Modernization, Social Identity, and Ethnic Conflict

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Abstract

The lack of a shared national identity is often blamed for ethnic conflict and low economic development. This raises the question: does a society's modernization (e.g., industrialization) lead to a shared identity, thereby bringing good outcomes in conflict and development? This paper theoretically examines the question using a contest model of conflict augmented with multiple production sectors and social identification.

The analysis shows that as modernization proceeds, a society shifts to an equilibrium with a universal national identity, a low level of conflict, and high output if national pride is high, ethnic differences are not salient in people's minds, resources are not abundant, or institutions are of good quality. Otherwise, the society shifts to an equilibrium with a universal ethnic identity and worse economic and political outcomes. The analysis suggests that nation-building policies play a critical role in overcoming the negative effects of modernization under the latter situation.

Keywords: ethnic conflict, social identity, modernization, nation-building policies, economic development

JEL classification numbers: D72, D74, O10, O20

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1 Introduction

Evidence suggests that ethnic divisions in a society lead to negative outcomes in civil conflict (Esteban, Mayoral, and Ray, 2012) and economic development (Montalvo and Reynal-Quero, 2005). The lack of a shared national identity, that is, the dominance of ethnic over national identity, is often blamed for the negative outcomes (Collier, 2009; Michalopoulos and Papaioannou, 2015). If shared national identity is important for positive outcomes, then how can it be realized? This study focuses on the effect of the modernization of a society, in particular, industrialization and resultant greater ethnic integration in the workplace. Does modernization contribute toward shared identity, as the classic thesis in political science argues (Deutsch, 1953; Gellner, 1964, 1983; Weber, 1979)? Or does it lead to heightened ethnic identity and conflict, as a competing thesis asserts (Melson and Wolpe, 1970; Bates, 1983)?

This paper explores the effect of modernization on social identity, ethnic conflict, and economic development theoretically. To do so, it develops a contest model of conflict augmented with multiple production sectors and social identification, drawing on the model by Sambanis and Shayo (2013).

In a standard contest model of conflict, groups compete for exogenous resources used to produce group-specific club goods (e.g., public services and infrastructure benefiting specific groups), the members of each group choose contributions to conflict, taking into account the cost and benefit of their action, and the total contributions of group members determine the amount of resources the group obtains.

Sambanis and Shayo (2013) augment the standard model with socio-psychological factors. Individual utility depends not only on the material cost and benefit of contributing to ethnic conflict but also negatively on the *perceived distance* between oneself and the group one identifies with (their ethnic group or the nation) and positively on the group's *status*. In other words, one bears a large cognitive cost when they are very different from other members of the group in relevant aspects, but takes pride in being a member when the group's status is high. These socio-psychological components are major determinants of social identification and intergroup behaviors, according to influential theories in social psychology (Tajfel and Turner, 1986; Turner et al., 1987) and empirical evidence (Manning and Roy, 2010; Sambanis, Skaperdas, and Wohlforth, 2015).[1,2] Importantly, because these components differ depending on whether one identifies with their ethnic group or the nation, the social identities of people influence individual contributions to conflict and thus the level of conflict. Further, social identity is *endogenously determined*: one chooses the identity that brings them higher utility. Hence, social identity and individual and aggregate outcomes interact with each other.

[1] The concept of perceived distance is the basis of a major social psychological theory, self-categorization theory (Turner et al., 1987). Intergroup status differences comprise major factors affecting intergroup behaviors such as conflict and discrimination, according to a closely related theory—social identity theory (Tajfel and Turner, 1986).

[2] Evidence suggests that perceived distance and status affect identity. For the United Kingdom, Manning and Roy (2010) find that non-whites, whose perceived distance to the nation seems to be greater than that of the whites, are less likely to think of themselves as British. They also find that immigrants from poorer and less democratic (i.e., lower status) countries assimilate faster into British identity. See footnote 15 for evidence from Sambanis, Skaperdas, and Wohlforth (2015).

To examine the effect of modernization on social identity, conflict, and development, this study modifies Sambanis and Shayo's model in two ways. First, private good production is introduced into the model. There are multiple sectors—ethnically segregated *traditional sectors* (e.g., traditional agriculture and the urban informal sector in the actual economy) and the ethnically integrated *modern sector*—and the individuals work in the sector that maximizes their utility. In the model, the modernization of a society is the sectoral shift of labor and production from the traditional sectors to the modern sector, and it is driven by the increased (total factor) productivity of the modern sector. Second, sectoral affiliation, along with ethnicity, is a component of the perceived distance. In other words, one feels strong discomfort when their ethnicity and sectoral affiliation, which represents the type of job (e.g., white collar, informal sector, and agricultural jobs) they have, differ significantly from those of a typical person of their identity group.

Although the present model is a simple extension of Sambanis and Shayo's model, it can analyze not only the interaction between social identity and conflict, the focus of Sambanis and Shayo (2013), but also the interactions of social identity with economic variables—sectoral compositions of labor and output and aggregate output. By examining how productivity-driven modernization changes these interactions, this paper shows how modernization affects social identity, conflict, and development and derives policy implications.

The model has multiple equilibria that differ in the proportion of individuals identifying with the nation. First, to see how social identification itself affects conflict and economic outcomes, different equilibria are compared for *given* parameters and exogenous variables.[3] The analysis shows that an equilibrium with more prevalent national identity has a lower level of conflict, higher modern sector shares in employment and production, and, under plausible conditions, higher levels of private good production and aggregate material payoff (the value of private and club good consumption net of the cost of conflict). In other words, national identity is associated not only with a lower level of conflict, shown in Sambanis and Shayo (2013), but also with higher modern sector shares and higher output.

Then, a driving force of modernization—the increased productivity of the modern sector—is introduced into the model. The productivity growth raises the modern sector wage, induces a higher proportion of workers to work in the sector, and increases the sector's output share. How the productivity-driven modernization affects identity, conflict, and development depends on the *status difference,* the difference between the status of the nation and that of ethnic groups. The national status and the status difference would be high when the people of a nation believe that they share a glorious history, rich culture, or a "right" sense of values because they feel proud of belonging to such a nation. When ethnic groups are distinctive in these aspects, the ethnic status is high and the status difference is low.

If the status difference is very high (low), everyone identifies with the nation (one's ethnic group) and the level of conflict is low (high) all the time. Otherwise, when the status difference is relatively high (low), the society typically shifts from an equilibrium in which the national identity

[3] Multiple or even all equilibria exist for certain values of parameters and exogenous variables.

is more prevalent among modern sector workers to an equilibrium with a universal national (ethnic) identity and a low (high) level of conflict. Given the productivity level, the society with a high (low) status difference has relatively large (small) modern sector shares and, under plausible conditions, relatively high (low) levels of private good production and aggregate material payoff. Hence, having sufficiently high national status relative to ethnic status is crucial for achieving good outcomes in development as well as in identity and conflict.[4] An exogenous change that makes ethnic differences less salient in people's minds also exerts effects similar to those of a rise in the national status. Similar results also hold for contested resources. Specifically, *given the status difference*, when the amount of contested resources is large (small), the society ends up in the equilibrium of a universal ethnic (national) identity. This indicates that both the abundance of resources and the absence of strong political and economic institutions (e.g., weak rule of law), which leads to abundant *contested* resources, are obstacles to good outcomes.

The above results are consistent with the classic thesis on the effects of modernization on identity in political science if the national status relative to the ethnic status is high, resources are not abundant, institutions are of good quality, or ethnic differences are not salient in people's minds. Otherwise, the results are consistent with the competing thesis. The results also have important policy implications. Under the former conditions, policies promoting modernization, such as policies stimulating the technological progress of the modern sector and the reform of institutions supporting the sector's activities, might be sufficient for achieving good outcomes. Conversely, under the latter conditions, they exert *negative* effects on national identity and conflict, thereby becoming less effective for development. This highlights the vital role of policies that raise the national status, improve institutional quality, or make ethnic differences less salient. Miguel (2004), Collier (2009), and Blouin and Mukand (2019), based on a case study or statistical analysis, argue that national identity is effectively strengthened through *nation-building policies*, such as school education and government propaganda that emphasize common history, culture, and values and the promotion of a national language.[5] These policies may be considered as ones that lift the national status or deemphasize ethnic differences and thus are critical under the adverse conditions.

This paper adds to the literature on contest models of conflict (Sambanis and Shayo, 2013;

[4] However, history or "luck" is also important if the status difference is not at an extreme level. For given parameters and exogenous variables, multiple equilibria could exist. If the initial equilibrium *happens to* be such that the national identity is relatively prevalent, the society tends to maintain a relatively strong national identity and relatively good outcomes in conflict and development.

[5] Miguel (2004) finds that the two neighboring rural districts of Tanzania and Kenya, which largely share geography, history, and a colonial institutional legacy, exhibit a sharp difference in the relationship between ethnic diversity and the local provision of public goods (school funds and infrastructure)—significantly negative for the Kenyan district and positive but insignificant for the Tanzanian district. He also finds that the relationship is insignificant for other local public finance outcomes for Tanzania (no comparable data for Kenya). He argues that sharply different ethnic policies in areas such as national language and public school education by post-independent governments contributed to differences in the strength of national identity and the above-mentioned relationship between the two countries. Collier (2009), drawing on episodes from Indonesia and Tanzania, argues that policies such as the promotion of a common language and the introduction of the school curriculum emphasizing common history fortified the national identity. Blouin and Mukand (2019), based on field and lab experiments in post-genocide Rwanda, argue that an exposure to government radio propaganda lowered ethnic salience and raised interethnic trust and cooperation.

Esteban and Ray, 2008; 2011; Besley and Persson, 2010; Sambanis, Skaperdas, and Wohlforth, 2015; Mariani, Mercier, and Verdier, 2018). Besley and Persson (2010) examine the interaction between conflict and development, as in this study, but focuses on capacities of the state to raise revenue and provide market-supporting services. Sambanis, Skaperdas, and Wohlforth (2015), partly drawing on Sambanis and Shayo (2013), present a model in which leaders may initiate interstate war anticipating that victory raises the national status and induces national identification beneficial to the people.

The paper also adds to the theoretical literature examining interactions between identity and economic or political behaviors (Fearon and Laitin, 2000; Akerlof and Kranton, 2000, 2010; Shayo, 2009; Benabou and Tirole, 2011; Bisin et al., 2011; Gennaioli and Tabellini, 2019; Grossman and Helpman, 2020).[6] By generalizing the pioneering work of Akerlof and Kranton (2000), Shayo (2009) constructs the basic framework, which motivates Sambanis and Shayo (2013) and this study, and applies it to analyze the political economy of income redistribution. Shayo's (2009) framework has been applied to various issues. For example, motivated by a recent reversal of trade policies in some western countries seemingly influenced by rises of populism and ethnic tensions, Grossman and Helpman (2020) construct a political economy model of trade policy with social identification and examine how policies are affected by changes in the identification patterns triggered by events such as increased ethnic tensions.

Further, the paper is related to theoretical work on nation-building policies, such as Alesina, Reich, and Riboni (2020), Almagro and Andrés-Cerezo (2020), and Alesina, Giuliano, and Reich (2021). Theses papers explicitly model the determination of nation-building polices and examine how the implemented policies depend on factors such as the threat of democratization and the initial distribution of identities.

Finally, the paper contributes to the literature theoretically examining the modernization of an economy, such as Lewis (1954), Banerjee and Newman (1998), Proto (2007), Vollrath (2009), and Yuki (2007, 2008, 2016). For analytical tractability, this study models the inefficient sectoral allocation of workers in a simplest manner and considers the modernization induced by exogenous productivity growth. Conversely, these studies model factors leading to the inefficient allocation more explicitly and examine economic mechanisms of modernization in detail.

The rest of the paper is organized as follows. Section 2 presents the model, Section 3 examines equilibria, and Section 4 analyzes the effects of modernization on identity, conflict, and development. Section 5 examines how the results in Section 4 are affected by the abundance of contested resources and discusses how the results depend on several assumptions. Section 6 concludes. Appendix A presents the existence conditions for equilibria, and Appendix B contains proofs.

2 Model

[6] In addition to the already mentioned work, recent empirical and experimental studies on identity in economics and political science include Chen and Li (2009); Benjamin, Choi, and Strickland (2010); Eifert, Miguel, and Posner (2010); Clots-Figueras and Masella (2013); Charnysh, Lucas, and Singh (2015); Cohn, Maréchal, and Noll (2015); Alenzuela and Michelson (2016); Benjamin, Choi, and Fisher (2016); and Wimmer (2017).

Consider a contest model of conflict augmented with private good production and social identification in which $n_e(\geq 2)$ ethnic groups contest for resources and a finite number L of individuals choose a contribution to conflict, a sector to work in, and a social group to identity with. The ethnic groups are assumed to be symmetric in every aspect (thus the population size of each group is L/n_e), for the sake of focus on a society without a dominant ethnic group and for analytical tractability.

2.1 Production

There are $n_e + 1$ sectors producing private goods: n_e ethnically segregated traditional sectors (sectors TJ, $J = 1, 2, ..., n_e$) and an ethnically integrated modern sector (sector M). The traditional sectors correspond to sectors using traditional or rudimentary technologies in the actual economy, such as traditional agriculture, the urban informal sector, and household production; the modern sector corresponds to sectors such as modern manufacturing and services.[7] The former sectors are more ethnically segregated than the latter sector (Ezcurra and Rodríguez-Pose, 2017, Beegle et al., 2014):[8] traditional agriculture is operated in largely ethnically homogeneous rural communities, and typical informal sector jobs are neighborhood jobs in ethnically segregated communities.

The production functions of sectors TJ ($J = 1, 2, ..., n_e$) and M are represented as

$$Y_{TJ} = A_T(L_{TJ})^{\alpha}, \ \alpha \in (0, 1), \tag{1}$$

$$Y_M = A_M \sum_{J=1}^{n_e} L_{MJ}, \tag{2}$$

where L_{TJ} and A_T are the number of workers in sector TJ and the sector's total factor productivity (TFP), respectively; L_{MJ} is the number of workers of ethnic group J in sector M, and A_M is the sector's TFP.[9] Decreasing returns to labor is assumed for sector TJ to capture the fact that labor productivity tends to fall quickly with the amount of labor input in traditional sectors owing to limited arable land (traditional agriculture), limited capital available to credit-constrained producers (the urban informal sector), or a decreasing degree of task specialization of each family member (household production).[10] The assumption reflects limited amounts of complementary production factors (except household production) in the real economy. Since such a constraint is not severe in the modern sector, for simplicity, constant returns to labor is assumed for sector M.

The wage rate is determined competitively in sector M. Conversely, in sector TJ, as in Lewis (1954) and many subsequent works modeling traditional sectors, labor income is determined so that the product is equally shared among the workers.[11] Thus, labor incomes in the sectors are

[7] The urban informal sector is a part of the urban economy comprising small-scale businesses supplying basic services (e.g., small shops and vendors selling commodities and meals) and basic manufacturing goods.

[8] Ezcurra and Rodríguez-Pose (2017) find that ethnic segregation tends to be lower in countries with higher proportions of people located in the urban area, where modern sector activities are concentrated, especially in developing countries. The review article by Beegle et al. (2014) argues that the majority of informal sector firms are individually- or family-owned and ethnic and kinship networks play an important role in organizing the sector.

[9] Each worker supplies a unit of labor inelastically.

[10] This is because the number of tasks performed by each family member increases as more production activities and thus labor input shift from the market to the household.

[11] This assumption reflects the fact that the typical production units of traditional sectors are family-run farms/firms

$$y_{TJ} = A_T(L_{TJ})^{\alpha-1}, \quad (3)$$

$$y_M = A_M. \quad (4)$$

In a simplest manner, this setting can generate the situation facing actual developing countries in which inefficiently many workers exist in traditional sectors and their shift to modern sectors raises the aggregate output (Gollin, Lagakos, and Waugh, 2014).[12]

Group J individuals are freely mobile between sector M and sector TJ; their sectoral distribution is determined so that their utilities in the two sectors, which are specified later, are equated.

2.2 Conflict

The ethnic groups contest for exogenous resources of value V used to produce group-specific club goods (e.g., public services and infrastructure benefiting particular groups).[13] The amount of resources each group acquires depends on the contributions to the conflict by members of each group. In particular, the contested resources are divided among the groups according to the following contest function:

$$\frac{V_J}{V} = \frac{F_J}{F} \text{ if } F > 0 \text{ and } = \frac{1}{n_e} \text{ if } F = 0, \quad (5)$$

where V_J denotes the resources acquired by group $J(J = 1, 2, ..., n_e)$; $F_J \equiv \sum_{i \in J} f_i$ denotes the total contributions or "efforts" by members of the group (f_i is the contribution by individual i); and $F \equiv \sum_{J=1}^{n_e} F_J$ represents the aggregate "efforts" in the society and is termed as the *level of conflict*. The contested resources represent not only material resources (such as natural resources) but also the government budget for ethnic-specific club goods.[14] The model describes a society in which the resource allocation among the groups is determined not by rule but by the consequences of violent *or non-violent* conflicts (such as rent-seeking activities), in which force, mass demonstrations, bribery, or lobbying are employed to influence the outcome.

Individual i contributing f_i to the conflict incurs a cost of $c(f_i)$, which, following Esteban and Ray (2011), takes the following form:

$$c(f_i) = \frac{1}{\theta}(f_i)^{\theta}, \ \theta \geq 2. \quad (6)$$

The restriction $\theta \geq 2$ is needed to prove some results, though $\theta > 1$ is enough for most results.

2.3 Utility

or households. Except for the results on the total output of private goods and the aggregate material payoff, the qualitative results below do not depend on this assumption.

[12] In the real economy, there exist other factors driving the inefficient allocation of workers, including limited access to quality education required for many modern sector jobs and inadequate access to capital to start a business in the sector. For analytical tractability, these factors are not modeled; however, their absence will not affect the results.

[13] The main results will not be affected by assuming that the resources are used to produce private goods.

[14] Precisely, when the contested resources represent the state budget for the club goods, taxation should be modeled. If the government imposes a lump-sum tax of the same amount on everyone, none of the results are affected. The only change is that the disposable incomes of individuals decrease by the tax payment.

As in Sambanis and Shayo (2013), individual utility depends not only on one's *material payoff* (the value of private and club good consumption minus the cost of conflict) positively but also negatively on the *perceived distance* to a *social group* with which one identifies (either their ethnic group or the nation) and positively on the *status* of the group. In other words, one incurs a mental cost when one is different from others of the group in relevant features but takes pride in belonging to the group when its status is high. These socio-psychological components, based on influential theories in social psychology (Tajfel and Turner, 1986; Turner et al., 1987), are the major factors affecting social identification in the society (Manning and Roy, 2010; Sambanis, Skaperdas, and Wohlforth, 2015).[15]

The *material payoff* of individual i who belongs to ethnic group J ($J = 1, 2, ..., n_e$) and works in sector K ($K = TJ, M$) is

$$\pi_i = y_K - \frac{1}{\theta}(f_i)^\theta + \delta V_J, \tag{7}$$

where δ measures the strength of preference for the group-specific club good.

The individual, who is characterized by two types of attributes, perceives the distance or proximity from a social group (ethnic group J or the nation N) based on the distance between his or her attributes and the average attributes of the group. The attributes are whether he or she belongs to (a) each ethnic group and (b) each traditional sector:

$$q_i^J = 1; \quad q_i^{J'} = 0 \text{ for } J' \neq J, \tag{8}$$

$$q_i^{TJ} = 1(=0) \text{ if } K = TJ(= M); \quad q_i^{TJ'} = 0 \text{ for } J' \neq J, \text{ where } J, J' = 1, 2, .., n_e. \tag{9}$$

For example, when the person belongs to ethnic group 2 and works in sector M, $q_i^2 = 1, q_i^{J'} = 0$ for $J' \neq 2$, and $q_i^{TJ'} = 0$ for any J'. Note that the ethnic attributes are fixed, while the sectoral attributes are determined endogenously by the sectoral choices of workers, which, as shown later, could affect their identity choices.

The *perceived distance* between individual i and social group G ($G = J, N$), on which the utility negatively depends, is represented by[16]

$$d_{iG} = \omega_e \sum_{J'=1}^{n_e} (q_i^{J'} - q_G^{J'})^2 + \omega_s \sum_{J'=1}^{n_e} (q_i^{TJ'} - q_G^{TJ'})^2, \tag{10}$$

where $q_G^{J'}$ and $q_G^{TJ'}$ ($J' = 1, 2, .., n_e$) are, respectively, average values of the ethnic and sectoral attributes of the group; ω_e and ω_s are weights on the respective types of attributes. For example, if

[15] For the United Kingdom, Manning and Roy (2010) find that the non-whites, whose perceived distance to the "average national" would be greater that of the whites, are less likely to think of themselves as British. They also find that immigrants from poorer and less democratic (i.e., lower status) countries assimilate faster into a British identity. Sambanis, Skaperdas, and Wohlforth (2015) present episodes suggesting that interstate wars affect social identity through the national status, including increased identification with the state in the USSR after the victory in the Second World War and the intensification of a common identity among southern Slavs (including Croats and Slovenes) after Serbian victories against the Ottoman Empire and Bulgaria in the Balkan Wars in 1912−1913.

[16] The concept of perceived distance was developed in cognitive psychology when studying how a person categorizes information that comes into his or her stimuli (Nosofsky, 1986). Turner et al. (1987) apply the concept to the categorization of people, including oneself, into social groups, when developing an influential social psychological theory, self-categorization theory. The theory tries to explain the psychological basis of social identification.

the person belongs to ethnic group 2, when $G = 2$, $q_2^2 = 1$, $q_2^{T2} = \frac{L_{T2}}{L_{M2}+L_{T2}} = \frac{L_{T2}}{L/n_e}$, and $q_2^{J'} = q_2^{TJ'} = 0$ for $J' \neq 2$, and when $G = N$, $q_N^{J'} = \frac{1}{n_e}$ and $q_N^{TJ'} = \frac{L_{TJ'}}{L}$ for any J'. The first term (henceforth called the *ethnic distance*) is the psychological cost the individual incurs owing to the difference in ethnicity between him or her and the "average member" of the group. The second term (*sectoral distance*) is the psychological cost owing to the difference in sectoral affiliation, which represents the type of occupation in the real economy.[17]

Following Sambanis and Shayo (2013), the weight on ethnic attributes ω_e is assumed to increase with the level of ethnic conflict F:

$$\omega_e = \eta_0 + \eta_1 F, \quad \eta_0 \geq 0, \ \eta_1 > 0, \tag{11}$$

The specification implies that an increase in the intensity of the ethnic conflict increases the salience of the ethnicity in people's minds. This is consistent with empirical evidence. A case study on the Yugoslav Wars in the 1990s conducted by Sambanis and Shayo (2013) cites evidence that the share of people identifying themselves as "Yugoslavs" declined significantly after the intensification of the conflict, despite episodes suggesting the lack of strong ethnic identities before the war. Using individual, county-level, and district-level data from Uganda and instrumental variable estimation, Rohner, Thoenig, and Zilibotti (2013) find that the proportion of those identifying more with their ethnic group than with the nation is higher in counties with a higher intensity of armed conflicts, after controlling for individual, ethnic, and spatial characteristics. Eifert, Miguel, and Posner (2010), based on 22 public opinion surveys from 10 African countries, find that one's closeness to a competitive presidential election is associated with ethnic identification.

The utility of an individual also depends positively on the *status of the social group* G ($G = J, N$) he or she identifies with, which is exogenous and denoted by S_G. Ethnic groups are symmetric, thus the level of their status is the same, which is denoted by

$$S_J = S_E \text{ for any } J. \tag{12}$$

The level of the *national status* S_N would be high when the people of a nation believe that they share a glorious history, rich culture, or a "right" sense of values or when a nation records commendable performance in international sports competitions because the people would feel proud of belonging to such a nation. Conversely, the level of the *ethnic status* S_E would be high when

[17] The reason sectoral affiliation is a component of the perceived distance is that the type of the job (modern sector or traditional sector job) of an individual seems to be an important factor affecting the individual's social identity. In the second round of the Afrobarometer, a multicounty African public attitude survey project, conducted in 2002-03 (https://www.afrobarometer.org/online-data-analysis/analyse-online), respondents are asked about the group with which they have the most belongingness, besides the nation. The highest proportion of respondents choose occupation (27.2%) rather than the language/tribe/ethnic group (22.6%). YouGov, albeit a British survey, has a similar question that, unlike the Afrobarometer, allows for multiple answers and the two most common answers are occupation (44%) and the area of residence (43%) (Shayo, 2009).

ethnic groups are distinctive in these dimensions.[18,19]

From these settings, the utility of individual i when he or she identifies with social group G is given by

$$u_{iG} = \pi_i - \beta d_{iG} + \gamma S_G, \qquad \beta, \gamma > 0. \tag{13}$$

The utility function implies that, *given* that an individual identifies with a particular social group, the utility increases with a decline in the perceived distance to the group; thus, the individual has an incentive to choose actions lowering the distance. For example, since the sectoral distance is a component of the perceived distance, other things being equal, an individual has an incentive to choose the same sector as that of the "average member" of the group.

However, social identification of an individual, that is, the group he or she identifies with, is *not fixed*. Between the nation and their ethnic group, one "chooses" the group bringing higher utility because of a higher material payoff, a shorter perceived distance, or higher status.[20] One's identity may change if levels of the variables affecting the utility directly or indirectly through choices of others change. For example, a rise in the level of conflict increases the ethnic distance, and thus could drive individuals to switch from the national to the ethnic identity.

2.4 Timing

Individuals play a two-stage game to maximize their utility. First, they decide in which sector to work (sector TJ or sector M for ethnic group J), which determines the traditional sector income (y_{TJ}) and the sectoral and aggregate outputs (Y_{TJ}, Y_M, and $Y \equiv Y_{TJ} + Y_M$). Then, that is, after L_{TJ} and L_{MJ} are settled, they simultaneously choose a group to identify with and the contribution to conflict f_i, which determines the level of conflict F, the allocation of the contested resources among the groups, and individual utilities. The timing of events reflects the following facts: the sectoral choice made earlier in life largely determines the sector of employment for the rest of the lives (given that the two sectors often require different levels or types of education and skills and are located in different places in the actual economy); in contrast, the social identity often changes over time, gradually (see footnote 15 for the evidence on immigrants) or suddenly triggered by events such as armed conflict and electoral competition (see the evidence following (11)). The solution concept applied is subgame perfect Nash equilibrium; thus, the game can be solved by backward induction.[21]

[18] Similar to works such as Grossman and Helpman (2020), status is an absolute measure. By contrast, in Shayo (2009) and Sambanis and Shayo (2013), status is a relative measure and is defined as the difference from the reference group. The main results remain unchanged under the alternative specification.

[19] To make the model manageable, unlike Sambanis and Shayo (2013) and Grossman and Helpman (2020), the status does not depend on the group's total or average material payoff (the sum or average of π_i). The results will not be affected by considering the economic status, as long as its importance in the utility is not very large.

[20] By assumption, one does not identify with the nation and their ethnic group simultaneously. Conversely, in the model of Grossman and Helpman (2020), an individual identifies with his or her class always and with the nation also if the additional identity increases the utility, where the utility depends on the sum of the perceived distance to and the status of each group with which the individual identifies. The present paper does not adopt this specification owing to the complexities associated with the perceived distance term and the difficulties when analyzing the model.

[21] Sambanis and Shayo (2013) apply the concept of social identity equilibrium to their one-shot game. The equilibrium is similar to standard Nash equilibrium, but the condition on the choice of identities is weaker. In this study, the concept of subgame perfect Nash equilibrium is used because it is more familiar and easier to apply. Shayo (2009)

3 Equilibria

The model assumes that ethnic groups are symmetric in every aspect; thus, the paper focuses on equilibria in which choices by all groups are symmetric.[22] These equilibria can be classified into two types: equilibria in which individuals from the same ethnic group share the same identity (*homogeneous identity equilibria*) and those in which they do not (*heterogeneous identity equilibria*). For the ease of exposition, first, the former type of equilibria is analyzed (Section 3.1); subsequently, the latter type of equilibria is analyzed and compared with the former type (Section 3.2).

To simplify the analysis, the following assumption, which is a sufficient condition for $f_i > 0$ and thus $F > 0$ to hold in all equilibria, is imposed.

$$\text{Assumption 1:} \quad \delta\frac{V}{L} > (\beta\eta_1)^{\frac{\theta}{\theta-1}}\left(\frac{n_e-1}{n_e}\right)^{\frac{1}{\theta-1}}. \tag{14}$$

3.1 Homogeneous identity equilibria

There exist two homogeneous identity equilibria: the equilibrium in which everyone identifies with their ethnic group (henceforth *equilibrium* (e)) and the one in which everyone identifies with the nation (henceforth *equilibrium* (n)).

3.1.1 Equilibrium (e)

Consider the second stage of the game in which the sectoral allocation of workers (L_{TJ} and L_{MJ}) is given. When individual i of ethnic group J ($J = 1, 2, ..., n_e$) in sector M identifies with his or her ethnic group, the individual chooses the contribution to conflict f_i to maximize the following utility (note $q_i^J = q_J^J = 1$, $q_i^{TJ} = 0$, $q_J^{TJ} = \frac{L_{TJ}}{L/n_e}$, and $q_i^{J'} = q_J^{J'} = q_i^{TJ'} = q_J^{TJ'} = 0$ for $J' \neq J$):

$$A_M - \frac{1}{\theta}(f_i)^\theta + \delta\frac{F_J}{F}V - \beta\omega_s\left(\frac{L_{TJ}}{L/n_e}\right)^2 + \gamma S_E. \tag{15}$$

From the first-order condition,

$$f_i = f_{i,e} \equiv \left(\delta\frac{F_{-J}}{F^2}V\right)^{\frac{1}{\theta-1}}, \text{ where } F_{-J} \equiv F - F_J. \tag{16}$$

When the individual is in sector TJ instead, he or she chooses f_i to maximize ($q_i^{TJ} = 1$)

$$A_T(L_{TJ})^{\alpha-1} - \frac{1}{\theta}(f_i)^\theta + \delta\frac{F_J}{F}V - \beta\omega_s\left(1 - \frac{L_{TJ}}{L/n_e}\right)^2 + \gamma S_E. \tag{17}$$

The solution for f_i is given by (16), as in the previous case.

Since all individuals identify with their ethnic groups, which are symmetric, by substituting $F_{-J} = \frac{n_e-1}{n_e}F$ and $f_i = F/L$ into (16), the equilibrium level of conflict F_e^* is obtained:

employs the Nash equilibrium concept to solve a one-shot game of social identity.

[22] There also exist subgame perfect Nash equilibria in which different ethnic groups make different choices, which are difficult to analyze.

$$F_e^* = \left(\delta \frac{n_e - 1}{n_e} \frac{V}{L}\right)^{\frac{1}{\theta}} L \text{ from } F_e^* = \left(\delta \frac{n_e - 1}{n_e} \frac{V}{F_e^*}\right)^{\frac{1}{\theta - 1}} L. \tag{18}$$

In the first stage, individuals choose sectors by taking into account the effects of their choices on the second stage. Assume that the following condition holds so that $L_{TJ} = \frac{L}{n_e}$ (i.e., everyone in group J chooses sector TJ) does not hold in equilibrium:

$$\text{Assumption 2:} \quad A_T \left(\frac{L}{n_e}\right)^{\alpha - 1} + \beta \omega_s < A_M. \tag{19}$$

Then, the sectoral allocation of workers is determined so that they are indifferent between the two sectors. From (15) and (17), the indifference condition for sectoral choices is

$$A_T (L_{TJ})^{\alpha - 1} - \beta \omega_s \left(1 - 2 n_e \frac{L_{TJ}}{L}\right) = A_M, \tag{20}$$

which gives the unique $(L_{TJ})_e^* \in (0, \frac{L}{n_e})$ that decreases with A_M and increases with A_T.[23]

3.1.2 Equilibrium (n)

Consider the second stage of the game in which the sectoral allocation of workers is given. When individual i of ethnic group J in sector M identifies with the nation, he or she chooses f_i to maximize the following utility ($\omega_e = \eta_0 + \eta_1 F$, $q_N^{J'} = \frac{1}{n_e}$, $q_N^{TJ'} = \frac{L_{TJ'}}{L}$ for any $J' = 1, 2, .., n_e$):

$$A_M - \frac{1}{\theta}(f_i)^\theta + \delta \frac{F_J}{F} V - \beta \left\{ (\eta_0 + \eta_1 F) \frac{n_e - 1}{n_e} + \omega_s \left[\left(\frac{L_{TJ}}{L}\right)^2 + \sum_{J' \neq J} \left(\frac{L_{TJ'}}{L}\right)^2 \right] \right\} + \gamma S_N. \tag{21}$$

From the first-order condition ($f_i > 0$ from (14)),

$$f_i = f_{i,n} \equiv \left(\delta \frac{F_{-J}}{F^2} V - \beta \eta_1 \frac{n_e - 1}{n_e}\right)^{\frac{1}{\theta - 1}}, \text{ where } F_{-J} \equiv F - F_J. \tag{22}$$

When the individual is in sector TJ instead, he or she chooses f_i to maximize

$$A_T (L_{TJ})^{\alpha - 1} - \frac{1}{\theta}(f_i)^\theta + \delta \frac{F_J}{F} V - \beta \left\{ (\eta_0 + \eta_1 F) \frac{n_e - 1}{n_e} + \omega_s \left[\left(1 - \frac{L_{TJ}}{L}\right)^2 + \sum_{J' \neq J} \left(\frac{L_{TJ'}}{L}\right)^2 \right] \right\} + \gamma S_N, \tag{23}$$

whose solution is given by (22).

Since all individuals identify with the nation and the groups are symmetric, by plugging $F_{-J} = \frac{n_e - 1}{n_e} F$ and $f_i = F/L$ into (22), the equilibrium level of conflict F_n^* is obtained as a solution for

$$F_n^* = \left[\frac{n_e - 1}{n_e} \left(\delta \frac{V}{F_n^*} - \beta \eta_1\right)\right]^{\frac{1}{\theta - 1}} L. \tag{24}$$

In the first stage, from (21) and (23), the indifference condition for sectoral choices equals

[23] The first derivative with respect to L_{TJ} of the LHS of (20) is $-(1-\alpha) A_T (L_{TJ})^{\alpha - 2} + \beta \omega_s \frac{2 n_e}{L}$, which equals $-\infty$ at $L_{TJ} = 0$ and equals 0 at $L_{TJ} = \left[\frac{(1-\alpha) A_T}{\beta \omega_s \frac{2 n_e}{L}}\right]^{\frac{1}{2 - \alpha}}$, and the second derivative equals $(2-\alpha)(1-\alpha) A_T (L_{TJ})^{\alpha - 3} > 0$. Thus, from (19) and the fact that the LHS of (20) at $L_{TJ} = 0$ equals $+\infty$, there exists a unique $L_{TJ} \in (0, \frac{L}{n_e})$ satisfying (20). The relationships of $(L_{TJ})_e^*$ with A_M and A_T are straightforward from the shape of the LHS of (20).

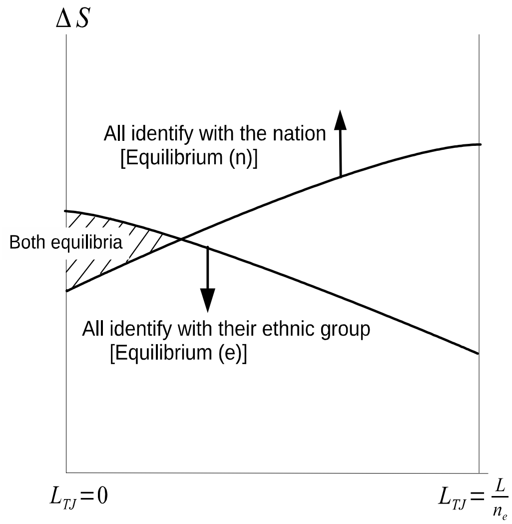

Figure 1: Regions in which homogeneous identity equilibria exist

$$A_T(L_{TJ})^{\alpha-1} - \beta\omega_s\left(1-2\frac{L_{TJ}}{L}\right) = A_M, \tag{25}$$

which gives the unique $(L_{TJ})_n^* \in (0, (L_{TJ})_e^*)$ that decreases with A_M and increases with A_T.[24]

3.1.3 Analysis

Based on Proposition A1 in Appendix A, Figure 1 shows combinations of L_{TJ} and $\Delta S \equiv S_N - S_E$ such that homogeneous identity equilibria exist. Equilibrium (n) exists when $(L_{TJ})_n^*$ and ΔS are in the region above the upward-sloping curve, and equilibrium (e) exists when $(L_{TJ})_e^*$ and ΔS are in the region on or below the downward-sloping curve, where values of $(L_{TJ})_n^*$ and $(L_{TJ})_e^*$ and, thus, their positions in the figure depend on exogenous variables such as A_M and A_T. Equilibrium (n) (equilibrium (e)) exists when ΔS is sufficiently high (low), that is, when the national status is high (low) enough or the ethnic status is low (high) enough. Unless ΔS is very high or very low, these equilibria exist only for small enough $(L_{TJ})_n^*$ and $(L_{TJ})_e^*$. In other words, homogeneous identity equilibria do not exist when the number of traditional sector workers is sufficiently large; this is because modern sector workers do not identify with their ethnic group as their sectoral distance to the "average co-ethnic" is quite larger than their distance to the "average national", and traditional sector workers do not identify with the nation as their sectoral distance to the "average national" is quite greater than their distance to the "average co-ethnic". Both equilibria exist when $(L_{TJ})_n^*$ and $(L_{TJ})_e^*$ are in the region with slant lines.[25]

How do the equilibria differ in conflict and economic outcomes? To isolate the effect of social identification, the next proposition compares them for *given* parameters and exogenous variables.

[24] From the comparison of the LHS of (25) with that of (20) and the discussion in footnote 23, it is clear that, when (19) is assumed, the unique solution $(L_{TJ})_n^* \in (0, (L_{TJ})_e^*)$ that decreases with A_M and increases with A_T exists.

[25] The reason for multiple equilibria is explained in Section 4.1 discussing the mechanism driving the main result.

Proposition 1 *For any given parameters and exogenous variables for which both homogeneous identity equilibria exist, the following holds.*

(i) The level of conflict is lower in equilibrium (n), that is, $F_n^* < F_e^*$.

(ii) L_{TJ} *and thus the proportion of workers in the traditional sectors are lower in equilibrium (n), that is,* $(L_{TJ})_n^* < (L_{TJ})_e^*$.

(iii) The output of private goods Y *is higher in equilibrium (n), if* α *(the parameter of the traditional sector production function) or* β *(the importance of the perceived distance in the utility) is not very high. The output is higher in equilibrium (e) if* α *or* β *is high.*

(iv) The aggregate material payoff is higher in equilibrium (n), unless α *or* β *is very high.*

Proof. See Appendix B. ■

People contribute less to conflict, and thus the level of conflict F is lower when they identify with the nation. This is because, in choosing f_i, they consider the undesirable effect of the conflict on the ethnic distance to the "average national": a higher F raises the weight on ethnicity, ω_e, thereby highlighting the ethnic differences among citizens and widening the distance. This result is shown in Sambanis and Shayo (2013) and is consistent with the aforementioned evidence (Eifert, Miguel, and Posner, 2010; Rohner, Thoenig, and Zilibotti, 2013).

What is new is the result on the sectoral distribution of workers and total output. For given parameters and exogenous variables, L_{TJ} and thus the share of workers in the traditional sectors are lower in equilibrium (n). The return to choosing the traditional sector of one's ethnic group is lower under national identity, because switching from the ethnically integrated modern sector to the segregated traditional sector increases the sectoral distance to the "average national", while under ethnic identity, it decreases the sectoral distance to the "average co-ethnic" (if $L_{TJ} > \frac{L}{2n_e}$, i.e., the majority is in the traditional sector) or increases the distance less (if $L_{TJ} < \frac{L}{2n_e}$).

The sectoral allocation of workers is generally inefficient: it does not maximize the total output of private goods. This is because labor income is greater than the marginal labor productivity in the traditional sectors (note decreasing returns to labor, $\alpha < 1$) *and* the sectoral distance term of the utility distorts sectoral choices by inducing workers to choose the same sector as that of the "average fellow" of their identity group. The former leads to too many workers in the traditional sectors, while the latter leads to too few workers in the traditional sectors in equilibrium (n) and too many (few) traditional sector workers in equilibrium (e) when $L_{TJ} > (<) \frac{L}{2n_e}$.

If α is not very high, the first effect dominates, and thus L_{TJ} is higher than the efficient level. Total output is higher in equilibrium (n) because L_{TJ} is lower, and thus closer to the efficient level than in equilibrium (e). The condition would be relevant to many developing countries, since a small α implies strong decreasing returns in the traditional sectors.[26] The same result holds for *any* α, if the importance of the perceived distance in the utility, β, is not very high, and thus the

[26] Recall that the decreasing returns to labor capture the fact that labor productivity tends to fall quickly with labor input owing to limited arable land (traditional agriculture), limited capital available to credit-constrained producers (the urban informal sector), or a decreasing degree of task specialization by each family member (household production).

effect of social identity on sectoral misallocation (the second effect) does not exceed the economic effect (the first effect), which is likely to be true. Conversely, Y is lower in equilibrium (n), if α or β is high enough that the second effect dominates. Finally, aggregate material payoff (the value of private and club good consumption net of the cost of conflict) is higher in equilibrium (n) because of the lower cost of conflict, unless α or β is very high, in which case it is lower owing to much lower Y in equilibrium (n).

To summarize, national identity is associated not only with a lower level of conflict but also with higher shares of the modern sector in employment and production; under plausible conditions, it is also associated with higher levels of private good production and aggregate material payoff.

3.2 Heterogeneous identity equilibria

Now, the paper examines the equilibria in which individuals of the same ethnic group have different identities. There are three heterogeneous identity equilibria: the equilibrium in which sector TJ workers identify with their ethnic group, and sector M workers identify with the nation (henceforth *equilibrium* (d); "d" denotes divided identities); the one in which sector M workers are divided over identities, and sector TJ workers identify with their ethnic group (henceforth *equilibrium* (Md)); and the one in which sector TJ workers are divided over identities, and sector M workers identify with the nation (henceforth *equilibrium* (Td)). Readers who are not interested in derivations of equations for these equilibria may directly go to the analysis in Section 3.2.4.

3.2.1 Equilibrium (d)

In the second stage in which the sectoral allocation of workers is given, sector TJ workers with an ethnic identity choose f_i to maximize (17), and the solution is given by (16), while sector M workers with a national identity choose f_i to maximize (21); the solution is given by (22).

Since the ethnic groups are symmetric, by substituting $F_{-J} = \frac{n_e-1}{n_e}F$ into (16) and (22) and plugging them into $F = f_{i,e}n_eL_{TJ}+f_{i,n}(L-n_eL_{TJ})$, the level of conflict F for given L_{TJ} is obtained:

$$F=\left(\frac{n_e-1}{n_e}\right)^{\frac{1}{\theta-1}}\left[\left(\delta\frac{V}{F}\right)^{\frac{1}{\theta-1}}n_eL_{TJ}+\left(\delta\frac{V}{F}-\beta\eta_1\right)^{\frac{1}{\theta-1}}(L-n_eL_{TJ})\right], \tag{26}$$

which increases with L_{TJ} and is denoted by $F_d(L_{TJ})$ (d is for "divided identities").

In the first stage, from (16), (17), (21), (22), and (26), the indifference condition for sectoral choices equals ($\Delta S \equiv S_N - S_E$)

$$\begin{aligned}
&A_T(L_{TJ})^{\alpha-1}-\tfrac{1}{\theta}\left(\delta\tfrac{n_e-1}{n_e}\tfrac{V}{F_d(L_{TJ})}\right)^{\frac{\theta}{\theta-1}}-\beta\omega_s\left(1-\tfrac{L_{TJ}}{L/n_e}\right)^2+\gamma S_E\\
&=A_M-\tfrac{1}{\theta}\left[\tfrac{n_e-1}{n_e}\left(\delta\tfrac{V}{F_d(L_{TJ})}-\beta\eta_1\right)\right]^{\frac{\theta}{\theta-1}}-\beta\left\{[\eta_0+\eta_1F_d(L_{TJ})]\tfrac{n_e-1}{n_e}+\omega_sn_e\left(\tfrac{L_{TJ}}{L}\right)^2\right\}+\gamma S_N\\
&\Leftrightarrow A_T(L_{TJ})^{\alpha-1}+\beta\left\{[\eta_0+\eta_1F_d(L_{TJ})]\tfrac{n_e-1}{n_e}+\omega_s\left[n_e\left(\tfrac{L_{TJ}}{L}\right)^2-\left(1-n_e\tfrac{L_{TJ}}{L}\right)^2\right]\right\}\\
&\quad-\tfrac{1}{\theta}\left(\tfrac{n_e-1}{n_e}\right)^{\frac{\theta}{\theta-1}}\left[\left(\delta\tfrac{V}{F_d(L_{TJ})}\right)^{\frac{\theta}{\theta-1}}-\left(\delta\tfrac{V}{F_d(L_{TJ})}-\beta\eta_1\right)^{\frac{\theta}{\theta-1}}\right]-\gamma\Delta S=A_M,
\end{aligned} \tag{27}$$

which gives the solution $(L_{TJ})^*_d \in (0, \frac{L}{n_e})$. Appendix B proves its uniqueness when $\theta = 2$.[27] The equilibrium level of conflict F^*_d equals $F_d((L_{TJ})^*_d)$.

3.2.2 Equilibrium (Md)

In the second stage, workers in sector TJ with an ethnic identity choose f_i to maximize (17), and the solution is given by (16); workers in sector M are indifferent between identifying with the nation, in which case f_i is chosen to maximize (21) and the solution is given by (22), and identifying with their ethnic group, in which case f_i is chosen to maximize (15) and the solution is given by (16).

Thus, the indifference condition for *identity* choices when ethnic groups are symmetric is

$$\begin{aligned}
&A_M - \frac{1}{\theta}(f_{i,e})^\theta + \delta\frac{F_J}{F}V - \beta\omega_s\left(\frac{L_{TJ}}{L/n_e}\right)^2 + \gamma S_E \\
&= A_M - \frac{1}{\theta}(f_{i,n})^\theta + \delta\frac{F_J}{F}V - \beta\left[(\eta_0+\eta_1 F)\frac{n_e-1}{n_e} + \omega_s n_e\left(\frac{L_{TJ}}{L}\right)^2\right] + \gamma S_N \qquad (28)\\
&\Leftrightarrow \beta\left[(\eta_0+\eta_1 F)\frac{n_e-1}{n_e} - \omega_s n_e(n_e-1)\left(\frac{L_{TJ}}{L}\right)^2\right] - \frac{1}{\theta}\left(\frac{n_e-1}{n_e}\right)^{\frac{\theta}{\theta-1}}\left[\left(\delta\frac{V}{F}\right)^{\frac{\theta}{\theta-1}} - \left(\delta\frac{V}{F}-\beta\eta_1\right)^{\frac{\theta}{\theta-1}}\right] = \gamma\Delta S. \qquad (29)
\end{aligned}$$

F in the above equation satisfies

$$\begin{aligned}
F &= f_{i,n}P_{M,n}(L-n_eL_{TJ}) + f_{i,e}\left[n_eL_{TJ} + (1-P_{M,n})(L-n_eL_{TJ})\right] \\
&= \left(\frac{n_e-1}{n_e}\right)^{\frac{1}{\theta-1}}\left\{\left(\delta\frac{V}{F}-\beta\eta_1\right)^{\frac{1}{\theta-1}}P_{M,n}(L-n_eL_{TJ}) + \left(\delta\frac{V}{F}\right)^{\frac{1}{\theta-1}}\left[n_eL_{TJ} + (1-P_{M,n})(L-n_eL_{TJ})\right]\right\}, \qquad (30)
\end{aligned}$$

where $P_{M,n}$ is the proportion of sector M workers identifying with the nation. Since the LHS of (29) decreases with L_{TJ} and increases with F, F satisfying (29) increases with L_{TJ}.

The indifference condition for sectoral choices in the first stage is given by (20) from (15) and (17), the same as equilibrium (e). Thus, the equilibrium level of L_{TJ}, $(L_{TJ})^*_{Md}$, equals $(L_{TJ})^*_e$, and the equilibrium level of conflict F^*_{Md} is obtained by plugging $(L_{TJ})^*_e$ into (29) and solving it for F.

3.2.3 Equilibrium (Td)

In the second stage, workers in sector M, who identify with the nation, choose f_i to maximize (21), and the solution is (22); workers in sector TJ are indifferent between identifying with the nation, in which case f_i is chosen to maximize (23) and the solution is given by (22), and identifying with their ethnic group, in which case f_i is chosen to maximize (17) and the solution is given by (16).

Thus, the indifference condition for *identity* choices when ethnic groups are symmetric is

$$\begin{aligned}
&A_T(L_{TJ})^{\alpha-1} - \frac{1}{\theta}(f_{i,e})^\theta + \delta\frac{F_J}{F}V - \beta\omega_s\left(1-\frac{L_{TJ}}{L/n_e}\right)^2 + \gamma S_E \\
&= A_T(L_{TJ})^{\alpha-1} - \frac{1}{\theta}(f_{i,n})^\theta + \delta\frac{F_J}{F}V - \beta\left\{\omega_e\frac{n_e-1}{n_e} + \omega_s\left[\left(1-\frac{L_{TJ}}{L}\right)^2 + (n_e-1)\left(\frac{L_{TK}}{L}\right)^2\right]\right\} + \gamma S_N \qquad (31)\\
&\Leftrightarrow \beta\left[(\eta_0+\eta_1 F)\frac{n_e-1}{n_e} + \omega_s(n_e-1)\frac{L_{TJ}}{L}\left(2-n_e\frac{L_{TJ}}{L}\right)\right] - \frac{1}{\theta}\left(\frac{n_e-1}{n_e}\right)^{\frac{\theta}{\theta-1}}\left[\left(\delta\frac{V}{F}\right)^{\frac{\theta}{\theta-1}} - \left(\delta\frac{V}{F}-\beta\eta_1\right)^{\frac{\theta}{\theta-1}}\right] = \gamma\Delta S. \qquad (32)
\end{aligned}$$

[27] When $\theta > 2$, the uniqueness of $(L_{TJ})^*_d$ cannot be proved, but whether $(L_{TJ})^*_d$ is unique does not affect the results below.

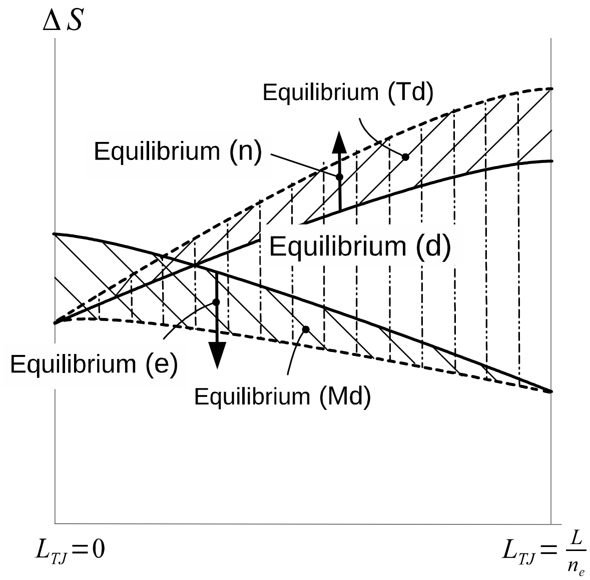

Figure 2: Regions in which heterogeneous and homogeneous identity equilibria exist

F in the above equation satisfies

$$F = f_{i,n}\left[P_{TJ,n}n_eL_{TJ} + (L - n_eL_{TJ})\right] + f_{i,e}\,(1 - P_{TJ,n})n_eL_{TJ}$$
$$= \left(\frac{n_e - 1}{n_e}\right)^{\frac{1}{\theta-1}}\left[\left(\delta\frac{V}{F} - \beta\eta_1\right)^{\frac{1}{\theta-1}}\left[P_{TJ,n}n_eL_{TJ} + (L - n_eL_{TJ})\right] + \left(\delta\frac{V}{F}\right)^{\frac{1}{\theta-1}}(1 - P_{TJ,n})n_eL_{TJ}\right], \quad (33)$$

where $P_{TJ,n}$ is the proportion of sector TJ workers identifying with the nation. F satisfying (32) decreases with L_{TJ} because the LHS of (32) increases with L_{TJ} and F. After the negative dependence of F on L_{TJ} is taken into account, $P_{TJ,n}$ increases with L_{TJ} from (33).

The indifference condition for sectoral choices in the first stage is given by (25) from (21) and (23), the same as equilibrium (n). The equilibrium level of L_{TJ}, $(L_{TJ})^*_{Td}$, equals $(L_{TJ})^*_n$, and the equilibrium level of conflict F^*_{Td} is obtained by substituting $(L_{TJ})^*_n$ into (32) and solving it for F.

3.2.4 Analysis

Based on Propositions A1 and A2 in Appendix A, Figure 2 illustrates combinations of L_{TJ} and ΔS such that the heterogeneous and homogeneous identity equilibria exist.[28] Equilibrium (d) exists when $((L_{TJ})^*_d,\ \Delta S)$ is in the region with triple-dashed double-dotted lines; equilibrium (Md) exists when $((L_{TJ})^*_{Md},\ \Delta S)$ is in the region with negatively-sloped lines; equilibrium (Td) exists when $((L_{TJ})^*_{Td},\ \Delta S)$ is in the region with positively-sloped lines.

The heterogeneous identity equilibria exist when ΔS is not at an extreme value: when the national status relative to the ethnic status is very high (low), everyone identifies with the nation

[28] The figure shows the case when ω_s (weight on sectoral attributes of the perceived distance) is relatively high and η_1 (strength of the effect of F on the weight on ethnic attributes ω_e) is relatively low. Figure 5 of Appendix A presents the case of relatively low ω_s and relatively high η_1. The basic features of the figures are similar.

(their ethnic group). Given ΔS, equilibrium (d) exists when $(L_{TJ})^*_d$ is large enough: when many people work in traditional sectors, traditional sector workers identify with their ethnic group mainly because the sectoral distance to the "average co-ethnic" is small, while modern sector workers identify with the nation mainly because the sectoral distance to the "average co-ethnic" is large. As can be seen from the presence of many overlapping regions, multiple equilibria often exist.[29] Equilibrium (Td) (equilibrium (Md)) exists only when equilibrium (n) (equilibrium (e)) exists.[30] Even all the equilibria can exist.[31]

In all the heterogeneous identity equilibria, modern sector workers are more (less) likely than traditional sector workers to identify with the nation (their ethnic group): when some workers in the traditional sectors (modern sector) identify with the nation (their ethnic group), all workers in the modern sector (traditional sectors) do the same.[32] Roughly speaking, this is because the sectoral distance to the "average national" for modern sector workers is smaller than their distance to the "average co-ethnic", and the opposite holds for traditional sector workers, given that the modern sector is ethnically integrated and the traditional sectors are ethnically segregated.[33] The result is consistent with Robinson (2014), who, using survey data from 16 African countries, finds that working in the modern sector is significantly and robustly associated with identifying more with the nation than with their ethnic group, after controlling for education, urban residence, gender, and group-level and country-level variables.[34,35]

How do equilibria differ in terms of conflict and economic outcomes? The next proposition examines the same issue as Proposition 1 for all the equilibria.

Proposition 2 *Consider any given parameters and exogenous variables for which multiple equilibria exist. Among these equilibria, the equilibrium with a higher proportion of individuals identifying with the nation has*

(*i*) *a lower level of conflict, i.e.,* $F^*_n < F^*_{Td} < F^*_d < F^*_{Md} < F^*_e$.

(*ii*) *a weakly lower share of traditional sector workers, i.e.,* $(L_{TJ})^*_n = (L_{TJ})^*_{Td} < (L_{TJ})^*_d < (L_{TJ})^*_{Md} = (L_{TJ})^*_e$.

[29] The reason for multiple equilibria is explained in Section 4.1 discussing the mechanism driving the main result.

[30] To be precise, this is because the region of equilibrium (Td) (equilibrium (Md)) is contained in that of equilibrium (n) (equilibrium (e)) *and* $(L_{TJ})^*_{Td} = (L_{TJ})^*_n$ ($(L_{TJ})^*_{Md} = (L_{TJ})^*_e$), which is proved in the next proposition.

[31] This happens, for example, when equilibrium L_{TJ} of all the equilibria are in the small triangular region on the left side of the figure, but it occurs more broadly, which can be inferred from (ii) of the next proposition.

[32] The proof of Proposition A2 in Appendix A formally shows the non-existence of equilibria in which modern sector workers are less likely than traditional sector workers to identify with the nation.

[33] The *total* perceived distance to the "average national" for the modern sector workers can be *greater* than their distance to the "average co-ethnic", owing to the greater ethnic distance to the "average national." However, the difference between the total distance under national identity and the one under ethnic identity for these workers is smaller than that for the traditional sector workers, implying their higher tendency to identify with the nation.

[34] Robinson (2014) classifies workers into two sectors based on their occupation: formal (modern) sector occupations include military/police, clerical worker, business person, professional worker, civil servant, and teacher, while the informal (traditional) sector occupations include subsistence farmer, informal manual labor, herder, and housewife.

[35] Conversely, Eifert, Miguel, and Posner (2010), based on surveys from 10 African countries, find that being a farmer or fisherman, whom they classify as traditional sector workers, is negatively correlated with ethnic identity. However, there is no option for the national identity in the surveys (other options are religious and class/occupational identities) and, unlike Robinson (2014), they classify those in the urban informal sector as *modern* sector workers.

(*iii*) *weakly greater output of private goods, i.e.,* $Y_n^* = Y_{Td}^* > Y_d^* > Y_{Md}^* = Y_e^*$, *if* α *or* β *is not very high; otherwise, weakly smaller output.*

(*iv*) *higher aggregate material payoff, unless* α *or* β *is very high.*

Proof. See Appendix B. ■

The level of conflict is lower when there is a higher proportion of individuals with a national identity, who contribute less to the conflict. Among heterogeneous identity equilibria, the conflict level is lowest in equilibrium (Td), in which sector M workers identify with the nation and sector TJ workers are divided over identities; it is highest in equilibrium (Md), in which sector M workers are divided over identities and sector TJ workers identify with their ethnic group. Among all the equilibria, equilibrium (n) and equilibrium (e) have the lowest and highest conflict levels, respectively.

This explanation presumes that, among the heterogeneous identity equilibria, the proportion of those identifying with the nation is highest in equilibrium (Td) and lowest in equilibrium (Md). The result on the share of traditional sector workers, $(L_{TJ})_n^* = (L_{TJ})_{Td}^* < (L_{TJ})_d^* < (L_{TJ})_{Md}^* = (L_{TJ})_e^*$, confirms the presumption. $(L_{TJ})_n^* < (L_{TJ})_d^* < (L_{TJ})_e^*$ holds because the proportion of those identifying with the nation, who gain less from choosing the traditional sector, is higher (lower) in equilibrium (d) than that in equilibrium (e) (equilibrium (n)). $(L_{TJ})_{Td}^* = (L_{TJ})_n^*$ ($(L_{TJ})_{Md}^* = (L_{TJ})_e^*$) holds for equilibrium (Td) (equilibrium (Md)), because individuals with a national identity (an ethnic identity) are in both sectors, and thus are indifferent between the sectors, as in equilibrium (n) (equilibrium (e)).

Finally, the result for the total output of private goods is similar to Proposition 1 and can be explained as before. Under plausible conditions, the total output is higher as the proportion of individuals having a national identity is higher, though $Y_{Td}^* = Y_n^*$ and $Y_{Md}^* = Y_e^*$ are true. Further, owing to the lower cost of conflict, aggregate material payoff is *strictly* higher when the proportion of those identifying with the nation is higher.

To summarize, results similar to Proposition 1 hold when heterogeneous identity equilibria are also considered: higher prevalence of national identity is associated not only with a lower level of conflict but also with higher labor and output shares of the modern sector and, under plausible conditions, the higher levels of private good output and aggregate material payoff. The results are consistent with the often-made argument (Collier, 2009; Michalopoulos and Papaioannou, 2015) that the dominance of the ethnic over national identity drives the poor performance in various dimensions, including conflict and development, in ethnically heterogeneous societies.[36] Note that a small share of modern sector workers under widespread ethnic identity implies a large intersectoral gap in earnings. This suggests that strong ethnic identity might partly explain the substantial gap in the average labor productivity between agriculture and non-agriculture in many developing countries, found by Gollin, Lagakos, and Waugh (2014) and others.

4 Main results

[36] Michalopoulos and Papaioannou (2015) base their argument on the findings (using Afrobarometer surveys covering 18−20 African countries) of positive relationships between national identification and a measure of state capacity in protecting property rights and between ethnic identification and a measure of the inefficiency of the legal system.

Section 3 compared different equilibria for given parameters and exogenous variables to examine the pure effect of social identification on outcomes. The set of realized equilibria, however, generally changes with values of exogenous variables, as indicated by Figures 1 and 2. Taking this into account, this section analyzes the effects of modernization on identity, conflict, and development.

Simple dynamics are introduced into the model by supposing that the TFP of sector M, A_M, increases over time, that is, $A_{M,t+1} > A_{M,t}$ for any period t. The TFP growth represents the technological progress of the modern sector and the improvement in quality of institutions supporting the sector's activities. It raises the modern sector income, the proportion of workers choosing the sector, and the sector's share in production. How does modernization driven by the productivity growth affect social identity, conflict, and output?[37] The next proposition shows that the effect differs depending on the *status difference* $\Delta S \equiv S_N - S_E$. Note that decreases in the salience of ethnicity and the contested resources have similar effects as an increase in ΔS, as shown later in Propositions 4 and 5.

Proposition 3 *Suppose that the TFP of sector M, A_M, increases over time, that is, $A_{M,t+1} > A_{M,t}$ for any period t.*

(i) If the status difference ΔS is very high (low), the society is always in equilibrium (n) (equilibrium (e)); thus, the level of conflict F is consistently low (high).

(ii) Otherwise, when ΔS is relatively high (low), the society shifts from a heterogeneous identity equilibrium to equilibrium (n) (equilibrium (e)) or stays in the latter equilibrium. For given parameters and exogenous variables, multiple equilibria can exist when A_M (L_{TJ}) is relatively low (high); thus, identity, conflict, and output differ depending on which equilibrium is realized.[38]

(iii) For a given A_M, when ΔS is high (low), the society is in an equilibrium with a high (low) proportion of people identifying with the nation and low (high) F. The equilibrium is characterized by relatively low (high) L_{TJ} and, unless α or β is very high, relatively high (low) levels of Y and aggregate material payoff.

Proof. See Appendix B. ■

If the status difference ΔS is at an extreme level, the society stays in the same equilibrium: when the national status relative to the ethnic status is very high (low), consistently, everyone identifies with the nation (their ethnic group) and the level of conflict F is low (high).

Otherwise, when ΔS is relatively high and the society starts with a heterogeneous identity equilibrium, it shifts from the equilibrium, in which modern sector workers are more likely than traditional sector workers to identify with the nation, to the one in which everyone identifies with the nation and the level of conflict is low; when ΔS is relatively low, it shifts to the equilibrium in which everyone identifies with their ethnic group and the level of conflict is high. In other words,

[37] Note that modernization is *not* the same as urbanization: traditional sectors correspond to the urban informal sector as well as traditional agriculture and household production in the real economy. Many developing countries have experienced rapid urbanization without significant modernization.

[38] Conversely, when ΔS is in the middle range, multiple equilibria exist for sufficiently *high* A_M (low L_{TJ}), and the society eventually reaches equilibrium (n), equilibrium (e), or equilibrium (Md). See Figure 4 below.

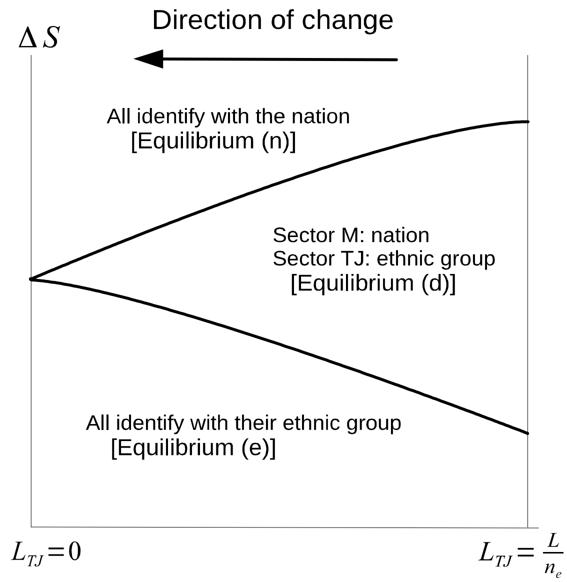

Figure 3: Equilibrium when $\eta_1 = 0$

when the status difference is relatively high, the social identity initially associated with modern sector workers eventually becomes the shared identity, while, when it is relatively low, the identity initially associated with the traditional sector workers becomes the common identity.

Although an increase in the modern sector productivity, A_M, always lowers L_{TJ} and raises the sector's share in production, for a *given* A_M (i.e., given levels of modern sector technology and of the quality of institutions supporting the sector's activities), the society with high (low) ΔS has relatively large (small) shares of the modern sector in employment and production and, under plausible conditions, relatively high (low) levels of private good output and aggregate material payoff. Hence, having sufficiently high national status relative to ethnic status is crucial for achieving good outcomes in development as well as in identity and conflict.

However, history or "luck" is also important, as long as the status difference is not at an extreme level. For *given* parameters and exogenous variables *including* ΔS, multiple equilibria can exist; thus, outcomes differ depending on which equilibrium is realized.[39] If the initial equilibrium *happens to* be such that a relatively high proportion of people identify with the nation, the society tends to maintain a relatively strong national identity and relatively good outcomes in conflict and development.[40]

4.1 Mechanism

[39] Multiple equilibria also exist in the model of Sambanis and Shyao (2013) when ΔS is not at an extreme level.

[40] When ΔS is in the middle range, multiple equilibria exist for sufficiently high A_M (footnote 38), and thus history or "luck" matters in the long-run. When ΔS is not in the middle range, the society shifts to a homogeneous identity equilibrium ((ii) of Proposition 3) and the effect of history or "luck" disappears eventually.

The result would be understood more easily by looking at the result when $\eta_1 = 0$ first, that is, when the weight on ethnic attributes ω_e in the perceived distance does not depend on the level of conflict F (see (11) in Section 2.3). In this case, F is the same in all the equilibria, and equilibrium is *unique* for given parameters and exogenous variables. Figure 3 illustrates how the realized equilibrium depends on ΔS and L_{TJ}. As A_M increases over time, equilibrium L_{TJ} decreases; thus, the society moves *leftward* in the figure.[41]

The realized equilibrium changes with increased productivity when the differential between the national status and the ethnic status is not at an extreme level. When the status difference is relatively high (low), the society shifts from the equilibrium in which sector M workers identify with the nation and sector TJ workers identify with their ethnic group to the one in which everyone identifies with the nation (their ethnic group). The growth of A_M increases the modern sector's income, and thus the proportion of workers in the sector. As a result, the national identity of modern sector workers becomes *weaker* in the sense that the utility gain from identifying with the nation rather than with their ethnic group decreases, given that the sectoral distance to the "average co-ethnic" of the workers decreases more than their distance to the "average national."[42] The modern sector workers, who found little affinity with their co-ethnics most of whom were in the rural agricultural region or in the urban informal sector, feel closer to their group, since there are more co-ethnics in occupations (and thus with lifestyles) similar to theirs. The ethnic identity of traditional sector workers also weakens because of a smaller proportion of their fellow group in their sector. In other words, the sectoral shift of labor associated with modernization shakes long-standing identities in *both sectors*. If the national status relative to the ethnic status is high, then the traditional sector workers change identities (since their utility gain from identifying with the ethnic group is small), and everyone identifies with the nation; otherwise, the modern sector workers change their identities and everyone identifies with their group.

When $\eta_1 > 0$, that is, when the weight on ethnic attributes ω_e increases with F, F is lower in an equilibrium with a higher proportion of people identifying with the nation (Proposition 2). Unlike when $\eta_1 = 0$, multiple equilibria can exist for given parameters and exogenous variables, and equilibria (Md) and (Td) can exist.[43] Multiple equilibria can arise because of the two-way positive causations between conflict and identity: when the level of conflict is high (low), ethnicity becomes more (less) salient in people's minds and they are more (less) likely to identify with their ethnic group; when the proportion of those with an ethnic identity is high (low), the level of conflict is high (low) because those with a national identity, who curb contributions to conflict from the concern that conflict increases the salience of ethnicity, and thus the distance to other ethnic groups, are small (large) in number. Equilibria (Md) and (Td) can exist since individual contribution to conflict f_i depends on identity when $\eta_1 > 0$: workers in one of the sectors can be indifferent between

[41] Positions of the dividing lines depend on parameters and several exogenous variables, but not on A_M and A_T.

[42] Since a higher proportion of workers is in the modern sector, both sectoral distances decrease. However, the former distance decreases more because of a greater change in the average sectoral attribute (see (15) and (21)).

[43] Remember that equilibrium (Md) (equilibrium (Td)) is the equilibrium in which sector M (sector TJ) workers are divided over identities, and sector TJ (sector M) workers share the ethnic (national) identity.

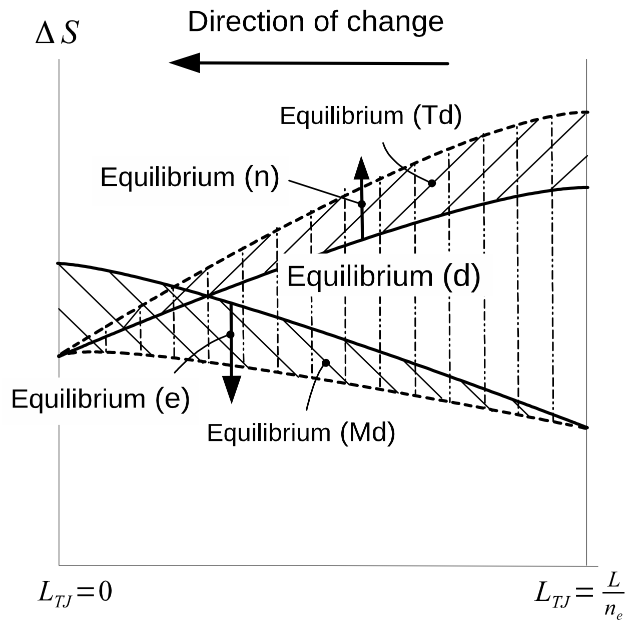

Figure 4: Equilibrium when $\eta_1 > 0$

the two identities only if the inter-identity difference in the level of socio-psychological components of the utility is counterbalanced by the inter-identity difference in the cost of conflict.

Figure 4, which is essentially the same as Figure 2, shows how realized equilibria depend on ΔS and L_{TJ} when $\eta_1 > 0$. Assume that the equilibrium realized initially is sustained in subsequent periods, as long as it continues to exist. Consider a typical situation in which the society starts with equilibrium (d). As long as it stays in this equilibrium, with the growth of A_M, L_{TJ} and thus the proportion of those with an ethnic identity decrease, which leads to a decline in the conflict level and increases in the modern sector's shares of employment and production, output, and material payoff. Eventually, the equilibrium ceases to exist and the society shifts to a different equilibrium. If ΔS is relatively high (low), it shifts to equilibrium (n) (equilibrium (e)) and the level of conflict falls (*rises*).[44,45] The shift to the modern sector and output growth continue. However, for a *given* level of the modern sector productivity, L_{TJ} is lower, and the modern sector shares, output, and material payoff are higher in the equilibrium of a universal national identity.

Multiple equilibria often exist, as can be seen from the presence of various overlapping regions in the figure. Suppose, for example, that the society starts with the region in which equilibria (e), (d), and (Md) exist. Depending on which equilibrium *happens to* be realized initially, social identity, conflict, sectoral compositions, and output differ in initial and subsequent periods (even in the long-run when ΔS is in the middle range): the outcomes are worst when the society starts

[44] The rise of F when ΔS is low may be interpreted as a *rise in non-violent conflict* such as rent-seeking activities, if the shift occurs at a relatively low L_{TJ}, that is, at a later stage of economic development.

[45] The shift changes F discontinuously, while it changes L_{TJ} continuously. When ΔS is in the intermediate range, as shown in the figure, the society shifts to equilibrium (n), equilibrium (e), or equilibrium (Md).

with equilibrium (e) and are best when it starts with equilibrium (d).

4.2 Discussion

As mentioned in the introduction, competing theses exist in political science on the effects of modernization on social identity (see Robinson, 2014, for a review). The classic thesis, based on the past experience of Europe, argues that modernization leads to widespread national identity at the expense of ethnic and other subnational identities (Deutsch, 1953; Gellner, 1964, 1983; Weber, 1979), while another influential thesis mainly focusing on Africa (the "second-generation" thesis) argues that modernization breeds ethnic identification (Melson and Wolpe, 1970; Bates, 1983).

Proposition 3 shows that when the differential between the national status and the ethnic status is relatively, but not extremely, high (low) and the society starts with a heterogeneous identity equilibrium, it shifts to the equilibrium with a universal national (ethnic) identity that is characterized by relatively good (bad) outcomes in conflict and development. Thus, the result is consistent with the classic view when the status difference is relatively high, while it is consistent with the competing view when it is relatively low, as far as the *relatively long-term effect* of modernization (the effect involving the equilibrium shift) is concerned.

Using survey data from 16 African countries, Robinson (2014) finds that GDP per capita is significantly related to individuals identifying more with the nation than with their ethnic group, after controlling for various variables. She interprets the evidence as suggesting that modernization (higher GDP per capita) leads to widespread national identity. Considering that it is based on the cross-sectional data from mostly poor African countries, the evidence may be regarded as capturing the *relatively short-term effect* in an economy with a low degree of modernization. Indeed, the effect of an increase in A_M *within a given equilibrium* is consistent with her interpretation, when the society is in a heterogeneous identity equilibrium (except equilibrium (Td)),[46] which is a likely situation when equilibrium L_{TJ} is relatively high. However, the evidence can also be interpreted differently, and certain results of the model are consistent with the alternative interpretations.[47]

Proposition 3 also shows that multiple equilibria can exist, and thus outcomes may depend on history or "luck" when the status difference is not at an extreme level. As Sambanis and Shayo (2013) argue, this is consistent with the evidence that nations similar in ethnic diversity, geography, economic conditions, and political institutions have diverse histories on the levels of ethnic conflict.

[46] In equilibrium (Td), F increases with a decrease in L_{TJ} from (32). Then, the number of those identifying with their ethnic group, $(1-P_{TJ,n})n_e L_{TJ}$, increases with a decrease in L_{TJ} from (33).

[47] The evidence is partly consistent with the story that, for a *given* modern sector productivity, national identity and modernization are positively related through the positive effects of the status difference on these variables (Figure 4 and Proposition 3 (iii)). In other words, when ΔS is higher, for *given* A_M, the society is in an equilibrium with a higher proportion of individuals identifying with the nation and lower L_{TJ} (and thus a higher degree of modernization). The evidence can also be partly explained by multiple equilibria because, for *given* parameters and exogenous variables, the national identity and modernization are positively related among different equilibria (Propositions 1 and 2). To distinguish the different stories empirically, it would be important to estimate regression models with enough control variables (including measures capturing ΔS and A_M) using longitudinal data, though such data are not available presently. The analysis using longitudinal data is also called for to examine empirically the relatively long-term effect of modernization, the focus of this study and the above-cited studies in political science.

What are the policy implications of the finding that having sufficiently high national status relative to the ethnic status is crucial for good outcomes? As mentioned in the introduction, Miguel (2004), Collier (2009), and Blouin and Mukand (2019) argue that *nation-building policies*, such as the promotion of a national language and school education and government propaganda that emphasize common history, culture, and values, can effectively fortify national identity. By drawing on the experience of Europe, classic modernization theories of nationalism also stress the importance of the unification of language and of the spread of common culture and values through education and universal military service for strengthening the national identity.

Some of the policies would enhance the national pride of the people; thus, they may be interpreted as policies increasing S_N and ΔS.[48] The analysis shows how they can reinforce national identity and produce good outcomes in conflict and development when ΔS is not high. The result implies that these policies are critical for achieving good outcomes in countries, such as many African countries, where the perceived national status seems to be low because of the lack of shared culture, history, and values. Without nation-building policies, policies promoting modernization (policies increasing A_M) would lead to widespread ethnic identities and a high level of conflict, which partially offset the direct positive effect of the policies on development. Ethnic-based policies such as education that emphasizes unique history, culture, and values of each ethnic group, and the promotion of ethnic languages may be considered as policies increasing S_E and lowering ΔS; thus, they will negatively affect national identity, conflict, and development. Conversely, the result suggests that, in countries with high perceived national status, policies promoting modernization, such as policies stimulating the technological progress of the modern sector and the reform of institutions supporting the sector's activities, might be enough to achieve good outcomes.

Alternatively, some of the nation-building policies, i.e., the promotion of a national language and education and propaganda emphasizing common values, may be interpreted as policies making ethnic differences less salient, that is, reducing ω_e, in the perceived distance. Indeed, the next proposition shows that a decrease in η_0 that lowers ω_e has similar effects to an increase in ΔS.

Proposition 4 *Suppose that A_M increases over time.*

(*i*) *If η_0 is very low (high), the society is always in equilibrium (n) (equilibrium (e)), and thus F is consistently low (high).*

(*ii*) *Otherwise, when η_0 is relatively low (high), the society shifts from a heterogeneous identity equilibrium to equilibrium (n) (equilibrium (e)) or stays in the latter equilibrium. For given parameters and exogenous variables, multiple equilibria can exist when A_M (L_{TJ}) is relatively low (high); thus, identity, conflict, and output differ depending on which equilibrium is realized.*

(*iii*) *For a given A_M, when η_0 is low (high), the society is in an equilibrium with a high (low) proportion of people identifying with the nation and low (high) F. The equilibrium is characterized by relatively low (high) L_{TJ} and, unless α or β is very high, relatively high (low) levels of Y and aggregate material payoff.*

[48] Conversely, Alesina, Giuliano, and Reich (2021) model a nation-building policy as the policy homogenizing costs individuals with heterogeneous preferences incur due to the preference distance from the government.

Proof. See Appendix B. ■

Graphically, this result holds because all the dividing lines of Figure 4 shift downward when there is a decline in η_0.

Empirical work seems to support the importance of nation-building policies in identity formation. Miguel (2004) and Blouin and Mukand (2019) base the aforementioned argument on a case study and an experimental study, respectively (see footnote 5 of the introduction). Clots-Figueras and Masella (2013) examine the effects of the introduction of a bilingual (Catalan and Spanish) education system in Catalonia and find that the number of years of exposure to the bilingual system is positively related to the strength of the Catalan identity and the propensity to vote for a party with a Catalanist platform.

5 Additional results and discussions

5.1 Effects of contested resources

This section examines the effects of contested resources, specifically how it affects conflict and influences the effects of modernization on identity, conflict, and development.

Proposition 5 *(i) The level of conflict F increases with the amount of contested resources V.*

(ii) Suppose that A_M increases over time. Then, the results similar to those of Proposition 4 hold when η_0 of the proposition is replaced with V.

Proof. See Appendix B. ■

The first result is standard and intuitive: an increase in the amount of contested resources leads to increased contributions to conflict and thus a rise in the level of conflict.

The second result states that similar results to Proposition 4 hold when η_0 of the proposition is replaced with V. Specifically, when the amount of contested resources is relatively, but not extremely, large (small), with an increase in the modern sector productivity, the society shifts from a heterogeneous identity equilibrium to equilibrium (e) (equilibrium (n)) or stays in the latter equilibrium. Thus, the abundance of contested resources prevents the society from achieving a universal national identity, a low level of conflict, high shares of the modern sector, and high output. Contested resources can be interpreted as being part of material resources (e.g., natural resources) and of the government budget for ethnic-specific club goods that are allocated among groups based not on rule but on the consequences of violent or non-violent conflicts (e.g., rent-seeking activities). Hence, the result suggests that both the abundance of resources and the absence of strong political and economic institutions (e.g., weak rule of law), which limit the role of rule-based allocations, are hindrances to desirable outcomes. It is consistent with the empirical work by Mehlum, Moene, and Torvik (2006), who find negative effects of natural resources on economic development when institutions are weak. The result reveals a novel mechanism interacting with social identity by which resources and institutions affect conflict and development.

5.2 Discussions on several assumptions

The model has several assumptions that are imposed to make the analysis tractable. This section briefly discusses how the results are affected and what new questions can be examined when these assumptions are replaced by more realistic ones. A more detailed analysis is available on the author's web page (http://www.econ.kyoto-u.ac.jp/~yuki/english.html).

5.2.1 Assumption on benefit of ethnic-specific club goods

In the model, every individual of an ethnic group is assumed to benefit equally from ethnic-specific club goods. In the real world, those with a national identity might benefit less from several ethnic-specific club goods, such as public spending on ethnic culture and language, than those with an ethnic identity. They may also benefit more from certain public goods, such as spending on common culture and language. Considering this factor, however, does not affect the results qualitatively.

Alternatively, compared to traditional sector workers, modern sector workers might benefit less from some ethnic-specific club goods, such as education, health services, and roads in areas specific groups are clustered in, and may benefit more from several public goods, such as legal services and scientific knowledge. Imposing a more realistic assumption in this aspect does not change the results summarized in Propositions 3−5. However, two results are qualitatively affected: an increase in A_M reduces F in a homogeneous identity equilibrium, and the level of conflict could fall even when the society shifts from a heterogeneous identity equilibrium to equilibrium (e).

5.2.2 Assumption of symmetric ethnic groups

Unlike in the model, ethnic groups are highly asymmetric in population size in many societies. One interesting question arising in such a society is whether individuals of a large ethnic group are more likely than those of the smaller groups to identify with the nation. Under the assumption that larger groups have higher status, individuals of a large group are more likely to have an ethnic identity if the group's status is sufficiently higher than that of the smaller groups; otherwise, they are more likely to have a national identity because of their smaller perceived distance under national identity. Hence, when the national status is relatively, but not extremely, low, the society shifts to an equilibrium with a universal ethnic identity for large groups and a universal national identity for the smaller groups in the former case, and vice versa in the latter case.

A model with asymmetric groups can be used to examine the relationship between measures of divisions of ethnic groups and conflict. In a contest model of conflict without socio-psychological factors, Esteban and Ray (2011) theoretically show that the level of conflict can be expressed as an increasing function of a linear combination of measures of ethnic divisions such as the polarization and fractionalization indexes. In addition, Esteban, Mayoral, and Ray (2012) empirically find that the intensity of civil conflict is significantly related to these measures. By contrast, an examination of the present model suggests that, with socio-psychological factors, F would generally depend on demographic variables as well as on the measures of ethnic divisions.

6 Conclusion

This paper developed a model to examine the effects of a society's modernization on social identity, ethnic conflict, and development. In the model, individuals of each ethnic group choose a sector to work in (between the modern and the group's traditional sectors), a social identity (between the national and ethnic identities), and a contribution to conflict. Identity, conflict, and sectoral choices of workers interact with each other in the model.

The analysis showed that a society with people having stronger pride in the nation or caring less about ethnic differences, less resources, and better institutions is in an equilibrium with a higher proportion of people identifying with the nation, a lower level of conflict, higher employment and production shares of the modern sector, and higher output. As modernization driven by the increased productivity of the modern sector proceeds, a society shifts to an equilibrium with a universal national identity and good outcomes in conflict and development if the national pride is high, ethnic differences are not salient in people's minds, resources are not abundant, or institutions are of good quality; otherwise, it shifts to an equilibrium with a universal ethnic identity and worse outcomes in the other dimensions. The result under the former (latter) situation is consistent with the classic (competing) thesis on the effect of modernization on identity in political science. The result suggests that, under the latter situation, policies that improve institutional quality, elevate the national pride, or deemphasize ethnic differences are crucial for achieving good outcomes. Nation-building policies, such as education and propaganda that emphasize common history and culture and the promotion of a national language, may be interpreted as policies raising the national pride or making ethnicity less pronounced; thus, they are pivotal under the latter situation.

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Appendix A Existence conditions of equilibria

This Appendix presents precise conditions (combinations of parameters and exogenous variables) under which each equilibrium exists. The propositions in this Appendix are the basis for Propositions 3−5 and Figures 1−4 in Sections 3 and 4.

A.1 Homogeneous identity equilibria

The next proposition presents the existence conditions for the two homogeneous identity equilibria. In the proposition, $\beta\Delta d[F,c_s] \equiv \beta\left[(\eta_0+\eta_1 F)\frac{n_e-1}{n_e}+c_s\omega_s\right]$ (c_s is an expression), $\Delta c(F) \equiv \frac{1}{\theta}\left(\frac{n_e-1}{n_e}\right)^{\frac{\theta}{\theta-1}}\left[\left(\delta\frac{V}{F}\right)^{\frac{\theta}{\theta-1}}-\left(\delta\frac{V}{F}-\beta\eta_1\right)^{\frac{\theta}{\theta-1}}\right]$, and $\Delta S \equiv S_N - S_E$.

Proposition A1 *(i) Equilibrium (e) exists iff* $L_{TJ} = (L_{TJ})^*_e$ *satisfies* $\gamma\Delta S \leq \beta\Delta d\left[F^*_e, -n_e(n_e-1)\left(\frac{L_{TJ}}{L}\right)^2\right] - \Delta c(F^*_e)$*, where the RHS of the equation decreases with* L_{TJ}*.*

(ii) Equilibrium (n) exists iff $L_{TJ} = (L_{TJ})^*_n$ *satisfies* $\gamma\Delta S > \beta\Delta d\left[F^*_n, (n_e-1)\frac{L_{TJ}}{L}\left(2-n_e\frac{L_{TJ}}{L}\right)\right] - \Delta c(F^*_n)$*, where the RHS of the equation increases with* L_{TJ}*.*[49]

Proof. See Appendix B. ■

$\gamma\Delta S = \beta\Delta d\left[F^*_e, -n_e(n_e-1)\left(\frac{L_{TJ}}{L}\right)^2\right] - \Delta c(F^*_e)$ and $\gamma\Delta S = \beta\Delta d\left[F^*_n, (n_e-1)\frac{L_{TJ}}{L}\left(2-n_e\frac{L_{TJ}}{L}\right)\right] - \Delta c(F^*_n)$ respectively, correspond to the downward- and upward-sloping solid curves of Figure 1 and other figures.

A.2 Heterogeneous identity equilibria

The next proposition presents the existence conditions for the heterogeneous identity equilibria.

Proposition A2 *(i) Equilibrium (d) exists iff* $L_{TJ} = (L_{TJ})^*_d$ *satisfies* $\beta\Delta d\left[F_d(L_{TJ}), -(n_e-1)n_e\left(\frac{L_{TJ}}{L}\right)^2\right] - \Delta c(F_d(L_{TJ})) < \gamma\Delta S \leq \beta\Delta d\left[F_d(L_{TJ}), (n_e-1)\frac{L_{TJ}}{L}\left(2-n_e\frac{L_{TJ}}{L}\right)\right] - \Delta c(F_d(L_{TJ}))$*, where* $F_d(L_{TJ})$ *is the solution for (26) and increases with* L_{TJ}*, the relation between the left-most side of the equation and* L_{TJ} *is positive for small* L_{TJ}*, and the right-most side of the equation increases with* L_{TJ}*.*

(ii) Equilibrium (Md) exists iff $L_{TJ} = (L_{TJ})^*_{Md} = (L_{TJ})^*_e$ *satisfies* $\beta\Delta d\left[F_d(L_{TJ}), -(n_e-1)n_e\left(\frac{L_{TJ}}{L}\right)^2\right] - \Delta c(F_d(L_{TJ})) < \gamma\Delta S < \beta\Delta d\left[F^*_e, -n_e(n_e-1)\left(\frac{L_{TJ}}{L}\right)^2\right] - \Delta c(F^*_e)$*, where the relation between the left-most side of the equation and* L_{TJ} *is positive for small* L_{TJ}*, and the right-most side of the equation decreases with* L_{TJ}*.*

(iii) Equilibrium (Td) exists iff $L_{TJ} = (L_{TJ})^*_{Td} = (L_{TJ})^*_n$ *satisfies* $\beta\Delta d\left[F^*_n, (n_e-1)\frac{L_{TJ}}{L}\left(2-n_e\frac{L_{TJ}}{L}\right)\right] - \Delta c(F^*_n) < \gamma\Delta S < \beta\Delta d\left[F_d(L_{TJ}), (n_e-1)\frac{L_{TJ}}{L}\left(2-n_e\frac{L_{TJ}}{L}\right)\right] - \Delta c(F_d(L_{TJ}))$*, where the left-most and right-most sides of the equation increase with* L_{TJ}*.*

Proof. See Appendix B. ■

$\gamma\Delta S = \beta\Delta d\left[F_d(L_{TJ}), -(n_e-1)n_e\left(\frac{L_{TJ}}{L}\right)^2\right] - \Delta c(F_d(L_{TJ}))$ and $\gamma\Delta S = \beta\Delta d\left[F_d(L_{TJ}), (n_e-1)\frac{L_{TJ}}{L}\left(2-n_e\frac{L_{TJ}}{L}\right)\right] - \Delta c(F_d(L_{TJ}))$ respectively, correspond to the lower and higher dotted lines of Figures 2 and 4.

Figures 2 and 4 illustrate combinations of L_{TJ} and ΔS such that each equilibrium exists when ω_s is relatively high and η_1 is relatively low.[50] Conversely, Figure 5 in this Appendix shows the combinations when ω_s is relatively low and η_1 is relatively high. Unlike Figures 2 and 4, the value of ΔS of the downward-sloping solid curve at $L_{TJ} = 0$ is greater than that of the upward-sloping

[49] At $L_{TJ} = 0$, the RHS of the condition is smaller than that of the condition of equilibrium (e).

[50] To be precise, Figures 2 and 4 illustrate the case when $\beta\Delta d\left[F^*_n, \frac{n_e-1}{n_e}\right] - \Delta c(F^*_n) > \beta\Delta d[F^*_e, 0] - \Delta c(F^*_e) \Leftrightarrow \beta\Delta d[F^*_n, 0] - \Delta c(F^*_n) > \beta\Delta d\left[F^*_e, -\frac{n_e-1}{n_e}\right] - \Delta c(F^*_e) \Leftrightarrow \beta\frac{n_e-1}{n_e}\omega_s > \beta\frac{n_e-1}{n_e}\eta_1(F^*_e - F^*_n) + [\Delta c(F^*_n) - \Delta c(F^*_e)]$ holds, where the LHS (RHS) of the first equation is γ times the value of ΔS of the upward-sloping (downward-sloping) solid curve at $L_{TJ} = \frac{L}{n_e}$ ($L_{TJ} = 0$), and the LHS (RHS) of the second equation is that of the lower dotted line at $L_{TJ} = 0$ ($L_{TJ} = \frac{L}{n_e}$). The LHS of the last equation increases with ω_s, and the RHS can be shown to increase with η_1.

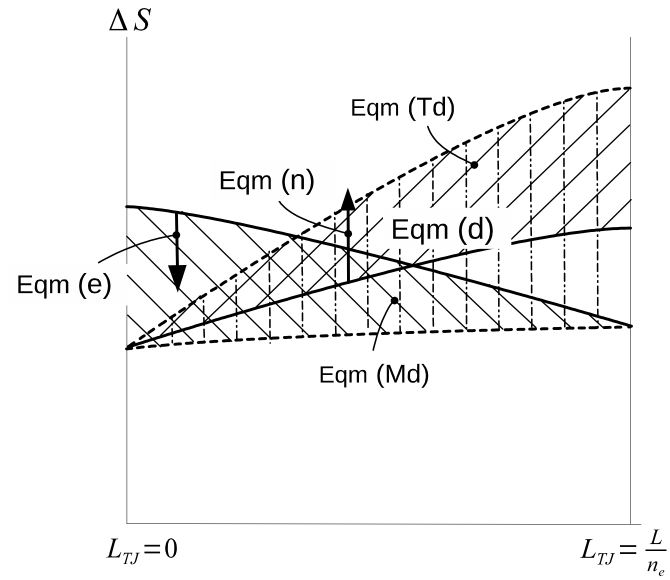

Figure 5: Equilibria when ω_s is relatively low and η_1 is relatively high

solid curve at $L_{TJ} = \frac{L}{n_e}$, and the value of ΔS of the lower dotted line at $L_{TJ} = 0$ is smaller than that at $L_{TJ} = \frac{L}{n_e}$. However, the basic features of Figure 5 are similar to those of Figures 2 and 4.

Appendix B Proofs

Proof of Proposition 1. (i) Evident from (18) and (24). (ii) Shown in footnotes 23 and 24.

(iii) Let Y_n^* (Y_e^*) be the output of private goods in equilibrium (n) ((e)). From (1) and (2),

$$Y_n^* > Y_e^* \Leftrightarrow A_T((L_{TJ})_n^*)^\alpha - A_M(L_{TJ})_n^* > A_T((L_{TJ})_e^*)^\alpha - A_M(L_{TJ})_e^*. \tag{34}$$

The derivative of $A_T(L_{TJ})^\alpha – A_M L_{TJ}$ with respect to L_{TJ} equals $\alpha A_T(L_{TJ})^{\alpha-1} – A_M$, which decreases with L_{TJ}. Thus, since $(L_{TJ})_e^* > (L_{TJ})_n^*$, (34) holds if $\alpha A_T((L_{TJ})_n^*)^{\alpha-1} – A_M \leq 0 \Leftrightarrow (L_{TJ})_n^* \geq \left(\frac{\alpha A_T}{A_M}\right)^{\frac{1}{1-\alpha}}$. This is always true when $\frac{\alpha A_T}{A_M} \leq 1$, hence the case $\frac{\alpha A_T}{A_M} > 1$ is considered in the following. Because $(L_{TJ})_n^*$ is the solution to (25), (34) holds if

$$\frac{A_M}{\alpha} - \beta\omega_s\left[1 - \frac{2}{L}\left(\frac{\alpha A_T}{A_M}\right)^{\frac{1}{1-\alpha}}\right] \geq A_M. \tag{35}$$

Hence, $Y_n^* > Y_e^*$ is true when β is not very large so that the inequality holds.

The derivative of the LHS of the above inequality with respect to α equals

$$-\frac{A_M}{\alpha^2} + \beta\omega_s \frac{2}{L}\frac{1}{1-\alpha}\left[\frac{1}{1-\alpha}\ln\left(\frac{\alpha A_T}{A_M}\right) + \frac{1}{\alpha}\right]\left(\frac{\alpha A_T}{A_M}\right)^{\frac{1}{1-\alpha}} \tag{36}$$

and the second derivative equals

$$\frac{2A_M}{\alpha^3} + \beta\omega_s \frac{2}{L}\left(\frac{1}{1-\alpha}\left[\frac{2}{(1-\alpha)^2}\ln\left(\frac{\alpha A_T}{A_M}\right) + \frac{1}{\alpha}\left(\frac{2}{1-\alpha} - \frac{1}{\alpha}\right)\right] + \left\{\frac{1}{1-\alpha}\left[\frac{1}{1-\alpha}\ln\left(\frac{\alpha A_T}{A_M}\right) + \frac{1}{\alpha}\right]\right\}^2\right)\left(\frac{\alpha A_T}{A_M}\right)^{\frac{1}{1-\alpha}}$$

$$= \frac{2A_M}{\alpha^3} + \beta\omega_s \frac{2}{L}\frac{1}{1-\alpha}\left\{\left[\frac{2}{(1-\alpha)^2}\ln\left(\frac{\alpha A_T}{A_M}\right) + \frac{1}{\alpha}\frac{3}{1-\alpha}\right] + \frac{1}{(1-\alpha)^2}\left[\frac{1}{1-\alpha}\ln\left(\frac{\alpha A_T}{A_M}\right) + \frac{2}{\alpha}\right]\ln\left(\frac{\alpha A_T}{A_M}\right)\right\}\left(\frac{\alpha A_T}{A_M}\right)^{\frac{1}{1-\alpha}} > 0, \quad (37)$$

Because (35) holds as $\alpha \to 0$, does not hold as $\alpha \to 1$, and the first derivative of the LHS of the equation is $-\infty$ as $\alpha \to 0$, and it increases as α goes up, there exists a $\underline{\alpha} \in (0,1)$ below which $(L_{TJ})^*_n \geq \left(\frac{\alpha A_T}{A_M}\right)^{\frac{1}{1-\alpha}}$ and thus $Y^*_n > Y^*_e$ hold.

By contrast, $Y^*_n < Y^*_e$ holds when $(L_{TJ})^*_e \leq \left(\frac{\alpha A_T}{A_M}\right)^{\frac{1}{1-\alpha}}$, which is always true if $\left(\frac{\alpha A_T}{A_M}\right)^{\frac{1}{1-\alpha}} \geq \frac{L}{n_e}$. Otherwise,

$$(L_{TJ})^*_e \leq \left(\frac{\alpha A_T}{A_M}\right)^{\frac{1}{1-\alpha}} \Leftrightarrow \frac{A_M}{\alpha} - \beta\omega_s\left[1 - 2n_e\left(\frac{\alpha A_T}{A_M}\right)^{\frac{1}{1-\alpha}}\frac{1}{L}\right] \leq A_M \text{ from (20).} \quad (38)$$

From the equation, when β is large enough, $(L_{TJ})^*_e \leq \left(\frac{\alpha A_T}{A_M}\right)^{\frac{1}{1-\alpha}}$ and thus $Y^*_n < Y^*_e$ hold. Also, from a similar reasoning as above, there exists a $\overline{\alpha} \in (0,1)$ ($> \underline{\alpha}$) above which $Y^*_n < Y^*_e$ hold. ■

Proof of Proposition 2. (i) $F^*_n < F^*_{Td}$ is from (24) and (30), $F^*_{Md} < F^*_e$ is from (18) and (33), and $F^*_{Td} < F^*_d < F^*_{Md}$ is from (26), (30), and (33) and $(L_{TJ})^*_{Td} < (L_{TJ})^*_d < (L_{TJ})^*_{Md}$ shown in (ii).

(ii) $(L_{TJ})^*_{Td} = (L_{TJ})^*_n$ and $(L_{TJ})^*_{Md} = (L_{TJ})^*_e$ are shown in Sections 3.2.2 and 3.2.3. From footnote 23, the LHS of (20), the indifference condition whose solution is $(L_{TJ})^*_e$, decreases with L_{TJ} for $L_{TJ} \leq (L_{TJ})^*_e$. Hence, $(L_{TJ})^*_d < (L_{TJ})^*_e$ holds, if the LHS of (27), the indifference condition whose solution is $(L_{TJ})^*_d$, is smaller than that of (20) at $L_{TJ} = (L_{TJ})^*_d$, which is true because

$$\beta\left\{(\eta_0 + \eta_1 F_d(L_{TJ}))\frac{n_e-1}{n_e} + \omega_s\left[n_e\left(\frac{L_{TJ}}{L}\right)^2 - \left(1 - n_e\frac{L_{TJ}}{L}\right)^2\right]\right\}$$
$$-\frac{1}{\theta}\left(\frac{n_e-1}{n_e}\right)^{\frac{\theta}{\theta-1}}\left[\left(\delta\frac{V}{F_d(L_{TJ})}\right)^{\frac{\theta}{\theta-1}} - \left(\delta\frac{V}{F_d(L_{TJ})} - \beta\eta_1\right)^{\frac{\theta}{\theta-1}}\right] - \gamma\Delta S \leq -\beta\omega_s\left(1 - 2n_e\frac{L_{TJ}}{L}\right) \quad (39)$$

$$\Leftrightarrow \beta\left[(\eta_0 + \eta_1 F_d(L_{TJ}))\frac{n_e-1}{n_e} - \omega_s n_e(n_e-1)\left(\frac{L_{TJ}}{L}\right)^2\right] - \frac{1}{\theta}\left(\frac{n_e-1}{n_e}\right)^{\frac{\theta}{\theta-1}}\left[\left(\delta\frac{V}{F_d(L_{TJ})}\right)^{\frac{\theta}{\theta-1}} - \left(\delta\frac{V}{F_d(L_{TJ})} - \beta\eta_1\right)^{\frac{\theta}{\theta-1}}\right] \leq \gamma\Delta S, \quad (40)$$

where the inequality holds from Proposition A2 (i).

From footnote 24, the shape of the LHS of (25), the indifference condition whose solution is $(L_{TJ})^*_n$, is similar to that of (20). Hence, $(L_{TJ})^*_d > (L_{TJ})^*_n$ holds if the LHS of (27) is greater than that of (25) at $L_{TJ} = (L_{TJ})^*_d$, which is true because

$$\beta\left\{(\eta_0 + \eta_1 F_d(L_{TJ}))\frac{n_e-1}{n_e} + \omega_s\left[n_e\left(\frac{L_{TJ}}{L}\right)^2 - \left(1 - n_e\frac{L_{TJ}}{L}\right)^2\right]\right\}$$
$$-\frac{1}{\theta}\left(\frac{n_e-1}{n_e}\right)^{\frac{\theta}{\theta-1}}\left[\left(\delta\frac{V}{F_d(L_{TJ})}\right)^{\frac{\theta}{\theta-1}} - \left(\delta\frac{V}{F_d(L_{TJ})} - \beta\eta_1\right)^{\frac{\theta}{\theta-1}}\right] - \gamma\Delta S \geq -\beta\omega_s\left(1 - \frac{2L_{TJ}}{L}\right) \quad (41)$$

$$\Leftrightarrow \beta\left[(\eta_0 + \eta_1 F_d(L_{TJ}))\frac{n_e-1}{n_e} + \omega_s(n_e-1)\frac{L_{TJ}}{L}\left(2 - n_e\frac{L_{TJ}}{L}\right)\right] - \frac{1}{\theta}\left(\frac{n_e-1}{n_e}\right)^{\frac{\theta}{\theta-1}}\left[\left(\frac{\delta V}{F_d(L_{TJ})}\right)^{\frac{\theta}{\theta-1}} - \left(\frac{\delta V}{F_d(L_{TJ})} - \beta\eta_1\right)^{\frac{\theta}{\theta-1}}\right] \geq \gamma\Delta S, \quad (42)$$

where the inequality holds from Proposition A2 (i).

(iii) Let Y^*_d be the output of private goods in equilibrium (d). From (1) and (2), $Y^*_n > Y^*_d > Y^*_e \Leftrightarrow A_T((L_{TJ})^*_n)^\alpha - A_M(L_{TJ})^*_n > A_T((L_{TJ})^*_d)^\alpha - A_M(L_{TJ})^*_d > A_T((L_{TJ})^*_e)^\alpha - A_M(L_{TJ})^*_e$. Since $(L_{TJ})^*_n < (L_{TJ})^*_d < (L_{TJ})^*_e$, if β is small enough that (35) in the proof of Proposition 1 (iii) holds, $A_T(L_{TJ})^\alpha - A_M L_{TJ}$

decreases with L_{TJ} for $L_{TJ} \geq (L_{TJ})^*_n$ and thus $Y^*_n > Y^*_d > Y^*_e$ is true. If β is large enough that (38) holds, $A_T(L_{TJ})^\alpha - A_M L_{TJ}$ increases with L_{TJ} for $L_{TJ} \leq (L_{TJ})^*_e$ and thus $Y^*_n < Y^*_d < Y^*_e$ is true. As for the result on α, the corresponding proof of Proposition 1 applies since $(L_{TJ})^*_e > (L_{TJ})^*_d > (L_{TJ})^*_n$ holds. $Y^*_{Td} = Y^*_n$ and $Y^*_{Md} = Y^*_e$ are evident from $(L_{TJ})^*_{Td} = (L_{TJ})^*_n$ and $(L_{TJ})^*_{Md} = (L_{TJ})^*_e$.

(iv) The total cost of conflict is (L_n and L_e are numbers of those identifying with the nation and their ethnic group) $\frac{1}{\theta}\left[(f_{i,n})^\theta L_n + (f_{i,e})^\theta L_e\right] = \frac{1}{\theta}\left[(f_{i,n})^{\theta-1} f_{i,n} L_n + (f_{i,e})^{\theta-1} f_{i,e} L_e\right] = \frac{1}{\theta}\frac{n_e-1}{n_e}\left[\left(\delta\frac{V}{F} - \beta\eta_1\right) f_{i,n} L_n + \delta\frac{V}{F} f_{i,e} L_e\right] = \frac{1}{\theta}\frac{n_e-1}{n_e}[\delta V - \beta\eta_1 f_{i,n} L_n]$, where the second equality is from (16), (22), and $F_{-J} = \frac{n_e-1}{n_e}F$. It decreases with L_n since $f_{i,n}$ increases with L_n from (22) and Proposition 2 (i). From this and (iii), aggregate material payoff increases with L_n unless α or β is very high. ■

Proof of Proposition 3. (i) Figure 2 and Figure 5 of Appendix A, which are based on Propositions A1 and A2, illustrate combinations of L_{TJ} and ΔS such that each equilibrium exists. First, fix values of exogenous variables including A_M. Equilibrium (n) exists if $((L_{TJ})^*_n, \Delta S)$ is above the upward-sloping solid line. It is the unique equilibrium if $(L_{TJ}, \Delta S)$ of any other equilibrium is not in the region in which such equilibrium exists. $(L_{TJ})^*_n = (L_{TJ})^*_{Td} < (L_{TJ})^*_d (< (L_{TJ})^*_e = (L_{TJ})^*_{Md})$ from Proposition 2. Hence, from the figures, the condition for the unique equilibrium when A_M is low is that $((L_{TJ})^*_d, \Delta S)$ is above the upper dotted line. Now, let A_M increase and thus $(L_{TJ})^*_n$ and values of L_{TJ} of other equilibria decrease over time. Suppose that an equilibrium realized initially is sustained in subsequent periods, as long as the equilibrium continues to exist. Then, because the upper dotted line is upward-sloping, the society is always in equilibrium (n) if $((L_{TJ})^*_d, \Delta S)$ in the initial period lies above the upper dotted line.

From a similar reasoning, the society is always in equilibrium (e) if the initial $((L_{TJ})^*_d, \Delta S)$ lies below the lower dotted line and ΔS is lower than the level on the line at $L_{TJ} = 0$. The latter condition is needed since the shape of the line is unclear except it is upward-sloping for small L_{TJ}.

(ii) If the conditions of (i) do not hold, when ΔS is relatively large, equilibria (d), (n) [if $((L_{TJ})^*_n, \Delta S)$ is above the upward-sloping solid line], and (Td) [if $((L_{TJ})^*_{Td}, \Delta S) = ((L_{TJ})^*_n, \Delta S)$ is in the region with positively-sloped lines] exist for large equilibrium L_{TJ} and thus small A_M, while equilibrium (n) exists for large A_M; when ΔS is relatively small, equilibria (d), (e) [if $((L_{TJ})^*_e, \Delta S)$ is below the downward-sloping solid line], and (Md) [if $((L_{TJ})^*_{Md}, \Delta S) = ((L_{TJ})^*_e, \Delta S)$ is in the region with negatively-sloped lines] exist for small A_M, while equilibrium (e) exists for large A_M.

(iii) From the figures, as ΔS decreases, realized equilibria change in the following order: (n), (Td), (d), (Md), (e). $(L_{TJ})^*_n = (L_{TJ})^*_{Td} < (L_{TJ})^*_{Md} = (L_{TJ})^*_e$ from Proposition 2, since their values do not depend on ΔS. Because equilibrium $(L_{TJ})^*_d$ must satisfy the condition of Proposition A2 (i), from the second inequality of the condition and (27), $A_T((L_{TJ})^*_d)^{\alpha-1} + \beta\Delta d\left[F_d((L_{TJ})^*_d), -\left(1 - \frac{2(L_{TJ})^*_d}{L}\right)\right] \leq A_M$, thus $(L_{TJ})^*_n \leq (L_{TJ})^*_d$ for any ΔS at which $(L_{TJ})^*_d$ exists (< except at highest ΔS). Similarly, from the first inequality of the condition and (27), $A_T((L_{TJ})^*_d)^{\alpha-1} + \beta\Delta d\left[F_d((L_{TJ})^*_d), -\left(1 - \frac{2(L_{TJ})^*_d}{L}\right)\right] > A_M$, thus $(L_{TJ})^*_d < (L_{TJ})^*_e$. The result on the national identity follows from the result on L_{TJ}. Since equilibrium (Td) [(Md)] exists only in the region equilibrium (n) [(e)] exists, $F^*_n < F^*_{Td}$ [$F^*_{Md} < F^*_e$] from Proposition 2. Since $(L_{TJ})^*_{Td} < (L_{TJ})^*_d$ [$(L_{TJ})^*_{Md} > (L_{TJ})^*_d$], $F^*_{Td} < F^*_d$ [$F^*_d < F^*_{Md}$] from (26) and (33) [(30)]. Given these results, the remaining results can be proved similarly to Proposition 2. ■

Proof of Proposition 4. It is enough to prove that the expressions on the opposite side of $\gamma\Delta S$ of the equilibrium conditions—the LHSs of (56) and (65) in the proof of Proposition A1 and of (69) and (70) in the proof of Proposition A2—increase with η_0, that is, the dividing lines in Figure 4 shift upward with an increase in η_0. As for the homogeneous identity equilibria, since F does not depend on η_0, the result is straightforward from (56) and (65). As for equilibrium (d), since F is independent of η_0 for given L_{TJ} from (26), the result is straightforward from (69) and (70). The expressions of the conditions of the remaining equilibria are same as one of these expressions. ■

Proof of Proposition 5. (i) Straightforward from the equation determining F of each equilibrium, (18), (24), (26), (27), (29), and (32).

(ii) It is enough to prove that the expressions on the opposite side of $\gamma\Delta S$ of the equilibrium conditions—the LHSs of (56) and (65) in the proof of Proposition A1 and of (69) and (70) in the proof of Proposition A2—increase with V.

[Equilibrium (e)] The derivative of the LHS of (56) with respect to V is, from (18),

$$\begin{aligned}&\tfrac{1}{\theta}(V)^{-1}\left\{\beta\eta_1\tfrac{n_e-1}{n_e}F_e^*-\left(\tfrac{n_e-1}{n_e}\right)^{\frac{\theta}{\theta-1}}\left[\left(\delta\tfrac{V}{F_e^*}\right)^{\frac{1}{\theta-1}}-\left(\delta\tfrac{V}{F_e^*}-\beta\eta_1\right)^{\frac{1}{\theta-1}}\right]\delta\tfrac{V}{F_e^*}\right\}\\&>\tfrac{1}{\theta}(V)^{-1}\left\{\beta\eta_1\tfrac{n_e-1}{n_e}F_e^*-\left(\tfrac{n_e-1}{n_e}\right)^{\frac{\theta}{\theta-1}}\left(\delta\tfrac{V}{F_e^*}\right)^{\frac{\theta}{\theta-1}}\right\}=\tfrac{1}{\theta}(V)^{-1}\tfrac{n_e-1}{n_e}F_e^*\left(\beta\eta_1-\tfrac{1}{L}\delta\tfrac{V}{F_e^*}\right).\end{aligned}\tag{43}$$

For $F_e^*=\left(\delta\frac{n_e-1}{n_e}\frac{V}{F_e^*}\right)^{\frac{1}{\theta-1}}L$ and $F_n^*=\left[\frac{n_e-1}{n_e}\left(\delta\frac{V}{F_n^*}-\beta\eta_1\right)\right]^{\frac{1}{\theta-1}}L$ not to be too close, $\beta\eta_1$ must be of a similar order of magnitude to $\delta\frac{V}{F_n^*}$ and $\delta\frac{V}{F_e^*}$. Then, $\beta\eta_1-\frac{1}{L}\delta\frac{V}{F_e^*}>0$ and the derivative is positive.

[Equilibrium (n)] Since $\frac{dF_n^*}{dV}=\frac{\frac{1}{\theta-1}\delta}{\frac{\theta}{\theta-1}\delta\frac{V}{F_n^*}-\beta\eta_1}$ and $\frac{d(\frac{V}{F_n^*})}{dV}=\frac{\frac{1}{F_n^*}\left(\delta\frac{V}{F_n^*}-\beta\eta_1\right)}{\frac{\theta}{\theta-1}\delta\frac{V}{F_n^*}-\beta\eta_1}$ from (24), the derivative of the LHS of (65) with respect to V is,

$$\begin{aligned}&\frac{\frac{1}{\theta-1}\delta}{\frac{\theta}{\theta-1}\delta\frac{V}{F_n^*}-\beta\eta_1}\left\{\beta\eta_1\tfrac{n_e-1}{n_e}-\left(\tfrac{n_e-1}{n_e}\right)^{\frac{\theta}{\theta-1}}\left[\left(\delta\tfrac{V}{F_n^*}\right)^{\frac{1}{\theta-1}}-\left(\delta\tfrac{V}{F_n^*}-\beta\eta_1\right)^{\frac{1}{\theta-1}}\right]\tfrac{1}{F_n^*}\left(\delta\tfrac{V}{F_n^*}-\beta\eta_1\right)\right\}\\&=\frac{\frac{1}{\theta-1}\delta}{\frac{\theta}{\theta-1}\delta\frac{V}{F_n^*}-\beta\eta_1}\tfrac{n_e-1}{n_e}\left\{\beta\eta_1-\tfrac{1}{L}\left[\left(\delta\tfrac{V}{F_n^*}\right)^{\frac{1}{\theta-1}}-\left(\delta\tfrac{V}{F_n^*}-\beta\eta_1\right)^{\frac{1}{\theta-1}}\right]\left(\delta\tfrac{V}{F_n^*}-\beta\eta_1\right)^{1-\frac{1}{\theta-1}}\right\}\\&>\frac{\frac{1}{\theta-1}\delta}{\frac{\theta}{\theta-1}\delta\frac{V}{F_n^*}-\beta\eta_1}\tfrac{n_e-1}{n_e}\left(\beta\eta_1-\tfrac{1}{L}\delta\tfrac{V}{F_n^*}\right)>0.\end{aligned}\tag{44}$$

[Equilibrium (d)] From (26),

$$\frac{dF_d(L_{TJ})}{dV}=\frac{\frac{\delta}{\theta-1}\frac{1}{F_d(L_{TJ})}\left(\frac{n_e-1}{n_e}\right)^{\frac{1}{\theta-1}}\left[\left(\delta\frac{V}{F_d(L_{TJ})}\right)^{\frac{1}{\theta-1}-1}n_eL_{TJ}+\left(\delta\frac{V}{F_d(L_{TJ})}-\beta\eta_1\right)^{\frac{1}{\theta-1}-1}(L-n_eL_{TJ})\right]}{1+\frac{\delta}{\theta-1}\frac{V}{[F_d(L_{TJ})]^2}\left(\frac{n_e-1}{n_e}\right)^{\frac{1}{\theta-1}}\left[\left(\delta\frac{V}{F_d(L_{TJ})}\right)^{\frac{1}{\theta-1}-1}n_eL_{TJ}+\left(\delta\frac{V}{F_d(L_{TJ})}-\beta\eta_1\right)^{\frac{1}{\theta-1}-1}(L-n_eL_{TJ})\right]},\tag{45}$$

$$\frac{d\left(\frac{V}{F_d(L_{TJ})}\right)}{dV}=\frac{\frac{1}{F_d(L_{TJ})}}{1+\frac{\delta}{\theta-1}\frac{V}{[F_d(L_{TJ})]^2}\left(\frac{n_e-1}{n_e}\right)^{\frac{1}{\theta-1}}\left[\left(\delta\frac{V}{F_d(L_{TJ})}\right)^{\frac{1}{\theta-1}-1}n_eL_{TJ}+\left(\delta\frac{V}{F_d(L_{TJ})}-\beta\eta_1\right)^{\frac{1}{\theta-1}-1}(L-n_eL_{TJ})\right]}.\tag{46}$$

Thus, the derivative of the LHS of (69) or (70) with respect to V is,

$$\frac{dF_d(L_{TJ})}{dV}\frac{n_e-1}{n_e}\left[\beta\eta_1-\frac{\left(\delta\frac{V}{F_d(L_{TJ})}\right)^{\frac{1}{\theta-1}}-\left(\delta\frac{V}{F_d(L_{TJ})}-\beta\eta_1\right)^{\frac{1}{\theta-1}}}{\left(\delta\frac{V}{F_d(L_{TJ})}\right)^{\frac{1}{\theta-1}-1}n_eL_{TJ}+\left(\delta\frac{V}{F_d(L_{TJ})}-\beta\eta_1\right)^{\frac{1}{\theta-1}-1}(L-n_eL_{TJ})}\right], \tag{47}$$

where the expression inside the large square bracket is greater than

$$\beta\eta_1-\frac{\left(\delta\frac{V}{F_d(L_{TJ})}\right)^{\frac{1}{\theta-1}}}{\left(\delta\frac{V}{F_d(L_{TJ})}\right)^{\frac{1}{\theta-1}-1}n_eL_{TJ}+\left(\delta\frac{V}{F_d(L_{TJ})}-\beta\eta_1\right)^{\frac{1}{\theta-1}-1}(L-n_eL_{TJ})}>\beta\eta_1-\frac{1}{L}\delta\frac{V}{F_d(L_{TJ})}>0. \tag{48}$$

Finally, from these results, the expressions on the opposite side of $\gamma\Delta S$ of the conditions of equilibria (Md) and (Td), (72) and (74), too increase with V. ■

Proof of Proposition A1. (i) Equilibrium (e): [Sector M] The utility of individual i of ethnic group J in sector M equals, from (15) and (16),

$$A_M-\frac{1}{\theta}(f_{i,e})^\theta+\delta\frac{F_J}{F}V-\beta\omega_s\left(\frac{L_{TJ}}{L/n_e}\right)^2+\gamma S_E. \tag{49}$$

If he deviates and identifies with the nation, from (21) and (22), the highest utility he gets is

$$A_M-\frac{1}{\theta}(\widetilde{f}_{i,n})^\theta+\delta\frac{\widetilde{F}_J}{\widetilde{F}}V-\beta\left\{(\eta_0+\eta_1\widetilde{F})\frac{n_e-1}{n_e}+\omega_s\left[\left(\frac{L_{TJ}}{L}\right)^2+\sum_{J'\neq J}\left(\frac{L_{TJ'}}{L}\right)^2\right]\right\}+\gamma S_N, \tag{50}$$

where $\widetilde{f}_{i,n}=\left[\delta\frac{F_{-J}}{(\widetilde{F})^2}V-\beta\eta_1\frac{n_e-1}{n_e}\right]^{\frac{1}{\theta-1}}$, $\widetilde{F}_J=\widetilde{f}_{i,n}+\left(\frac{L}{n_e}-1\right)f_{i,e}$, and $\widetilde{F}=\widetilde{F}_J+F_{-J}$.

When L is large enough, the deviation by one player affects $\widetilde{F}_J$ and $\widetilde{F}$ very little, thus (50) is approximated very well by the following equation that is marginally larger than the original one.

$$A_M-\frac{1}{\theta}(f_{i,n})^\theta+\delta\frac{F_J}{F}V-\beta\left\{(\eta_0+\eta_1F)\frac{n_e-1}{n_e}+\omega_s\left[\left(\frac{L_{TJ}}{L}\right)^2+\sum_{J'\neq J}\left(\frac{L_{TJ'}}{L}\right)^2\right]\right\}+\gamma S_N,\text{ where } f_{i,n} \text{ is given by (22).} \tag{51}$$

Thus, from (49) and (51), the deviation is not profitable if

$$\beta\left\{(\eta_0+\eta_1F)\frac{n_e-1}{n_e}+\omega_s\left[\sum_{J'\neq J}\left(\frac{L_{TJ'}}{L}\right)^2-({n_e}^2-1)\left(\frac{L_{TJ}}{L}\right)^2\right]\right\}-\frac{1}{\theta}\left[(f_{i,e})^\theta-(f_{i,n})^\theta\right]\geq\gamma\Delta S. \tag{52}$$

[Sector TJ] The utility of individual i of ethnic group J in sector TJ is, from (17) and (16),

$$A_T(L_{TJ})^{\alpha-1}-\frac{1}{\theta}(f_{i,e})^\theta+\delta\frac{F_J}{F}V-\beta\omega_s\left(1-\frac{L_{TJ}}{L/n_e}\right)^2+\gamma S_E \tag{53}$$

If he deviates and identifies with the nation, from (23) and (22), the highest utility is well approximated by

$$A_T(L_{TJ})^{\alpha-1}-\frac{1}{\theta}(f_{i,n})^\theta+\delta\frac{F_J}{F}V-\beta\left\{(\eta_0+\eta_1F)\frac{n_e-1}{n_e}+\omega_s\left[\left(1-\frac{L_{TJ}}{L}\right)^2+\sum_{J'\neq J}\left(\frac{L_{TJ'}}{L}\right)^2\right]\right\}+\gamma S_N. \tag{54}$$

The deviation is not profitable if

$$\beta\left((\eta_0+\eta_1 F)\frac{n_e-1}{n_e}+\omega_s\left\{\sum_{J'\neq J}\left(\frac{L_{TJ'}}{L}\right)^2+(n_e-1)\frac{L_{TJ}}{L}\left[2-(n_e+1)\frac{L_{TJ}}{L}\right]\right\}\right)-\frac{1}{\theta}[(f_{i,e})^\theta-(f_{i,n})^\theta]\geq\gamma\Delta S. \quad (55)$$

[Equilibrium condition] If (52) holds, so does (55). Thus, (52) is the condition for the existence of equilibrium (e). Since ethnic groups are symmetric, (52) becomes

$$\beta\left[(\eta_0+\eta_1 F_e^*)\frac{n_e-1}{n_e}-\omega_s n_e(n_e-1)\left(\frac{L_{TJ}}{L}\right)^2\right]-\frac{1}{\theta}\left(\frac{n_e-1}{n_e}\right)^{\frac{\theta}{\theta-1}}\left[\left(\delta\frac{V}{F_e^*}\right)^{\frac{\theta}{\theta-1}}-\left(\delta\frac{V}{F_e^*}-\beta\eta_1\right)^{\frac{\theta}{\theta-1}}\right]\geq\gamma\Delta S \quad (56)$$

$$\Leftrightarrow\gamma\Delta S\leq\beta\Delta d\left[F_e^*,-n_e(n_e-1)\left(\frac{L_{TJ}}{L}\right)^2\right]-\Delta c(F_e^*), \quad (57)$$

where F_e^* is given by (18), and $\Delta d[\cdot]$ and $\Delta c(\cdot)$ are defined just before the proposition. Hence, equilibrium (e) exists iff $L_{TJ}=(L_{TJ})_e^*$, the solution for (20), satisfies (57). The RHS of the condition decreases with L_{TJ}.

(ii) Equilibrium (n): [Sector TJ] The utility of a group J worker in sector TJ is, from (23) and (22),

$$A_T(L_{TJ})^{\alpha-1}-\frac{1}{\theta}(f_{i,n})^\theta+\delta\frac{F_J}{F}V-\beta\left\{(\eta_0+\eta_1 F)\frac{n_e-1}{n_e}+\omega_s\left[\left(1-\frac{L_{TJ}}{L}\right)^2+\sum_{J'\neq J}\left(\frac{L_{TJ'}}{L}\right)^2\right]\right\}+\gamma S_N. \quad (58)$$

If he deviates and identifies with his group, from (17) and (16), the highest utility he gets is

$$A_T(L_{TJ})^{\alpha-1}-\frac{1}{\theta}(\widehat{f}_{i,e})^\theta+\delta\frac{\widehat{F}_J}{\widehat{F}}V-\beta\omega_s\left(1-\frac{L_{TJ}}{L/n_e}\right)^2+\gamma S_E, \quad (59)$$

$$\text{where } \widehat{f}_{i,e}=\left[\delta\frac{F_{-J}}{(\widehat{F})^2}V\right]^{\frac{1}{\theta-1}},\ \widehat{F}_J=\widehat{f}_{i,e}+\left(\frac{L}{n_e}-1\right)f_{i,n},\ \text{and } \widehat{F}=\widehat{F}_J+F_{-J}.$$

When L is large enough, the deviation affects $\widehat{F}_J$ and $\widehat{F}$ very little, thus (59) is approximated very well by the following equation that is marginally smaller than the original one.

$$A_T(L_{TJ})^{\alpha-1}-\frac{1}{\theta}(f_{i,e})^\theta+\delta\frac{F_J}{F}V-\beta\omega_s\left(1-\frac{L_{TJ}}{L/n_e}\right)^2+\gamma S_E,\ \text{where } f_{i,e} \text{ is given by (16).} \quad (60)$$

Thus, the deviation is not profitable if[51]

$$\gamma\Delta S>\beta\left((\eta_0+\eta_1 F)\frac{n_e-1}{n_e}+\omega_s\left\{\sum_{J'\neq J}\left(\frac{L_{TJ'}}{L}\right)^2+(n_e-1)\frac{L_{TJ}}{L}\left[2-(n_e+1)\frac{L_{TJ}}{L}\right]\right\}\right)-\frac{1}{\theta}\left[(f_{i,e})^\theta-(f_{i,n})^\theta\right]. \quad (61)$$

[Sector M] The utility of individual i of ethnic group J in sector M equals, from (21) and (22),

$$A_M-\frac{1}{\theta}(f_{i,n})^\theta+\delta\frac{F_J}{F}V-\beta\left\{(\eta_0+\eta_1 F)\frac{n_e-1}{n_e}+\omega_s\left[\left(\frac{L_{TJ}}{L}\right)^2+\sum_{J'\neq J}\left(\frac{L_{TJ'}}{L}\right)^2\right]\right\}+\gamma S_N. \quad (62)$$

From (15) and (16), the highest utility of deviating from the equilibrium is well approximated by,

$$A_M-\frac{1}{\theta}(f_{i,e})^\theta+\delta\frac{F_J}{F}V-\beta\omega_s\left(\frac{L_{TJ}}{L/n_e}\right)^2+\gamma S_E. \quad (63)$$

The deviation is not profitable if

$$\gamma\Delta S>\beta\left\{(\eta_0+\eta_1 F)\frac{n_e-1}{n_e}+\omega_s\left[\sum_{J'\neq J}\left(\frac{L_{TJ'}}{L}\right)^2-(n_e^2-1)\left(\frac{L_{TJ}}{L}\right)^2\right]\right\}-\frac{1}{\theta}\left[(f_{i,e})^\theta-(f_{i,n})^\theta\right]. \quad (64)$$

[51] The equation must hold with strict inequality because the deviant's approximate utility is marginally smaller than the true utility.

[Equilibrium condition] If (61) holds, so does (64). Thus, (61) is the condition for the existence of equilibrium (n). Since ethnic groups are symmetric, (61) becomes

$$\gamma\Delta S > \beta\left[(\eta_0+\eta_1 F_n^*)\frac{n_e-1}{n_e}+\omega_s(n_e-1)\frac{L_{TJ}}{L}\left(2-n_e\frac{L_{TJ}}{L}\right)\right]-\frac{1}{\theta}\left(\frac{n_e-1}{n_e}\right)^{\frac{\theta}{\theta-1}}\left[\left(\delta\frac{V}{F_n^*}\right)^{\frac{\theta}{\theta-1}}-\left(\delta\frac{V}{F_n^*}-\beta\eta_1\right)^{\frac{\theta}{\theta-1}}\right] \quad (65)$$

$$\Leftrightarrow \gamma\Delta S > \beta\Delta d\left[F_n^*,(n_e-1)\frac{L_{TJ}}{L}\left(2-n_e\frac{L_{TJ}}{L}\right)\right]-\Delta c(F_n^*), \quad (66)$$

where F_n^* is the solution for (24). Hence, equilibrium (n) exists iff $L_{TJ}=(L_{TJ})_n^*$, the solution for (25), satisfies (66). The RHS of (66) increases with L_{TJ}, and its value at $L_{TJ}=0$ is smaller than that of the corresponding condition of equilibrium (e), (57), since $\Delta c'(\cdot)<0$ and $F_n^*<F_e^*$. ■

Proof of Proposition A2. [Proof that no other heterogeneous identity equilibria exist] If workers in sector M weakly prefer to identify with their ethnic group, from (56) of the proof of Proposition A1, the following must hold in a symmetric equilibrium:

$$\beta\left[(\eta_0+\eta_1 F)\frac{n_e-1}{n_e}-\omega_s n_e(n_e-1)\left(\frac{L_{TJ}}{L}\right)^2\right]-\frac{1}{\theta}\left(\frac{n_e-1}{n_e}\right)^{\frac{\theta}{\theta-1}}\left[\left(\delta\frac{V}{F}\right)^{\frac{\theta}{\theta-1}}-\left(\delta\frac{V}{F}-\beta\eta_1\right)^{\frac{\theta}{\theta-1}}\right]\geq\gamma\Delta S. \quad (67)$$

If workers in sector TJ weakly prefer to identify with the nation, from (65) of the proof of Proposition A1, the following must hold in a symmetric equilibrium:

$$\gamma\Delta S\geq\beta\left[(\eta_0+\eta_1 F)\frac{n_e-1}{n_e}+\omega_s(n_e-1)\frac{L_{TJ}}{L}\left(2-n_e\frac{L_{TJ}}{L}\right)\right]-\frac{1}{\theta}\left(\frac{n_e-1}{n_e}\right)^{\frac{\theta}{\theta-1}}\left[\left(\delta\frac{V}{F}\right)^{\frac{\theta}{\theta-1}}-\left(\delta\frac{V}{F}-\beta\eta_1\right)^{\frac{\theta}{\theta-1}}\right]. \quad (68)$$

Both conditions cannot hold simultaneously, thus such situation does not arise in equilibrium.

(i) Equilibrium (d): [Sector M] Because sector M workers identify with the nation, the condition for them not to deviate from the equilibrium is given by (64) as in equilibrium (n). Since groups are symmetric, the equation becomes

$$\beta\left[(\eta_0+\eta_1 F_d(L_{TJ}))\frac{n_e-1}{n_e}-\omega_s(n_e-1)n_e\left(\frac{L_{TJ}}{L}\right)^2\right]-\frac{1}{\theta}\left(\frac{n_e-1}{n_e}\right)^{\frac{\theta}{\theta-1}}\left[\left(\delta\frac{V}{F_d(L_{TJ})}\right)^{\frac{\theta}{\theta-1}}-\left(\delta\frac{V}{F_d(L_{TJ})}-\beta\eta_1\right)^{\frac{\theta}{\theta-1}}\right]<\gamma\Delta S, \quad (69)$$

$$\Leftrightarrow \beta\Delta d\left[F_d(L_{TJ}),-(n_e-1)n_e\left(\frac{L_{TJ}}{L}\right)^2\right]-\Delta c(F_d(L_{TJ}))<\gamma\Delta S,$$

where $F_d(L_{TJ})$ is the solution for (26) and $F_d'(L_{TJ})>0$. The relation between the LHS of (69) and L_{TJ} is generally ambiguous but positive for small L_{TJ}, since the derivative at $L_{TJ}=0$ is positive.

[Sector TJ] Because sector TJ workers identify with their ethnic group, the non-deviation condition is given by (55) as in equilibrium (e). Since groups are symmetric, the equation becomes

$$\gamma\Delta S\leq\beta\left[(\eta_0+\eta_1 F_d(L_{TJ}))\frac{n_e-1}{n_e}+\omega_s(n_e-1)\frac{L_{TJ}}{L}\left(2-n_e\frac{L_{TJ}}{L}\right)\right]-\frac{1}{\theta}\left(\frac{n_e-1}{n_e}\right)^{\frac{\theta}{\theta-1}}\left[\left(\delta\frac{V}{F_d(L_{TJ})}\right)^{\frac{\theta}{\theta-1}}-\left(\delta\frac{V}{F_d(L_{TJ})}-\beta\eta_1\right)^{\frac{\theta}{\theta-1}}\right], \quad (70)$$

$$\Leftrightarrow \gamma\Delta S\leq\beta\Delta d\left[F_d(L_{TJ}),(n_e-1)\frac{L_{TJ}}{L}\left(2-n_e\frac{L_{TJ}}{L}\right)\right]-\Delta c(F_d(L_{TJ})), \quad (71)$$

where the RHS increases with L_{TJ} since $\Delta c'(\cdot)<0$ and $F_d'(L_{TJ})>0$.

[Equilibrium condition] Hence, equilibrium (d) exists iff $L_{TJ}=(L_{TJ})_d^*$, the solution for (27), satisfies $\beta\Delta d\left[F_d(L_{TJ}),-(n_e-1)n_e\left(\frac{L_{TJ}}{L}\right)^2\right]-\Delta c(F_d(L_{TJ}))<\gamma\Delta S\leq\beta\Delta d\left[F_d(L_{TJ}),(n_e-1)\frac{L_{TJ}}{L}\left(2-n_e\frac{L_{TJ}}{L}\right)\right]-$

$\Delta c(F_d(L_{TJ}))$. The right-most side of the condition increases with L_{TJ}, while the relation between the left-most side and L_{TJ} is generally ambiguous but is positive for small L_{TJ}.

(ii) Equilibrium (Md): [Sector M] As shown in Section 3.2.2, the following indifference condition for identity choices of sector M workers must hold

$$\beta\left[(\eta_0+\eta_1 F)\frac{n_e-1}{n_e}-\omega_s n_e(n_e-1)\left(\frac{L_{TJ}}{L}\right)^2\right]-\frac{1}{\theta}\left(\frac{n_e-1}{n_e}\right)^{\frac{\theta}{\theta-1}}\left[\left(\delta\frac{V}{F}\right)^{\frac{\theta}{\theta-1}}-\left(\delta\frac{V}{F}-\beta\eta_1\right)^{\frac{\theta}{\theta-1}}\right]=\gamma\Delta S, \tag{29}$$

$$\text{where } F=\left(\frac{n_e-1}{n_e}\right)^{\frac{1}{\theta-1}}\left\{\left(\delta\frac{V}{F}-\beta\eta_1\right)^{\frac{1}{\theta-1}}P_{M,n}(L-n_eL_{TJ})+\left(\delta\frac{V}{F}\right)^{\frac{1}{\theta-1}}[n_eL_{TJ}+(1-P_{M,n})(L-n_eL_{TJ})]\right\}. \tag{30}$$

Given L_{TJ}, the LHS of (29) increases with F and F satisfying (30) decreases with $P_{M,n}$. Hence, F and $P_{M,n}$ satisfying both equations exist iff

$$\beta\left[(\eta_0+\eta_1 F_d(L_{TJ}))\frac{n_e-1}{n_e}-\omega_s n_e(n_e-1)\left(\frac{L_{TJ}}{L}\right)^2\right]-\frac{1}{\theta}\left(\frac{n_e-1}{n_e}\right)^{\frac{1}{\theta-1}}\left[\left(\delta\frac{V}{F_d(L_{TJ})}\right)^{\frac{\theta}{\theta-1}}-\left(\delta\frac{V}{F_d(L_{TJ})F}-\beta\eta_1\right)^{\frac{\theta}{\theta-1}}\right]$$
$$<\gamma\Delta S<\beta\left[(\eta_0+\eta_1 F_e^*)\frac{n_e-1}{n_e}-\omega_s n_e(n_e-1)\left(\frac{L_{TJ}}{L}\right)^2\right]-\frac{1}{\theta}\left(\frac{n_e-1}{n_e}\right)^{\frac{1}{\theta-1}}\left[\left(\delta\frac{V}{F_e^*}\right)^{\frac{\theta}{\theta-1}}-\left(\delta\frac{V}{F_e^*}-\beta\eta_1\right)^{\frac{\theta}{\theta-1}}\right], \tag{72}$$

where F_e^* is given by (18), $F_d(L_{TJ})$ is given by (26), and $F_d'(L_{TJ})>0$. The LHS of the first inequality is same as (69) in (i), thus the relation with L_{TJ} is positive for small L_{TJ} but generally ambiguous.

[Sector TJ] Because workers in sector TJ identify with their ethnic group, the non-deviation condition is given by (55) as in equilibrium (e). Since groups are symmetric, the condition becomes

$$\beta\left[(\eta_0+\eta_1 F)\frac{n_e-1}{n_e}+\omega_s(n_e-1)\frac{L_{TJ}}{L}\left(2-n_e\frac{L_{TJ}}{L}\right)\right]-\frac{1}{\theta}\left(\frac{n_e-1}{n_e}\right)^{\frac{\theta}{\theta-1}}\left[\left(\delta\frac{V}{F}\right)^{\frac{\theta}{\theta-1}}-\left(\delta\frac{V}{F}-\beta\eta_1\right)^{\frac{\theta}{\theta-1}}\right]\geq\gamma\Delta S, \tag{73}$$

where F is the solution for (29). When (72) and thus (29) hold, the condition holds clearly.

[Equilibrium condition] Hence, from (72), equilibrium (Md) exists iff $L_{TJ}=(L_{TJ})^*_{Md}=(L_{TJ})^*_e$ satisfies $\beta\Delta d\left[F_d(L_{TJ}),-(n_e-1)n_e\left(\frac{L_{TJ}}{L}\right)^2\right]-\Delta c(F_d(L_{TJ}))<\gamma\Delta S<\beta\Delta d\left[F_e^*,-n_e(n_e-1)\left(\frac{L_{TJ}}{L}\right)^2\right]-\Delta c(F_e^*)$, where the relation between the LHS of the first inequality and L_{TJ} is positive for small L_{TJ} but generally ambiguous, and the RHS of the second inequality decreases with L_{TJ}.

(iii) Equilibrium (Td): [Sector TJ] From Section 3.2.3, the following condition must hold:

$$\beta\left[(\eta_0+\eta_1 F)\frac{n_e-1}{n_e}+\omega_s(n_e-1)\frac{L_{TJ}}{L}\left(2-n_e\frac{L_{TJ}}{L}\right)\right]-\frac{1}{\theta}\left(\frac{n_e-1}{n_e}\right)^{\frac{\theta}{\theta-1}}\left[\left(\delta\frac{V}{F}\right)^{\frac{\theta}{\theta-1}}-\left(\delta\frac{V}{F}-\beta\eta_1\right)^{\frac{\theta}{\theta-1}}\right]=\gamma\Delta S, \tag{32}$$

$$\text{where } F=\left(\frac{n_e-1}{n_e}\right)^{\frac{1}{\theta-1}}\left[\left(\delta\frac{V}{F}-\beta\eta_1\right)^{\frac{1}{\theta-1}}[P_{TJ,n}n_eL_{TJ}+(L-n_eL_{TJ})]+\left(\delta\frac{V}{F}\right)^{\frac{1}{\theta-1}}(1-P_{TJ,n})n_eL_{TJ}\right]. \tag{33}$$

Given L_{TJ}, the LHS of (32) increases with F, and F satisfying (33) decreases with $P_{TJ,n}$. Hence, F and $P_{TJ,n}$ satisfying both equations exist iff

$$\beta\left[(\eta_0+\eta_1 F_n^*)\frac{n_e-1}{n_e}+\omega_s(n_e-1)\frac{L_{TJ}}{L}\left(2-n_e\frac{L_{TJ}}{L}\right)\right]-\frac{1}{\theta}\left(\frac{n_e-1}{n_e}\right)^{\frac{\theta}{\theta-1}}\left[\left(\delta\frac{V}{F_n^*}\right)^{\frac{\theta}{\theta-1}}-\left(\delta\frac{V}{F_n^*}-\beta\eta_1\right)^{\frac{\theta}{\theta-1}}\right]<\gamma\Delta S$$
$$<\beta\left[(\eta_0+\eta_1 F_d(L_{TJ}))\frac{n_e-1}{n_e}+\omega_s(n_e-1)\frac{L_{TJ}}{L}\left(2-n_e\frac{L_{TJ}}{L}\right)\right]-\frac{1}{\theta}\left(\frac{n_e-1}{n_e}\right)^{\frac{\theta}{\theta-1}}\left[\left(\delta\frac{V}{F_d(L_{TJ})}\right)^{\frac{\theta}{\theta-1}}-\left(\delta\frac{V}{F_d(L_{TJ})}-\beta\eta_1\right)^{\frac{\theta}{\theta-1}}\right], \tag{74}$$

where F_n^* is given by (24) and $F_d(L_{TJ})$ is given by (26) and increases with L_{TJ}. Both the LHS of the first inequality and the RHS of the second inequality increase with L_{TJ}.

[Sector M] Because workers in sector M identify with the nation, the non-deviation condition is given by (64) as in equilibrium (n). Since groups are symmetric, the condition becomes

$$\gamma\Delta S > \beta\left[(\eta_0+\eta_1 F)\frac{n_e-1}{n_e} - \omega_s n_e(n_e-1)\left(\frac{L_{TJ}}{L}\right)^2\right] - \frac{1}{\theta}\left(\frac{n_e-1}{n_e}\right)^{\frac{\theta}{\theta-1}}\left[\left(\delta\frac{V}{F}\right)^{\frac{\theta}{\theta-1}} - \left(\delta\frac{V}{F}-\beta\eta_1\right)^{\frac{\theta}{\theta-1}}\right], \tag{75}$$

where F is obtained from (32) and (33). When (74) and thus (32) hold, this condition holds.

[Equilibrium condition] Hence, equilibrium (Td) exists iff $L_{TJ} = (L_{TJ})^*_{Td} = (L_{TJ})^*_n$ satisfies $\beta\Delta d\left[F_n^*, (n_e-1)\frac{L_{TJ}}{L}\left(2-n_e\frac{L_{TJ}}{L}\right)\right] - \Delta c(F_n^*) < \gamma\Delta S < \beta\Delta d\left[F_d(L_{TJ}), (n_e-1)\frac{L_{TJ}}{L}\left(2-n_e\frac{L_{TJ}}{L}\right)\right] - \Delta c(F_d(L_{TJ}))$ from (74), where the left-most side and the right-most side of the condition increase with L_{TJ}. ■

Proof of the uniqueness of $(L_{TJ})^*_d$. The derivative of the LHS of (27) with respect to L_{TJ} equals

$$\begin{aligned} &-(1-\alpha)A_T(L_{TJ})^{\alpha-2} + 2\beta\omega_s\frac{n_e}{L}\left[1-(n_e-1)\frac{L_{TJ}}{L}\right] \\ &+\frac{n_e-1}{n_e}\left\{\beta\eta_1 + \frac{\delta}{\theta-1}\frac{V}{(F_d(L_{TJ}))^2}\left(\frac{n_e-1}{n_e}\right)^{\frac{1}{\theta-1}}\left[\left(\delta\frac{V}{F_d(L_{TJ})}\right)^{\frac{1}{\theta-1}} - \left(\delta\frac{V}{F_d(L_{TJ})}-\beta\eta_1\right)^{\frac{1}{\theta-1}}\right]\right\} F_d'(L_{TJ}), \end{aligned} \tag{76}$$

where, from (26),

$$F_d'(L_{TJ}) = \frac{\left(\frac{n_e-1}{n_e}\right)^{\frac{1}{\theta-1}}\left[\left(\delta\frac{V}{F_d(L_{TJ})}\right)^{\frac{1}{\theta-1}} - \left(\delta\frac{V}{F_d(L_{TJ})}-\beta\eta_1\right)^{\frac{1}{\theta-1}}\right] n_e}{1+\frac{\delta}{\theta-1}\frac{V}{(F_d(L_{TJ}))^2}\left(\frac{n_e-1}{n_e}\right)^{\frac{1}{\theta-1}}\left[\left(\delta\frac{V}{F_d(L_{TJ})}\right)^{\frac{1}{\theta-1}-1} n_e L_{TJ} + \left(\delta\frac{V}{F_d(L_{TJ})}-\beta\eta_1\right)^{\frac{1}{\theta-1}-1}(L-n_e L_{TJ})\right]} > 0. \tag{77}$$

The second derivative of the LHS of (27) with respect to L_{TJ} equals

$$(2-\alpha)(1-\alpha)A_T(L_{TJ})^{\alpha-3} - 2\beta\omega_s\frac{n_e(n_e-1)}{L^2} + \frac{d(\text{third term of (76)})}{dL_{TJ}}. \tag{78}$$

From footnote 23, the derivative of the LHS of (20) with respect to L_{TJ} is negative for $L_{TJ} \leq (L_{TJ})^*_e$, that is, $-(1-\alpha)A_T(L_{TJ})^{\alpha-2} + 2\beta\omega_s\frac{n_e}{L} < 0$. Since $(L_{TJ})^*_d < (L_{TJ})^*_e$ from Proposition 2 (ii), this is true for $L_{TJ} \leq (L_{TJ})^*_d$. Hence, the first two terms of (78) is positive for $L_{TJ} \leq (L_{TJ})^*_d$, because it is greater than $(1-\alpha)A_T(L_{TJ})^{\alpha-3}\left[(2-\alpha) - L_{TJ}\frac{n_e-1}{L}\right] > 0$. The last term of (78) too is positive for $L_{TJ} \leq (L_{TJ})^*_d$ when $\theta = 2$, since the second line of (76) equals

$$\frac{n_e-1}{n_e}\beta\eta_1\left[1+\delta\frac{V}{(F_d(L_{TJ}))^2}\frac{n_e-1}{n_e}\right]\frac{\frac{n_e-1}{n_e}\beta\eta_1 n_e}{1+\delta\frac{V}{(F_d(L_{TJ}))^2}\frac{n_e-1}{n_e}L}, \tag{79}$$

which clearly increases with L_{TJ}. Hence, the second derivative of the LHS of (27) is positive for $L_{TJ} \leq (L_{TJ})^*_d$. Further, the LHS of (27) is always lower than that of (20) whose solution is unique $(L_{TJ})^*_e$. Therefore, $(L_{TJ})^*_d$ is unique when $\theta = 2$. ■