Research in International Business and Finance 62 (2022) 101754

Contents lists available at ScienceDirect

Research in International Business and Finance

journal homepage: www.elsevier.com/locate/ribaf

Full length article

Asymmetric volatility dynamics in cryptocurrency markets on multi-time scales

Shinji Kakinaka *, Ken Umeno

Department of Applied Mathematics and Physics, Graduate School of Informatics, Kyoto University, Yoshida-honmachi, Sakyo-ku, Kyoto, 606-8501, Japan

ARTICLE INFO

Keywords:
Asymmetric volatility effect
Fractal regression analysis
Cryptocurrency markets
Scale-dependent correlations

ABSTRACT

This study investigates the scale-dependent structure of asymmetric volatility effect in six representative cryptocurrencies: Bitcoin, Ethereum, Ripple, Litecoin, Monero, and Dash. By developing the dynamical approach of DFA-based fractal regression analysis, we detect whether the volatility of price changes is positively or negatively related to return shocks at different time scales. We find that the asymmetric volatility phenomenon varies by scale and cryptocurrency, and the structure is time-varying. Contrary to what is typically observed in equity markets, minor currencies show an "inverse" asymmetric volatility effect at relatively large scales, where positive shocks (good news) have a greater impact on volatility than negative shocks (bad news). The consequences are discussed in the context of who is trading in the market and heterogeneity of the investors.

1. Introduction

Since its first interaction in 2009 (Nakamoto, 2008) and undergoing two price bubbles in 2013 and 2017, Bitcoin and other cryptocurrencies have attracted the attention of a wide community, including online traders, economic actors, and academic researchers. Unlike other assets, cryptocurrencies are built on block-chain technology that operates without a central bank or single administrator while ensuring anonymity in transactions, incredible security, and cross-border payments open at all times. With these benefits, they have become a booming economy with market capitalization on its rising trend reaching more than 1900 billion US dollars in total by January 2022. Besides their rapid growth, research on cryptocurrency markets has become more active. Numerous studies reveal that despite the unique system, cryptocurrency price fluctuations also exhibit stylized facts similar to what is recognized in stock and commodity markets, such as long-range dependence and long memory in volatility (Bariviera et al., 2017; Bouri et al., 2018; Cheah et al., 2018), fat-tails in price distribution, multifractality, and scaling properties (Jiang et al., 2018; Takaishi, 2018; Zhang et al., 2019). Nevertheless, cryptocurrencies tend to have more distinctive features, i.e., they tend to be more volatile (Bariviera et al., 2017; Alvarez-Ramirez et al., 2018), more inefficient, and more complex due to significant long-memory and stronger multifractality both in price and volatility (Al-Yahyaee et al., 2018; da Silva Filho et al., 2018; Telli and Chen, 2020), and also exhibit fatter tails (Begušić et al., 2018). Other than these stylized facts, Bitcoin is uncorrelated with traditional assets and is suggested as a useful hedging tool with similar abilities to gold (Dyhrberg, 2016a,b). A "hedge" is an asset that is uncorrelated or negatively correlated with another asset or portfolio, whereas a "diversifier" is an asset that is positively but not perfectly correlated with another asset or portfolio (Diniz-Maganini et al., 2021). Bitcoin shows a property of a solid "safe-haven", defined as an asset that functions as a hedge not on average but in particular cases only, i.e., during the periods of market stress (Bouri et al., 2017b).

* Corresponding author.
E-mail address: kakinaka.shinji.35e@st.kyoto-u.ac.jp (S. Kakinaka).

https://doi.org/10.1016/j.ribaf.2022.101754
Received 3 March 2022; Received in revised form 26 July 2022; Accepted 13 August 2022
Available online 22 August 2022

This property indicates that the combination of financial assets with Bitcoin could help reduce the correlation levels and risks of a portfolio in times of market turmoil. Moreover, a vast application of fractal and nonlinear theory-based methods to analyzing cryptocurrency time series has shed light on the underlying physical mechanisms of their market dynamics.

Many of the contributions above arise from the Fractal Market Hypothesis (FMH) proposed by Peters (1994), extending the widely acknowledged Efficient Market Hypothesis (EMH). According to the FMH, financial time series exhibit properties relevant to fractals that appear similar or self-repeating within sets of time series when viewed at different scales. This is due to the different valuations for information flows among investment horizons, which justifies sudden spikes in market volatility and lack of market liquidity during crashes. Therefore, the FMH compensates for scaling factors the EMH fails to capture. The dynamical approach of detrended fluctuation analysis (DFA) of Peng et al. (1994) has become a widely utilized tool in analyzing fractal structures of a financial time series and scaling factors of its long-range correlation in a nonlinear manner. Later on, detrended cross-correlation analysis (DCCA) was developed from DFA to reveal cross-correlation features (Podobnik and Stanley, 2008). By putting DFA and DCCA techniques together, Zebende (2011) introduced the DCCA coefficient to measure the degree of cross-correlation for each specific scale quantitatively. This work promoted the development of the field into the investigation of multi-time scale dependencies of highly complex financial series.[1] By translating the standard regression analysis into the DFA and DCCA-based language, Kristoufek (2015) proposed the fractal regression analysis that enables quantifying scale-dependent interactions between time series. The method provides richer information than the standard regression framework in that it deals with complexity and nonlinearity in dynamical systems, as well as the actual response of the series at multi-time scales. The fractal regression analysis was then extended to the case of two impulse series, namely the DFA-based bivariate regression analysis (Wang et al., 2018). A further development was made recently by Tilfani et al. (2022), where multivariate regression model in the fractal framework is illustrated for financial applications. In effect, these multi-time scale approaches are practical for modeling heterogeneity in economic and financial systems and their underlying scale structure (Kakinaka and Umeno, 2021; Tilfani et al., 2022).

One of the economic behaviors well-established in the finance literature is that volatility of financial series responds asymmetrically to return shocks (Black, 1976; Bollerslev et al., 2009; Bentes, 2018). The so-called "leverage effect", also known as the "asymmetric volatility effect", is described in stock markets that bad news (negative return shocks) increase the volatility by more than good news (positive return shocks) (Schwert, 1989; Cheung and Ng, 1992). The concept of this asymmetric effect has been theoretically studied and modeled utilizing various statistical volatility models (e.g., GARCH-type models), and how asymmetric responses are produced is generally demonstrated by factors built on the traditional framework of EMH (Black, 1976; Christie, 1982). However, the asymmetric volatility effect has not been well-studied in terms of the fractal framework. Financial time series as well as cryptocurrencies are rather inefficient and are likely to represent remarkable properties of fractality associated with multi-time scales. Motivated by evidence of significant scale-dependent consequences in cryptocurrencies and limited empirical evidence of asymmetric volatility effect due to the markets' short history, the objective of this paper is to examine whether and why volatility responds asymmetrically in cryptocurrency markets, in a more precise manner accounting for scaling dependence of the market behavior given the high complexity. We take advantage of the bivariate fractal regression analysis of Wang et al. (2018) to detect whether the volatility of price change is positively or negatively related to its return shocks at different time scales. The approach allows us to consider FMH-based features of asymmetric volatility that cannot be captured by the conventional models.

Following several works that discover a reversed leverage among early emergence periods of cryptocurrency markets (Bouri et al., 2017a; Baur and Dimpfl, 2018; Cheikh et al., 2020; Kakinaka and Umeno, 2021), this paper also finds evidence of the presence of an "inverse" asymmetric volatility effect in cryptocurrency markets — contrary to conventional markets, the return volatility is higher when a positive shock occurs. The consequences are discussed in the context of who is trading in the market and heterogeneity of the investors. According to Glosten and Milgrom (1985) and Easley et al. (1996), the trading of informed and uninformed investors leads to different traces in the return process due to the inefficiency of market information. In particular, informed traders do not generate auto-correlation, while uninformed traders drive serial correlation in the return process, thereby increasing the volatility (Avramov et al., 2006). Utilizing this idea, we will examine under what market conditions uninformed investors dominate the market and discuss the asymmetric factors of our empirical findings.

We reconfirm that except for the prominent markets of Bitcoin and Ethereum, the minor markets substantially exhibit a positive relation between return shocks and volatility. More interestingly, our dynamical fractal approach reveals further the multi-time scale components of the asymmetric volatility phenomenon under different time scales. Since asymmetric response may be time-varying (Takaishi, 2021), we also attempt to examine inverse/non-inverse effects under different data periods. Our approach can be an alternative to existing models, providing a new view based on investors' speculative trading and how they are the source of asymmetry, which could play a crucial role in investment decisions, pricing, risk management, and monetary policy.

The rest of this paper is organized as follows. Section 2 provides general information and literature reviews relevant to asymmetric volatility in cryptocurrency markets. Section 3 introduces the datasets used in this study. Section 4 describes the methodology associated with analyzing asymmetric volatility. Section 5 presents the results and empirical findings we have reached. Finally, Section 6 draws the main conclusions.

[1] See the review literature of Watorek et al. (2021) for more information and about multi-time scale properties in cryptocurrency price fluctuations.

2. Literature review

The negative response of volatility to return shocks was originally referred to the financial leverage, because the decrease of stock prices naturally brings in the rise in firm's leverage, making the stock become riskier and increasing its volatility (Black, 1976; Christie, 1982; Schwert, 1989; Cheung and Ng, 1992). However, many papers warn us that such an effect should not be associated with financial leverage. Hens and Steude (2009) explain the effect in an experimental stock market under a controlled setting where students are given instructions to trade artificial securities with each other using an electronic trading system with no financial leverage. They find that the effect can be observed even in the absence of financial leverage. Similar results are presented by Hasanhodzic and Lo (2019), where they confirm the existence of leverage effect in all-equity-financed firms having no debt. The volatility feedback is another possible factor that explains the negative correlation between volatility and expected rate of return (Campbell and Hentschel, 1992). In response to favorable information (good news), the increase in return is mitigated by the effect of price decline due to increased risk. In response to unfavorable information (bad news), the price decline due to increased risk is added, magnifying the decrease in the rate of return. Nevertheless, since both factors of financial leverage and volatility feedback are built on the EMH, they do not fully explain the leverage effect. Namely, the inefficiency of market information can also give rise to asymmetric volatility, so it has arisen as an important factor.[2] The informational inefficiency imposes different impacts on the return process, i.e., asymmetric volatility occurs when investors trade based on noise rather than information. Black (1976) calls them "noise traders" because they overreact to information and trade irrationally. These uninformed investors affect liquidity to the market (Easley et al., 1996) and generate asymmetric fluctuations in market prices (Avramov et al., 2006; Baur and Dimpfl, 2018). In this context, noise traders happen to be dominant after negative shocks in many of the conventional markets, where higher volatility follows.

On the other hand, some markets show an inverse asymmetric volatility phenomenon where higher volatility follows after positive shocks. Chen and Mu (2021) reveal the presence of an inverse leverage in a wide range of commodity markets, including agricultural products, energy, industrial metals, and precious metals, except for crude oil. Kliber (2016) finds evidence of inverse leverage in the sovereign credit default swap spreads (sCDS) for the countries of Portugal, Poland, Greece, and Slovenia. The author refers it to the Prospect Theory of Tversky and Kahneman (1992), where the decision making of investors is explained under different risk conditions. Under several assumptions, the author justifies that inverse-leveraged sCDS in the above cases are due to market participants feeling that the probability of default is higher than the one implied by the spread.

In respect of cryptocurrencies, a number of studies have attempted to model the behavior of volatility. Whether volatility of price changes is positively or negatively related to return shocks has traditionally been modeled by making explicit the conditional variance of returns. Katsiampa (2017) compares several competing GARCH-type models and concludes that the Component GARCH (CGARCH), a model that allows a short-run and a long-run component of conditional variance, provides the optimal fit level of Bitcoin data for the period between July 2010 and October 2016. It is worth noting that during the period, the log-likelihood value under the CGARCH model is higher than that of under its asymmetric model, the asymmetric component GARCH (ACGARCH) model. This result implies that asymmetric models are not always the most appropriate to explain Bitcoin volatility, and thus symmetric models can sometimes provide a better explanation. The information criteria for diagnosing model selection discussed in the literature also support the findings that the (symmetric) CGARCH model presents a plausible fit.

However, asymmetric models under other GARCH-type models generally outperform symmetric models in many cases, and they have the potential to address significant asymmetry in volatility. On implementing the well-known GJR-GARCH model of Glosten et al. (1993) and the EGARCH model of Nelson (1991), Bouri et al. (2017a) investigate the relation between price returns and volatility changes in the Bitcoin market against various world currencies and test if there is a difference in the asymmetric structure before and after the price crash of 2013. They report that before the crash, positive returns helped increase the conditional variance more than negative returns but not after the crash, suggesting that positive (inverse) asymmetric volatility effect is relevant to a safe-haven property of Bitcoin rather than the financial leverage or volatility feedback. Cheikh et al. (2020) attempt to capture different impacts of positive versus negative shocks regarding a flexible intermediate state between variance regimes. They employ the smooth transition GARCH (ST-GARCH) model and investigate four representative cryptocurrencies of Bitcoin, Ethereum, Ripple, and Litecoin using data from April 28, 2013, to December 1, 2018. They find a positive relationship between return shocks and volatility for the majority of currencies except for the case of Ethereum, where no asymmetry can be detected under the ST-GARCH, EGARCH, threshold GJR-GARCH, and threshold GARCH (ZGARCH) models. Using a wide range of data periods available through August 2018, Baur and Dimpfl (2018) test the existence of an asymmetric volatility effect in as many as 20 major and minor cryptocurrencies by employing the TGARCH model in addition to the quantile autoregressive model (QAR). They find that positive shocks increase volatility more than negative shocks for most cases, with the most notable exceptions being the two largest currencies, Bitcoin and Ethereum. They attempt to explain the phenomenon in terms of informed and uninformed (noise) traders' trading activities — asymmetry is due to uninformed traders' herding and buying activity boosted by the fear of missing out (FOMO) on rising cryptocurrency prices, as well as the pump-and-dump schemes. In this context, the authors argue that the two largest Bitcoin and Ethereum play a special and different role because, in the most mature peer-to-peer currencies, the market is dominated by informed traders who have the ability to reduce some of the prominent asymmetry generated by uninformed traders.

Fakhfekh and Jeribi (2020) refer to the importance of focusing on long-memory properties of time series in finding the most optimum model or sets for depicting volatility. By introducing fractionally integrated models of FIGARCH and FIEGARCH, they

[2] In the study of Antoniou et al. (1998), they reject the traditional leverage effect and conclude that information inefficiency in markets is the cause of asymmetry in volatility.

take into account long-memory factors in the conditional heteroskedasticity variance towards modeling sixteen of the most popular cryptocurrencies' volatility. Under the period from August 7, 2017, to December 12, 2018, they apply fourteen GARCH specifications, including typical asymmetric GARCH-type models, with different error distributions. They conclude that, in general, the TGARCH and EGARCH models provide the best model explanation, although the best model fit varies across cryptocurrencies. They also report the presence of an inverse asymmetric volatility effect, more or less in line with other relevant studies. Mensi et al. (2019a) examine structural break impacts on the dual long memory of Bitcoin and Ethereum using four different ARFIMA-GARCH family models, specifically GARCH, FIGARCH, FIAPARCH, and HYGARCH models. By considering long-memory and structural breaks, they find that dual long memory exists in Bitcoin and Ethereum returns and volatility. Their results indicate that market returns and volatility do not follow a random pattern, and thus EMH does not hold for cryptocurrencies. They also point out that FIGARCH with structural breaks is comparatively superior to other models for volatility forecasting.

Al-Yahyaee et al. (2018) show evidence of long-memory feature as well as multifractality in the Bitcoin market and compare the level of market efficiency to gold, stock, and foreign exchange markets by applying the dynamical approach of multifractal detrended fluctuation analysis (MFDFA) proposed by Kantelhardt et al. (2002), which is a method that generalizes the DFA to multifractality. They find evidence of the market being more inefficient with stronger multifractality than other assets. With the use of A-MFDFA, which extends the MFDFA to capture asymmetric structures, Liu and Chen (2018) examine the asymmetric volatility of the dry bulk shipping market brought by the financial crisis. They also demonstrated the fractal method's usefulness in illustrating asymmetric characteristics of long-range correlation, multifractality, and many other data properties in financial time series. In the same framework, with high-frequency Bitcoin and Ethereum data, Mensi et al. (2019b) examine long-memory, asymmetric multifractality, and time-varying efficiency to reveal different market patterns between downside and upside trends. They clarify that both Bitcoin and Ethereum are highly inefficient because different scaling laws and asymmetric fractal patterns exist in the price dynamics, supporting the FMH. Both markets are more inefficient when moving downwards relative to when they are moving upward. Other studies use the MF-ADCCA, an extension of A-MFDFA, in estimating the scaling factor of asymmetric long-range cross-correlations between time series. In the literature of Cao and Xie (2021), the authors highlight the long-memory and asymmetric multifractal characteristics of cross-correlations between cryptocurrencies and Chinese financial markets. These stylized facts are also evident between leading cryptocurrencies, leading conventional currencies (Kristjanpoller and Bouri, 2019), and equity ETFs (Kristjanpoller et al., 2020). The above studies suggest that cryptocurrency markets represent a complex system that can generate asymmetry in its inter-relationship with other financial markets.

Kakinaka and Umeno (2021) utilize the fractal method of MF-ADCCA to investigate asymmetric cross-correlation between price return and return volatility in cryptocurrency markets from June 1, 2016, to December 28, 2020. The literature further quantifies the multi-time scale strength of asymmetric cross-correlation by employing the asymmetric DCCA coefficient at various scales. They report that stronger cross-correlation appears in the downtrend market for the major coins of Bitcoin and Ethereum. In contrast, stronger cross-correlation appears in the uptrend market for the more minor coins of Ripple and Litecoin. One of the main contributions of the work to the field is that they established an approach to investigate dynamical properties of long-range dependent processes of return and volatility at a specific scale, which extends the discussion to the economic phenomenon of asymmetric volatility effect on multi-time scales.

Although the idea of using fractal analysis towards detecting the asymmetric volatility effect under different time scales is demonstrated in Kakinaka and Umeno (2021), its interpretation is limited in terms of correlation coefficients defined between −1 and 1. In our study we will also utilize fractal analysis, but unlike earlier studies, the asymmetric effect is associated with the actual effect of an economic variable (return shocks) on another (volatility) rather than simply the strength of correlation between variables. Our study is also different from other volatility models, i.e., GARCH models, in that we take into account the multi-time scale structure between the variables, while the scaling property is ignored in these conventional models. The multi-time scale fractal regression analysis we implement is complementary to other existing methods built on long-memory, fractality, and scale-dependent processes and works excellent with modeling heterogeneity in economic and financial variables (Tilfani et al., 2022). As the detection of asymmetric volatility effect is crucial towards deciding portfolio positions, modeling its heterogeneity expectations in terms of multi-time scales may provide additional views from past studies. Our study deepens the interdisciplinary understanding of the connection between economic behavior and stylized facts that emerged from the field of physics.

3. Data

We collect price data from https://poloniex.com/, one of the largest cryptocurrency exchanges with various cryptocurrencies available. By using the public API, we obtain high-frequency 5-minute interval closing price of Bitcoin (BTC), Ethereum (ETH), Ripple (XRP), Litecoin (LTC), Monero (XMR), and Dash (DASH) for the period from 2016/06/02 to 2021/09/25.[3] The data include the period of the boom in 2017 when cryptocurrency prices experienced a substantial increase. The choice of cryptocurrencies is the large ones with a market capitalization of $50 million or more as of June 2016, when they began to show growth in more active online trading (except for XMR, which had a market capitalization of only $10 million but has grown rapidly to over $200 million within a year, so we also select this cryptocurrency). Since cryptocurrencies do not belong to a particular country or an institution, the closing price data we use is based on the Coordinated Universal Time (UTC). Note that the "closing" price here does not indicate that the market itself closes (cryptocurrency markets are open 24–7).

[3] Due to data availability, all cryptocurrencies in our analyses are against Tether (USDT), a currency designed to maintain the same value as the US dollar.

Table 1
Descriptive statistics and moments of the return series r_t. For the Jarque–Bera test statistic, *** indicates significance at the 1% level.

	BTC	ETH	XRP	LTC	XMR	DASH
Mean (%)	0.2254	0.2758	0.2624	0.179	0.2854	0.1545
Median (%)	0.236	0.1126	−0.1312	−0.0411	0.2024	0.0356
Std. Dev. (%)	4.2175	5.7697	7.5973	6.0541	6.4134	6.2678
Max. (%)	23.814	25.274	104.61	60.051	59.249	45.668
Min. (%)	−50.435	−58.697	−68.039	−47.796	−54.466	−47.595
Skewness	−0.9362	−0.6665	1.8394	0.4671	0.2289	0.3336
Kurtosis	13.086	8.756	30.195	12.043	12.126	8.7689
J.B.	14132***	6344.2***	74829***	11800***	11909***	6254.7***

Table 2
Descriptive statistics and moments of the realized volatility measure $\sqrt{\mathrm{RV}_t}$.

	BTC	ETH	XRP	LTC	XMR	DASH
Median	0.0354	0.0496	0.0592	0.0573	0.0621	0.0642
Max.	0.3192	0.4320	1.7104	0.4100	0.4160	0.4183
Min.	0.0070	0.0112	0.0094	0.0131	0.0141	0.0075
Skewness	2.8323	3.1065	8.2063	2.7228	2.5840	2.3125
Kurtosis	13.943	16.779	150.56	12.697	10.354	9.8965

Barndorff-Nielsen and Shephard (2002) propose to use the realized volatility (RV), i.e., intraday square returns, as a proxy of the daily volatility series:

$$\mathrm{RV}_t = \sum_j r_{t,t_j}^2, \tag{1}$$

where r_{t,t_j} denotes intraday returns, i.e., the log-difference of price calculated from high frequent sample intervals, and t_j denotes the jth value on day t. It is widely known that when sample intervals are set closer to zero and infinite numbers of intraday returns are summed up, the realized volatility estimator converges to the integrated volatility σ_t^2, which is a former standard measure used in various sets of studies. We use 5-minute intervals since such a sampling base is a reasonable choice for avoiding strong bias driven by extremely high frequencies and thus maintaining an accurate measure of volatility (Bandi and Russell, 2006; Liu et al., 2015). The daily return series are calculated as the log difference in prices shown as

$$r_t = \ln p_t - \ln p_{t-1}, \tag{2}$$

where p_t denotes the price at day t. We equally have 1941 return and volatility observations for each cryptocurrency.

We show in Fig. 1 the return series r_t and the volatility measure of $\sqrt{\mathrm{RV}_t}$ for each cryptocurrency, along with their descriptive statistics (see Tables 1 and 2). Note that for $\sqrt{\mathrm{RV}_t}$ we do not show the mean, standard deviation, and the Jarque–Bera test since the data is far from stationary and normality. All cryptocurrency returns present similar positive mean values to some extent; however, higher moments tend to differ among the major and relatively minor coins. Negative skewness is observed for BTC and ETH, whereas positive skewness and larger standard deviation are found for the others. This is a consequence of BTC and ETH being more exposed to negative returns, while other coins are more exposed to volatile positive returns. Kurtosis values of the investigated cryptocurrencies are all well above 3, suggesting that the distribution of returns is highly leptokurtic, having a broader or flatter shape with fatter tails. The Jarque–Bera test reconfirms its significant deviation from normality. In Table 2, we see how the extreme events of XRP led to very high skewness and kurtosis of the volatility indicator.

4. Methodology

4.1. DFA-based bivariate regression estimator

By combining DFA with the standard bivariate linear regression method, we estimate the scale-dependent regression coefficients between return and volatility series in cryptocurrency markets, which are considered to exhibit non-stationary and complex behaviors (Telli and Chen, 2020; Watorek et al., 2021). The idea of this method is based initially on the work of Kristoufek (2015), where they propose to replace part of the standard least squares regression procedure with a multi-time scale procedure to analyze nonlinear dependence between a response series and an impulse series at different levels of scales. In the same vein, Wang et al. (2018) developed the method designed for the case of two impulse series against the response series and proposed the DFA-based bivariate regression analysis.

We slightly modify the DFA process of the above DFA-based bivariate regression analysis of Wang et al. (2018). We follow the initial approach of Podobnik and Stanley (2008), where they derive the fluctuation functions relying on *overlapping* segments of the dataset instead of *non-overlapping* segments. Although the process requires more segments to be averaged over the fluctuation functions, it avoids the significant variance of the estimates due to the small number of sample segments to be averaged.

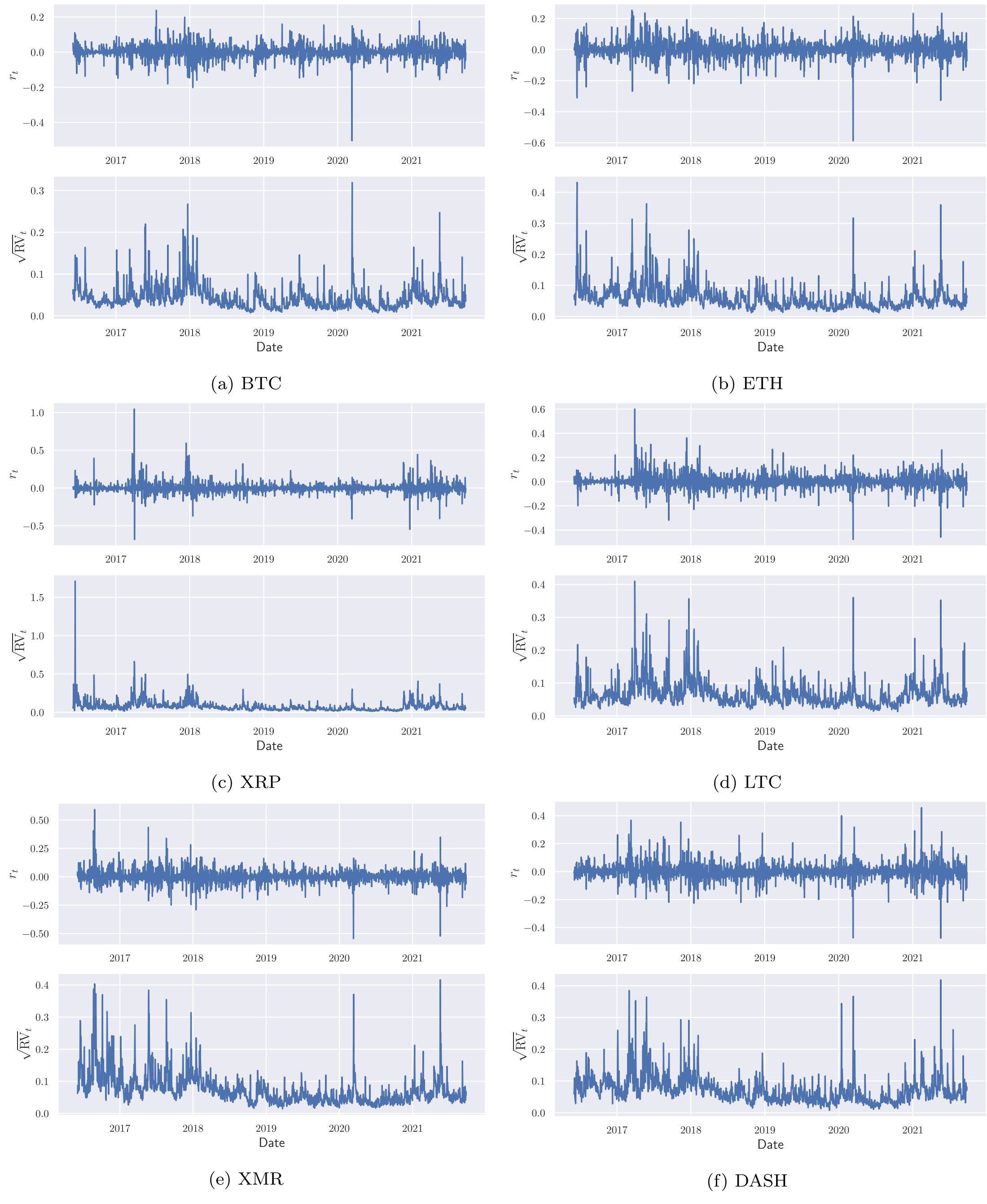

Fig. 1. Daily return and volatility series of (a) Bitcoin, (b) Ethereum, (c) Ripple, (d) Litecoin, (e) Monero, and (f) Dash, for the investigated period (2016/06/02 to 2021/09/25).

The critical point of these fractal regression methods is to use the scale-dependent variance and covariance derived from the "detrended function", instead of the standard variance form. For a given time series $\{x_t\}$ with length N, we split its cumulative sum, or in other words the profile series, $X(t) = \sum_{i=1}^{t} x_i$, for $t = 1, 2, \ldots, N$, into $N - s$ overlapping segments of length $s + 1$. The degree-2 polynomial fits $\tilde{X}(t)$ are used to detrend $X(t)$ for each segment, and then calculate the detrended variance function for each segment defined as

$$f_{XX}^2(s, v) = \frac{1}{s+1} \sum_{t=v}^{v+s} \left[X(t) - \tilde{X}(t)\right]^2. \tag{3}$$

By averaging $f^2_{XX}(s,v)$ over all the segments, we get the fluctuation function, or the scale-dependent variance

$$F^2_{XX}(s) = \frac{1}{N-s}\sum_{v=1}^{N-s} f^2_{XX}(s,v). \tag{4}$$

The scale-dependent covariance can be derived in a similar way by the detrended covariance function of bivariate series $\{x_t\}$ and $\{y_t\}$ with the same length determined as

$$F^2_{XY}(s) = \frac{1}{N-s}\sum_{v=1}^{N-s} f^2_{XY}(s,v), \tag{5}$$

where

$$f^2_{XY}(s,v) = \frac{1}{s+1}\sum_{t=v}^{v+s}\left[X(t)-\tilde{X}(t)\right]\left[Y(t)-\tilde{Y}(t)\right]. \tag{6}$$

Note that Eqs. (5) and (6) can take positive as well as negative values.

To illustrate the dependences of bivariate series, we consider a bivariate linear regression model

$$Z_t = \beta_0 + \beta_1 X_t + \beta_2 Y_t + \varepsilon_t \ (t = 1,\dots,N), \tag{7}$$

where Z_t is a response variable, X_t and Y_t are impulse variables, and ε_t is a Gaussian error term with zero mean. Partial regression coefficients β_1 and β_2 characterize the dependence of response variables on impulse variables. In accordance with the standard OLS method and replacing variance (covariance) with scale-dependent variance (covariance), we get the scale-dependent coefficient estimators as follows:

$$\hat{\beta}_1^{\mathrm{DFA}}(s) = \frac{F^2_{XZ}(s)F^2_{YY}(s) - F^2_{YZ}(s)F^2_{XY}(s)}{F^2_{XX}(s)F^2_{YY}(s) - \left[F^2_{XY}(s)\right]^2}, \tag{8}$$

$$\hat{\beta}_2^{\mathrm{DFA}}(s) = \frac{F^2_{YZ}(s)F^2_{XX}(s) - F^2_{XZ}(s)F^2_{XY}(s)}{F^2_{XX}(s)F^2_{YY}(s) - \left[F^2_{XY}(s)\right]^2}. \tag{9}$$

By using the scale-dependent residual $\hat{\varepsilon}_t(s) = Z_t - \hat{\beta}_1^{\mathrm{DFA}}(s)X_t - \hat{\beta}_2^{\mathrm{DFA}}(s)Y_t - \left\langle Z_t - \hat{\beta}_1^{\mathrm{DFA}}(s)X_t - \hat{\beta}_2^{\mathrm{DFA}}(s)Y_t\right\rangle$, we can calculate the fluctuation function $F^2_{\varepsilon\varepsilon}(s)$ in the same manner as Eq. (4), and the variance of the above coefficients can be estimated as below:

$$\mathrm{var}\left[\hat{\beta}_1^{\mathrm{DFA}}(s)\right] = \frac{1}{N-3}\frac{F^2_{YY}(s)F^2_{\varepsilon\varepsilon}(s)}{F^2_{XX}(s)F^2_{YY}(s) - \left[F^2_{XY}(s)\right]^2}, \tag{10}$$

$$\mathrm{var}\left[\hat{\beta}_2^{\mathrm{DFA}}(s)\right] = \frac{1}{N-3}\frac{F^2_{XX}(s)F^2_{\varepsilon\varepsilon}(s)}{F^2_{XX}(s)F^2_{YY}(s) - \left[F^2_{XY}(s)\right]^2}. \tag{11}$$

Now that the coefficients are estimated for some specific scale s, we can do the same procedure under other scales by changing s. It is worth noting that Fan and Wang (2020) introduces a similar approach of DMA-based bivariate regression estimator, which uses the centered moving average technique when detrending the profile series, i.e., $X(t)$ in Eq. (3). Since the centered DMA analysis requires some reference to future data, we focus on the DFA that can be carried on with the data at hand.[4]

In this study, we use the DFA and DCCA fluctuation functions to implement the fractal regression analysis. Discussions based on the fractal regression analysis provide a more transparent view of multi-time scale connections between the variables because the regression model intends to design the actual dependence rather than just looking at the strength of its correlation.[5] Moreover, it plays a role in meeting the need to regard asymmetric effects of asymmetric volatility by devising the fractal regression model presented in the following subsection.

4.2. Modeling asymmetric volatility behavior with fractal regression analysis

We provide an alternative approach to model asymmetric effects of return shocks on volatility. To clarify the dependence at different time scales, we develop the DFA-based bivariate fractal regression and construct the following regression model:

$$Z_t = \beta_0 + \beta_1 X_t + \beta_2 Y_t + \varepsilon_t \ (t = 2,\dots,N), \tag{12}$$

$$\begin{cases} X_t = \frac{|r_{t-1}|}{\sqrt{\mathrm{RV}_{t-1}}} \\ Y_t = \frac{r_{t-1}}{\sqrt{\mathrm{RV}_{t-1}}} \\ Z_t = \ln \mathrm{RV}_t \end{cases}$$

[4] In general, the DFA-based methods are powerful and robust tools for determining remarkable stylized facts of long-range correlations and scale dependencies in one's series and across other series.

[5] Zebende (2011) uses the DFA and DCCA fluctuation functions to calculate the cross-correlation coefficient at multi-time scales defined as $\rho_{\mathrm{DCCA}}(s) = \frac{F^2_{XY}(s)}{F_{XX}(s)F_{YY}(s)}$.

where RV_t is the realized volatility series in Eq. (1), r_t is the return series in Eq. (2), and $|r_t|$ is the absolute value of return series. The independent variable $\frac{r_{t-1}}{\sqrt{RV_{t-1}}}$ represents the positive and negative return shocks relative to volatility, in the previous time step. The independent variable $\frac{|r_{t-1}|}{\sqrt{RV_{t-1}}}$ represents its magnitude—the larger the value, the greater the impact, i.e., impact of shocks relative to volatility. In other words, these variables are filtered by volatility. This filtering procedure removes short range dependencies and reduces possible volatility bias among different time windows (Tilfani et al., 2019). The information on market sign enables to model potential asymmetry of the impact on volatility. Asymmetric response of positive and negative shocks can be quantified through the regression coefficients β_1 and β_2; a positive shock in the market is responsible for the increase in volatility as much as $(\beta_1 + \beta_2)\frac{|r_{t-1}|}{\sqrt{RV_{t-1}}}$, whereas a negative shock in the market is responsible for the increase in volatility as much as $(\beta_1 - \beta_2)\frac{|r_{t-1}|}{\sqrt{RV_{t-1}}}$. Therefore, among the coefficients, β_2 determines the asymmetric volatility behavior. Significantly positive (negative) values of β_2 in the model imply that positive (negative) shocks increase volatility by more than shocks whose sign is opposite.

Rewriting Eq. (12) as

$$\ln RV_t = \beta_0 + \beta_1 \frac{|r_{t-1}| + \gamma r_{t-1}}{\sqrt{RV_{t-1}}} + \varepsilon_t, \tag{13}$$

where $\gamma = \frac{\beta_2}{\beta_1}$, appears to resemble the structure of the conventional EGARCH model, which is one of the most common GARCH model capable of investigating asymmetric effects. The EGARCH model and our fractal regression model are similar to some extent. For both the impulse variable of our regression model and the process of return innovations in the EGARCH model, the sign of past return shocks yields separate effects on volatility, and for the response variable, the logarithm of the volatility relaxes the positiveness constraint of model coefficients and allows values to be negative. While the EGARCH model describes the volatility and variance of the current error term or innovation conditioned to previous error terms and innovations, the DFA-based regression model aims to demonstrate the nonlinear dependence of volatility with lagged return series across different time scales. The conditional volatility term is not included in our model because the DFA-based method accounts for the nonlinear elements of volatility, such as long-range dependence. Since the whole history of returns are already incorporated into the fractal regression model in terms of long-range correlations, taking additional lagged return innovations may be inappropriate in fractal regressions. The fractal regression framework addresses the dynamic behavior, even in simple regression models. Under a specific time scale of s, the model of Eq. (12) can be expressed as

$$\ln RV_t = \beta_0(s) + \beta_1(s)\frac{|r_{t-1}|}{\sqrt{RV_{t-1}}} + \beta_2(s)\frac{r_{t-1}}{\sqrt{RV_{t-1}}} + \varepsilon_t(s) \ (t = 2, \dots, N), \tag{14}$$

where the scales can be interpreted as investment horizons. For each specific scale, we can estimate the model coefficients separately. Our model provides a new view of how good and bad news affect the volatility on multi-time scales and how they differ across investment horizons when detecting asymmetric volatility.

5. Results and discussions

5.1. Scaling dependencies of asymmetric volatility effect in cryptocurrency markets

After we calculate return and realized volatility series from 5-minute interval cryptocurrency data as presented in Section 3, we analyze the multi-time scale property of the return-volatility structure regarding its asymmetry following the procedures in Section 4. We show in Fig. 2 the estimated values of the scale-dependent coefficients $\hat{\beta}_1^{\mathrm{DFA}}(s)$ and $\hat{\beta}_2^{\mathrm{DFA}}(s)$ of model Eq. (12) for each cryptocurrency. We also depict together with colored ranges the 95% confidence intervals of the estimates. Clearly, the coefficients are not monotonous — the dependence of the series oscillates at different time scales. The coefficients $\hat{\beta}_1^{\mathrm{DFA}}(s)$ are always significantly above zero in all cases, as expected, since the standardized absolute return (corresponding impulse variable) and the volatility index (response variable) are both a representation of volatility. The coefficient value increases as scales become larger. We find that the impact of good and bad news to volatility, $\hat{\beta}_1^{\mathrm{DFA}}(s)+\hat{\beta}_2^{\mathrm{DFA}}(s)$ and $\hat{\beta}_1^{\mathrm{DFA}}(s)-\hat{\beta}_2^{\mathrm{DFA}}(s)$, always stay positive (Fig. 3). In our model, as long as $\hat{\beta}_1^{\mathrm{DFA}}(s)+\hat{\beta}_2^{\mathrm{DFA}}(s) > 0$ and $\hat{\beta}_1^{\mathrm{DFA}}(s)-\hat{\beta}_2^{\mathrm{DFA}}(s) > 0$ satisfy, only the sign of coefficient $\hat{\beta}_2^{\mathrm{DFA}}(s)$ sufficiently determines the asymmetric reaction. Therefore, we focus on $\hat{\beta}_2^{\mathrm{DFA}}(s)$ to discuss whether negative or positive price movements have more impact on volatility. We find different asymmetric features among the investigated cryptocurrencies and that they can be broadly classified into three categories; positive, negative, and both positive and negative effects.[6]

First of all, we find that $\hat{\beta}_2^{\mathrm{DFA}}(s)$ are always negative for the two major currencies of BTC and ETH, indicating the presence of an asymmetric volatility effect regardless of time scale — negative news has a greater impact on volatility increment than positive news at all scales. This leverage effect is consistent with what is commonly observed in stock markets (Jeribi et al., 2015; Fakhfekh et al., 2016; Bentes, 2018), however, its origin should not be associated with financial leverage (Hens and Steude, 2009; Hasanhodzic and Lo, 2019). More generally, such an asymmetric phenomenon has its explanation on the background of who trades and how they transact in practice (Black, 1976; Antoniou et al., 1998). This is especially true in cryptocurrency markets (Baur and Dimpfl, 2018),

[6] To check the stability of the results we have also performed an analysis using volume-weighted averaged daily prices (VWAP) in case the asymmetric volatility effect is a consequence of market illiquidity. By taking the ratio of the value of cryptocurrency traded to the total volume of daily transactions, the trading prices are averaged out, thus reducing market illiquidity. We report no notable changes compared to the results for the closing price case, indicating that illiquidity does not greatly contribute to asymmetric volatility effect (see Figs. B.6 and B.7 in Appendix B).

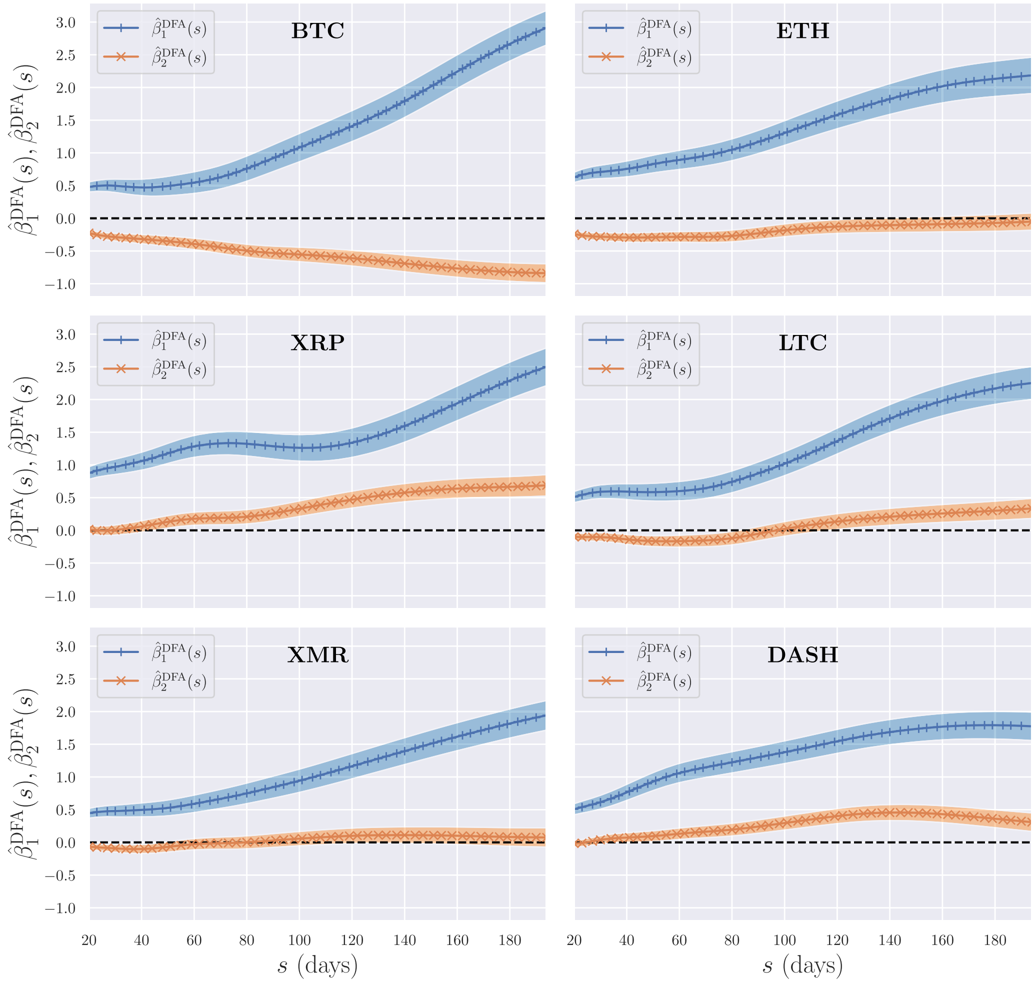

Fig. 2. DFA-based bivariate regression estimates of cryptocurrency series. The coefficients $\hat{\beta}_1^{\mathrm{DFA}}(s)$ and $\hat{\beta}_2^{\mathrm{DFA}}(s)$ are shown for each cryptocurrency; BTC, ETH, XRP, LTC, XMR, and DASH. The colored ranges denote 95% confidence intervals calculated as $\hat{\beta}_i^{\mathrm{DFA}}(s) \pm 2\sqrt{\mathrm{var}(\hat{\beta}_i^{\mathrm{DFA}}(s))}$, for $i = 1, 2$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

since a "financial" explanation of the effect may be challenging. Traces of market price fluctuations are a consequence of informed traders correcting market asymmetries caused by irrational transactions and herding behavior of uninformed traders. In this regard, uninformed traders are responsible for the striking rises in volatility, so speculative investments of uninformed traders' in BTC and ETH are more active when the markets experience negative return shocks. We find that the effect is remarkable, especially in BTC.

However, a completely opposite effect can be observed for the relatively minor cryptocurrencies of XRP and DASH. As shown in the figure, the coefficients $\hat{\beta}_2^{\mathrm{DFA}}(s)$ turn out positive. This outcome indicates the presence of an "inversed" asymmetric volatility effect in these markets, meaning that positive returns affect volatility more than negative returns. The rise in volatility can be interpreted as being due to uninformed traders, this time reacting to positive news. We can confirm this effect across scales. According to Baur and Dimpfl (2018), the pump-and-dump scheme, telling people to buy a particular currency, and the fear of missing out, not feeling that they are taking full advantage of information towards future prices, are in the background of this reaction. These attributes can remarkably drive to raise cryptocurrency prices, and as a result, volatility will increase more than in rising markets compared to falling markets. In addition, the relatively small market size of XRP and DASH may attribute to increasing volatility especially when the market is in its rising trend with soaring prices. Such markets tend to be susceptible to attracting more uninformed traders to speculative investment where informed traders become less capable of exerting pressure on reducing market volatility.

Interestingly, the remaining LTC and XMR currencies exhibit a composite structure with different signs of asymmetric volatility. As shown in Fig. 2, the coefficients $\hat{\beta}_2^{\mathrm{DFA}}(s)$ oscillate around zero and do not constantly take the same sign — they can be either positive or negative depending on which time scale we focus. LTC and XMR have in common that $\hat{\beta}_2^{\mathrm{DFA}}(s)$ take negative values for $s < 80$ and positive values for $s > 100$. Although the values are close to zero and the volatility effect seems to be almost absent, either positive or negative return news can lead to larger volatility increments. Asymmetric volatility effect may be present on scales smaller than approximately three months, but on larger scales, the effect is slightly reversed or, in the worst case, disappear. The

Fig. 3. The impact of good and bad news to volatility, $\hat{\beta}_1^{\mathrm{DFA}}(s) + \hat{\beta}_2^{\mathrm{DFA}}(s)$ and $\hat{\beta}_1^{\mathrm{DFA}}(s) - \hat{\beta}_2^{\mathrm{DFA}}(s)$, respectively. We show the results for each cryptocurrency; BTC, ETH, XRP, LTC, XMR, and DASH.

findings imply that uninformed investors who seek short-term horizons play a prominent role in downside markets to amplify the asymmetry of volatility effect.

Although the scale-dependent regression coefficients seem to be mostly well above or below zero value, including their confidence intervals (orange ranges in Fig. 2), they may be an outcome where asymmetric behavior is absent. The theoretical value of $\hat{\beta}_2^{\mathrm{DFA}}(s) = 0$ can be calculated only for an infinitely long time series. As long as datasets are finite, empirical estimations can vary due to sample size effects, and even if there is no dependence between the variables, the coefficient estimates can be far from zero to some extent. To assure whether the regression relationship under different scales is genuine or not, we employ the statistical test of Wang et al. (2018) that tests the absence of dependencies between Z and X or Y of the regression model $Z_t = \beta_0 + \beta_1 X_t + \beta_2 Y_t + \varepsilon_t$, i.e., the null hypothesis of $\beta_i(s) = 0$ for $i = 1, 2$. They introduce the scale-dependent t-statistics defined as $t_i(s) = \frac{\hat{\beta}_i^{\mathrm{DFA}}(s) - \beta_i}{\sqrt{\mathrm{var}(\hat{\beta}_i^{\mathrm{DFA}}(s))}}$, similar to the t-statistics used in the standard regression analysis. The subject series are firstly shuffled in practice, and then the scale-dependent t-statistics of the regression coefficients are calculated. This procedure is repeated many times to carry out the computation of scale-dependent critical values $t^{\mathrm{c}}(s)$ based on Podobnik et al. (2011), defined such that the integral of the probability distribution function of $t_i(s)$, $[-t^{\mathrm{c}}(s), t^{\mathrm{c}}(s)]$, is equal to $1 - \alpha$, where α denotes confidence level. Thereby, we can determine the range of $t_i(s)$ that can be considered statistically significant under some specific time scale. We shuffle $Z_t = \ln \mathrm{RV}_t$ (the volatility series) and $Y_t = r_{t-1}/\sqrt{\mathrm{RV}_{t-1}}$ (return shocks series) while setting $X_t = |Y_t|$ in order to correspond to our model in Eq. (12). In this way, positive and negative return shocks are always associated with their magnitude (impulse variables) while their correlation with volatility (response variable) is destroyed, thus $\beta_2 = 0$. We calculate the empirical values of $t_2(s)$ and perform the hypothesis test of $\beta_2(s) = 0$ by repeating 1000 times the procedure of calculating the scale-dependent t-statistics from the shuffled series. For robustness check, we also use the bootstrap method to test whether the $\beta_2(s)$ coefficients are significantly different from 0. For each scale we compute

Table 3
Estimation results of the EGARCH model; $\ln\sigma_t^2 = \omega + \gamma_1 \frac{|r_{t-1}|}{\sigma_{t-1}} + \gamma_2 \frac{r_{t-1}}{\sigma_{t-1}} + \alpha \ln\sigma_{t-1}^2$ and $r_t = \epsilon_t\sigma_t$, where σ_t^2 is the conditional variance at time t, and ϵ_t denotes an error term with i.i.d. standard Gaussian noise. Standard errors of estimates are reported in parentheses. The asymmetric parameter γ_2 is shown in bold. Note that ***, **, and * denote 1%, 5%, and 10% significance levels, respectively.

	BTC	ETH	XRP	LTC	XMR	DASH
ω	−0.5935***	−0.6653***	−1.1289***	−0.3691***	−0.4701***	−0.6078***
	(0.0582)	(0.0646)	(0.0410)	(0.0276)	(0.0419)	(0.0500)
γ_1	0.2256***	0.2724***	0.4895***	0.1772***	0.2574***	0.3224***
	(0.0184)	(0.0170)	(0.0179)	(0.0123)	(0.0146)	(0.0184)
γ_2	**−0.0514***	**−0.0068**	**0.0924***	**0.0196***	**0.0292***	**0.0333***
	(0.0066)	(0.0087)	(0.0103)	(0.0060)	(0.0054)	(0.0075)
α	0.9317***	0.9187***	0.8554***	0.9562***	0.9487***	0.9330***
	(0.0074)	(0.0097)	(0.0060)	(0.0036)	(0.0063)	(0.0073)

the quantile of the coefficient and its confidence interval is obtained. Details of the procedure and the estimation results can be found in Appendix A.

We depict in Fig. 4 the values of $t_2(s)$ with the scale-dependent critical values at the 5% and 10% levels of significance ($\alpha = 0.1, 0.05$). If the value is not within the critical band, the hypothesis can be rejected. We find that in BTC, the values of $t_2(s)$ are always lower than the critical band, so the absence of an asymmetric response is rejected and hence asymmetric volatility effect exists for all scales. However, in other cryptocurrencies, it is not always assured whether asymmetric reaction exists across scales. For instance, we find in ETH that $t_2(s)$ stay way below the band only for $s < 90$ (three months). In other words, asymmetric volatility effect is significant on scales shorter than three months but not for scales larger than that. In XRP and DASH, which appeared to show an inverse asymmetric volatility effect, we find that $t_2(s)$ stay mainly within the critical band for small scales. An inverse effect can be confirmed on mid-scales around $s = 120$ (four months) or more, where the scale-dependent t-statistics go over the upper critical bound of 95% confidence level. For other scales (small and large scales) the corresponding t-statistics are not rigorously statistically significant, however, they are rather close to the upper critical values suggesting that good news in XRP and DASH tends to help increase volatility slightly more than bad news on average. The results tell us the possibility of an interesting story in which the uninformed traders' herding is dominant in rising markets but not in falling markets, and that its effect is strongest at mid-term horizons. Investors seem to have speculative expectations of mid- to long-term growth in these markets. In LTC and XMR, which may present both signs of asymmetric volatility, $t_2(s)$ fall out of the lower 95% critical band for scales smaller than approximately $s = 30$ (one month). Therefore, the null hypothesis is rejected and assures the presence of an asymmetric volatility effect only at small scales. The result implies that herding of uninformed traders on shorter time horizons has significantly increased the volatility in falling markets. On the other hand, for large scales of $s > 30$ (one month), we find no statistical evidence of asymmetric volatility, neither positive nor negative. This indicates that the inverse asymmetric volatility phenomenon (positive effect) at large scales is likely to be simply within the statistical accident of size effects.

In addition, we present in Table 3 the asymmetry results estimated by the EGARCH(1,1) model to check the differences and similarities with the results estimated by our model. Since the scaling properties are not considered in the EGARCH model, the sign of γ_2 alone represents the overall asymmetric response of volatility to good and bad news. As expected, a significant negative effect ($\gamma_2 < 0$) is found for BTC. We also find negative in ETH, but the effect is insignificant. All the other cryptocurrencies show an inverse asymmetric volatility effect ($\gamma_2 > 0$), and their coefficients are found to be statistically significant. These results tell us that the traditional model provides us a plausible overall picture of asymmetry, but fails to address the heterogeneous effect among scales — for example, the negative effect we have confirmed in LTC for small scales cannot be detected by the EGARCH model. Our model thus helps us find some interesting results throughout the various investment horizons.

5.2. Time-varying properties of scale-dependent asymmetric volatility

As the cryptocurrency market heads to maturity with more active online trades, the asymmetric volatility response to return shocks may vary due to the change in informed and uninformed traders' behavior. In effect, using the asymmetric GARCH models, Takaishi (2021) reports that Bitcoin exhibits different signs of asymmetric volatility for other historical periods and infers that such an underlying time-varying property may be one of the reasons why a constant picture is not observed in the Bitcoin market. Therefore, we examine how the asymmetry and scaling factors changed through the evolution of cryptocurrency history, including other representative cryptocurrencies.

We focus on analyzing two periods, from 2016/6/2 to 2019/4/30 and 2018/11/1 to 2021/9/25, with data long enough to run our fractal regression analysis.[7] They include typical cryptocurrency bubbles and crashes, the first occurring in late 2017 to early 2018

[7] The first sub-period corresponds to one year before and after the bubble and crash periods of 2017 to 2018, in order to also consider the periods when prices are in a stable state. Remarkable rises and falls in prices in 2021 are ongoing, and the second sub-period is set to have roughly the same data length as the first sub-period.

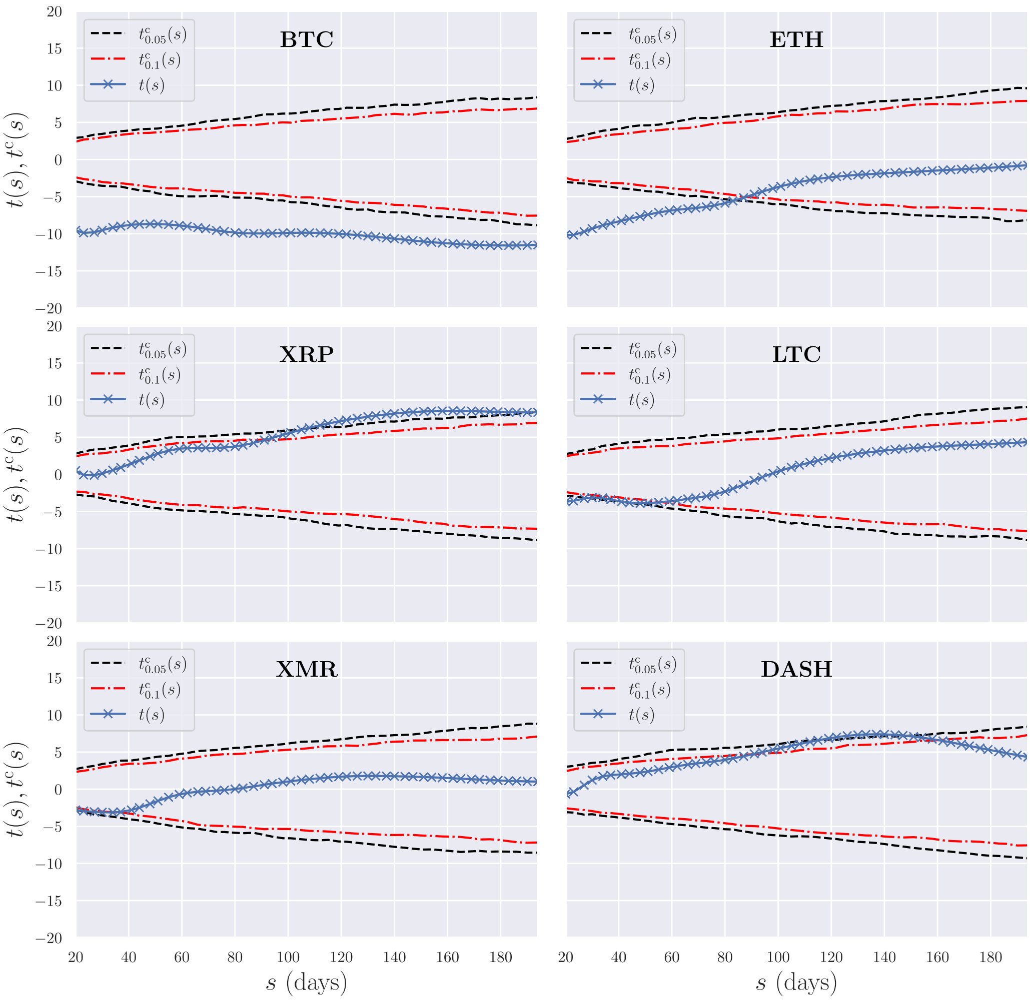

Fig. 4. Statistic significance tests of the DFA-based bivariate regression estimates. Among the synthetic distribution of the scale-dependent t-statistics of the coefficient $\hat{\beta}_2^{\mathrm{DFA}}(s)$, the black dashed line indicates the critical values of the 95% band, and the red dash-dotted line indicates the critical values of the 90% band.

and the second in 2021. Cryptocurrencies during these periods have experienced remarkable rises and intense falls in prices. In such situations, the existence of noise traders cannot be ignored, and it is clear that they, in part, play an essential role in influencing the asymmetric behavior of market volatility in the short- and long-term. In Tables 4 and 5, we show the DFA-based bivariate regression coefficient estimates associated with the t-statistics for each period. We consider the following investment horizon settings; short-term ($s = 30$), mid-term ($s = 60$), and long-term scales ($s = 120$). During the first period (Table 4), the scale-dependent coefficients $\hat{\beta}_2^{\mathrm{DFA}}(s)$ of BTC stay negative for all investment horizons, indicating that BTC volatility is higher following negative return shocks on whatever scale. A similar outcome is found in ETH, although the coefficient is insignificant for long-term scales. For the minor coins of XRP and DASH, the coefficients turn out to be slightly positive for all scales. This indicates that volatility may rather be higher following positive return shocks, however, the coefficients are generally not statistically significant. The remaining minor coins of LTC and XMR show no remarkable evidence of asymmetry; on the one hand, we find negative coefficients for short- and mid-term scales but a positive coefficient for long-term scales. This trend is in line with the findings mentioned earlier in Figs. 2–4.

The results so far provide not so much a different picture from that of using the entire period. However, we find different asymmetric effects and scaling dependencies in the second period (Table 5). All scale-dependent coefficients $\hat{\beta}_2^{\mathrm{DFA}}(s)$ and their scale-dependent t-statistics $t_2(s)$ of BTC are closer to zero, meaning that compared to the first period, the degree of asymmetry became weaker.[8] Noteworthy, the BTC market still holds a significant asymmetric volatility effect, and also, the ETH does. More importantly, in the relatively minor cryptocurrencies, traces of the inverse asymmetric volatility effect is no longer present on most scales because these markets are prone to show negative coefficients more often than in the first period. The shift from positive

[8] Urquhart (2016) provides empirical evidence that Bitcoin is an inefficient market but may be in the process of moving towards an efficient market.

Table 4
DFA-based bivariate regression estimates of cryptocurrency series for the period of 2016/6/2 to 2019/4/30, corresponding to the first cryptocurrency boom and crash. The scale-dependent coefficients $\hat{\beta}_1^{DFA}(s)$ and $\hat{\beta}_2^{DFA}(s)$ associated with their variance (×10^{-3}; shown in parenthesis) are shown with the scale-dependent t-statistics $t_2(s)$. We show the results for the cases of short-term scales ($s = 30$ days), mid-term scales ($s = 60$ days), and long-term scales ($s = 120$ days). Note that *,**, and *** denotes 90%, 95%, and 99% confidence level, respectively.

	BTC			ETH			XRP		
	$s = 30$	$s = 60$	$s = 120$	$s = 30$	$s = 60$	$s = 120$	$s = 30$	$s = 60$	$s = 120$
$\hat{\beta}_1^{DFA}(s)$	0.517	1.061	2.313	0.920	1.462	2.428	1.055	1.519	1.051
	(5.542)	(11.50)	(22.05)	(5.296)	(9.439)	(18.59)	(7.113)	(12.60)	(25.33)
$\hat{\beta}_2^{DFA}(s)$	−0.391	−0.792	−0.884	−0.300	−0.501	−0.174	0.048	0.024	0.595
	(1.829)	(3.226)	(6.126)	(1.954)	(3.096)	(4.783)	(3.082)	(4.712)	(9.316)
$t_2(s)$	−9.148***	−13.94***	−11.30***	−6.776***	−9.001***	−2.521	0.865	0.344	6.163*
	LTC			XMR			DASH		
	$s = 30$	$s = 60$	$s = 120$	$s = 30$	$s = 60$	$s = 120$	$s = 30$	$s = 60$	$s = 120$
$\hat{\beta}_1^{DFA}(s)$	0.677	0.970	1.655	0.804	1.101	1.795	0.636	1.101	1.869
	(6.315)	(11.97)	(18.54)	(4.386)	(9.503)	(22.31)	(5.082)	(9.105)	(19.35)
$\hat{\beta}_2^{DFA}(s)$	−0.089	−0.289	0.241	−0.041	−0.045	0.558	0.013	0.038	0.456
	(2.420)	(4.296)	(7.311)	(1.771)	(4.160)	(8.627)	(1.916)	(3.202)	(7.451)
$t_2(s)$	−1.803	−4.408*	2.816	−0.963	−0.699	6.011*	0.302	0.671	5.277*

Table 5
DFA-based bivariate regression estimates of cryptocurrency series for the period of 2018/11/01 to 2021/9/25, corresponding to the second cryptocurrency boom and crash.

	BTC			ETH			XRP		
	$s = 30$	$s = 60$	$s = 120$	$s = 30$	$s = 60$	$s = 120$	$s = 30$	$s = 60$	$s = 120$
$\hat{\beta}_1^{DFA}(s)$	0.584	0.876	0.876	0.603	0.791	0.836	0.539	0.624	0.775
	(1.057)	(1.515)	(1.956)	(1.191)	(1.486)	(2.028)	(0.598)	(0.727)	(1.051)
$\hat{\beta}_2^{DFA}(s)$	−0.122	−0.123	−0.405	−0.214	−0.193	−0.324	−0.008	0.059	0.005
	(0.588)	(1.151)	(2.707)	(0.760)	(1.372)	(2.310)	(0.595)	(1.086)	(1.376)
$t_2(s)$	−5.016***	−3.632	−7.787**	−7.775***	−5.208**	−6.741*	−0.317	1.784	−0.138
	LTC			XMR			DASH		
	$s = 30$	$s = 60$	$s = 120$	$s = 30$	$s = 60$	$s = 120$	$s = 30$	$s = 60$	$s = 120$
$\hat{\beta}_1^{DFA}(s)$	0.701	0.889	1.046	0.686	1.138	1.187	0.764	1.127	1.164
	(1.224)	(1.384)	(1.792)	(1.758)	(2.051)	(2.278)	(1.240)	(1.339)	(1.812)
$\hat{\beta}_2^{DFA}(s)$	−0.093	−0.150	−0.158	−0.078	−0.042	−0.262	−0.004	−0.015	0.051
	(0.708)	(1.157)	(2.272)	(1.013)	(1.906)	(2.919)	(0.761)	(1.177)	(1.889)
$t_2(s)$	−3.498**	−4.421*	−3.318	−2.459	−0.953	−4.842	−0.163	−0.430	1.165

to negative asymmetric effect, in addition to the reduction of asymmetry in major cryptocurrencies, is in good agreement with the argument that cryptocurrency markets are steadily heading towards maturity (Drozdz et al., 2018). As the market matures, informed investors will be more dominant, helping to reduce market asymmetry.

The asymmetric results among different time periods may be attributed to the safe-haven property of cryptocurrencies and their change in recent times. Bouri et al. (2017a) demonstrate that any evidence of an inverse asymmetric volatility in cryptocurrency markets may point towards a safe-haven property. When cryptocurrency prices rise in periods of financial turmoil in which traditional market prices (e.g., stock prices) fall, investors interpret this as an increase in macroeconomic environment and uncertainty. In this situation, investors (in particular the uninformed) buy cryptocurrencies and transmit the increased volatility of the stock market to cryptocurrency markets. On the contrary, when cryptocurrency prices fall in periods of rising stock prices, uninformed investors consider that the uncertainty of macroeconomic environment is low. They thereby transmit the decreased volatility of stock markets to cryptocurrency markets, which operates to mitigate downside market risks in cryptocurrencies and prevents volatility from rising. Accordingly, the existence of an inverse leverage during the first sub-period justifies the possibility that they are a consequence of cryptocurrencies acting as a safe-haven against leveraged traditional assets.[9] However, the potential is lost in the second period, and thereby the market can no longer be associated with the safe-haven property. The market has grown to show asymmetric outcomes similar to those generally seen in mature markets. In this context, the results warn financial risk managers that using cryptocurrencies on the route to maturity for hedging requires careful investigation of their dynamic interdependence between return and volatility. It is expected that the discussion will be further developed by investigating the connection of cryptocurrencies to global traditional markets.

[9] In fact, the cryptocurrency crash of 2017 and 2018 is said to be detached from the global financial system and thus the market is uncorrelated with traditional markets.

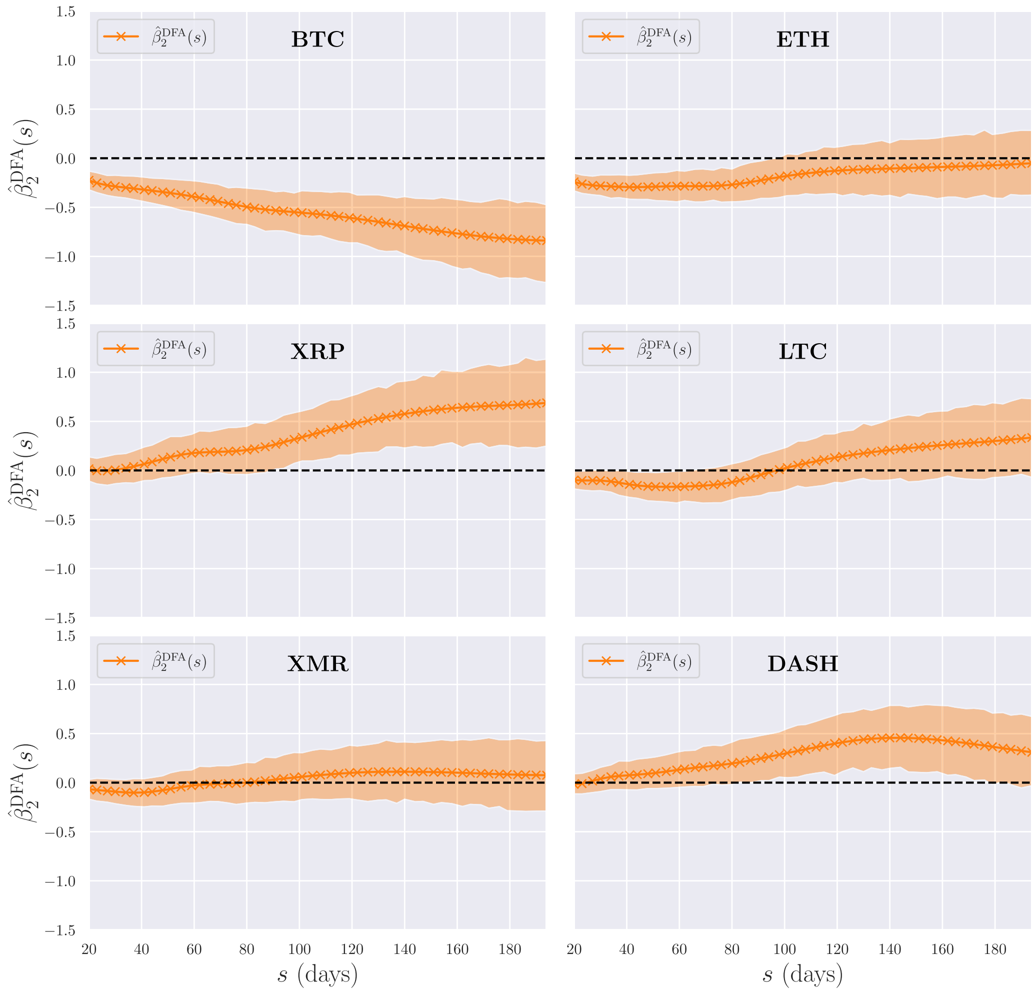

Fig. A.5. Bootstrapped 95% confidence interval of DFA-based bivariate regression.

Moreover, one can extend the DFA-based fractal regression model to address another critical component of financial time series – multifractal characteristics – by extending the DFA to the MFDFA. A deeper investigation of multifractal dynamics in the return-volatility nexus may further develop the understanding of complex behaviors of volatility in cryptocurrency markets and how return shocks play an essential role in producing asymmetric responses in the market. These are left for future work.

6. Conclusion

This paper develops a fractal regression framework to evaluate how return volatility responds asymmetrically to return shocks in six representative cryptocurrency markets of Bitcoin, Ethereum, Ripple, Litecoin, Monero, and Dash. We make two major contributions in this paper. First, we reveal the presence of a scaling-dependent structure in the asymmetric relationship between return shocks and return volatility — volatility of cryptocurrencies can negatively or positively be influenced by return shocks dependent on time scales, i.e., investment horizons. We focus on discussing the asymmetric volatility effect and its inverse effect using the fractal regression analysis, which allows us to quantify the scaling factors of dependencies between the series. The proposed fractal regression model has its advantage in modeling heterogeneity between asymmetric shocks and between large and small scales. In this sense, our approach is more general than the traditional models that do not account for multi-time scales. The findings illustrate that the asymmetry of volatility effect is determined not only by proximate return shocks but also by shocks across scales. The empirical results present a more precise insight that regardless of scales, the major BTC and ETH show a strong asymmetric volatility effect (negative effect), where negative shocks tend to have a greater impact on volatility. On the contrary, for some specific ranges of mid-term scales, minor cryptocurrencies (especially XRP and DASH) show an inverse asymmetric volatility effect (positive effect), where positive shocks tend to have a greater impact on volatility. The reason is discussed in the context of uninformed traders dominating the market in different situations. The impact of informed short sellers trading during the downside market is worth future discussion, since it is another possibility that could affect volatility.

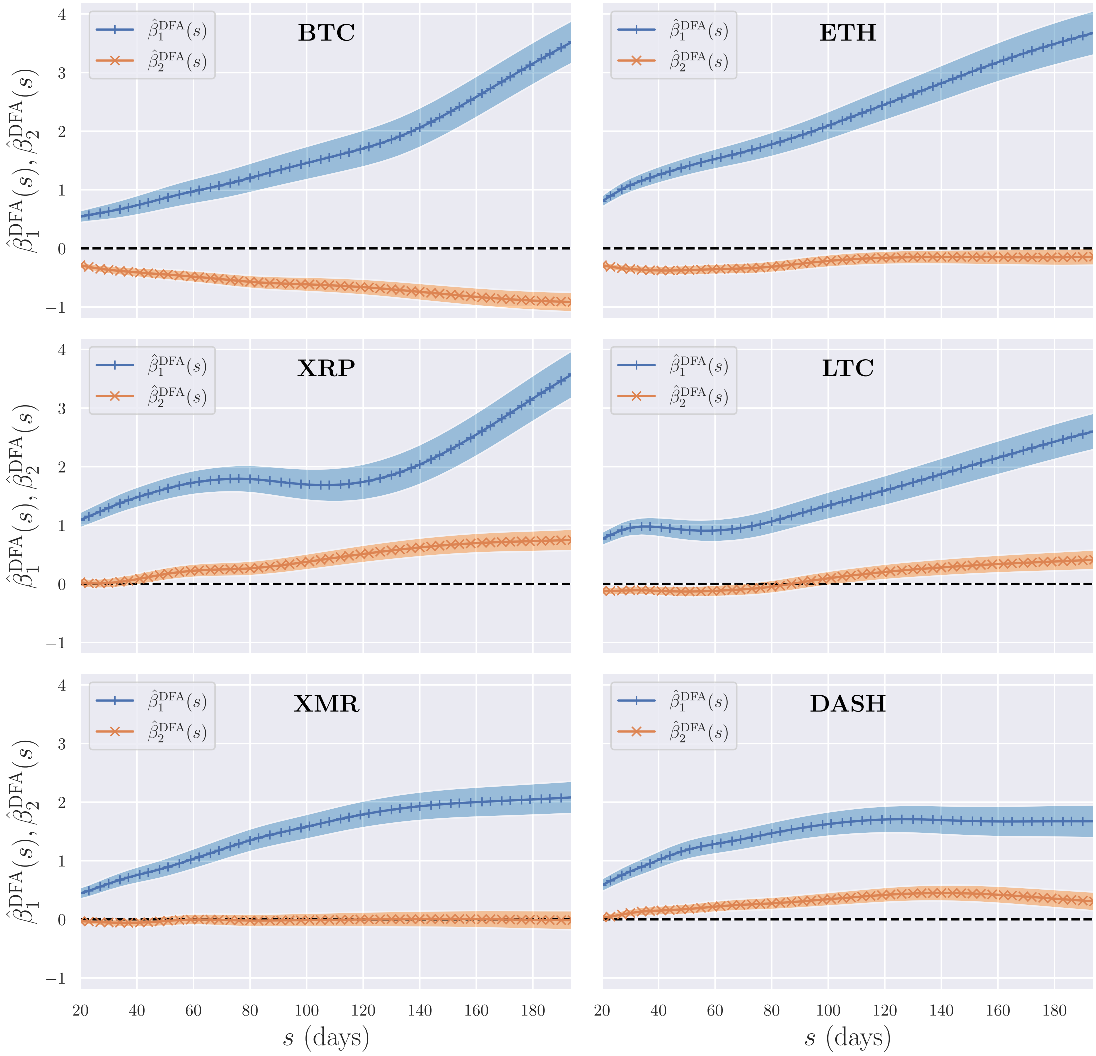

Fig. B.6. DFA-based bivariate regression estimates of cryptocurrency series based on VWAP. The coefficients $\hat{\beta}_1^{\mathrm{DFA}}(s)$ and $\hat{\beta}_2^{\mathrm{DFA}}(s)$ are shown for each cryptocurrency; BTC, ETH, XRP, LTC, XMR, and DASH. The colored ranges denote 95% confidence intervals calculated as $\hat{\beta}_i^{\mathrm{DFA}}(s) \pm 2\sqrt{\mathrm{var}(\hat{\beta}_i^{\mathrm{DFA}}(s))}$, for $i = 1, 2$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Second, we study how the scale-dependent asymmetric volatility effect in cryptocurrencies changed by experiencing two prominent bubbles and crashes. Under the proposed fractal regression model, we highlight the asymmetric outcomes for the two periods. Volatility in major cryptocurrencies is higher following a negative return shock for both periods. In minor cryptocurrencies, the features tend to be time-varying, where the effect shifted from positive to negative for a wide range of scales. This negative effect is consistent with that reported in recent major cryptocurrencies and other traditional financial assets. The reduction of such an asymmetric effect reveals traces of the markets' increasing maturity with a larger predominance of informed traders mitigating the effect of uninformed traders' herding.

To sum up, our approach has the ability to explain the asymmetric return-volatility relationship in addition to their scaling-dependencies. Since understanding the features of volatility plays a crucial role in various determinants in real-world finance, our findings should be of interest to academic researchers, market investors, and policymakers.

CRediT authorship contribution statement

Shinji Kakinaka: Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Data curation, Writing – original draft, Writing – review & editing, Visualization. **Ken Umeno:** Validation, Supervision.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

Acknowledgments

This research was supported by the Japan Society for the Promotion of Science (JSPS) KAKENHI Grant Number JP21J15805.

Appendix A. Bootstrap confidence interval of scale-dependent regression

After obtaining $\hat{\beta}_i^{\mathrm{DFA}}(s)$ and the error $\varepsilon_t(s)$ from the model of Eq. (12) (Eq. (14)), we resample the error $\varepsilon_t^*(s)$ and construct a new data Z_t^* using $\hat{\beta}_i^{\mathrm{DFA}}(s)$ and $\varepsilon_t^*(s)$. We apply once again the fractal regression and run the procedure 1000 times. Then we obtain a collection of estimated coefficients, so the quantile of $\beta_2^{\mathrm{DFA}}(s)$ can be calculated. Below in Fig. A.5, the 95% confidence intervals across scales for each cryptocurrency are depicted. If $\hat{\beta}_2^{\mathrm{DFA}}(s) = 0$ is out of the range in orange, the coefficient is significantly different from 0. We confirm that the results are very similar to those obtained using the scale-dependent t-statistics presented in Fig. 4.

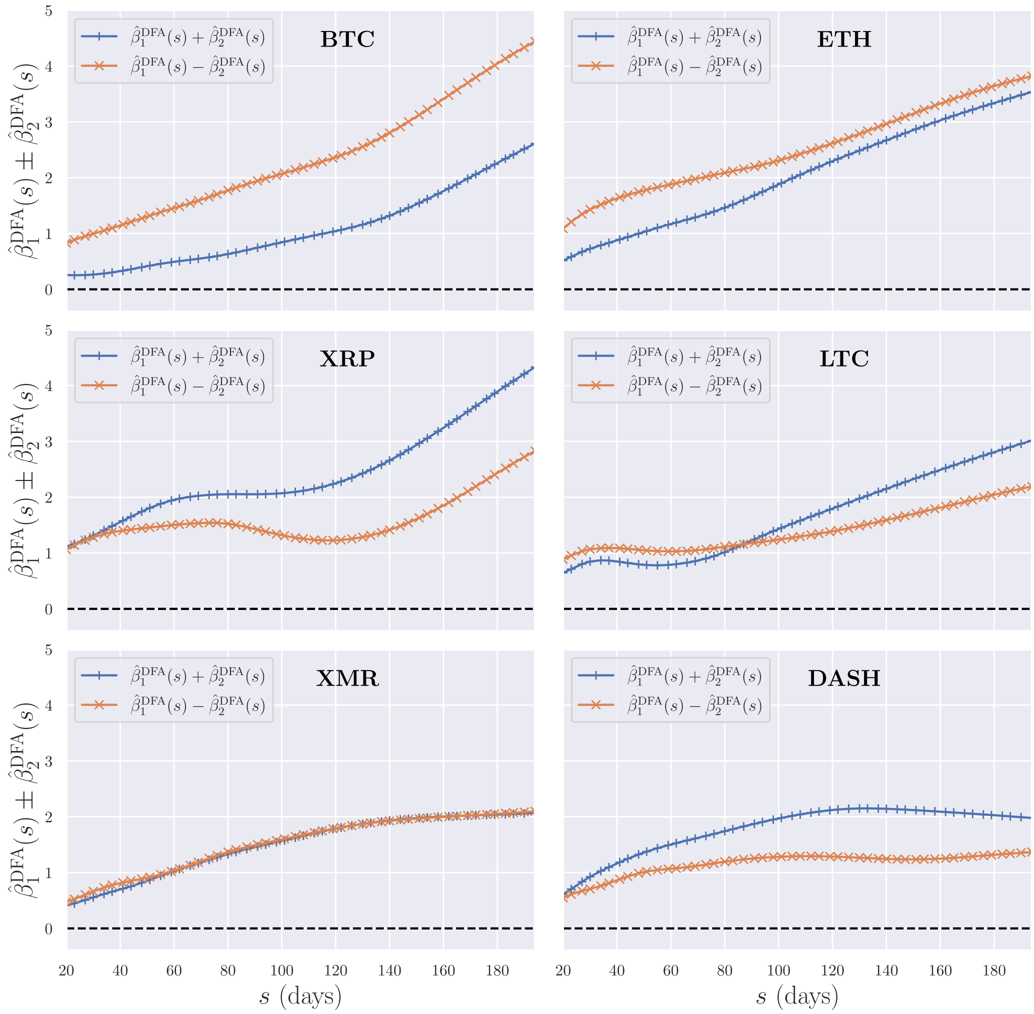

Fig. B.7. The impact of good and bad news to volatility, $\hat{\beta}_1^{\mathrm{DFA}}(s) + \hat{\beta}_2^{\mathrm{DFA}}(s)$ and $\hat{\beta}_1^{\mathrm{DFA}}(s) - \hat{\beta}_2^{\mathrm{DFA}}(s)$, respectively. We show the results based on VWAP for each cryptocurrency; BTC, ETH, XRP, LTC, XMR, and DASH.

Appendix B. Estimation results using volume-weighted average daily prices

See Figs. B.6 and B.7.

References

Al-Yahyaee, K.H., Mensi, W., Yoon, S.-M., 2018. Efficiency, multifractality, and the long-memory property of the Bitcoin market: A comparative analysis with stock, currency, and gold markets. Finance Res. Lett. 27, 228–234. http://dx.doi.org/10.1016/j.frl.2018.03.017.

Alvarez-Ramirez, J., Rodriguez, E., Ibarra-Valdez, C., 2018. Long-range correlations and asymmetry in the Bitcoin market. Physica A 492, 948–955. http://dx.doi.org/10.1016/j.physa.2017.11.025.

Antoniou, A., Holmes, P., Priestley, R., 1998. The effects of stock index futures trading on stock index volatility: An analysis of the asymmetric response of volatility to news. J. Futures Mark. (1986-1998) 18 (2), 151.

Avramov, D., Chordia, T., Goyal, A., 2006. The impact of trades on daily volatility. Rev. Financ. Stud. 19 (4), 1241–1277. http://dx.doi.org/10.1093/rfs/hhj027.

Bandi, F.M., Russell, J.R., 2006. Separating microstructure noise from volatility. J. Financ. Econ. 79 (3), 655–692. http://dx.doi.org/10.1016/j.jfineco.2005.01.005.

Bariviera, A.F., Basgall, M.J., Hasperué, W., Naiouf, M., 2017. Some stylized facts of the Bitcoin market. Physica A 484, 82–90. http://dx.doi.org/10.1016/j.physa.2017.04.159.

Barndorff-Nielsen, O.E., Shephard, N., 2002. Estimating quadratic variation using realized variance. J. Appl. Econometrics 17 (5), 457–477. http://dx.doi.org/10.1002/jae.691.

Baur, D.G., Dimpfl, T., 2018. Asymmetric volatility in cryptocurrencies. Econom. Lett. 173, 148–151. http://dx.doi.org/10.1016/j.econlet.2018.10.008.

Begušić, S., Kostanjčar, Z., Stanley, H.E., Podobnik, B., 2018. Scaling properties of extreme price fluctuations in Bitcoin markets. Physica A 510, 400–406. http://dx.doi.org/10.1016/j.physa.2018.06.131.

Bentes, S.R., 2018. Is stock market volatility asymmetric? A multi-period analysis for five countries. Physica A 499, 258–265. http://dx.doi.org/10.1016/j.physa.2018.02.031.

Black, F., 1976. Studies of stock market volatility changes. In: 1976 Proceedings of the American Statistical Association Business and Economic Statistics Section. American Statistical Association.

Bollerslev, T., Kretschmer, U., Pigorsch, C., Tauchen, G., 2009. A discrete-time model for daily S&P500 returns and realized variations: Jumps and leverage effects. J. Econometrics 150 (2), 151–166. http://dx.doi.org/10.1016/j.jeconom.2008.12.001.

Bouri, E., Azzi, G., Dyhrberg, A.H., 2017a. On the return-volatility relationship in the Bitcoin market around the price crash of 2013. Economics 11 (1), http://dx.doi.org/10.5018/economics-ejournal.ja.2017-2.

Bouri, E., Gil-Alana, L.A., Gupta, R., Roubaud, D., 2018. Modelling long memory volatility in the Bitcoin market: Evidence of persistence and structural breaks. Int. J. Finance Econ. 24 (1), 412–426. http://dx.doi.org/10.1002/ijfe.1670.

Bouri, E., Molnár, P., Azzi, G., Roubaud, D., Hagfors, L.I., 2017b. On the hedge and safe haven properties of Bitcoin: Is it really more than a diversifier? Finance Res. Lett. 20, 192–198. http://dx.doi.org/10.1016/j.frl.2016.09.025.

Campbell, J.Y., Hentschel, L., 1992. No news is good news: An asymmetric model of changing volatility in stock returns. J. Financ. Econ. 31 (3), 281–318. http://dx.doi.org/10.1016/0304-405x(92)90037-x.

Cao, G., Xie, W., 2021. The impact of the shutdown policy on the asymmetric interdependence structure and risk transmission of cryptocurrency and China's financial market. North Am. J. Econ. Finance 58, 101514. http://dx.doi.org/10.1016/j.najef.2021.101514.

Cheah, E.-T., Mishra, T., Parhi, M., Zhang, Z., 2018. Long memory interdependency and inefficiency in bitcoin markets. Econom. Lett. 167, 18–25. http://dx.doi.org/10.1016/j.econlet.2018.02.010.

Cheikh, N.B., Zaied, Y.B., Chevallier, J., 2020. Asymmetric volatility in cryptocurrency markets: New evidence from smooth transition GARCH models. Finance Res. Lett. 35, 101293. http://dx.doi.org/10.1016/j.frl.2019.09.008.

Chen, Y.-F., Mu, X., 2021. Asymmetric volatility in commodity markets. J. Commodity Mark. 22, 100139. http://dx.doi.org/10.1016/j.jcomm.2020.100139.

Cheung, Y.-W., Ng, L.K., 1992. Stock price dynamics and firm size: An empirical investigation. J. Finance 47 (5), 1985–1997. http://dx.doi.org/10.1111/j.1540-6261.1992.tb04693.x.

Christie, A., 1982. The stochastic behavior of common stock variances value, leverage and interest rate effects. J. Financ. Econ. 10 (4), 407–432. http://dx.doi.org/10.1016/0304-405x(82)90018-6.

da Silva Filho, A.C., Maganini, N.D., de Almeida, E.F., 2018. Multifractal analysis of Bitcoin market. Physica A 512, 954–967. http://dx.doi.org/10.1016/j.physa.2018.08.076.

Diniz-Maganini, N., Diniz, E.H., Rasheed, A.A., 2021. Bitcoin's price efficiency and safe haven properties during the COVID-19 pandemic: A comparison. Res. Int. Bus. Finance 58, 101472. http://dx.doi.org/10.1016/j.ribaf.2021.101472.

Drozdz, S., Gebarowski, R., Minati, L., Oswiecimka, P., Watorek, M., 2018. Bitcoin market route to maturity? Evidence from return fluctuations, temporal correlations and multiscaling effects. Chaos 28 (7), 071101. http://dx.doi.org/10.1063/1.5036517.

Dyhrberg, A.H., 2016a. Bitcoin, gold and the dollar – A GARCH volatility analysis. Finance Res. Lett. 16, 85–92. http://dx.doi.org/10.1016/j.frl.2015.10.008.

Dyhrberg, A.H., 2016b. Hedging capabilities of bitcoin. Is it the virtual gold? Finance Res. Lett. 16, 139–144. http://dx.doi.org/10.1016/j.frl.2015.10.025.

Easley, D., Kiefer, N.M., O'hara, M., Paperman, J.B., 1996. Liquidity, information, and infrequently traded stocks. J. Finance 51 (4), 1405–1436. http://dx.doi.org/10.1111/j.1540-6261.1996.tb04074.x.

Fakhfekh, M., Hachicha, N., Jawadi, F., Selmi, N., Cheffou, A.I., 2016. Measuring volatility persistence for conventional and Islamic banks: An FI-EGARCH approach. Emerg. Mark. Rev 27, 84–99. http://dx.doi.org/10.1016/j.ememar.2016.03.004.

Fakhfekh, M., Jeribi, A., 2020. Volatility dynamics of crypto-currencies' returns: Evidence from asymmetric and long memory GARCH models. Res. Int. Bus. Finance 51, 101075. http://dx.doi.org/10.1016/j.ribaf.2019.101075.

Fan, Q., Wang, F., 2020. Detrending-moving-average-based bivariate regression estimator. Phys. Rev. E 102 (1), http://dx.doi.org/10.1103/physreve.102.012218.

Glosten, L.R., Jagannathan, R., Runkle, D.E., 1993. On the relation between the expected value and the volatility of the nominal excess return on stocks. J. Finance 48 (5), 1779–1801. http://dx.doi.org/10.1111/j.1540-6261.1993.tb05128.x.

Glosten, L.R., Milgrom, P.R., 1985. Bid, ask and transaction prices in a specialist market with heterogeneously informed traders. J. Financial Econ. 14 (1), 71–100. http://dx.doi.org/10.1016/0304-405x(85)90044-3.

Hasanhodzic, J., Lo, A.W., 2019. On black's leverage effect in firms with no leverage. J. Portfolio Manag. 46 (1), 106–122. http://dx.doi.org/10.3905/jpm.2019.46.1.106.

Hens, T., Steude, S.C., 2009. The leverage effect without leverage. Finance Res. Lett. 6 (2), 83–94. http://dx.doi.org/10.1016/j.frl.2009.01.002.

Jeribi, A., Fakhfekh, M., Jarboui, A., 2015. Tunisian revolution and stock market volatility: evidence from FIEGARCH model. In: Dr Scott Beyer, D.G.J., Johnson, D.R. (Eds.), Managerial Finance 41 (10), 1112–1135. http://dx.doi.org/10.1108/mf-12-2014-0310.

Jiang, Y., Nie, H., Ruan, W., 2018. Time-varying long-term memory in Bitcoin market. Finance Res. Lett. 25, 280–284. http://dx.doi.org/10.1016/j.frl.2017.12.009.

Kakinaka, S., Umeno, K., 2021. Exploring asymmetric multifractal cross-correlations of price–volatility and asymmetric volatility dynamics in cryptocurrency markets. Physica A 581, 126237. http://dx.doi.org/10.1016/j.physa.2021.126237.

Kantelhardt, J.W., Zschiegner, S.A., Koscielny-Bunde, E., Havlin, S., Bunde, A., Stanley, H., 2002. Multifractal detrended fluctuation analysis of nonstationary time series. Physica A 316 (1–4), 87–114. http://dx.doi.org/10.1016/s0378-4371(02)01383-3.

Katsiampa, P., 2017. Volatility estimation for Bitcoin: A comparison of GARCH models. Econom. Lett. 158, 3–6. http://dx.doi.org/10.1016/j.econlet.2017.06.023.

Kliber, A., 2016. The leverage effect puzzle: the case of European sovereign credit default swap market. Rev. Derivatives Res. 19 (3), 217–235. http://dx.doi.org/10.1007/s11147-016-9121-3.

Kristjanpoller, W., Bouri, E., 2019. Asymmetric multifractal cross-correlations between the main world currencies and the main cryptocurrencies. Physica A 523, 1057–1071. http://dx.doi.org/10.1016/j.physa.2019.04.115.

Kristjanpoller, W., Bouri, E., Takaishi, T., 2020. Cryptocurrencies and equity funds: Evidence from an asymmetric multifractal analysis. Physica A 545, 123711. http://dx.doi.org/10.1016/j.physa.2019.123711.

Kristoufek, L., 2015. Detrended fluctuation analysis as a regression framework: Estimating dependence at different scales. Phys. Rev. E 91 (2), http://dx.doi.org/10.1103/physreve.91.022802.

Liu, J., Chen, F., 2018. Asymmetric volatility varies in different dry bulk freight rate markets under structure breaks. Physica A 505, 316–327. http://dx.doi.org/10.1016/j.physa.2018.02.165.

Liu, L.Y., Patton, A.J., Sheppard, K., 2015. Does anything beat 5-minute RV? A comparison of realized measures across multiple asset classes. J. Econometrics 187 (1), 293–311. http://dx.doi.org/10.1016/j.jeconom.2015.02.008.

Mensi, W., Al-Yahyaee, K.H., Kang, S.H., 2019a. Structural breaks and double long memory of cryptocurrency prices: A comparative analysis from Bitcoin and Ethereum. Finance Res. Lett. 29, 222–230. http://dx.doi.org/10.1016/j.frl.2018.07.011.

Mensi, W., Lee, Y.-J., Al-Yahyaee, K.H., Sensoy, A., Yoon, S.-M., 2019b. Intraday downward/upward multifractality and long memory in Bitcoin and Ethereum markets: An asymmetric multifractal detrended fluctuation analysis. Finance Res. Lett. 31, 19–25. http://dx.doi.org/10.1016/j.frl.2019.03.029.

Nakamoto, S., 2008. Bitcoin: A peer-to-peer electronic cash system. Decentralized Bus. Rev. 21260.

Nelson, D.B., 1991. Conditional heteroskedasticity in asset returns: A new approach. Econometrica 59 (2), 347. http://dx.doi.org/10.2307/2938260.

Peng, C.-K., Buldyrev, S.V., Havlin, S., Simons, M., Stanley, H.E., Goldberger, A.L., 1994. Mosaic organization of DNA nucleotides. Phys. Rev. E 49 (2), 1685–1689. http://dx.doi.org/10.1103/physreve.49.1685.

Peters, E.E., 1994. Fractal Market Analysis: Applying Chaos Theory to Investment and Economics. Vol. 24, John Wiley & Sons.

Podobnik, B., Jiang, Z.-Q., Zhou, W.-X., Stanley, H.E., 2011. Statistical tests for power-law cross-correlated processes. Phys. Rev. E 84 (6), http://dx.doi.org/10.1103/physreve.84.066118.

Podobnik, B., Stanley, H.E., 2008. Detrended cross-correlation analysis: A new method for analyzing two nonstationary time series. Phys. Rev. Lett. 100 (8), http://dx.doi.org/10.1103/physrevlett.100.084102.

Schwert, G.W., 1989. Why does stock market volatility change over time? J. Finance 44 (5), 1115–1153. http://dx.doi.org/10.1111/j.1540-6261.1989.tb02647.x.

Takaishi, T., 2018. Statistical properties and multifractality of Bitcoin. Physica A 506, 507–519. http://dx.doi.org/10.1016/j.physa.2018.04.046.

Takaishi, T., 2021. Time-varying properties of asymmetric volatility and multifractality in Bitcoin. In: Segovia, J.E.T. (Ed.), PlOS ONE 16 (2), e0246209. http://dx.doi.org/10.1371/journal.pone.0246209.

Telli, Ş., Chen, H., 2020. Multifractal behavior in return and volatility series of Bitcoin and gold in comparison. Chaos Solitons Fractals 139, 109994. http://dx.doi.org/10.1016/j.chaos.2020.109994.

Tilfani, O., Ferreira, P., Boukfaoui, M.Y.E., 2019. Building multi-scale portfolios and efficient market frontiers using fractal regressions. Phys. A: Stat. Mech. Appl. 532, 121758. http://dx.doi.org/10.1016/j.physa.2019.121758.

Tilfani, O., Kristoufek, L., Ferreira, P., Boukfaoui, M.Y.E., 2022. Heterogeneity in economic relationships: Scale dependence through the multivariate fractal regression. Physica A 588, 126530. http://dx.doi.org/10.1016/j.physa.2021.126530.

Tversky, A., Kahneman, D., 1992. Advances in prospect theory: Cumulative representation of uncertainty. J. Risk Uncertainty 5 (4), 297–323. http://dx.doi.org/10.1007/bf00122574.

Urquhart, A., 2016. The inefficiency of Bitcoin. Econom. Lett. 148, 80–82. http://dx.doi.org/10.1016/j.econlet.2016.09.019.

Wang, F., Wang, L., Chen, Y., 2018. A DFA-based bivariate regression model for estimating the dependence of PM2.5 among neighbouring cities. Sci. Rep. 8 (1), http://dx.doi.org/10.1038/s41598-018-25822-w.

Watorek, M., Drożdż, S., Kwapień, J., Minati, L., Oświecimka, P., Stanuszek, M., 2021. Multiscale characteristics of the emerging global cryptocurrency market. Phys. Rep. 901, 1–82. http://dx.doi.org/10.1016/j.physrep.2020.10.005.

Zebende, G.F., 2011. DCCA cross-correlation coefficient: Quantifying level of cross-correlation. Physica A 390 (4), 614–618. http://dx.doi.org/10.1016/j.physa.2010.10.022.

Zhang, X., Yang, L., Zhu, Y., 2019. Analysis of multifractal characterization of Bitcoin market based on multifractal detrended fluctuation analysis. Physica A 523, 973–983. http://dx.doi.org/10.1016/j.physa.2019.04.149.