is possible that situations can occur in which any two axes of the three revolute pairs do not intersect at a point, or in which two axes intersect at the same point, but the remaining axis does not pass through that point, or in which all axes intersect at the same point. For simplicity, we will assume that the axes of the revolute pairs satisfy the latter two geometrical conditions. In general, each of the revolute pair axes should remain on the same plane as the reciprocal screw, while each prismatic pair axis should be perpendicular to that of the reciprocal screw.

Considering the above conditions, the possible chain structures are shown in Fig. 5, where new structures are obtained if the order of the pairs is altered properly. The screw $\$_i$ indicated by a dashed line represents the axis of the i-th pair. The screw $\$_r$ indicated by a red line refers to a wrench that is unique to the chain twist system. Here, $\underline{\mathrm{RR}}$ refers to the axes of revolute pairs that intersect at a point, $\overline{\mathrm{RR}}$ means the axes of these revolute pairs that are in parallel, $\mathrm{R}_{\perp\mathrm{P}}$ refers to the axis of a revolute pair that is perpendicular to that of the neighboring prismatic pair, while $\mathrm{P}_{\perp\mathrm{R}}$ indicates the opposite configuration. However, the structures shown in Fig. 5(b), (d), and (g) are inappropriate because the direction of the 1-system wrench must be in parallel with the ground.

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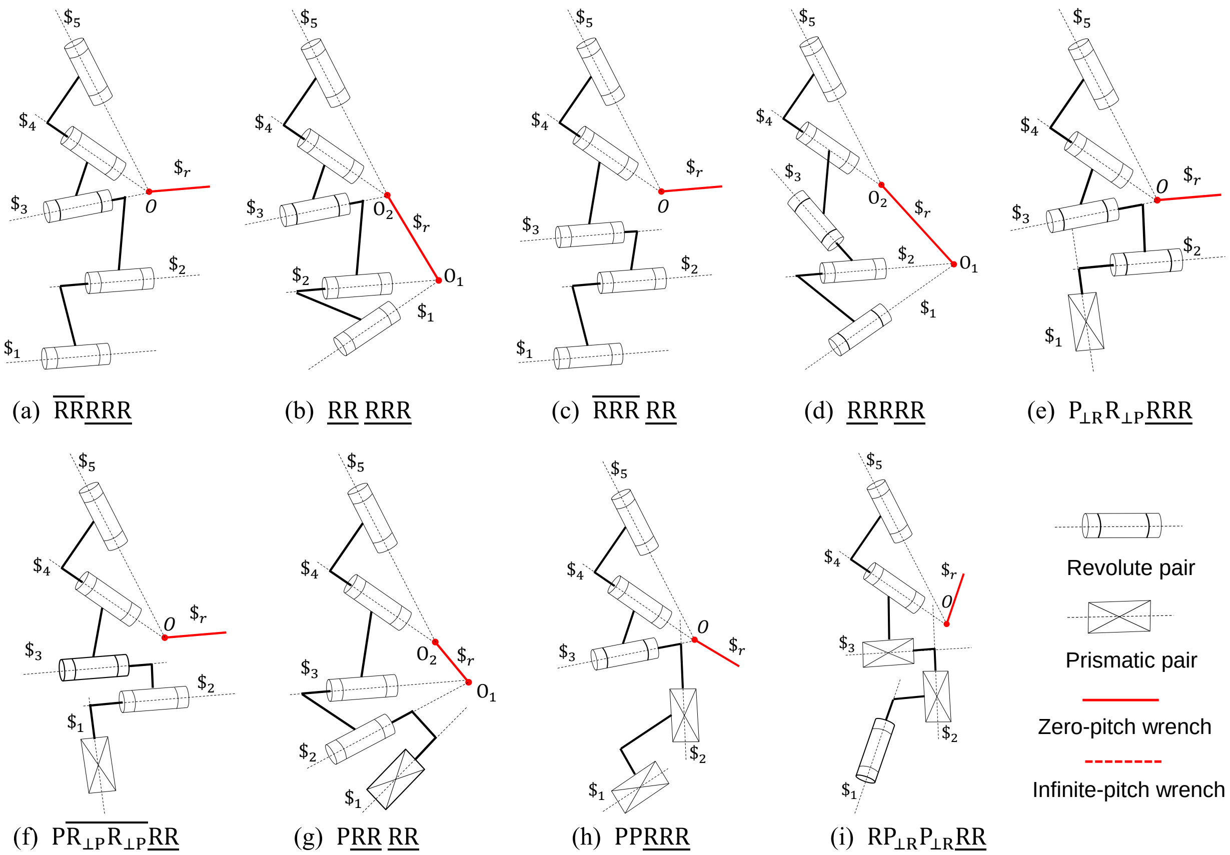

Fig. 5 Possible connecting chain structures with one force constraint.

3.1.1.2. Chain with two constraints

When the chain wrench system is a 2-system, as shown in Fig. 4(b), the chain twist system is composed of four independent 1-DOF pairs. When the wrench system is composed of two zero-pitch wrenches, the maximum number of prismatic pairs in the connecting chain is one. Additionally, two or three of the revolute pairs should intersect at a common point to ensure there is no infinite-pitch screw in the chain wrench system. The reciprocal screws should pass through the intersection point. If they do not, they are either in the same plane as the remaining revolute pair or are perpendicular to the prismatic pair. Therefore, the composition possibilities of each chain are 4R and 3R1P, as shown in Fig. 6. Note that $\$_{r2}$ in Fig. 6(b) is on the intersection line of the plane formed by $\$_1$ and $\$_2$ and the plane formed by $\$_3$ and $\$_4$. However, the structure in Fig. 6(b) is unsuitable because it is unable to keep the one wrench direction being stably in parallel with the ground.

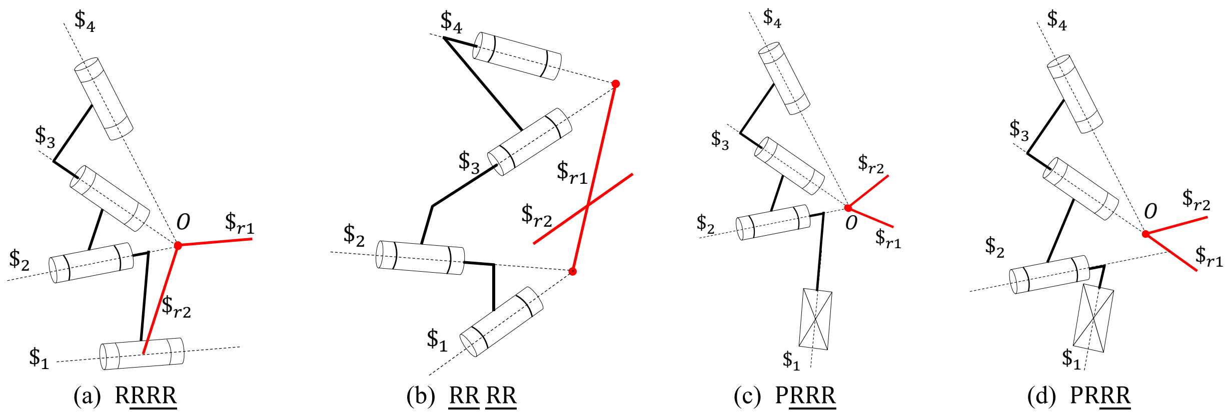

Fig. 6 Possible connecting chain structures with two force constraints.

When the chain wrench system is a 2-system composed of an infinite-pitch wrench and a zero-pitch wrench, the

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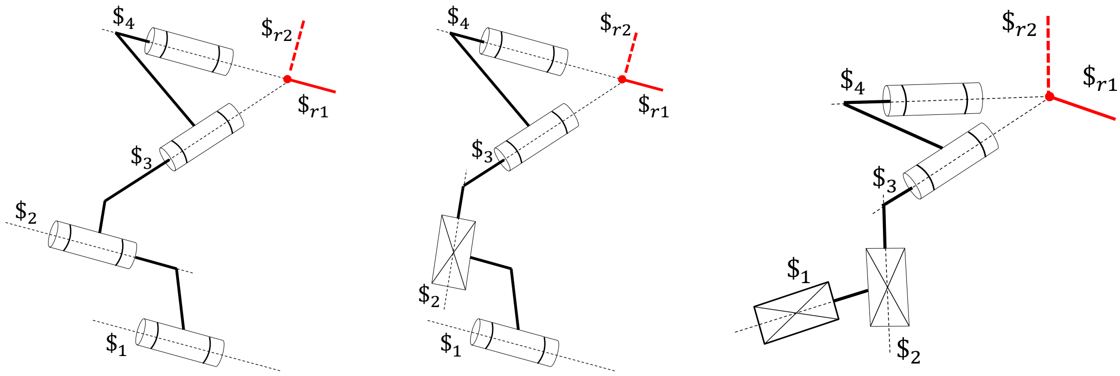
maximum number of axis-intersected revolute pairs in the connecting chain must be two in order to ensure there is one infinite-pitch screw in the chain wrench system. The remaining revolute pairs must be in parallel with one of the axis-intersected revolute pairs, while the prismatic pair axes should be perpendicular to the zero-pitch reciprocal screw, as stipulated by Rules A2 and A3 in Appendix A. Therefore, the composition possibilities of each chain are 4R, 3R1P, and 2R2P, as shown in Fig. 7.

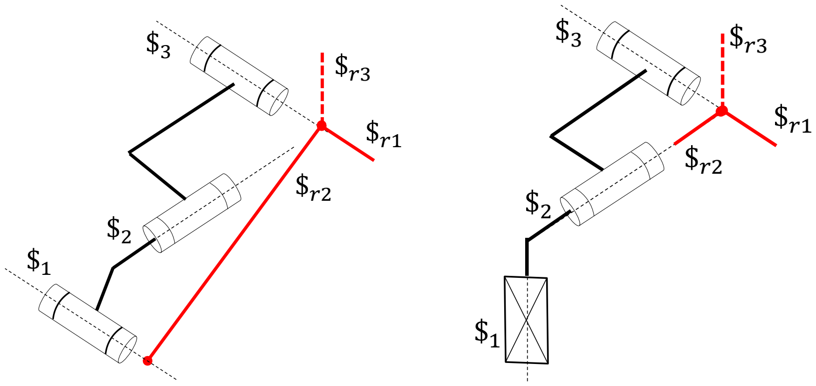


(a) $\overline{\mathrm{RR}}\underline{\mathrm{RR}}$ (b) $\mathrm{R}_{\perp\mathrm{P}}\mathrm{P}_{\perp\mathrm{R}}\underline{\mathrm{RR}}$ (c) $\mathrm{PP}\underline{\mathrm{RR}}$

Fig. 7 Possible chain structures with two constraints composed of a force and a couple constraint.

3.1.1.3. Chain with three constraints

When the chain wrench system is a 3-system, as shown in Fig. 4(c), the chain twist system is composed of three independent 1-DOF pairs, and the maximum number of prismatic pairs in the connecting chain is one to ensure there are two zero-pitch screws in the chain wrench system. Therefore, the composition possibilities of the connecting chain are 3R and 2R1P, as shown in Fig. 8. The axes of intersected revolute pairs reside in a horizontal plane, while the axis of the remaining revolute pair is in parallel with one of the axis-intersected revolute pairs shown in Fig. 8(a), and the axis of the remaining prismatic pair is perpendicular to that horizontal plane, as shown in Fig. 8(b).



(a) $\mathrm{R}\underline{\mathrm{RR}}$ (b) $\mathrm{P}\underline{\mathrm{RR}}$

Fig. 8 Possible connecting chain structures with three constraints.

3.1.2 Analysis of the enumerated chain structures

According to Gruebler's equation, when each connecting chain imposes one, two, or three constraints on the output platform, the DOFs of the MPM (excluding the mobile base) are 3, 0, and -3, respectively. This result indicates that using chain structures with three constraints to construct a 6-DOF MPM is unfeasible, even though the other two structure types could potentially realize the desired MPM mobility by integrating them with the driving unit.

On the other hand, the above DOF evaluation is based on the assumption that there is no overconstraint. An overconstrained mechanism is one that has more DOFs in the actual motion than are given by Gruebler's equation. Next, we analyze the enumerated structure while considering overconstrained characteristics. In general, an overconstraint condition occurs when pair axes are in specified relationships, such as when the axis of a prismatic pair in a connecting

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chain is in parallel with those in other chains, when the axes of the revolute pair are collinear with each other among three connecting chains, when the axes of all revolute pairs intersect at a common point, and so on. In the MPM discussed here, the conditions related to revolute pairs are not always satisfied due to the three-fold symmetric structure of the entire MPM and the motion constraints of the enumerated chain structures. Nevertheless, the condition regarding the prismatic pair can be satisfied in some situations, as will be discussed below.

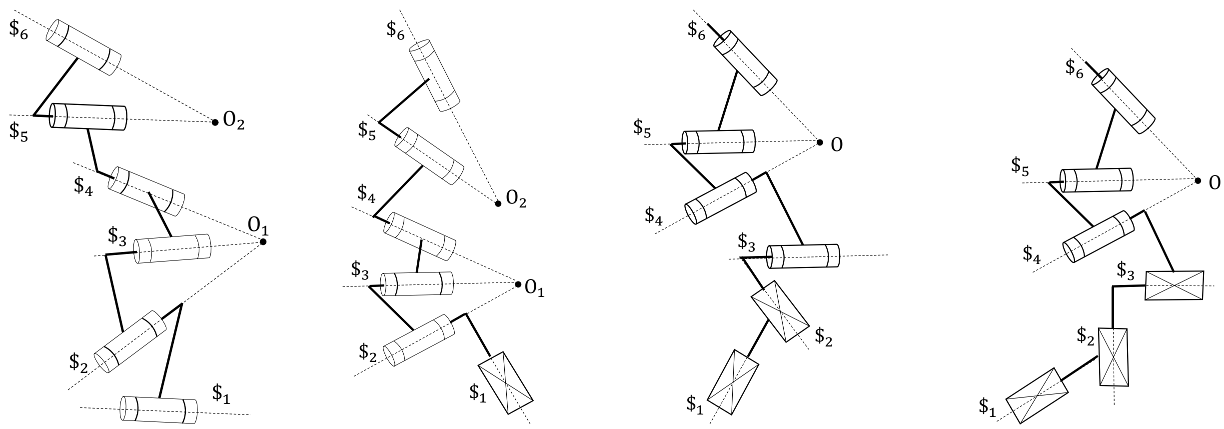
If only one constraint is imposed on the output platform by each connecting chain, the overconstrained mechanism exists at the structures shown in Fig. 5(e), (f), (h), and (i) when the axis of the prismatic pair is perpendicular to the ground. For the structures shown in Fig. 5(e) and (f), the overconstrained issue is easily solved when the axis of the prismatic pair is in parallel with the ground. With regards to the structures shown in Fig. 5(h) and (i), the overconstrained issue cannot be solved unless one prismatic pair axis is arranged obliquely. However, in Fig. 5(i), it can be seen that the angle of the prismatic pair in relation to the ground cannot be fixed due to the revolute pair, which means that the overconstrained configuration changes during operation, which is unacceptable for an MPM.

When two constraints are imposed on the output platform by each connecting chain, the overconstrained mechanism exists at the structures shown in Figs. 6(c), 6(d), 7(b), and 7(c) when the prismatic pair is perpendicular to the ground. For the structures shown in Fig. 6(c) and (d), the overconstrained issue is also solved if the axis of the prismatic pair is set obliquely. The structure shown in Fig. 7(b) permits the angle of the prismatic pair relative to the ground to be changeable due to the revolute pair, which means that the overconstrained characteristics can vary. Hence, this structure is also unsuitable for use by an MPM. As for the structures shown in Fig. 7(c), at least one prismatic pair should be arranged obliquely to avoid the overconstrained situation.

In the case of the connecting chain with three constraints, the structure shown in Fig. 8(b) produces the overconstrained mechanism when the prismatic pair is vertical. However, even when the overconstrained situation occurs, the available DOFs obtained are not sufficient to construct an MPM.

3.2 Using additional connecting chain to limit undesired motion

Since the output platform has 6-DOF mobility if the connecting chains do not impose any wrench on it, it is possible to realize the RxRyTz motion by constraining other undesired motions. First, the possible connecting chain structure that imposes no constraint on the output platform is discussed below. In this case, the number of the revolute pairs is more than three because no infinite-pitch wrench is imposed on the output platform. Therefore, the composition possibilities of the connecting chain are 6R, 5R1P, 4R2P, and 3R3P. Additionally, since three revolute pairs must intersect at a point to ensure there is no infinite-pitch wrench, the remaining pairs are arranged to ensure they do not create a zero-pitch wrench. Considering these conditions, the possible connecting chain structures are shown in Fig. 9. With regards to the structures shown in Fig. 9(a) and (b), the unique reciprocal screw of the two intersected revolute pair sets ($\$_2$ and $\$_3$; $\$_4$, $\$_5$, and $\$_6$) passes through points O_1 and O_2. If the last revolute pair is not coplanar with line O_1O_2, as shown in Fig. 9(a), or if it is not perpendicular to the prismatic pair shown in Fig. 9(b), there is no screw reciprocal to the chain twist. With regards to the structure shown in Fig. 9(c), the screw reciprocal to the four revolute pair screws exists in the plane formed by point O and the axis of pair $\$_3$. If the axis of the remaining prismatic pair is not perpendicular to that plane, there is no wrench imposed on the output platform. With regards to structures shown in Fig. 9(d), the three prismatic pairs axes are not coplanar, which means there is no zero-pitch wrench constraining the output platform.



(a) RRRR RR (b) PRRR RR (c) PPRRRR (d) PPPRRR

Fig. 9 Possible connecting chain structures with 6-system twist.

Next, the additional connecting chain, which is required to impose three constraints on the output platform, is discussed. The structural possibilities of this additional chain are the same as those shown in Fig. 8. However, with regards to the structure shown in Fig. 8(a), the direction of the wrench $\$_{r2}$ changes from instant to instant when the revolute pair $\$_1$ rotates. Therefore, only the structure shown in Fig. 8(b) is suitable for use as the additional chain.

Finally, the DOF of this type of solution is analyzed. Without the overconstrained situation, the MPM with three connecting chains and one additional chain (excluding the mobile base) has three DOFs. The overconstrained mechanism exists at the structures shown in Fig. 9(b), (c), and (d), providing a prismatic pair axis is perpendicular to the ground. To avoid this overconstrained situation, the prismatic pairs should be arranged in parallel with or obliquely to the ground.

4. Type synthesis of mobile base

In this section, the possible structures of the mobile base are discussed based on the screw theory. Here, the mobile base is seen as a virtual planar parallel mechanism composed of three driving units.

4.1 Mobility analysis

Since the position and orientation of the mobile base need to be changeable to solve the limited workspace problem and achieve the spatial motion of the output platform, both 3-DOF planar motion and 3-DOF relative motion are necessary to achieve 6-DOF mobility for the mobile base. Hence, the mobile base twist system for the former motion is written by the following equation.

$$\begin{cases} \hat{\$}_1 = [0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0] \\ \hat{\$}_2 = [0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0] \\ \hat{\$}_3 = [0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0] \end{cases} \tag{1}$$

In Eq. (1), $\hat{\$}_1$, $\hat{\$}_2$, and $\hat{\$}_3$ represent the rotation around z_b axis, translation along the x_b axis, and translation along the y_b axis, respectively, referring to the coordinate system shown in Fig. 2. The wrench system imposed on the mobile base can be obtained easily via using the geometrical method of determining the reciprocal screw, which is the same as its twist system. Therefore, the wrench imposed on the mobile base by the driving units is a 3-system.

$$\begin{cases} \hat{\$}_1^r = [0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0] \\ \hat{\$}_2^r = [0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0] \\ \hat{\$}_3^r = [0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0] \end{cases} \tag{2}$$

When applying the above wrench system to the mobile base, the twist of each driving unit can be 1-, 2-, or 3-system, and the twist can be written by one of the following screws or a linear combination of them.

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$$\begin{cases} \hat{\$}_1^t = [0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0] \\ \hat{\$}_2^t = [0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0] \\ \hat{\$}_3^t = [0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0] \end{cases} \tag{3}$$

Since the total number of the actuators is normally equal to the DOFs of the MPM, the input-symmetric structures require two active DOFs for each driving unit. Therefore, the driving unit twist is either a 2- or a 3-system. In the 2-system case, the driving unit twist is divided into two types. One is the linear combination of $\hat{\$}_1^t$ with $\hat{\$}_2^t$ or $\hat{\$}_3^t$, which represents the rotation about the z_b axis and translation along the x_b- or y_b-axis direction, while the other is the linear combination of $\hat{\$}_2^t$ with $\hat{\$}_3^t$, which represents the translation along x_b- and y_b-axis directions. On the other hand, the 3-system twist is also feasible if only two twist motions are active and the third is passive.

4.2 Type synthesis of driving unit

In this section, the type synthesis of the driving unit is discussed in relation to the number of wheels.

4.2.1 Driving unit with one wheel

Various wheel mechanisms, such as the conventional wheel and omnidirectional wheel (Campion et al., 1996; Komori et al., 2016; Taheri and Zhao, 2020; Terakawa et al., 2019a), have been reported in previous studies. Here, the conventional wheel mechanism contains the fixed wheel, centered orientable wheel, and off-centered orientable wheel, while the omnidirectional wheel mechanism contains the omni wheel, active omni wheel, Mecanum wheel, and spherical wheel. We assume that all of these wheels are in contact with the ground at a point and do not slip. Therefore, all types of wheels can be equivalent to the combination of a revolute pair and one or two prismatic pairs. For example, the fixed wheel is equivalent to the combination of a passive revolute pair that is perpendicular to the ground and an active prismatic pair that is in parallel with the wheel plane. The passive revolute pair becomes actuatable when an active steering axis is added like the centered or off-centered orientable wheel. Similarly, the omni wheel or Mecanum wheel is equivalent to the combination of a passive revolute pair perpendicular to the ground, a passive prismatic pair perpendicular or oblique to the wheel plane, and an active prismatic pair in parallel with the wheel plane. Although regular omni wheel or Mecanum wheel only has one active DOF, they become actively orientable when a steering axis is added mechanically. The active omni wheel can perform active translations in two directions (Komori and Matsuda, 2018; Terakawa, 2019b). The typical spherical wheel, which is equivalent to one active revolute pair and two active prismatic pairs, has three active DOFs.

Wheel motions can be expressed by screws, as shown in Table 1. The actuatable motion of the typical configuration is denoted by the blue frame in the figure. All wheels, except for the fixed wheel and the simple omni wheel or Mecanum wheel, can obtain at least two active DOFs. Therefore, based on the above discussion, they can even function as driving units by themselves. However, it should be noted that only two DOFs of the spherical wheel can be active when it is used to form a driving unit.

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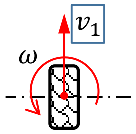
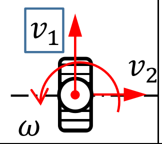
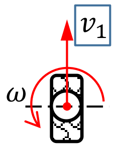
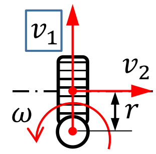
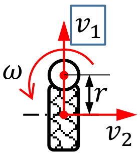
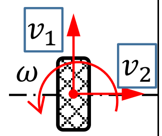
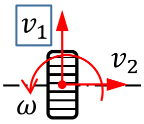
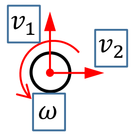
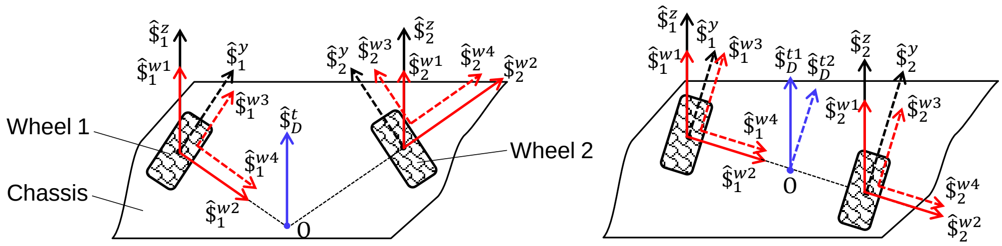
Table 1 Mobility of wheel mechanism represented by screw.

Wheel mechanism	Mobility	Unit twist screw	Wheel mechanism	Mobility	Unit twist screw
Fixed wheel		[0 0 1 0 0 0] [0 0 0 1 0 0]	Centered orientable omni wheel		[0 0 1 0 0 0] [0 0 0 1 0 0] [0 0 0 0 1 0]
Centered orientable wheel		[0 0 1 0 0 0] [0 0 0 1 0 0]	Off-centered orientable omni wheel		[0 0 1 r 0 0] [0 0 0 1 0 0] [0 0 0 0 1 0]
Off-centered orientable wheel		[0 0 1 r 0 0] [0 0 0 1 0 0] [0 0 0 0 1 0]	Active omni wheel		[0 0 1 0 0 0] [0 0 0 1 0 0] [0 0 0 0 1 0]
Omni wheel or Mecanum wheel		[0 0 1 0 0 0] [0 0 0 1 0 0] [0 0 0 0 1 0]	Spherical wheel		[0 0 1 0 0 0] [0 0 0 1 0 0] [0 0 0 0 1 0]

4.2.2 Driving unit with two wheels

Next, the structure of a two-wheeled driving unit is discussed. First, we assume that each wheel has only one active DOF. In this situation, the wheels, ground, and chassis of the driving unit form a closed-loop mechanism that has two chains. When the driving unit is equipped with two fixed wheels, each wheel is equivalent to the combination of a passive revolute pair and an active prismatic pair, as shown in Fig. 10. The pair revolving around the axis perpendicular to the ground is defined as screw $\hat{\$}_i^z$, while screw $\hat{\$}_i^y$ represents the prismatic pair along the wheel plane. The index i represents wheel i and $i = 1, 2$. Each wheel chain forms a 4-system wrench to the chassis, which is composed of two zero-pitch screws ($\hat{\$}_i^{w1}$ and $\hat{\$}_i^{w2}$) shown as red solid lines, and two infinite-pitch screws ($\hat{\$}_i^{w3}$ and $\hat{\$}_i^{w4}$) shown as red dashed lines.

The twist screw of the chassis can be obtained by calculating the reciprocal product of the abovementioned wrench screws. When the axles of the wheels intersect at a point, as shown in Fig. 10(a), there is only one twist screw $\hat{\$}_D^t$ that is reciprocal to the wrench system. This indicates that the driving unit is only able to rotate about the axis perpendicular to the ground. When the wheel axles are lying on the same line, as shown in Fig. 10(b), two twist screws ($\hat{\$}_D^{t1}$ and $\hat{\$}_D^{t2}$) can be obtained by the reciprocal product. This indicates that the driving unit can not only rotate about the vertical axis but also travel along the direction in parallel with the wheel planes. Therefore, to obtain the 2-DOF mobility when the driving unit is equipped with two fixed wheels, the wheel axles should be collinear.



(a) Intersection arrangement (b) Collinear arrangement

Fig. 10 Screws formed in driving units with two fixed wheels.

When a driving unit is equipped with two centered orientable wheels, the wrench system provided by the wheels at

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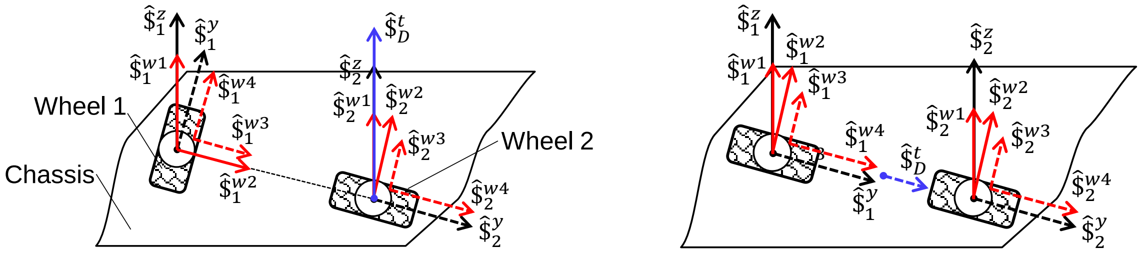
each instant is similar to that provided by fixed wheels. However, unlike a fixed wheel, a center orientable wheel is steerable, which means the wrench system provided by the wheels changes from moment to moment. For example, when both wheels are rotated actively, the driving unit can rotate around the steering axis of wheel 2 under the situation shown in Fig. 11(a), while it can travel along the wheel plane direction in the situation shown in Fig. 11(b). However, in this case, the steering of the wheels should be passive due to the assumption that each wheel has one active DOF, which means that switching between Figs. 11(a) and 11(b) conditions is involuntary. Hence, this configuration is unsuitable.

On the other hand, this problem can be solved when wheel 1 is actively steerable, and wheel 2 rotates actively. In such cases, excluding the situation shown in Fig. 11(a), the driving unit can perform a translation when the wheel planes are in parallel with each other, as shown in Fig. 11(b), and perform a circular motion when the wheel planes are oblique to each other. This means that wheel 2 does not need to be orientable. Therefore, a centered orientable wheel should be combined with a fixed wheel to form a driving unit in the same manner as a driving unit with an off-centered orientable wheel.



(a) Intersection arrangement (b) Parallel configuration

Fig. 11 Screws formed in driving units with two centered orientable wheels.

When a driving unit is equipped with two omnidirectional wheels that have three DOFs, such as an omni wheel, the twist screw system representing the wheel contains a zero-pitch screw $\hat{\$}_i^z$ and two infinite-pitch screws $\hat{\$}_i^x$ and $\hat{\$}_i^y$. The wrench system provided by each wheel chain is a 3-system containing three independent screws shown by red lines in Fig. 12. By calculating the reciprocal product of the wrench system formed by both wheels, the mobility of the driving unit is given by $\hat{\$}_D^{t1}$, $\hat{\$}_D^{t2}$, and $\hat{\$}_D^{t3}$, as shown by the blue line in the figure. This figure also shows that the driving unit has three DOFs that contain one rotation and two translations. However, since one of three DOFs becomes uncontrollable due to the passive motion represented by the infinite-pitch screw $\hat{\$}_i^x$, one of the omnidirectional wheels must be replaced with a fixed wheel to form a driving unit.

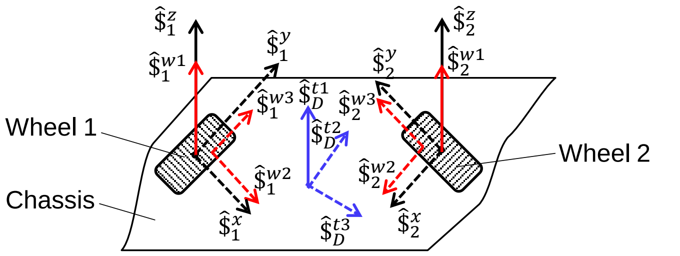


Fig. 12 Screws formed in driving units with two omnidirectional wheels.

Overall, the possible driving unit structures are shown in Fig. 13. They are composed of either a single wheel, such as the centered orientable wheel, or two wheels, such as two independent drive fixed wheels. According to the above discussion, driving units can be divided into two types. The first, of which there are two types, is equivalent to the combination of two active prismatic pairs and includes a single active omni wheel, as shown in Fig. 13(e), and a spherical wheel, as shown in Fig. 13(f). The second, of which there are eight types, combines active revolute and prismatic pairs. Both structures enable the mobile base to realize both 3-DOF planar and 3-DOF relative movements.

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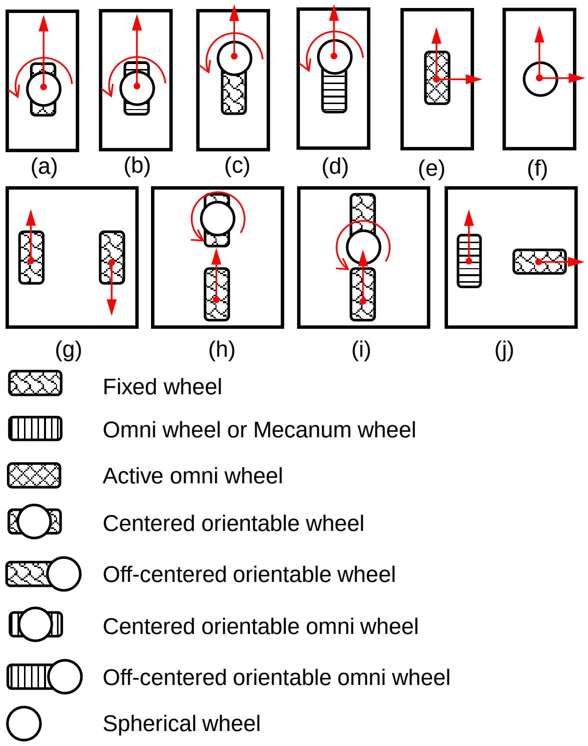

Fig. 13 Possible driving unit structures.

5 Integration of connecting chain and mobile base

In this section, the integration methods of the connecting chain and mobile base are discussed, and the MPMs are synthesized.

5.1 Connecting scheme

As analyzed in Section 3, some of the enumerated connecting chain structures bring redundant DOFs to the MPM. These are the one-constraint chain with or without overconstraint, the two-constraint chain with or without overconstraint, and the no-constraint chain with or without overconstraint. To construct an MPM properly, such redundant DOFs must be constrained but kept movable at the same time. To address this issue, the appropriate number of pairs in the connecting chains are made activate by using the driving units.

When each connecting chain imposes only one constraint on the output platform without overconstraint, the end pair of each connecting chain can be actuated by the driving unit. Then, the posture of the output platform is uniquely decided by the input motions of the three driving units. If an overconstraint situation occurs, the redundant DOFs exceed three, at which point they can no longer be actuated by the three driving units. Therefore, a one-constraint chain with overconstraint is unsuitable for use by the MPM.

When each connecting chain imposes two constraints on the output platform without overconstraint, the equivalent pair screws representing the motions of the driving units should be reciprocal to at least one of the wrenches. If the equivalent pair screw is reciprocal to just one of the wrenches, the wrench imposed on the output platform is reduced from two to one by the addition of the driving unit. In this situation, activation is unnecessary, and the connecting chain is only required to connect with the driving unit. If the equivalent pair screw is reciprocal to both wrenches, it is regarded as a surplus pair in the chain and prevents the input motion of the driving unit from converting into the spatial motion of the output platform. At this time, the end pair of the connecting chain must be actuated by the driving unit. The end pair must also be actuated when the connecting chains constitute an overconstrained mechanism.

When no constraint is imposed on the output platform, an additional connecting chain is added to limit the undesired motions. Without overconstraint, the three connecting chains, excluding the additional chain, are needed to join with the driving unit, and the end pair must be actuated. With overconstraint, the redundant DOF exceeds the capability of the driving units, and the MPM cannot be constructed.

Based on the above analysis, the connecting chain structures are divided into Groups A, B, and C. For the structures

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in Groups A and C, it is necessary to use the motion of the driving unit to actuate the end pair of the connecting chain. In contrast, for the Group B structures, it is only necessary to join the driving unit with the connecting chain directly. The 14 enumerated types of connecting chain structures are summarized and classified in Table 2.

Table 2 Summary of the enumerated connecting chain structures.

Method	Constraint situation	Structure	Classification
Applying singularity configuration	One constraint (without overconstraint)	Fig. 5(a), (c), (e), (f), and (h)	Group A
	Two constraints (with overconstraint or wrench overlap)	Fig. 6(c) and (d); Fig. 7(a) and (c)	
	Two constraints (without overconstraint)	Fig. 6(a), (c), and (d); Fig. 7(a) and (c)	Group B
Using additional connecting chain	No constraint (without overconstraint)	Fig. 9(a), (b), (c), and (d)	Group C

5.2 Actuating mechanism

As discussed in the previous section, the end pairs of the connecting chain in Groups A and C must be powered by the driving units. The mobility of the driving unit is composed of either two planar translations or translation and rotation. Therefore, the translation or rotation motion of the driving unit must be capable of being used as the driving source to power the end pair of the connecting chain. When the end pair is a prismatic pair, its sliding direction is assumed to be either in parallel with or perpendicular to the ground. When the end pair is a revolute pair, its axis is assumed to be in parallel with the ground. This is because if the end pair axis is perpendicular to the ground, the zero-pitch wrench formed by the connecting chain must be perpendicular to the ground as well. However, the output platform would be unable to translate along the vertical direction under this condition. Therefore, it was necessary to consider a transmission mechanism to convert the rotation or translation of the driving unit to the revolute pair or prismatic pair of the connecting chain. Examples of the transmission mechanisms according to the configuration of the end pair are shown in Fig. 14. The transmission mechanism, such as bevel gear, worm drive, orthogonal double-slider mechanism, rack and pinion, and helical transmission, is used to change motion direction or motion form.

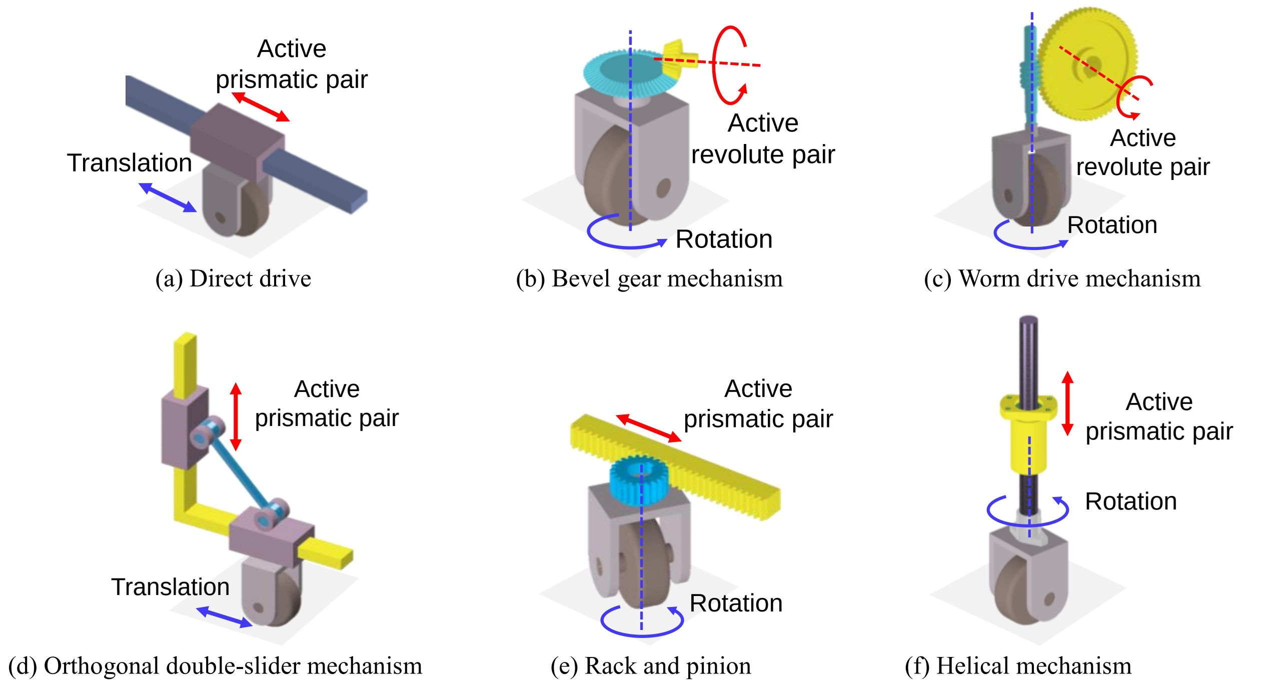

Fig. 14 Examples of transmission mechanisms between driving unit and connecting chain.

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The possible actuating mechanisms for the driving unit to drive the end pair of the connecting chain are summarized in Table 3 based on the pair configurations and the input motion of the driving unit. For example, when the translation motion of the driving unit is applied as the driven source, the orthogonal double-slider mechanism is used to change the motion direction if the end pair is a vertical prismatic pair. Alternatively, when the rotation motion of the driving unit is applied to drive the vertical prismatic pair, the helical mechanism is used to change the rotation motion to the translation motion.

Table 3 Driving unit actuating mechanisms

End pair type of connecting chain	Feasible motion of driving unit		Corresponding connecting chain structure
	Translation	Rotation	
Prismatic pair (in parallel with floor)	Driving directly	Rack and pinion	Fig. 5(e), (f), and (h); Fig. 9(b), (c), and (d)
Prismatic pair (perpendicular to floor)	Orthogonal double-slider mechanism	Helical mechanism	Fig. 6(c) and (d); Fig. 7(c)
Revolute pair (in parallel with floor)	Rack and pinion	Bevel gear, worm drive	Fig. 5(a) and (c); Fig. 7(a); Fig. 9(a)

Overall, there are 13 feasible connecting chain structures for Groups A and C. For each structure, ten and eight types of the driving unit can feasibly use translation motion and rotation motion, respectively, to actuate the end pair of the connecting chain. Hence, the number of the possible structures for the MPM in Group A is $13 \times 18 = 234$. On the other hand, five types of connecting chain structures and ten types of driving units are feasible for Group B, which means it has $5 \times 10 = 50$ possible structure combinations for the MPM. Therefore, there are 284 types of MPM structures in total.

5.3 Examples of mobile parallel manipulator

Based on the above integration method, we can obtain several novel MPM structures. One example of an MPM with one chain constraint, which integrates the connecting chain shown in Fig. 5(e) with the driving unit shown in Fig. 13(a), is shown in Fig. 15. The connecting chain structure, which is composed of a prismatic pair, a revolute pair, and a spherical pair, is classified as belonging to Group A. The centered orientable wheels, where the orientation and the rotation of the wheel are active, are applied as the driving units. The end prismatic pair is directly actuated by the translation motion of the driving unit. When the driving units travel along the rail, the spatial motion of the output platform is achieved. The steering and rotation of the driving unit are controlled to realize the planar motions of the output platform. Therefore, this proposed MPM can achieve 6-DOF motion.

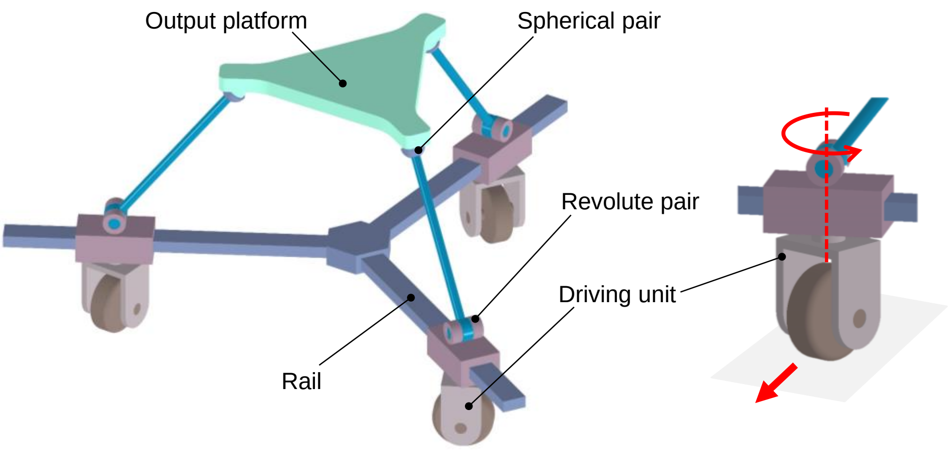

Fig. 15 3-PRS MPM.

Figure 16 shows another example of Group A, in which the connecting chain shown in Fig. 7(a) is integrated with the driving unit shown in Fig. 13(g). A worm drive mechanism is used as the transmission mechanism to change the vertical rotation motion of the driving unit into horizontal rotation motion of the end revolute pair. Two revolute pairs and one universal pair provide the link that connects the gear axis and the output platform. When the driving units rotate at a fixed location, the rotation direction is altered and transmitted to produce the spatial movement of the output platform. Meanwhile, the cooperative motions of the driving units make it possible for the entire MPM to move to any location. Therefore, this mechanism also functions as a 6-DOF MPM.

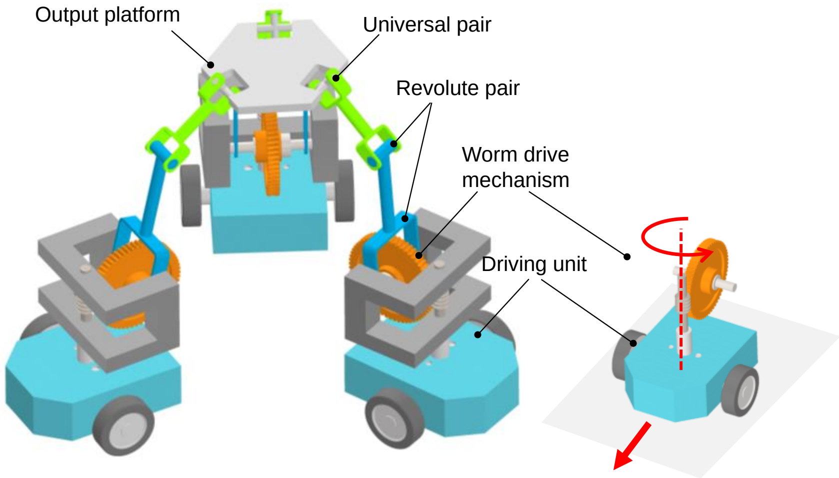

Fig. 16 3-RRU MPM.

Figure 17 shows an example of MPM that integrates the connecting chain in Group B as shown in Fig. 6(a) with the driving unit shown in Fig. 13(e). An active omni wheel, which can move freely in the forward and lateral directions, is applied as the driving unit. The end pair of the connecting chain is simply connected with the driving unit because the spherical pair and revolute pair form a 4-system chain twist without overconstraint. As a result, when the driving units move in the lateral direction, the output platform realizes the RxRyTz spatial motion. Additionally, by cooperatively moving the driving units forward or backward, the entire MPM can move in any desired direction. Therefore, this MPM has 6 DOFs, including 3-DOF spatial mobility and 3-DOF planar mobility.

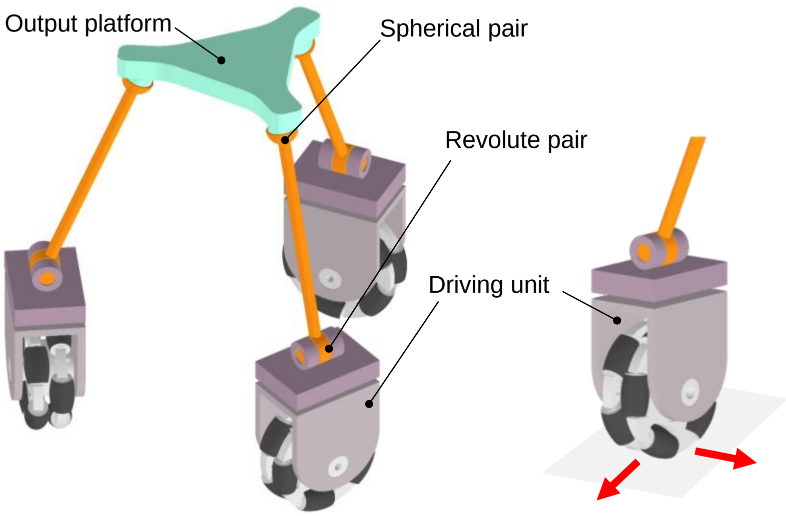

Fig. 17 3-RS MPM.

An example of MPM applying the structure in Group C is shown in Fig. 18, in which the connecting chain shown in Fig. 9(b) is integrated with the driving unit shown in Fig. 13(a). Here, an additional chain is added to limit the undesired motions. The connecting chain is composed of a prismatic pair, a spherical pair, and a universal pair. The additional chain uses the combination of a prismatic pair and a universal pair to connect the mobile base with the output platform. The driving unit is the same as that shown in Fig. 15. Under the limitation of the additional chain, the spatial motion and planar motion of the output platform are achieved in the same way as the MPM shown in Fig. 15.

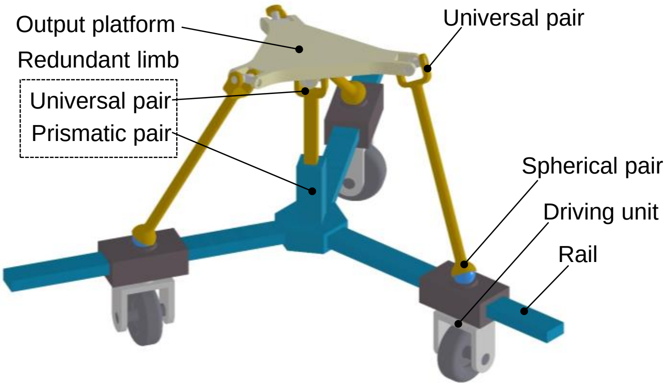

Fig. 18 MPM with an additional chain.

6. Conclusion

The MPM was proposed to solve the limited workspace issue of conventional parallel mechanisms. However, previous investigations about MPMs focused primarily on the kinematic of some specified MPMs and rarely addressed their type synthesis methods. With this in mind, this study tried to find out the possible 6-DOF MPM structures by constructing a type synthesis method. The following results were obtained:

1) We proposed a 6-DOF MPM synthesis method based on the screw theory. Based on the consideration that 6-DOF mobility of MPM is divided into 3-DOF planar motion and 3-DOF spatial motion, and that the planar motions of the driving units are transmitted to realize these motions through the connecting chains, we divided the type synthesis of the entire MPM into the type synthesis of the driving unit and connecting chain.
2) For the synthesis of the connecting chain, two solutions were discussed for achieving 3-DOF spatial motions: applying singularity configuration to reduce DOFs and adding an additional chain to restrict undesired motions. Next, the possible connecting chain structures were enumerated and analyzed based on both solutions, and 14 types of feasible connecting chains were selected for further review.
3) To achieve the synthesis of the driving unit, it is seen as a planar mechanism which makes it possible to use the same type-synthesis method as the connecting chain to obtain feasible structures. As a result of this synthesis, a total of 10 types of driving units matching the mobility requirement were obtained.
4) A method of integrating driving unit and connecting chain was proposed, and the connecting chain structures were classified into three groups based on our DOF analysis. From the results, we proved that the end pairs of connecting chain structures in two groups should be powered by driving units via the actuating mechanism, and then discussed the possible actuating mechanisms based on connecting chain structures and the motion of the driving unit used as the driving source. Ultimately, 284 possible types of MPM structures were suggested.
5) Four MPMs were introduced as examples. All of these MPMs were theoretically capable of realizing the 6-DOF mobility, in which the planar motion and spatial motion of the output platform are realized by the planar motions and relative motions of the driving units, respectively, as intended.

Although this study proposed a type synthesis method for 6-DOF MPMs, the group coupling in the output motion was not discussed sufficiently. We will consider this issue in our future studies.

Appendix A: Brief view of screw theory

A brief review of the screw theory is introduced in this section. A unit screw can be written by the following equation.

$$\hat{\$} = \begin{bmatrix} \boldsymbol{s} \\ \lambda \boldsymbol{s} + \boldsymbol{s}_0 \times \boldsymbol{s} \end{bmatrix} = [L \quad M \quad N \quad P \quad Q \quad R]^T \tag{A1}$$

In Eq. (A1), $\boldsymbol{s}$ is a unit vector that defines the direction of the screw axis, $\boldsymbol{s_0}$ defines the position of a point on the screw axis that refers to the reference coordinate system, λ is the pitch of the screw, and L, M, N, P, Q, and R are the Plücker coordinates of the screw. When $\lambda = 0$, the screw is a zero-pitch screw, while an infinite-pitch screw exists when $\lambda = \infty$. These two types of screws are given as following equation.

$$\hat{\$} = \begin{cases} \begin{bmatrix} \boldsymbol{s} \\ \boldsymbol{s}_0 \times \boldsymbol{s} \end{bmatrix} & \text{if } \lambda = 0 \\ \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{s} \end{bmatrix} & \text{if } \lambda = \infty \end{cases} \tag{A2}$$

When two screws satisfy the following equation, they are reciprocal to each other.

$$\$_r \circ \$ = S_{r4}S_1 + S_{r5}S_2 + S_{r6}S_3 + S_{r1}S_4 + S_{r2}S_5 + S_{r3}S_6 = 0 \tag{A3}$$

In Eq. (A3), $\$ = [S_1 \quad S_2 \quad S_3 \quad S_4 \quad S_5 \quad S_6]$ and $\$_r = [S_{r1} \quad S_{r2} \quad S_{r3} \quad S_{r4} \quad S_{r5} \quad S_{r6}]$. The geometrical conditions for the reciprocal screws are as follows:

Rule A1. Two zero-pitch screws are reciprocal to each other unless they are coplanar.

Rule A2. Two infinite-pitch screws are always reciprocal to each other regardless of their positions and orientations.

Rule A3. A zero-pitch screw is reciprocal to an infinite-pitch screw only when they are perpendicular to each other.

The screw is applied to represent the instantaneous motions of or constraints imposed on a rigid body, which are called a twist and a wrench, respectively. Their Plücker coordinates are as following equation.

$$\hat{\$} = \begin{cases} [\omega_x \quad \omega_y \quad \omega_z \quad v_x \quad v_y \quad v_z] & \text{if the screw represents a twist} \\ [f_x \quad f_y \quad f_z \quad m_x \quad m_y \quad m_z] & \text{if the screw represents a wrench} \end{cases} \tag{A4}$$

When the screw is a twist, the first three components represent the rotation motion, while the last three components represent the translational motion. When the screw is a wrench, the first three components represent the force constraint, while the latter three components represent the couple constraint.

A twist system with n linearly independent screws is called an n-system, the corresponding wrench system forms a $(6 - n)$-system.

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