

THE GHOST EFFECT IN THE CONTINUUM LIMIT FOR A VAPOR–GAS MIXTURE AROUND CONDENSED PHASES: ASYMPTOTIC ANALYSIS OF THE BOLTZMANN EQUATION

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ABSTRACT

A binary mixture of a vapor and a noncondensable gas around arbitrarily shaped condensed phases of the vapor is considered. Its steady behavior in the continuum limit (the limit where the Knudsen number vanishes) is investigated on the basis of kinetic theory in the case where the condensed phases are at rest, and the mixture is in a state at rest with a uniform pressure at infinity when an infinite domain is considered. A systematic asymptotic analysis of the Boltzmann equation with kinetic boundary condition is carried out for small Knudsen numbers, and the system of fluid-dynamic type equations and their appropriate boundary conditions that describes the behavior in the continuum limit is derived. The system shows that the flow of the mixture vanishes in the continuum limit, but the vanishing flow gives a finite effect on the behavior of the mixture in this limit. This is an example of the ghost effect discovered recently by Sone and coworkers [e.g., Y. Sone *et al.*, *Phys. Fluids* **8**, 628 and 3403 (1996); Y. Sone, in *Rarefied Gas Dynamics*, edited by C. Shen (Peking University Press, Beijing, 1997), p. 3]. It is shown that there are several new source factors of the ghost effect that are peculiar to a gas mixture, i.e., that originate from the nonuniformity of the concentration.

1 INTRODUCTION

The Navier–Stokes system (the conservation equations for the mass, momentum, and energy with Newton’s law of stress and Fourier’s law of heat flow and

their boundary conditions of nonslip or nonjump type) is commonly accepted as the correct system for describing the behavior of a gas under ordinary conditions or in the continuum limit (i.e., the limit where the Knudsen number, the mean free path of the gas molecules divided by the characteristic length of the system, vanishes). According to the system, the temperature field in a gas at rest with a uniform pressure (e.g., the gas in a resting vessel with a steady but arbitrary temperature distribution in the absence of external forces) is determined by the steady heat-conduction equation with the nonjump condition on the boundary. However, recent study¹ based on kinetic theory revealed that the heat-conduction equation (thus, the Navier–Stokes system) does not give the correct temperature field in the gas even in the continuum limit. In this study, a systematic asymptotic analysis of the Boltzmann equation and its boundary condition was carried out, and the system of fluid-dynamic type equations and their boundary conditions that gives the correct temperature field in a gas at rest in the continuum limit was derived. On the basis of the system, the cause of the invalidity of the Navier–Stokes system was clarified. These findings gave kinetic theory a new important role of checking the validity of the classical fluid dynamics (for a gas) in the continuum limit.

The invalidity of the heat-conduction equation is due to the fact that the flows caused by the effect of gas rarefaction (or more precisely, the gas flows of the order of the Knudsen number²) give a finite effect on the temperature field even in the continuum limit where the flows themselves vanish. This effect was termed the ghost effect³ because it is an effect of nonexistent flows in the world of continuum fluid dynamics (see also Ref. 4). In a certain situation, the ghost effect also appears in the velocity field as well as the temperature field. Such an example is shown in the cylindrical Couette flow of a vapor between two coaxial circular cylinders consisting of its condensed phase.⁵

In a single-component gas, as was clarified in Ref. 1, the flow of the order of the Knudsen number, which is the source of the ghost effect, generally consists of the following three types of flow: the flow caused by the temperature gradient along the boundary (thermal creep flow),^{6–9} that by the thermal stress in the gas (nonlinear thermal stress flow),^{10,11,1} and that by the boundary motion (in its surface)¹² with a

speed of the order of the Knudsen number.¹³ In a mixture of gases, there seem to be other types of flow that can cause the ghost effect naturally in the continuum limit, such as the flows due to the diffusion, concentration stress,^{14,11} and concentration gradient along the boundary (diffusion slip),¹⁵⁻¹⁸ in addition to the three types of flow mentioned above. Therefore, the ghost effect is expected to manifest itself in a wider class of problems. In fact, we have demonstrated the effect in a simple one-dimensional problem^{19,20} where it is absent for a single-component gas.²¹ To be more specific, we have considered the behavior of a mixture of vapors and noncondensable gases between two parallel condensed phases. In this problem, the flow caused by evaporation and condensation vanishes in the continuum limit. Nevertheless, it gives a finite effect on the motion of the mixture along the condensed phases (when one of the condensed phases is moving in its surface) as well as on the temperature of the total mixture in the limit.

In the present paper, we try to extend the study in Refs. 19 and 20 to the general geometry. That is, we consider a mixture of a vapor and a noncondensable gas around condensed phases of the vapor of arbitrary shapes and investigate the steady behavior of the mixture in the continuum limit on the basis of kinetic theory in the following situation: the condensed phases are at rest, the noncondensable gas is present everywhere in the domain, and the mixture is in a state at rest with a uniform pressure at infinity when an infinite domain is considered. As will be seen, the flow of the mixture vanishes and the ghost effect appears in this situation. The aim of the present study is to obtain the whole picture of the ghost effect in the mixture. But in the course of the study, the gas flows caused by the effect of gas rarefaction will also be clarified. The problem considered here is also an extension of the problem of Ref. 1 to a mixture. The natural extension of Ref. 1 would be a mixture of noncondensable gases, not the mixture of a vapor and a noncondensable gas. The reason why we choose the latter is that in this case the nonuniformity of the concentrations of the components and thus the ghost effect peculiar to a mixture appears more clearly. We will also discuss the former case briefly.

The paper is organized as follows. After the statement of the problem and its formulation in Sec. 2, we carry out a systematic asymptotic analysis of the Boltz-

mann equation for small Knudsen numbers in Sec. 3, where we derive the fluid-dynamic type equations and their boundary conditions for the distributions of the temperature and number densities in the continuum limit as well as the flow of the mixture of the order of the Knudsen number. Then, in Sec. 4, we discuss the features of the flow and the ghost effect in the continuum limit. Some special situations as well as extensions to more general cases are also discussed in Secs. 5–7.

2 PROBLEM AND BASIC EQUATIONS

Consider a binary mixture of a vapor (say, A -component) and a noncondensable gas (say, B -component) around condensed phases of the vapor of arbitrary but smooth shapes (therefore, evaporation and condensation of the A -component may take place on the surfaces of the condensed phases). There is no external force in the system. Clarify the steady behavior of the mixture in the continuum limit, where the Knudsen number tends to zero, on the basis of kinetic theory in the following situation: (i) the condensed phases are at rest; (ii) the noncondensable gas is present everywhere in the domain; and (iii) the mixture is in a state at rest with a uniform pressure at infinity when an infinite domain is considered.

To answer this question, we will investigate the steady behavior of the mixture for small Knudsen numbers on the basis of the Boltzmann equation for hard-sphere molecules assuming the complete condensation condition for the vapor molecules and the diffuse reflection condition for the noncondensable-gas molecules on the surfaces of the condensed phases [see the sentences below Eq. (2.6b) for the details of the boundary conditions].

We first introduce some reference quantities: L is the reference length of the system, T_0 the reference temperature, n_0 the reference number density, and $p_0 = kn_0T_0$ the reference pressure, where k is the Boltzmann constant. Further, m^α and d^α represent the mass and the diameter of a molecule of α -component ($\alpha = A, B$), respectively.

Let us denote the rectangular space coordinates by x_iL , the molecular velocity by $\zeta_i(2kT_0/m^A)^{1/2}$, and the velocity distribution function of α -component

by $n_0(2kT_0/m^A)^{-3/2}f^\alpha$ ($\alpha = A, B$). Then the Boltzmann equation for a binary mixture in the present steady case is written as²²⁻²⁴

$$\zeta_i \frac{\partial f^\alpha}{\partial x_i} = \frac{2}{\sqrt{\pi}} \frac{1}{\text{Kn}} \sum_{\beta=A,B} K^{\beta\alpha} \hat{j}^{\beta\alpha}(f^\beta, f^\alpha), \quad (\alpha = A, B), \quad (2.1)$$

$$\hat{j}^{\beta\alpha}(f, g) = \frac{1}{4\sqrt{2\pi}} \int [f(\zeta'_{*i})g(\zeta'_i) - f(\zeta_{*i})g(\zeta_i)] |a_j \hat{V}_j| d\Omega(a_i) d\zeta_*, \quad (2.2)$$

$$\zeta'_i = \zeta_i + \frac{\hat{\mu}^{\beta\alpha}}{\hat{m}^\alpha} (a_j \hat{V}_j) a_i, \quad \zeta'_{*i} = \zeta_{*i} - \frac{\hat{\mu}^{\beta\alpha}}{\hat{m}^\beta} (a_j \hat{V}_j) a_i, \quad (2.3a)$$

$$\hat{V}_i = \zeta_{*i} - \zeta_i, \quad d\zeta_* = d\zeta_{*1} d\zeta_{*2} d\zeta_{*3}, \quad (2.3b)$$

$$K^{\beta\alpha} = (\hat{d}^{\beta\alpha})^2, \quad \hat{d}^{\beta\alpha} = \frac{\hat{d}^\alpha + \hat{d}^\beta}{2}, \quad \hat{\mu}^{\beta\alpha} = \frac{2\hat{m}^\alpha \hat{m}^\beta}{\hat{m}^\alpha + \hat{m}^\beta}, \quad (2.3c)$$

$$\text{Kn} = \frac{\ell_0}{L}, \quad \ell_0 = \frac{1}{\sqrt{2\pi}(d^A)^2 n_0}, \quad (2.3d)$$

$$\hat{m}^\alpha = m^\alpha/m^A, \quad \hat{d}^\alpha = d^\alpha/d^A, \quad (2.3e)$$

where a_i is a unit vector, ζ_{*i} the variable of integration corresponding to ζ_i , and $d\Omega(a_i)$ the solid angle element in the direction of a_i . The domain of integration in Eq. (2.2) is the whole space of ζ_{*i} and all directions of a_i . In Eq. (2.3d), ℓ_0 is the mean free path of a molecule of A -component when it is in the equilibrium state at rest with molecular number density n_0 , and Kn is the corresponding Knudsen number, which represents the degree of rarefaction of the system. Here and in what follows (except APPENDIX C), the Greek letters α and β are used to represent the labels A and B of the components. Since no confusion is expected, the notes such as $\alpha = A, B$ in Eq. (2.1) will mostly be omitted below.

We now denote the temperature of the surfaces of the condensed phases by $T_0 \hat{T}_w$, their velocity by $u_{wi}(2kT_0/m^A)^{1/2}$, and the saturation pressure of the vapor at temperature $T_0 \hat{T}_w$ by $p_0 \hat{p}_w^A$. Since we are considering the case where the condensed phases are at rest in the continuum limit $\text{Kn} \rightarrow 0$, we assume that

$$u_{wi} = O(\text{Kn}). \quad (2.4)$$

Further, since the problem is steady, $u_{wi} n_i = 0$ is assumed, where n_i is the unit vector normal to the boundary pointing to the gas region. Then the boundary

conditions are written as

$$f^\alpha = \sigma_w^\alpha \hat{T}_w^{-3/2} \left(\frac{\hat{m}^\alpha}{\pi} \right)^{3/2} \exp \left(-\frac{\hat{m}^\alpha (\zeta_i - u_{wi})^2}{\hat{T}_w} \right), \quad \zeta_i n_i > 0, \quad (2.5)$$

with

$$\sigma_w^A = \hat{p}_w^A / \hat{T}_w, \quad (2.6a)$$

$$\sigma_w^B = -2 \left(\frac{\pi \hat{m}^B}{\hat{T}_w} \right)^{1/2} \int_{\zeta_i n_i < 0} \zeta_i n_i f^B d\zeta, \quad (2.6b)$$

where $d\zeta = d\zeta_1 d\zeta_2 d\zeta_3$. Equation (2.5) with $\alpha = A$ means that the vapor molecules leaving the surfaces obey the corresponding part of the Maxwellian distribution characterized by \hat{T}_w , u_{wi} , and \hat{p}_w^A (complete condensation condition). On the other hand, Eq. (2.5) with $\alpha = B$ means that the noncondensable gas molecules reflected by the surfaces obey the corresponding part of the Maxwellian distribution characterized by \hat{T}_w and u_{wi} and that there is no net mass flow of this component across the surfaces (diffuse reflection).

Next, we introduce macroscopic variables as the moments of the velocity distribution functions. For each component, we define its number density $n_0 \hat{n}^\alpha$, density $n_0 m^A \hat{\rho}^\alpha$, flow velocity $u_i^\alpha (2kT_0/m^A)^{1/2}$, temperature $T_0 \hat{T}^\alpha$, partial pressure $p_0 \hat{p}^\alpha$, stress tensor $p_0 \hat{p}_{ij}^\alpha$, and heat-flow vector $p_0 (2kT_0/m^A)^{1/2} \hat{q}_i^\alpha$ by²⁵

$$\begin{aligned} \hat{n}^\alpha &= \int f^\alpha d\zeta, & \hat{\rho}^\alpha &= \hat{m}^\alpha \int f^\alpha d\zeta (= \hat{m}^\alpha \hat{n}^\alpha), \\ u_i^\alpha &= \frac{1}{\hat{n}^\alpha} \int \zeta_i f^\alpha d\zeta, & \hat{T}^\alpha &= \frac{2}{3} \frac{\hat{m}^\alpha}{\hat{n}^\alpha} \int (\zeta_i - u_i^\alpha)^2 f^\alpha d\zeta, \\ \hat{p}^\alpha &= \frac{2}{3} \hat{m}^\alpha \int (\zeta_i - u_i^\alpha)^2 f^\alpha d\zeta (= \hat{n}^\alpha \hat{T}^\alpha), \\ \hat{p}_{ij}^\alpha &= 2 \hat{m}^\alpha \int (\zeta_i - u_i^\alpha)(\zeta_j - u_j^\alpha) f^\alpha d\zeta, \\ \hat{q}_i^\alpha &= \hat{m}^\alpha \int (\zeta_i - u_i^\alpha)(\zeta_j - u_j^\alpha)^2 f^\alpha d\zeta. \end{aligned} \quad (2.7)$$

Here and in what follows, the domain of integration with respect to ζ_i is its whole space, unless otherwise stated. On the other hand, for the total mixture, the number density $n_0 \hat{n}$, density $n_0 m^A \hat{\rho}$, flow velocity $u_i (2kT_0/m^A)^{1/2}$, temperature $T_0 \hat{T}$, pressure $p_0 \hat{p}$, stress tensor $p_0 \hat{p}_{ij}$, and heat-flow vector $p_0 (2kT_0/m^A)^{1/2} \hat{q}_i$ are defined

by

$$\begin{aligned}
\hat{n} &= \int \sum_{\beta=A,B} f^\beta d\zeta, & \hat{\rho} &= \int \sum_{\beta=A,B} \hat{m}^\beta f^\beta d\zeta, \\
u_i &= \frac{1}{\hat{\rho}} \int \zeta_i \sum_{\beta=A,B} \hat{m}^\beta f^\beta d\zeta, & \hat{T} &= \frac{2}{3\hat{n}} \int (\zeta_i - u_i)^2 \sum_{\beta=A,B} \hat{m}^\beta f^\beta d\zeta, \\
\hat{p} &= \frac{2}{3} \int (\zeta_i - u_i)^2 \sum_{\beta=A,B} \hat{m}^\beta f^\beta d\zeta (= \hat{n}\hat{T}), \\
\hat{p}_{ij} &= 2 \int (\zeta_i - u_i)(\zeta_j - u_j) \sum_{\beta=A,B} \hat{m}^\beta f^\beta d\zeta, \\
\hat{q}_i &= \int (\zeta_i - u_i)(\zeta_j - u_j)^2 \sum_{\beta=A,B} \hat{m}^\beta f^\beta d\zeta.
\end{aligned} \tag{2.8}$$

Thus the macroscopic variables for the total mixture are expressed in terms of those for individual components as follows:

$$\begin{aligned}
\hat{n} &= \sum_{\beta=A,B} \hat{n}^\beta, & \hat{\rho} &= \sum_{\beta=A,B} \hat{\rho}^\beta, & \hat{\rho}u_i &= \sum_{\beta=A,B} \hat{\rho}^\beta u_i^\beta, \\
\hat{p} &= \sum_{\beta=A,B} \left[\hat{p}^\beta + \frac{2}{3} \hat{\rho}^\beta (u_i^\beta - u_i)^2 \right], \\
\hat{p}_{ij} &= \sum_{\beta=A,B} \left[\hat{p}_{ij}^\beta + 2\hat{\rho}^\beta (u_i^\beta - u_i)(u_j^\beta - u_j) \right], \\
\hat{q}_i &= \sum_{\beta=A,B} \left[\hat{q}_i^\beta + \hat{p}_{ij}^\beta (u_j^\beta - u_j) + \frac{3}{2} \hat{p}^\beta (u_i^\beta - u_i) + \hat{\rho}^\beta (u_i^\beta - u_i)(u_j^\beta - u_j)^2 \right].
\end{aligned} \tag{2.9}$$

3 ASYMPTOTIC ANALYSIS

In this section, we carry out asymptotic analysis of the boundary-value problem (2.1)–(2.6) for small Knudsen numbers in the situation consistent with the continuum limit described in the first paragraph of Sec. 2. Following Refs. 26–30 and 20 as a guideline, we derive the fluid-dynamic type equations and their appropriate boundary conditions. For convenience, we use the following small parameter ϵ :

$$\epsilon = \frac{\sqrt{\pi}}{2} \text{Kn}, \tag{3.1}$$

instead of the Knudsen number Kn in the analysis.

3.1 Hilbert Solution

Let us first seek the moderately varying solutions f_H^α [$\partial f_H^\alpha / \partial x_i = O(f_H^\alpha)$] of the Boltzmann equation (2.1) in a power series of ϵ :

$$f_H^\alpha = f_{H0}^\alpha + f_{H1}^\alpha \epsilon + f_{H2}^\alpha \epsilon^2 + \dots \quad (3.2)$$

Correspondingly, the macroscopic variables are expanded as

$$\begin{aligned} h_H^\alpha &= h_{H0}^\alpha + h_{H1}^\alpha \epsilon + h_{H2}^\alpha \epsilon^2 + \dots, \\ h_H &= h_{H0} + h_{H1} \epsilon + h_{H2} \epsilon^2 + \dots, \\ (h &= \hat{n}, \hat{p}, \hat{\rho}, \hat{T}, u_i, \hat{p}_{ij}, \text{ or } \hat{q}_i), \end{aligned} \quad (3.3)$$

where h_H^α and h_H are defined by Eqs. (2.7) and (2.8) with f^α replaced by f_H^α , and expansion coefficients h_{Hm}^α and h_{Hm} are obtained by substituting the expansions (3.2) and (3.3) into the definitions of h_H^α and h_H . This solution (or expansion) is called the Hilbert solution (or expansion). Substituting Eq. (3.2) into Eq. (2.1), we obtain the following series of integral equations:

$$\sum_{\beta=A,B} K^{\beta\alpha} \hat{J}^{\beta\alpha}(f_{H0}^\beta, f_{H0}^\alpha) = 0, \quad (3.4)$$

$$\begin{aligned} &\sum_{\beta=A,B} K^{\beta\alpha} [\hat{J}^{\beta\alpha}(f_{Hm}^\beta, f_{H0}^\alpha) + \hat{J}^{\beta\alpha}(f_{H0}^\beta, f_{Hm}^\alpha)] \\ &= \zeta_i \frac{\partial f_{Hm-1}^\alpha}{\partial x_i} - \sum_{\beta=A,B} K^{\beta\alpha} \sum_{n=1}^{m-1} \hat{J}^{\beta\alpha}(f_{Hm-n}^\beta, f_{Hn}^\alpha), \end{aligned} \quad (3.5)$$

where $m = 1, 2, \dots$ and $\sum_1^0 = 0$ in Eq. (3.5) with $m = 1$. Equation (3.4) is the system of nonlinear integral equations for f_{H0}^α , while Eq. (3.5) is that of inhomogeneous linear integral equations for f_{Hm}^α . The series of equations can, in principle, be solved successively from the lowest order.

The solution of Eq. (3.4) is given by local equilibrium distributions,²³ which can be expressed as

$$f_{H0}^\alpha = \hat{n}_{H0}^\alpha \hat{T}_{H0}^{-3/2} \left(\frac{\hat{m}^\alpha}{\pi} \right)^{3/2} \exp \left(- \frac{\hat{m}^\alpha (\zeta_i - u_{iH0})^2}{\hat{T}_{H0}} \right), \quad (3.6)$$

by the use of the leading-order terms \hat{n}_{H0}^α , u_{iH0} , and \hat{T}_{H0} of the expansions (3.3). For this distribution, the flow velocity and the temperature of each component are

the same as those of the total mixture:

$$u_{iH0}^\alpha = u_{iH0}, \quad (3.7a)$$

$$\hat{T}_{H0}^\alpha = \hat{T}_{H0}. \quad (3.7b)$$

In addition, the following relations hold:

$$\hat{p}_{ijH0}^\alpha = \hat{p}_{H0}^\alpha \delta_{ij}, \quad \hat{q}_{iH0}^\alpha = 0, \quad (3.8a)$$

$$\hat{p}_{ijH0} = \hat{p}_{H0} \delta_{ij}, \quad \hat{q}_{iH0} = 0. \quad (3.8b)$$

The homogeneous system corresponding to Eq. (3.5) has nontrivial solutions (f_{H0}^A, cf_{H0}^B) (c : arbitrary constant), $(\hat{m}^A \zeta_i f_{H0}^A, \hat{m}^B \zeta_i f_{H0}^B)$, and $(\hat{m}^A \zeta_j^2 f_{H0}^A, \hat{m}^B \zeta_j^2 f_{H0}^B)$. Therefore, in order that the system (3.5) has a solution, its inhomogeneous terms should satisfy the compatibility conditions:

$$\int [\text{R.H.S. of Eq. (3.5)}] d\zeta = 0, \quad (3.9a)$$

$$\sum_{\alpha=A,B} \int \hat{m}^\alpha \begin{pmatrix} \zeta_j \\ \zeta_k^2 \end{pmatrix} [\text{R.H.S. of Eq. (3.5)}] d\zeta = 0, \quad (3.9b)$$

which are reduced to

$$\frac{\partial}{\partial x_i} \int \zeta_i f_{Hm-1}^\alpha d\zeta = 0, \quad (3.10a)$$

$$\frac{\partial}{\partial x_i} \sum_{\beta=A,B} \hat{m}^\beta \int \zeta_i \zeta_j f_{Hm-1}^\beta d\zeta = 0, \quad (3.10b)$$

$$\frac{\partial}{\partial x_i} \sum_{\beta=A,B} \hat{m}^\beta \int \zeta_i \zeta_k^2 f_{Hm-1}^\beta d\zeta = 0, \quad (3.10c)$$

where $m = 1, 2, \dots$. Then, the solution to Eq. (3.5) can be expressed by the sum of the solution of their homogeneous equations (the linear combination of the nontrivial solutions), which contains \hat{n}_{Hm}^α , u_{iHm} , and \hat{T}_{Hm} as well as \hat{n}_{Hn}^α , u_{iHn} , and \hat{T}_{Hn} ($n < m$), and a particular solution, which contains derivatives of \hat{n}_{Hn}^α , u_{iHn} , and \hat{T}_{Hn} ($n < m$). The substitution of the explicit form of f_{Hm}^α into Eq. (3.10) for $m = 1, 2, \dots$ gives a set of partial differential equations for the macroscopic variables [more precisely, that for coefficient functions in the Hilbert expansion (3.3) of the macroscopic variables]. This is the set of fluid-dynamic type equations.

In the present analysis, we further assume that

$$u_{iH0}^B \equiv 0. \quad (3.11)$$

Since the situation (ii) in the first paragraph of Sec. 2 corresponds to the assumption that $\hat{n}_{H0}^B \neq 0$ everywhere, Eqs. (3.7a) and (3.11) lead to

$$u_{iH0} = u_{iH0}^A \equiv 0, \quad (3.12)$$

everywhere. [If there is a region where $\hat{n}_{H0}^B \equiv 0$, u_{iH0}^B does not have a sense and thus u_{iH0}^A in general does not vanish there.³¹ Such a region may exist. In the present study, however, we only consider the case without such a region.] In this situation, the analysis for the Hilbert expansion can be carried out consistently, and the result is also consistent with the situation (iii) in the first paragraph of Sec. 2. [As will be seen in Sec. 3.2, the matching with the boundary condition can also be made consistently under the condition (2.4).]

We here summarize the explicit form of the fluid-dynamic type equations that are necessary to determine the leading-order solution f_{H0}^α under the condition (3.11) and (3.12):

$$\frac{\partial \hat{p}_{H0}}{\partial x_i} = 0, \quad (3.13)$$

$$\frac{\partial \hat{p}_{H1}}{\partial x_i} = 0, \quad (3.14)$$

$$\frac{\partial}{\partial x_i} (\hat{n}_{H0}^\alpha u_{iH1}^\alpha) = 0, \quad (3.15)$$

$$\hat{\rho}_{H0} u_{iH1} \frac{\partial u_{jH1}}{\partial x_i} = -\frac{1}{2} \frac{\partial \hat{p}_{ijH2}}{\partial x_i}, \quad (3.16)$$

$$\frac{\partial}{\partial x_i} (\hat{p}_{H0} u_{iH1}) = -\frac{2}{5} \frac{\partial \hat{q}_{iH1}}{\partial x_i}, \quad (3.17)$$

where

$$u_{iH1}^A - u_{iH1}^B = -\frac{\hat{T}_{H0}^{1/2}}{\hat{n}_{H0}} \frac{\hat{D}_{AB}}{X_{H0}^A X_{H0}^B} \left(\frac{\partial X_{H0}^A}{\partial x_i} + k_T \frac{\partial \ln \hat{T}_{H0}}{\partial x_i} \right), \quad (3.18)$$

$$\begin{aligned}
\hat{p}_{ijH2} = \hat{p}_{H2}\delta_{ij} & - \hat{\mu}\hat{T}_{H0}^{1/2}\overline{\frac{\partial u_{iH1}}{\partial x_j}} + \frac{\hat{\Upsilon}_1}{\hat{p}_{H0}}\overline{\frac{\partial\hat{T}_{H0}}{\partial x_i}\frac{\partial\hat{T}_{H0}}{\partial x_j}} + \frac{1}{\hat{n}_{H0}}\overline{\frac{\partial}{\partial x_i}\left(\hat{\Upsilon}_2\frac{\partial\hat{T}_{H0}}{\partial x_j}\right)} \\
& + \frac{\hat{T}_{H0}}{\hat{n}_{H0}}\hat{\Upsilon}_3\overline{\frac{\partial X_{H0}^A}{\partial x_i}\frac{\partial X_{H0}^A}{\partial x_j}} + \frac{1}{\hat{n}_{H0}}\hat{\Upsilon}_4\overline{\frac{\partial X_{H0}^A}{\partial x_i}\frac{\partial\hat{T}_{H0}}{\partial x_j}} \\
& + \frac{\hat{T}_{H0}}{\hat{n}_{H0}}\overline{\frac{\partial}{\partial x_i}\left(\hat{\Upsilon}_5\frac{\partial X_{H0}^A}{\partial x_j}\right)}, \tag{3.19}
\end{aligned}$$

$$\begin{aligned}
\hat{q}_{iH1} & = -\hat{T}_{H0}^{3/2}\left(\frac{\hat{D}_T}{X_{H0}^A X_{H0}^B}\frac{\partial X_{H0}^A}{\partial x_i} + \hat{\lambda}'\frac{\partial\ln\hat{T}_{H0}}{\partial x_i}\right) + \frac{5}{2}\sum_{\beta=A,B}\hat{p}_{H0}^\beta(u_{iH1}^\beta - u_{iH1}) \\
& = -\hat{\lambda}\hat{T}_{H0}^{1/2}\frac{\partial\hat{T}_{H0}}{\partial x_i} + k_T\hat{p}_{H0}(u_{iH1}^A - u_{iH1}^B) + \frac{5}{2}\sum_{\beta=A,B}\hat{p}_{H0}^\beta(u_{iH1}^\beta - u_{iH1}) \\
& = -\hat{\lambda}\hat{T}_{H0}^{1/2}\frac{\partial\hat{T}_{H0}}{\partial x_i} + [k_T\hat{p}_{H0} + \frac{5}{2}\hat{\rho}_{H0}^{-1}(\hat{p}_{H0}^A\hat{\rho}_{H0}^B - \hat{\rho}_{H0}^A\hat{p}_{H0}^B)](u_{iH1}^A - u_{iH1}^B), \tag{3.20}
\end{aligned}$$

and the bar indicates

$$\overline{A_{ij}} = A_{ij} + A_{ji} - (2/3)A_{kk}\delta_{ij}.$$

Here, the following relations also hold:

$$\begin{aligned}
\hat{p}_{H0}^\alpha & = \hat{n}_{H0}^\alpha\hat{T}_{H0}, \quad \hat{\rho}_{H0}^\alpha = \hat{m}^\alpha\hat{n}_{H0}^\alpha, \\
\hat{n}_{H0} & = \sum_{\beta=A,B}\hat{n}_{H0}^\beta, \quad \hat{p}_{H0} = \sum_{\beta=A,B}\hat{p}_{H0}^\beta, \\
\hat{\rho}_{H0} & = \sum_{\beta=A,B}\hat{\rho}_{H0}^\beta, \quad \hat{\rho}_{H0}u_{iH1} = \sum_{\beta=A,B}\hat{\rho}_{H0}^\beta u_{iH1}^\beta, \tag{3.21}
\end{aligned}$$

and X_{H0}^α , which is the concentration of each component based on \hat{n}_{H0}^α , is defined by

$$X_{H0}^\alpha = \hat{n}_{H0}^\alpha/\hat{n}_{H0}, \quad \left(\sum_{\beta=A,B}X_{H0}^\beta = 1\right). \tag{3.22}$$

The \hat{D}_{AB} , \hat{D}_T , $\hat{\lambda}'$, $\hat{\lambda}$, k_T , $\hat{\mu}$, $\hat{\Upsilon}_1$, $\hat{\Upsilon}_2$, $\hat{\Upsilon}_3$, $\hat{\Upsilon}_4$, and $\hat{\Upsilon}_5$ in Eqs. (3.18)–(3.20) are functions of X_{H0}^A (or X_{H0}^B); and $\hat{\mu}$, $\hat{\lambda}$, \hat{D}_{AB} , \hat{D}_T , and k_T correspond to the viscosity, thermal conductivity, mutual diffusion coefficient, thermal diffusion coefficient, and thermal-diffusion ratio, respectively (see APPENDIX A for their definitions). Equations (3.13) and (3.14) indicate that \hat{p}_{H0} and \hat{p}_{H1} are constant (these constants are determined by conditions proper to concrete problems). We can, in principle, express u_{iH1}^A , u_{iH1}^B , and \hat{q}_{iH1} in terms of \hat{n}_{H0}^A , \hat{n}_{H0}^B , \hat{T}_{H0} , and u_{iH1} by using Eqs. (3.18) and (3.20)–(3.22) and express \hat{p}_{ijH2} in terms of \hat{n}_{H0}^A , \hat{n}_{H0}^B , \hat{T}_{H0} , u_{iH1} , and \hat{p}_{H2} by

using Eqs. (3.19), (3.21), and (3.22). Substituting these expressions into Eqs. (3.15)–(3.17), using $\hat{\rho}_{H0} = \hat{m}^A \hat{n}_{H0}^A + \hat{m}^B \hat{n}_{H0}^B$, and noting that \hat{n}_{H0}^B is expressed in terms of \hat{n}_{H0}^A and \hat{T}_{H0} [i.e., $(\hat{n}_{H0}^A + \hat{n}_{H0}^B)\hat{T}_{H0} = \hat{p}_{H0} = \text{const}$], we find that Eqs. (3.15)–(3.17) give six equations for six unknown functions \hat{n}_{H0}^A , \hat{T}_{H0} , u_{iH1} , and \hat{p}_{H2} . But we retain the form of Eqs. (3.15)–(3.20) for convenience in comparing the equations with the classical fluid-dynamic equations. It is noted here that the explicit expressions of \hat{p}_{ijH1} , \hat{p}_{ijH2} , and \hat{q}_{iH1} are $\hat{p}_{ijH1} = \hat{p}_{H1}\delta_{ij}$, Eq. (3.19), and Eq. (3.20), respectively.

Equation (3.21) is derived from the definition of h_{H0}^α , h_{H0} , etc. [Eq. (3.3)] with Eqs. (3.7b), (3.11), and (3.12). Equation (3.13) follows from Eq. (3.10b) with $m = 1$ [Eqs. (3.10a) and (3.10c) with $m = 1$ are satisfied automatically]; Eqs. (3.15) (with u_{iH1}^α having been eliminated in the way mentioned above), (3.14), and (3.17) (with \hat{q}_{iH1} having been eliminated) follow from Eqs. (3.10a), (3.10b), and (3.10c) with $m = 2$, respectively; Eq. (3.16) (with \hat{p}_{ijH2} having been eliminated) follows from Eq. (3.10b) with $m = 3$. The explicit form of f_{H1}^α , which has been used in deriving these equations [except Eq. (3.13)], is given in APPENDIX A [Eq. (A1)].³²

Equations (3.13)–(3.22) formally form a closed set of partial differential equations, from which some important information can be obtained. However, the functional forms of the coefficients \hat{D}_{AB} , \hat{D}_T , etc. with respect to X_{H0}^A , which are essentially determined by Eqs. (A2), (A3), etc., are not explicit. In order to apply the set (with its boundary condition derived in Sec. 3.2) to practical problems, we need to obtain the functional forms approximately or numerically. The approximate results for the coefficients corresponding to $\hat{\mu}$, $\hat{\lambda}$, \hat{D}_{AB} , and \hat{D}_T , based on the first term or the first few terms of polynomial expansions of the solutions of Eqs. (A2), (A3), etc., are given in, e.g., Refs. 23 and 33. In the meantime, a database that gives accurate numerical values of $\hat{\mu}$, $\hat{\lambda}$, \hat{D}_{AB} , and \hat{D}_T immediately for arbitrary X_{H0}^A was constructed recently.³⁴

3.2 Knudsen-Layer Correction and Slip Boundary Condition

In the previous subsection, we obtained the Hilbert solution putting the boundary condition aside. In this subsection we try to obtain the solution sat-

isfying the boundary condition. In accordance with Eq. (2.4), let us assume the following form for the dimensionless velocity u_{wi} of the surfaces of the condensed phases (or the boundary, for short):

$$u_{wi} = U_i \epsilon. \quad (3.23)$$

Then, the boundary condition (2.5) can be written as

$$f^\alpha = \sigma_w^\alpha \hat{T}_w^{-3/2} \left(\frac{\hat{m}^\alpha}{\pi} \right)^{3/2} \exp \left(-\frac{\hat{m}^\alpha \zeta_i^2}{\hat{T}_w} \right) \left(1 + \frac{2\hat{m}^\alpha \zeta_i U_i}{\hat{T}_w} \epsilon + \dots \right), \quad \zeta_i n_i > 0. \quad (3.24)$$

Now let us assume that \hat{n}_{H0}^A and \hat{T}_{H0} take the following values on the boundary:

$$\hat{n}_{H0}^A = \hat{p}_w^A / \hat{T}_w, \quad (3.25a)$$

$$\hat{T}_{H0} = \hat{T}_w. \quad (3.25b)$$

Then we find that f_{H0}^α given by Eq. (3.6) with Eq. (3.12) satisfies the boundary condition (3.24) at the leading order.

However, at higher orders of ϵ , the Hilbert solution cannot generally be made to satisfy the boundary condition for the same reason as in the case of a single component gas (see e.g., Refs. 1, 5, and 30). Therefore, we seek the solution satisfying the boundary condition in the following form:

$$f^\alpha = f_H^\alpha + f_K^\alpha, \quad (3.26)$$

where f_K^α , which is called the Knudsen-layer correction, is a correction term to the Hilbert solution near the boundary. More precisely, f_K^α is assumed to have the length scale of variation of the order of ϵ (or the mean free path in the physical space) in the direction normal to the boundary, i.e., $n_j \partial f_K^\alpha / \partial x_j = O(f_K^\alpha / \epsilon)$, and to be appreciable only in the thin layer with thickness of the order of ϵ adjacent to the boundary. We now introduce the new coordinate system (η, s_1, s_2) defined by

$$x_i = \epsilon \eta n_i(s_1, s_2) + x_{wi}(s_1, s_2), \quad (3.27)$$

where x_{wi} represents the boundary (the surfaces of the condensed phases), η is the stretched coordinate normal to the boundary, and s_1 and s_2 are the coordinates on

the boundary, and consider f_K^α to be a function of η , s_1 , and s_2 [$\partial f_K^\alpha / \partial \eta = O(f_K^\alpha)$], as well as ζ_i , vanishing rapidly as $\eta \rightarrow \infty$. We further expand f_K^α in a power series of ϵ as

$$f_K^\alpha = f_{K1}^\alpha \epsilon + f_{K2}^\alpha \epsilon^2 + \dots, \quad (3.28)$$

where the expansion is assumed to start from the first order because f_{H0}^α satisfies the boundary condition at the zeroth order. Substituting Eq. (3.26) with Eq. (3.28) into Eqs. (2.1)–(2.6) and taking into account the properties of f_H^α as well as f_K^α , we obtain the equations and boundary conditions for $f_{K_m}^\alpha$ ($m = 1, 2, \dots$). Here we only give those for f_{K1}^α :

$$\zeta_i n_i \frac{\partial f_{K1}^\alpha}{\partial \eta} = \sum_{\beta=A,B} K^{\beta\alpha} \left(\hat{J}^{\beta\alpha}([f_{H0}^\beta]_w, f_{K1}^\alpha) + \hat{J}^{\beta\alpha}(f_{K1}^\beta, [f_{H0}^\alpha]_w) \right), \quad (3.29)$$

$$f_{K1}^\alpha = f_{H0}^\alpha \left(\kappa_w^\alpha + 2\hat{m}^\alpha \bar{\zeta}_i (U_i - u_{iH1}) \hat{T}_w^{-1/2} - (\hat{m}^\alpha \bar{\zeta}^2 - \frac{5}{2}) \hat{T}_{H1} / \hat{T}_w \right. \\ \left. + \bar{\zeta}_i A^\alpha(\bar{\zeta}) \frac{1}{\hat{p}_{H0}} \frac{\partial \hat{T}_{H0}}{\partial x_i} + \sum_{\beta=A,B} \bar{\zeta}_i D^{(\beta)\alpha}(\bar{\zeta}) \frac{1}{\hat{n}_{H0}} \frac{\partial X_{H0}^\beta}{\partial x_i} \right), \\ \zeta_i n_i > 0, \quad \text{at } \eta = 0, \quad (3.30a)$$

$$f_{K1}^\alpha \rightarrow 0, \quad \eta \rightarrow \infty, \quad (3.30b)$$

with

$$\kappa_w^A = -\hat{p}_{H1}^A / \hat{p}_w^A, \quad (3.31a)$$

$$\kappa_w^B = -(\pi \hat{m}^B)^{1/2} (u_{iH1} n_i) \hat{T}_w^{-1/2} - \frac{1}{2} \hat{T}_{H1} / \hat{T}_w \\ + (\pi \hat{m}^B)^{1/2} \sum_{\beta=A,B} \hat{\Delta}_{B\beta} \frac{1}{\hat{n}_{H0}} \frac{\partial X_{H0}^\beta}{\partial x_i} n_i \\ + (\pi \hat{m}^B)^{1/2} \hat{D}_{TB} \frac{1}{\hat{p}_{H0}} \frac{\partial \hat{T}_{H0}}{\partial x_i} n_i \\ - 2(\pi \hat{m}^B)^{1/2} \hat{T}_w^{-1/2} \int_{\zeta_i n_i < 0} \zeta_i n_i (\hat{n}_{H0}^B)^{-1} f_{K1}^B d\zeta. \quad (3.31b)$$

Here, $[]_w$ represents the value of the function in the square brackets on the boundary (i.e., at $\eta = 0$); $\bar{\zeta}_i = \zeta_i / \hat{T}_w^{1/2}$ and $\bar{\zeta} = (\bar{\zeta}_j^2)^{1/2}$; $\hat{\Delta}_{B\beta}$ and \hat{D}_{TB} are defined in Eq. (A5b). The condition (3.25) has been used in Eqs. (3.30) and (3.31). Integrating both sides of Eq. (3.29) over the whole ζ_i space and taking Eq. (3.30b) into account, we have

$u_{iK1}^\alpha n_i = 0$, where $u_{iK1}^\alpha = (1/[\hat{n}_{H0}^\alpha]_w) \int \zeta_i f_{K1}^\alpha d\zeta$ is the contribution of the Knudsen-layer part to the ϵ -order velocity [i.e., $u_i^\alpha = (u_{iH1}^\alpha + u_{iK1}^\alpha)\epsilon + \dots$]; therefore, for the B -component, we have

$$u_{iH1}^B n_i = 0, \quad (3.32)$$

on the boundary (note that $u_i^B n_i = 0$ on the boundary because of the diffuse reflection and that $u_{iH0}^B \equiv 0$).

Equations (3.29)–(3.31) form a half-space boundary-value problem of the linearized Boltzmann equation for a binary mixture (see APPENDIX B). The boundary condition (3.30a) contains the boundary values of the Hilbert parts (the part corresponding to the Hilbert solution) of the macroscopic variables and those of their derivatives. As in the case of a single-component gas, the solution f_{K1}^α exists only when these boundary values satisfy certain relations (see APPENDIX B for the details of this point). Part of these relations, together with Eqs. (3.25) and (3.32), give the boundary conditions for the fluid-dynamic type equations (3.15)–(3.22). More specifically, the solution to Eqs. (3.29)–(3.31) can be obtained as the sum of the solutions of five fundamental half-space problems of the linearized Boltzmann equation for a binary mixture, that is, the problems of (i) evaporation and condensation,³⁵ (ii) temperature jump,^{36,37} (iii) partial pressure jump,³⁶ (iv) thermal creep,^{17,38} and (v) diffusion slip.^{15–18} The (iii) and (v) are peculiar to a mixture, whereas the (i), (ii), and (iv) are well-known problems for a single-component gas [see, e.g., Refs. 39 and 40 for (i), Ref. 41 for (ii), and Refs. 6–8 for (iv)]. Corresponding to this separation of the problem, the relation among the boundary values mentioned above can be obtained separately. Here, we note that only the relations associated with the problems (iv) and (v) play the role of the boundary condition for Eqs. (3.15)–(3.22) and that the others are related to the higher-order fluid-dynamic type equations which are not obtained in this paper. The relations associated with the problems (iv) and (v) give the velocity slip for u_{iH1} of the following form:

$$(u_{iH1} - U_i)t_i = -\hat{T}_{H0}^{1/2} \left(b_7 \frac{1}{\hat{p}_{H0}} \frac{\partial \hat{T}_{H0}}{\partial x_i} t_i + b_9 \frac{1}{\hat{n}_{H0}} \frac{\partial X_{H0}^A}{\partial x_i} t_i \right), \quad (3.33)$$

on the boundary. Here, t_i is a unit vector tangential to the boundary. The b_7 and b_9 , the coefficient of thermal creep and that of diffusion slip, respectively, are the

functions of X_{H0}^A (or X_{H0}^B) on the boundary.

To summarize, Eqs. (3.25), (3.32), and (3.33) form the boundary conditions for the fluid-dynamic type equations (3.15)–(3.22).⁴² By solving this system, we obtain the overall flow u_{iH1} (of the order of ϵ or Kn) induced in the gas (note that it is subject to the Knudsen-layer correction near the boundary) as well as the number densities \hat{n}_{H0}^α and the temperature \hat{T}_{H0} required to determine the ϵ^0 -order Hilbert solution, Eq. (3.6) with $u_{iH0} \equiv 0$.

When the region of the mixture extends to infinity, we have to impose the conditions at infinity that are consistent with the fluid-dynamic type equations. It follows from Eqs. (3.11)–(3.14) that $u_{iH0}^\alpha = u_{iH0} \equiv 0$, $\hat{p}_{H0} = \text{const}$, and $\hat{p}_{H1} = \text{const}$ should hold at infinity. This restriction is consistent with the situation (iii) in the first paragraph of Sec. 2.

4 FLOW INDUCED IN THE GAS AND THE GHOST EFFECT IN THE CONTINUUM LIMIT

In the previous section, we were able to carry out the asymptotic analysis consistently under the assumptions (2.4) [or (3.23)] and (3.11)⁴³ and derived the fluid-dynamic type system for the ϵ -order flow as well as the number densities and temperature of the ϵ^0 order. On the basis of the system, we first discuss the cause of the flow (Sec. 4.1) and then the behavior of the mixture (i.e., the ghost effect) in the continuum limit (Sec. 4.2).

4.1 Flow Induced in the Mixture

It is obvious that the boundary condition (3.33) causes the ϵ -order flow, u_{iH1} and thus u_{iH1}^α . If the boundary is moving in its surface [with a speed of $O(\epsilon)$], i.e., $U_i \neq 0$, then there is a motion of the gas mixture along the boundary. Next, if the temperature of the boundary varies along it, then the temperature gradient $(\partial\hat{T}_{H0}/\partial x_i)t_i$ takes a nonzero value on the boundary because of Eq. (3.25b), and therefore a flow is induced along it [the first term on the R.H.S. of Eq. (3.33)]. This

phenomenon, which is called the thermal creep, has extensively been studied for a single-component gas.⁶⁻⁹ Finally, if the saturation vapor pressure \hat{p}_w^A varies along the boundary, then the concentration X_{H0}^A of the vapor is not uniform along it because of Eq. (3.25) and the relation $\hat{p}_{H0} = (\hat{n}_{H0}^A + \hat{n}_{H0}^B)\hat{T}_{H0} = \text{const.}$ Therefore, a flow is induced along the boundary [the second term on the R.H.S. of Eq. (3.33)]. This flow, caused by the nonuniformity of the concentration along the boundary, is called the diffusion slip¹⁵⁻¹⁸ and is peculiar to gas mixtures. In summary, the boundary condition (3.33) contains the following three factors that cause the flow of $O(\epsilon)$:

- I. the motion of the boundary
- II. the thermal creep
- III. the diffusion slip

If the condensed phases are at rest and if \hat{T}_w and \hat{p}_w^A are uniform on the surface of each condensed phase, the boundary condition (3.33) causes no flow.

Various factors causing the ϵ -order flow are also contained in the momentum equation, Eq. (3.16) with Eq. (3.19). That is, in addition to the Navier–Stokes stress [the first and second terms of the R.H.S. of Eq. (3.19)], the stress \hat{p}_{ijH2} contains the terms consisting of the product or the derivative of the temperature gradient $\partial\hat{T}_{H0}/\partial x_j$, the product or the derivative of the concentration gradient $\partial X_{H0}^A/\partial x_j$, and the product of the temperature and the concentration gradients. The divergences of these terms appear in the momentum equation and can be the cause of the motion of the mixture. The sum of the terms containing \hat{Y}_1 and \hat{Y}_2 in Eq. (3.19) is called the thermal stress and that of the terms containing \hat{Y}_3 and \hat{Y}_5 the concentration stress. The fact that the thermal stress can cause a gas motion was pointed out in Ref. 10 for a single-component gas (the nonlinear thermal stress flow; see also Refs. 11 and 1). In the case of a mixture, the concentration stress can also induce a flow,^{14,11} even if the temperature of the mixture \hat{T}_{H0} is uniform. In addition to the thermal and concentration stresses, there is a stress term due to the cross effect of the temperature and concentration gradients [the term containing \hat{Y}_4 in Eq. (3.19)]. To summarize, the momentum equation contains the following three factors causing the flow of $O(\epsilon)$:

IV. the thermal stress

V. the concentration stress

VI. the stress by the cross effect of the temperature and concentration gradients

In the case of a mixture, however, there is another important cause for the ϵ -order flow, that is,

VII. the diffusion

represented by Eq. (3.18). If the concentration X_{H0}^A is not uniform, the difference $u_{iH1}^A - u_{iH1}^B$ between the flow velocities of the A and B -components is not zero, and therefore a flow of the mixture u_{iH1} appears generally. The flow caused by the concentration stress mentioned above cannot, in general, be separated from that caused by the diffusion. In one-dimensional problems, such as the mixture between two plane condensed phases^{19,20} and that between two coaxial cylindrical condensed phases, the flow u_{iH1} is determined by Eqs. (3.15), (3.17), (3.18), and (3.20)–(3.22) and the boundary conditions (3.25) and (3.32) [Eq. (3.33) reduces to $u_{iH1}t_i = 0$ (on the boundary) in this case, which is automatically satisfied because of the symmetry of the problem]. That is, the flow is determined only by the diffusion. In this case, Eqs. (3.16) and (3.19) play a secondary role of determining \hat{p}_{H2} from the known u_{iH1} , \hat{T}_{H0} , \hat{n}_{H0}^A , etc. The nonuniformity of the temperature also causes the diffusion, which is known as the thermal diffusion [the second term on the R.H.S. of Eq. (3.18)].

4.2 Behavior in the Continuum Limit

Let us now investigate the behavior of the mixture in the continuum limit, i.e., the limit where ϵ (or Kn) tends to zero. If we take the limit recalling that the velocities u_i^α and thus u_i are of $O(\epsilon)$ [i.e., $u_i^\alpha = (u_{iH1}^\alpha + u_{iK1}^\alpha)\epsilon + \dots$], we have $(\hat{n}^\alpha, \hat{p}^\alpha, \hat{T}^\alpha, \hat{n}, \hat{p}, \hat{T}) \rightarrow (\hat{n}_{H0}^\alpha, \hat{p}_{H0}^\alpha, \hat{T}_{H0}^\alpha, \hat{n}_{H0}, \hat{p}_{H0}, \hat{T}_{H0})$ and $(u_i^\alpha, u_i) \rightarrow (0, 0)$. The flow vanishes in this limit. That is, no evaporation or condensation takes place, and each component is at rest. However, it should be noted that the limiting macroscopic variables $\hat{n}_{H0}^\alpha, \hat{T}_{H0}$, etc. are determined simultaneously with

u_{iH1}^α and u_{iH1} by the fluid-dynamic type system and that u_{iH1}^α and u_{iH1} , which are the expansion coefficients in Eq. (3.3), are independent of ϵ . This means the following. In spite of the fact that the flow itself vanishes in the continuum limit, it still affects other macroscopic quantities in this limit. Therefore, the treatment in which only the continuum limit is considered from the beginning (i.e., classical fluid dynamics) cannot describe the behavior in the continuum limit correctly. This effect is no other than the ghost effect that was found in Ref. 1 and has extensively been investigated in Refs. 3–5, 19, and 20.

In Sec. 4.1, we discussed the factors causing the ϵ -order flow. Since the ghost effect is caused by the vanishing ϵ -order flow in the continuum limit, all these factors are also the sources of the effect. The factors I, II, and IV in Sec. 4.1 have already been clarified in the case of a single-component gas in Ref. 1.⁴⁴ The other factors are peculiar to gas mixtures. The factor VII has been studied in Refs. 19 and 20, where the planar two-surface problem for a mixture of vapor(s) and noncondensable gas(es) is considered. The III, V, and VI are the factors for the ghost effect first clarified in the present paper.

We here give some comments on the conventional treatment^{45,46} of the problem in the continuum limit on the basis of the classical fluid dynamics. The basic equations are the Navier–Stokes equations for a binary mixture with the flow of $O(\epsilon)$ being taken into account, namely, the equations corresponding to Eq. (3.13)–(3.22) with $\hat{Y}_1 = \hat{Y}_2 = \hat{Y}_3 = \hat{Y}_4 = \hat{Y}_5 = 0$ in Eq. (3.19). The associated boundary conditions are those corresponding to Eqs. (3.25), (3.32), and (3.33) with $U_i = b_7 = b_9 = 0$ (nonslip condition). In the framework of classical fluid dynamics, these boundary conditions are derived from experience. On the other hand, in the same framework, the flow of $O(\epsilon)$ is generally neglected as an effect of gas rarefaction. But, if the flow is neglected, i.e., u_{iH1}^α are put to be zero, the Navier–Stokes equations are not solvable under the boundary conditions (3.25), (3.32), and $u_{iH1}t_i = 0$. For the purpose of avoiding this difficulty, the flow of $O(\epsilon)$ is retained in the Navier–Stokes system in the conventional treatment, although it is inconsistent with the framework. In this way, the problem is somewhat made to be solvable. However, the system does not contain the factors I–VI for the ghost effect and is incomplete. Therefore, it

cannot describe the behavior in the continuum limit correctly. It happens to give the correct behavior in some special cases where the factors I–III are absent and IV–VI do not affect the (ϵ -order) velocity field. Such an example is given by the planar two-surface problem studied in Refs. 19 and 20.

5 MIXTURE OF MECHANICALLY IDENTICAL MOLECULES

We now consider a special case where the molecules of the A -component are mechanically identical with those of the B -component, i.e., $m^A = m^B$ and $d^A = d^B$. In this case, the functions A^α , B^α , and $D^{(\beta)\alpha}$ occurring in Eq. (A1) are reduced as follows:

$$\begin{aligned} A^A &= A^B = A(\zeta), & B^A &= B^B = B(\zeta), \\ D^{(A)B} &= D^{(B)A} = -(X_{H0}^A/X_{H0}^B)D^{(A)A} = -(X_{H0}^B/X_{H0}^A)D^{(B)B} = -C(\zeta), \end{aligned} \quad (5.1)$$

where $A(\zeta)$, $B(\zeta)$, and $C(\zeta)$ are the functions of ζ only and are defined in Appendix B of Ref. 20 [The $A(\zeta)$ and $B(\zeta)$ appear in connection with the thermal conductivity and viscosity for a single-component gas; see the references cited in Ref. 20]. As is readily seen from Eq. (A5), all the coefficients in Eqs. (3.18)–(3.20) become independent of X_{H0}^A , and moreover, k_T , \hat{D}_T , $\hat{\Upsilon}_3$, $\hat{\Upsilon}_4$, and $\hat{\Upsilon}_5$ vanish. More specifically, the coefficients become as follows:

$$\begin{aligned} \hat{\mu} &= \gamma_1, & \hat{\lambda} &= \hat{\lambda}' = \frac{5}{4}\gamma_2, & \hat{D}_{AB} &= \gamma_c, & k_T &= \hat{D}_T = 0, \\ \hat{\Upsilon}_1 &= \frac{1}{2}\gamma_7, & \hat{\Upsilon}_2 &= \frac{1}{2}\gamma_3, & \hat{\Upsilon}_3 &= \hat{\Upsilon}_4 = \hat{\Upsilon}_5 = 0, \end{aligned} \quad (5.2a)$$

with

$$\begin{aligned} \gamma_1 &= 1.270042, & \gamma_2 &= 1.922284, & \gamma_3 &= 1.947906, \\ \gamma_7 &= 0.188106, & \gamma_c &= 0.764215. \end{aligned} \quad (5.2b)$$

Here, the values of γ_1 , γ_2 , γ_3 , and γ_7 are taken from Ref. 1, and the value of γ_c from Ref. 20 (see also the references cited in these papers). As for the boundary condition (3.33), the coefficient b_7 reduces to the slip coefficient for the thermal creep for a single-component gas,⁸ and b_9 vanishes, i.e.,

$$b_7 = K_1, \quad b_9 = 0, \quad (5.3a)$$

with

$$K_1 = -0.6463. \quad (5.3b)$$

This special mixture is useful because it simplifies the structure of the Boltzmann equation and often retains the important features of the general mixture. In the present problem, however, the vanishing of six coefficients [see Eqs. (5.2a) and (5.3a)] simplifies the fluid-dynamic type system too much, so that all the new factors for the ghost effect, i.e., the factors III, V, and VI, disappear. In this sense, the mixture under consideration is a very special one and does not represent the general mixture in the present problem.

6 MIXTURE AROUND SOLID BODIES

Let us consider a binary gas mixture around ordinary solid bodies on the surfaces of which neither evaporation nor condensation takes place, i.e., the case where the A -component is also noncondensable. This is a natural extension of the problem of Ref. 1 to a binary mixture. Let us assume the diffuse reflection condition also for the A -component. Then, σ_w^A in Eq. (2.5) is given by Eqs. (2.6b) with the superscript B being replaced by A . The analysis of the Hilbert solution in Sec. 3.1, which was carried out irrespective of the boundary condition, is not affected by the change of the boundary condition. We should only note that, as Eq. (3.11), $u_{iH0}^A = u_{iH0}^B = u_{iH0} \equiv 0$ is assumed in the present case. As for the analysis of the Knudsen-layer correction in Sec. 3.2, a slight modification is necessary. We have to discard the relation (3.25a). Equation (3.29) for f_{K1}^α remains unchanged, but κ_w^A in Eq. (3.31a) is given by Eq. (3.31b) with the superscript B being replaced by A . Then, in the same way as we derived Eq. (3.32), we have

$$u_{iH1}^A n_i = 0, \quad (6.1)$$

on the boundary. This gives a boundary condition in place of Eq. (3.25a). The change of κ_w^A does not affect the slip boundary condition (3.33). To summarize, the fluid-dynamic type system in the present case consists of Eqs. (3.13)–(3.22) and the boundary conditions (3.25b), (3.32), (3.33), and (6.1).⁴⁷

The fluid-dynamic type system, therefore, retains all the terms that are the cause of the ϵ -order flow and thus that of the ghost effect in the continuum limit. However, the effect of the factors III and V–VII in Sec. 4.1, which are peculiar to a gas mixture, is generally weak for the reason described bellow. Let us consider the mixture in a closed domain or in an infinite domain with a uniform concentration and a uniform pressure at infinity. According to Refs. 23 and 34, the magnitude of the coefficient k_T in Eqs. (3.18) and (3.20) is, in most cases, considerably smaller than unity. Therefore, the terms including k_T may be neglected as the first approximation. Then, it can be shown that the solution with a uniform concentration is a compatible solution. In this situation, all the factors associated with the nonuniformity of the concentration, i.e., the factors III and V–VII, vanish, and the behavior of the mixture is essentially the same as that of a single-component gas investigated in Ref. 1. From this, it is expected that the term including k_T causes small nonuniformity of the concentration of $O(k_T)$ and that all the above factors have only weak effects.

Finally, it is noted that, when the condensed phases and the ordinary solid bodies coexist, the boundary conditions (3.25), (3.32), and (3.33) are to be used on the former and the conditions (3.25b), (3.32), (3.33), and (6.1) on the latter for Eqs. (3.13)–(3.22).

7 SOME GENERALIZATIONS

So far, we have considered the case of a binary mixture. The generalization of the analysis to a multicomponent mixture of vapors and noncondensable gases is rather straightforward. The resulting fluid-dynamic type system is summarized in APPENDIX C.

Here, we discussed the generalization of the boundary condition. In our analysis, we have assumed the complete condensation condition for the vapor [cf., Eqs. (2.5) and (2.6a)]. In this condition, all the vapor molecules impinging on the surfaces of the condensed phases are assumed to be absorbed. When the case where part of the impinging molecules are reflected on the surfaces is considered, the boundary condition that is the linear combination of the complete condensation and the

diffuse reflection [cf. the last sentence in the paragraph containing Eq. (2.5)] is often used (see, e.g., Ref. 30). This generalized condition is also expressed by Eq. (2.5), but with the following σ_w^A in place of Eq. (2.6a):

$$\sigma_w^A = \alpha_c \hat{p}_w^A / \hat{T}_w - 2(1 - \alpha_c) \left(\frac{\pi \hat{m}^A}{\hat{T}_w} \right)^{1/2} \int_{\zeta_i n_i < 0} \zeta_i n_i f^A d\zeta, \quad (7.1)$$

where α_c ($0 < \alpha_c \leq 1$) is called the condensation factor, and $\alpha_c = 1$ corresponds to the complete condensation. It can easily be shown that this generalization of the kinetic boundary condition for the A -component does not change the fluid-dynamic type equations (3.13)–(3.22) and their boundary conditions (3.25), (3.32), and (3.33).

8 CONCLUDING REMARKS

In the present study, we have tried to clarify the steady behavior of a binary mixture of a vapor and a noncondensable gas around condensed phases of the vapor (of arbitrary shapes) in the continuum limit in the situations (i)–(iii) described at the beginning of Sec. 2. For this purpose, we have carried out a systematic asymptotic analysis of the Boltzmann equation for small Knudsen numbers. As the result, we have shown that the solution in which the flow of the mixture appears at the first order of the Knudsen number [$O(\text{Kn})$] is a consistent solution, and we have derived the set of fluid-dynamic type equations and their appropriate boundary conditions that describes the overall flow of $O(\text{Kn})$ as well as the number densities and temperature of $O(1)$. On the basis of the system, we have shown that the ghost effect, which was discovered in Ref. 1 for a single-component gas, appears naturally in the present problem. That is, in spite of the fact that the flow vanishes in the continuum limit, the distributions of other macroscopic variables, such as the number densities and temperature, in this limit are still affected by the vanishing (or nonexistent) flow. We have examined the factors causing the ghost effect and shown that there are several new factors which are peculiar to gas mixtures in addition to the factors already known for a single-component gas^{1,3} and to the factor corresponding to the diffusion for a mixture.^{19,20} These new factors correspond to the diffusion slip and

the flows due to the concentration stress and to the stress by the cross effect of the temperature and concentration gradients. They are relevant to the nonuniformity of the concentration distribution of the mixture. We have also considered the corresponding problem for a mixture of noncondensable gases. Further, we have discussed the generalization of the result to a multicomponent mixture of the vapors and noncondensable gases as well as to a more general boundary condition.

In this paper, the formal part of the asymptotic analysis has been completed. The next step is to add the quantitative information, such as the numerical values of the coefficients included in the fluid-dynamic type equations and those of the coefficients in the slip boundary condition. These will be the subjects of our forthcoming papers, where some numerical examples demonstrating the ghost effect caused by the new factors will also be given.

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APPENDIX A EXPRESSION OF f_{H1}^α , RELATED INTEGRALS, AND TRANSPORT COEFFICIENTS

In addition to α and β , the letters γ and κ are also used to represent the labels A and B of the components in this appendix. Let us introduce the notations $\tilde{\zeta}_i = \zeta_i \hat{T}_{H0}^{-1/2}$ and $\tilde{\zeta} = (\tilde{\zeta}_j^2)^{1/2}$. Then the solution f_{H1}^α of Eq. (3.5) with $m = 1$ is

given by

$$f_{H1}^\alpha = f_{H0}^\alpha \left[\hat{p}_{H1}^\alpha / \hat{p}_{H0}^\alpha + 2\hat{m}^\alpha \tilde{\zeta}_i u_{iH1} \hat{T}_{H0}^{-1/2} + (\hat{m}^\alpha \tilde{\zeta}^2 - \frac{5}{2}) \hat{T}_{H1} \hat{T}_{H0}^{-1} - \tilde{\zeta}_i A^\alpha(\tilde{\zeta}) \frac{1}{\hat{p}_{H0}} \frac{\partial \hat{T}_{H0}}{\partial x_i} - \tilde{\zeta}_i \sum_{\beta=A,B} D^{(\beta)\alpha}(\tilde{\zeta}) \frac{1}{\hat{n}_{H0}} \frac{\partial X_{H0}^\beta}{\partial x_i} \right], \quad (\text{A1})$$

where $A^\alpha(\zeta)$ and $D^{(\beta)\alpha}(\zeta)$ are the solutions of the following integral equations:^{23,24}

$$\sum_{\beta=A,B} K^{\beta\alpha} X_{H0}^\beta \tilde{L}^{\beta\alpha}(\zeta_i A^\beta, \zeta_i A^\alpha) = -\zeta_i (\hat{m}^\alpha \zeta^2 - \frac{5}{2}), \quad (\text{A2})$$

subsidiary condition: $\sum_{\beta=A,B} \hat{m}^\beta X_{H0}^\beta I_4^\beta(A^\beta) = 0,$

$$\sum_{\beta=A,B} K^{\beta\alpha} X_{H0}^\alpha X_{H0}^\beta \tilde{L}^{\beta\alpha}(\zeta_i D^{(\gamma)\beta}, \zeta_i D^{(\gamma)\alpha}) = -\zeta_i \left(\delta_{\alpha\gamma} - \frac{\hat{m}^\alpha X_{H0}^\alpha}{\sum_{\beta=A,B} \hat{m}^\beta X_{H0}^\beta} \right), \quad (\text{A3})$$

subsidiary condition: $\sum_{\beta=A,B} \hat{m}^\beta X_{H0}^\beta I_4^\beta(D^{(\alpha)\beta}) = 0.$

Here, $\tilde{L}^{\beta\alpha}(f, g)$ and $I_n^\alpha(F)$ are defined by

$$\begin{aligned} \tilde{L}^{\beta\alpha}(f, g) &= [\hat{J}^{\beta\alpha}(f E^\beta, E^\alpha) + \hat{J}^{\beta\alpha}(E^\beta, g E^\alpha)] (E^\alpha)^{-1}, \\ I_n^\alpha(F) &= \frac{8\pi}{15} \int_0^\infty \zeta^n F(\zeta) E^\alpha(\zeta) d\zeta, \\ E^\alpha(\zeta) &= \left(\frac{\hat{m}^\alpha}{\pi} \right)^{3/2} \exp(-\hat{m}^\alpha \zeta^2). \end{aligned} \quad (\text{A4})$$

In Eqs. (A2) and (A3), the restriction $\sum_{\beta=A,B} X_{H0}^\beta = 1$ for the concentrations is not used. In other words, A^α and $D^{(\beta)\alpha}$ are defined for arbitrary positive X_{H0}^γ ($\gamma = A, B$). Therefore, A^α and $D^{(\beta)\alpha}$ are the functions of X_{H0}^γ and ζ , depending on \hat{m}^γ and \hat{d}^γ ($\gamma = A, B$).

Next we introduce the following functions $B^\alpha(\zeta)$, $D_1^{(\beta,\gamma)\alpha}(\zeta)$, $D_2^{(\beta,\gamma)\alpha}(\zeta)$, $A_1^\alpha(\zeta)$, $A_2^\alpha(\zeta)$, $D_{A1}^{(\beta)\alpha}(\zeta)$ and $D_{A2}^{(\beta)\alpha}(\zeta)$:

$$\begin{aligned} \sum_{\beta=A,B} K^{\beta\alpha} X_{H0}^\beta \tilde{L}^{\beta\alpha} \left((\zeta_i \zeta_j - \frac{1}{3} \zeta^2 \delta_{ij}) B^\beta, (\zeta_i \zeta_j - \frac{1}{3} \zeta^2 \delta_{ij}) B^\alpha \right) &= -2\hat{m}^\alpha (\zeta_i \zeta_j - \frac{1}{3} \zeta^2 \delta_{ij}), \\ \sum_{\beta=A,B} K^{\beta\alpha} X_{H0}^\beta \tilde{J}^{\beta\alpha}(\zeta_i D^{(\gamma)\beta}, \zeta_j D^{(\kappa)\alpha}) &= \zeta_i \zeta_j D_1^{(\gamma,\kappa)\alpha} + D_2^{(\gamma,\kappa)\alpha} \delta_{ij}, \\ \sum_{\beta=A,B} K^{\beta\alpha} X_{H0}^\beta \tilde{J}^{\beta\alpha}(\zeta_i A^\beta, \zeta_j A^\alpha) &= \zeta_i \zeta_j A_1^\alpha + A_2^\alpha \delta_{ij}, \end{aligned}$$

$$\sum_{\beta=A,B} K^{\beta\alpha} X_{H0}^\beta [\tilde{J}^{\beta\alpha}(\zeta_i A^\beta, \zeta_j D^{(\gamma)\alpha}) + \tilde{J}^{\beta\alpha}(\zeta_i D^{(\gamma)\beta}, \zeta_j A^\alpha)] = \zeta_i \zeta_j D_{A1}^{(\gamma)\alpha} + D_{A2}^{(\gamma)\alpha} \delta_{ij},$$

where the operator $\tilde{J}^{\beta\alpha}$ is defined as

$$\tilde{J}^{\beta\alpha}(f, g) = \hat{J}^{\beta\alpha}(fE^\beta, gE^\alpha)/E^\alpha.$$

The comments below Eq. (A4) also apply to the above definitions. That is, $B^\alpha(\zeta)$, $D_1^{(\beta,\gamma)\alpha}(\zeta)$, $D_2^{(\beta,\gamma)\alpha}(\zeta)$, $A_1^\alpha(\zeta)$, $A_2^\alpha(\zeta)$, $D_{A1}^{(\beta)\alpha}(\zeta)$ and $D_{A2}^{(\beta)\alpha}(\zeta)$ are the functions of X_{H0}^κ and ζ , depending on \hat{m}^κ and \hat{d}^κ ($\kappa = A, B$).

By the use of $A^\alpha(\zeta)$, $D^{(\beta)\alpha}(\zeta)$, and these functions, the coefficients in Eqs. (3.18)–(3.20) are defined as:

$$\begin{aligned} \hat{D}_{AB} &= X_{H0}^A X_{H0}^B (\hat{\Delta}_{AA} + \hat{\Delta}_{BB} - \hat{\Delta}_{AB} - \hat{\Delta}_{BA}), \\ \hat{D}_T &= X_{H0}^A X_{H0}^B (\hat{D}_{TA} - \hat{D}_{TB}), \quad \hat{\mu} = \sum_{\beta=A,B} \hat{m}^\beta X_{H0}^\beta I_6^\beta(B^\beta), \\ \hat{\lambda}' &= \frac{5}{2} \sum_{\beta=A,B} X_{H0}^\beta I_4^\beta([\hat{m}^\beta \zeta^2 - \frac{5}{2}]A^\beta), \\ k_T &= \frac{\hat{D}_T}{\hat{D}_{AB}}, \quad \hat{\lambda} = \hat{\lambda}' - k_T \frac{\hat{D}_T}{X_{H0}^A X_{H0}^B}, \\ \hat{\Upsilon}_1 &= \frac{1}{2} \sum_{\beta=A,B} X_{H0}^\beta I_6^\beta(B^\beta [A_1^\beta + (\hat{m}^\beta \zeta^2 - 3)A^\beta - \frac{1}{2}\zeta \frac{\partial A^\beta}{\partial \zeta}]), \\ \hat{\Upsilon}_2 &= \frac{1}{2} \sum_{\beta=A,B} X_{H0}^\beta I_6^\beta(A^\beta B^\beta), \\ \hat{\Upsilon}_3 &= \hat{\Upsilon}_3^{(A,A)} + \hat{\Upsilon}_3^{(B,B)} - \hat{\Upsilon}_3^{(A,B)} - \hat{\Upsilon}_3^{(B,A)}, \\ \hat{\Upsilon}_4 &= \hat{\Upsilon}_4^{(A)} - \hat{\Upsilon}_4^{(B)}, \quad \hat{\Upsilon}_5 = \hat{\Upsilon}_5^{(A)} - \hat{\Upsilon}_5^{(B)}, \end{aligned} \tag{A5a}$$

where

$$\begin{aligned} \hat{\Delta}_{\alpha\beta} &= \frac{5}{2} I_4^\alpha(D^{(\beta)\alpha}), \quad \hat{D}_{T\alpha} = \frac{5}{2} I_4^\alpha(A^\alpha), \\ \hat{\Upsilon}_3^{(\alpha,\gamma)} &= \frac{1}{2} \sum_{\beta=A,B} X_{H0}^\beta I_6^\beta(B^\beta D_1^{(\alpha,\gamma)\beta} - D^{(\alpha)\beta} \frac{\partial B^\beta}{\partial X_{H0}^\gamma}), \\ \hat{\Upsilon}_4^{(\alpha)} &= \frac{1}{2} \sum_{\beta=A,B} X_{H0}^\beta I_6^\beta(B^\beta [D_{A1}^{(\alpha)\beta} + (\hat{m}^\beta \zeta^2 - 2)D^{(\alpha)\beta} - \frac{\zeta}{2} \frac{\partial D^{(\alpha)\beta}}{\partial \zeta}] - A^\beta \frac{\partial B^\beta}{\partial X_{H0}^\alpha}), \\ \hat{\Upsilon}_5^{(\alpha)} &= \frac{1}{2} \sum_{\beta=A,B} X_{H0}^\beta I_6^\beta(B^\beta D^{(\alpha)\beta}). \end{aligned} \tag{A5b}$$

Here the subsidiary coefficients $\hat{\Delta}_{\alpha\beta}$ and $\hat{D}_{T\alpha}$ have the following properties:

$$\hat{\Delta}_{\alpha\beta} = \hat{\Delta}_{\beta\alpha}, \quad \sum_{\beta=A,B} \hat{m}^\beta X_{H0}^\beta \hat{\Delta}_{\alpha\beta} = 0, \quad \sum_{\beta=A,B} \hat{m}^\beta X_{H0}^\beta \hat{D}_{T\beta} = 0. \quad (\text{A6})$$

In addition, $\hat{D}_{T\alpha}$ can also be expressed in terms of $D^{(\alpha)\beta}$ as

$$\hat{D}_{T\alpha} = \frac{5}{2} \sum_{\beta=A,B} X_{H0}^\beta I_4^\beta ([\hat{m}^\beta \zeta^2 - \frac{5}{2}] D^{(\alpha)\beta}). \quad (\text{A7})$$

To be consistent with Eqs. (3.18) and (3.19), the restriction $\sum_{\beta=A,B} X_{H0}^\beta = 1$ have been imposed in Eq. (A5). Therefore, all the coefficients defined by Eq. (A5) are the functions of X_{H0}^A .

The coefficients $\hat{\mu}$, $\hat{\lambda}$, \hat{D}_{AB} , \hat{D}_T , and k_T are directly related to the conventional transport coefficients of a binary mixture; that is, if we follow the definition in Ref. 23, the viscosity μ , the thermal conductivity λ , the mutual diffusion coefficient D_{AB} , the thermal diffusion coefficient D_T , and the thermal-diffusion ratio k_T are expressed as

$$\begin{aligned} \mu &= (\sqrt{\pi}/2) \hat{\mu} p (2kT/m^A)^{-1/2} \ell, & \lambda &= \sqrt{\pi} \hat{\lambda} p (2kT/m^A)^{-1/2} (k/m^A) \ell, \\ D_{AB} &= (\sqrt{\pi}/2) \hat{D}_{AB} (2kT/m^A)^{1/2} \ell, & D_T &= (\sqrt{\pi}/2) \hat{D}_T (2kT/m^A)^{1/2} \ell, \\ k_T &= D_T/D_{AB}, \end{aligned} \quad (\text{A8})$$

with $\ell = [\sqrt{2}\pi(d^A)^2 n]^{-1}$. Here we have identified the macroscopic variables with the corresponding leading-order terms of the Hilbert solution, i.e., $p = p_0 \hat{p}_{H0}$, $T = T_0 \hat{T}_{H0}$, and $n = n_0 \hat{n}_{H0}$.

APPENDIX B THE KNUDSEN-LAYER PROBLEM FOR A MIXTURE

Let us consider the boundary-value problem (3.29)–(3.31) for the Knudsen-layer correction f_{K1}^α . We put

$$f_{K1}^\alpha(\eta, s_1, s_2, \zeta_i) = [\hat{n}_{H0}^\alpha]_w \hat{T}_w^{-3/2} E^\alpha(\bar{\zeta}) \Psi^\alpha(\bar{\eta}, s_1, s_2, \bar{\zeta}_i), \quad (\text{B1})$$

where

$$\bar{\eta} = [\hat{n}_{H0}]_w \eta, \quad (\text{B2})$$

and E^α is defined by Eq. (A4). Then, if we change the notation as

$$\bar{\zeta}_i \rightarrow \zeta_i, \quad (\bar{\zeta} \rightarrow \zeta), \quad \bar{\eta} \rightarrow \eta, \quad (\text{B3})$$

Eqs. (3.29)–(3.31) are rewritten as

$$\zeta_i n_i \frac{\partial \Psi^\alpha}{\partial \eta} = \sum_{\beta=A,B} K^{\beta\alpha} [X_{H0}^\beta]_w \tilde{L}^{\beta\alpha}(\Psi^\beta, \Psi^\alpha), \quad (\text{B4})$$

$$\begin{aligned} \Psi^\alpha &= \kappa_w^\alpha + 2\hat{m}^\alpha \zeta_i (U_i - u_{iH1}) \hat{T}_w^{-1/2} - (\hat{m}^\alpha \zeta^2 - \frac{5}{2}) \hat{T}_{H1} / \hat{T}_w \\ &+ \zeta_i A^\alpha(\zeta) \frac{1}{\hat{p}_{H0}} \frac{\partial \hat{T}_{H0}}{\partial x_i} + \sum_{\beta=A,B} \zeta_i D^{(\beta)\alpha}(\zeta) \frac{1}{\hat{n}_{H0}} \frac{\partial X_{H0}^\beta}{\partial x_i}, \\ &\zeta_i n_i > 0, \quad \text{at } \eta = 0, \end{aligned} \quad (\text{B5a})$$

$$\Psi^\alpha \rightarrow 0, \quad \eta \rightarrow \infty, \quad (\text{B5b})$$

where κ_w^A is given by Eq. (3.31a), κ_w^B by Eq. (3.31b) with the last term in R.H.S. being replaced by

$$-2(\pi \hat{n}^B)^{1/2} \int_{\zeta_i n_i < 0} \zeta_i n_i \Psi^B E^B d\zeta, \quad (\text{B6})$$

and $\tilde{L}^{\beta\alpha}$ is the linearized collision operator defined in Eq. (A4).

Now let us consider the following boundary-value problem in a half space ($x_1 > 0$) of the linearized Boltzmann equation for a binary mixture:

$$\zeta_1 \frac{\partial \phi^\alpha}{\partial x_1} = \sum_{\beta=A,B} K^{\beta\alpha} C^\beta \tilde{L}^{\beta\alpha}(\phi^\beta, \phi^\alpha), \quad (\alpha = A, B), \quad (\text{B7a})$$

$$\phi^\alpha = c^\alpha + \hat{m}^\alpha c_2 \zeta_2 + \hat{m}^\alpha c_3 \zeta_3 + \hat{m}^\alpha c_4 \zeta_i^2 + g^\alpha(\zeta_i), \quad \zeta_1 > 0, \quad x_1 = 0, \quad (\text{B7b})$$

$$\phi^\alpha \rightarrow 0, \quad \text{as } x_1 \rightarrow \infty,$$

where c^α , c_2 , c_3 , and c_4 are constants and $g^\alpha(\zeta_i)$ are given functions. The C^α are given constants corresponding to the concentration of the α -component at $x_1 = 0$ and satisfying the relation

$$\sum_{\beta=A,B} C^\beta = 1.$$

We assume that the following statement holds: The solution $\phi^\alpha(x_1, \zeta_i)$ of the problem exists if and only if the constants c^α , c_2 , c_3 , and c_4 take a special set of values,

and the solution and the constants are unique. This is a conjecture based on the corresponding theorem for a single-component gas first proposed by Grad⁴⁸ and proved by Bardos *et al.*⁴⁹ for hard-sphere molecules (the theorem has been generalized to other molecular models^{50,51}).

The application of the conjecture to our original Knudsen-layer problem (B4) and (B5) can be made in the same way as in the single-component case.³⁰ That is, we find by comparison that $\kappa_w^\alpha + (5/2)(\hat{T}_{H1}/\hat{T}_w)$, $2(U_i - u_{iH1})\hat{T}_w^{-1/2}t_i$, $2(U_i - u_{iH1})\hat{T}_w^{-1/2}\tilde{t}_i$, and $-\hat{T}_{H1}/\hat{T}_w$ correspond to c^α , c_2 , c_3 , and c_4 , respectively, and $-2\hat{m}^\alpha\zeta_j n_j(u_{iH1}n_i)\hat{T}_w^{-1/2}$ plus the last two terms in R.H.S. of Eq. (B5a) corresponds to $g^\alpha(\zeta_i)$. Here, t_i and \tilde{t}_i are two orthogonal unit vectors tangential to the boundary, and the expression for κ_w^B [Eq. (3.31b)] is put aside. As the result, κ_w^α and \hat{T}_{H1} are expressed as linear combinations of $\hat{T}_w^{-1/2}u_{iH1}n_i$, $\hat{p}_{H0}^{-1}(\partial\hat{T}_{H0}/\partial x_i)n_i$, and $\hat{n}_{H0}^{-1}(\partial X_{H0}^A/\partial x_i)n_i$; and $\hat{T}_w^{-1/2}(U_i - u_{iH1})t_i$ is expressed as a linear combination of $\hat{p}_{H0}^{-1}(\partial\hat{T}_{H0}/\partial x_i)t_i$, and $\hat{n}_{H0}^{-1}(\partial X_{H0}^A/\partial x_i)t_i$. The latter relation gives the slip boundary condition (3.33). At the same time, the solution Ψ^α is expressed as the linear combination of $\hat{T}_w^{-1/2}u_{iH1}n_i$, $\hat{p}_{H0}^{-1}(\partial\hat{T}_{H0}/\partial x_i)n_i$, etc. If we insert the solution Ψ^B in Eq. (3.31b) with the last term replaced by Eq. (B6), we have another expression of κ_w^B in terms of the linear combination of $\hat{T}_w^{-1/2}u_{iH1}n_i$, $\hat{p}_{H0}^{-1}(\partial\hat{T}_{H0}/\partial x_i)n_i$, etc. Eliminating κ_w^B from the two expressions, we obtain a certain relation, which reduces to Eq. (3.32).

APPENDIX C EXTENSION TO A MULTICOMPONENT MIXTURE

The formal extension of the analysis and results shown in the previous sections to a multicomponent mixture is rather straightforward. Suppose that there are N components in the mixture, where the 1st to N_1 -th components are vapors and the $(N_1 + 1)$ -th to N -th components are noncondensable gases (i.e., the condensed phases are composed of N_1 different substances, and their N_1 vapors are contained in the gaseous mixture). Then, all the equations shown in the previous sections can be extended to this case with slight modification, which is listed below.

- i). Regard the dimensional molecular mass m^A and diameter d^A in Sec. 2 as those of the first component.
- ii). Regard the superscripts α , β , γ , and κ as running from 1 to N .
- iii). Replace the summation $\sum_{\beta=A,B}$ (or $\sum_{\alpha=A,B}$) by $\sum_{\beta=1,\dots,N}$ (or $\sum_{\alpha=1,\dots,N}$).
- iv). Replace Eqs. (3.18)–(3.20) by the following equations:

$$u_{iH1}^\alpha = u_{iH1} - \frac{\hat{T}_{H0}^{1/2}}{\hat{n}_{H0}} \left(\sum_{\beta=1}^N \hat{\Delta}_{\alpha\beta} \frac{\partial X_{H0}^\beta}{\partial x_i} + \hat{D}_{T\alpha} \frac{\partial \ln \hat{T}_{H0}}{\partial x_i} \right), \quad (\text{C1a})$$

$$\begin{aligned} \hat{p}_{ijH2} = \hat{p}_{H2} \delta_{ij} & - \hat{\mu} \hat{T}_{H0}^{1/2} \frac{\partial \overline{u_{iH1}}}{\partial x_j} + \frac{\hat{\Upsilon}_1}{\hat{p}_{H0}} \frac{\partial \overline{\hat{T}_{H0}}}{\partial x_i} \frac{\partial \overline{\hat{T}_{H0}}}{\partial x_j} + \frac{1}{\hat{n}_{H0}} \frac{\partial}{\partial x_i} \left(\hat{\Upsilon}_2 \frac{\partial \overline{\hat{T}_{H0}}}{\partial x_j} \right) \\ & + \frac{\hat{T}_{H0}}{\hat{n}_{H0}} \sum_{\beta,\gamma=1}^N \hat{\Upsilon}_3^{(\beta,\gamma)} \frac{\partial \overline{X_{H0}^\beta}}{\partial x_i} \frac{\partial \overline{X_{H0}^\gamma}}{\partial x_j} + \frac{1}{\hat{n}_{H0}} \sum_{\beta=1}^N \hat{\Upsilon}_4^{(\beta)} \frac{\partial \overline{X_{H0}^\beta}}{\partial x_i} \frac{\partial \overline{\hat{T}_{H0}}}{\partial x_j} \\ & + \frac{\hat{T}_{H0}}{\hat{n}_{H0}} \frac{\partial}{\partial x_i} \left(\sum_{\beta=1}^N \hat{\Upsilon}_5^{(\beta)} \frac{\partial \overline{X_{H0}^\beta}}{\partial x_j} \right), \end{aligned} \quad (\text{C1b})$$

$$\begin{aligned} \hat{q}_{iH1} & = -\hat{T}_{H0}^{3/2} \left(\sum_{\beta=1}^N \hat{D}_{T\beta} \frac{\partial X_{H0}^\beta}{\partial x_i} + \hat{\lambda}' \frac{\partial \ln \hat{T}_{H0}}{\partial x_i} \right) + \frac{5}{2} \sum_{\beta=1}^N \hat{p}_{H0}^\beta (u_{iH1}^\beta - u_{iH1}) \\ & = -\hat{\lambda} \hat{T}_{H0}^{1/2} \frac{\partial \overline{\hat{T}_{H0}}}{\partial x_i} + \sum_{\beta=1}^N (k_{T\beta} \hat{p}_{H0} + \frac{5}{2} \hat{p}_{H0}^\beta) (u_{iH1}^\beta - u_{iH1}), \end{aligned} \quad (\text{C1c})$$

where $k_{T\alpha}$ and $\hat{\lambda}$ are defined by

$$\begin{aligned} \hat{D}_{T\alpha} & = \sum_{\beta=1}^N k_{T\beta} \hat{\Delta}_{\alpha\beta} \quad \text{with} \quad \sum_{\beta=1}^N k_{T\beta} = 0, \\ \hat{\lambda} & = \hat{\lambda}' - \sum_{\beta=1}^N k_{T\beta} \hat{D}_{T\beta}, \end{aligned}$$

and the other coefficients $\hat{\Delta}_{\alpha\beta}$, $\hat{D}_{T\alpha}$, $\hat{\lambda}'$, $\hat{\mu}$, $\hat{\Upsilon}_1$, $\hat{\Upsilon}_2$, $\hat{\Upsilon}_3^{(\alpha,\beta)}$, $\hat{\Upsilon}_4^{(\alpha)}$, and $\hat{\Upsilon}_5^{(\alpha)}$ are defined by Eq. (A5) with the extended superscripts (see also APPENDIX A in Ref. 20). They are functions of $N-1$ concentrations out of X_{H0}^γ ($\gamma = 1, \dots, N$) [see the note below Eq. (A7)]. When the molecules of all the components are

mechanically identical, Eq. (5.2) holds for $\hat{\mu}$, $\hat{\lambda}$, $\hat{\lambda}'$, $\hat{\Upsilon}_1$, and $\hat{\Upsilon}_2$, and the other coefficients are

$$k_{T\alpha} = \hat{D}_{T\alpha} = 0, \quad \hat{\Delta}_{\alpha\beta} = -\gamma_c(1 - \delta_{\alpha\beta}/X_{H0}^\alpha), \quad \hat{\Upsilon}_3^{(\alpha,\beta)} = \hat{\Upsilon}_4^{(\alpha)} = \hat{\Upsilon}_5^{(\alpha)} = 0.$$

v). Regard the superscript A as running from 1 to N_1 and B as running from $N_1 + 1$ to N in Eqs. (2.6), (3.11), (3.12), (3.25), (3.31), (3.32), (6.1), (7.1), and (B6).

vi). Replace Eq. (3.33) by

$$(u_{iH1} - U_i)t_i = -\hat{T}_{H0}^{1/2} \left(b_7 \frac{1}{\hat{p}_{H0}} \frac{\partial \hat{T}_{H0}}{\partial x_i} t_i + \frac{1}{\hat{n}_{H0}} \sum_{\beta=1}^N b_9^{(\beta)} \frac{\partial X_{H0}^\beta}{\partial x_i} t_i \right), \quad (\text{C2})$$

where b_7 and $b_9^{(\alpha)}$ depend on $N - 1$ concentrations out of X_{H0}^β ($\beta = 1, \dots, N$) on the boundary. For a mixture of mechanically identical molecules, Eq. (5.3) holds for b_7 and $b_9^{(\alpha)} = 0$.

We note here that in the physical situation under consideration, the parameter $p_0 \hat{p}_w^\alpha$ should be a hypothetical pressure of the α -th vapor prescribed by (modified) Raoult's law, not the saturation pressure of the vapor at temperature $T_0 \hat{T}_w$ (see Ref. 20 for the detail). However, it is seen from the above extension that there is no essential difference between the behavior of the multicomponent mixture and that of the binary mixture. Therefore, the classification of the source factors for the ghost effect in Sec. 4 is also valid for the multicomponent mixture.

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- [43] In fact, if the assumptions (3.11) and (3.23) are not compatible with the physical problem under consideration, the analysis cannot be performed consistently. The compatibility does not exclude the possibility of a solution with a flow of $O(1)$, but the presence of such a solution is unlikely in the situation without external forces.
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ERRATUM

**THE GHOST EFFECT IN THE
CONTINUUM LIMIT FOR A VAPOR–GAS
MIXTURE AROUND CONDENSED
PHASES: ASYMPTOTIC ANALYSIS OF
THE BOLTZMANN EQUATION**

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The above article was previously printed in *Transport Theory and Statistical Physics*, 30(2&3), pp. 205–237, with errors. The corrected text is listed below.

In the third line from the bottom on page 206, “Knudsen number [2]” should read “Knudsen number [2]”.

In Eq. (2.2) on page 209, the $d\zeta_*$ should read $d\zeta_*$.

The second equation of Eq. (2.3b) on page 209 should read $d\zeta_* = d\zeta_{*1}d\zeta_{*2}d\zeta_{*3}$.

The $d\zeta$ in Eqs. (2.6b), (2.7), (2.8), (3.9a), (3.9b), (3.10a)–(3.10c), (3.31b), (7.1), and (B6), in the fourth line on page 210, in the tenth line on page 218, and in the twenty-fourth line on page 235 should read $d\zeta$.

In the first line on page 214, the second “Section 2” should read “Section 3.2”.

In the seventh line from the bottom on page 218, “(15–18)” should read “[15–18]”.

In the thirteenth line on page 223, “(8)” should read “[8]”.

In the fifteenth line on page 225, “the fluid-dynamic type Eqs” should read “the fluid-dynamic type equations”.

Eq. (B7b) on page 230 should read as follows.

$$\begin{aligned} \phi^\alpha &= c^\alpha + \hat{m}^\alpha c_2 \zeta_2 + \hat{m}^\alpha c_3 \zeta_3 + \hat{m}^\alpha c_4 \zeta_i^2 + g^\alpha(\zeta_i), \quad \zeta_1 > 0, \quad x_1 = 0, \\ \phi^\alpha &\rightarrow 0, \quad \text{as } x_1 \rightarrow \infty, \end{aligned} \tag{B7b}$$

In the last line on page 230, “Eq. (B6a)” should read “Eq. (B6)”.

All these errors, as well as many other minor errors that are not listed above, were a result of mistakes in the printing process and not the fault of the authors.