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Bounding of earthquake response via critical double impulse for efficient optimal design of viscous dampers for elastic-plastic moment frames

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Abstract
An input velocity adjustment method of the critical double impulse (DI) is presented for efficient design of viscous dampers for elastic-plastic moment frames. The input velocity is adjusted so that the input energy to the lowest eigenmode under the critical DI is equal to those under the selected recorded ground motions. This adjustment makes the critical DI work as the active earthquake. The response bounding property of the critical DI is supported by, (1) the multimodal property of the critical DI, (2) the usual excitation of the lowest mode response under recorded ground motions, (3) the large proportion of the instantaneous input energy to the total input energy in the critical DI. The proposed method can treat not only pulse-like near-fault ground motions but also ground motions of random nature. To validate the proposed method, the input energies and the maximum interstory drifts under the recorded ground motions and the critical DI with the adjusted input velocity are investigated for elastic single-degree-of-freedom (SDOF) models, elastic proportionally damped MDOF models, and elastic-plastic proportionally damped MDOF models. The optimization method presented in the previous paper is extended so that the critical DI can be treated for the design of viscous dampers for elastic-plastic moment frames.

Keywords
double impulse, elastic-plastic moment frame, optimization, response bounding, viscous damper

1. Introduction
Extensive research on optimal damper placement has been accumulated so far.1–5 Zhang and Soong6 presented a simple algorithm of damper allocation using response indices. Takewaki7 proposed an optimality criterion-based approach for minimizing the sum of amplitudes of the transfer functions evaluated at the undamped fundamental natural frequency of a structural system subject to a constraint on the sum of the added damping coefficients. Silvestri and Trombetti8 investigated the properties of several damper optimization techniques. Whittle et al.9 compared several optimization methods through numerical examples, especially in terms of effectiveness for reducing seismic performance objectives, usability, and computational load. Kanno10 tackled a mixed-integer programming problem for optimal allocation of viscous dampers. De Dominic and Ricciardi11 developed an energy-based stochastic approach for optimizing nonlinear fluid viscous damper placement. Terazawa and Takeuchi12 incorporated the generalized response spectrum analysis into damper optimization, and Terazawa et al.13 used it for the optimal design of base isolation systems with nonlinear oil dampers. Although most of the above-mentioned researches treated elastic responses of structural frames, elastic-plastic responses of structural frames are incorporated into the damper optimization problems by a limited number of researchers.14–21

It is well known that the selection of design earthquake ground motions affects the optimization results for damper design. The use of multiple ground motions for optimal damper design leads to the design with high reliability. However, it greatly increases the total computational load. The use of the ‘active earthquake’, which bounds or maximizes the structural response under the selected motions, saves the computational load and enhances the reliabilities for various
Although the energy spectrum may help to choose one of the motions as the active earthquake, the chosen motion may not always maximize the response. The instantaneous input energy (or energy rate) should also be considered for the selection because it also affects the maximum responses. From these points of view, this paper is aimed at presenting an input velocity adjustment method of a double impulse (DI) to work as the active earthquake. DI was introduced by Kojima and Takewaki to capture the critical responses under fling-step near-fault motions for SDOF elastic-plastic models. Although the critical elastic-plastic responses have been mainly investigated for various models, this paper focuses on the following characteristics of DI: (1) the duration is short and the computational load is small for the time-history response analysis, (2) the sensitivity-based optimization is easily applicable because of the simple expression of the input, (3) the use of DI leads to a damper design which is effective for the multi-eigenmodes because DI has multiple frequency components and recorded ground motions usually do not have, (4) the total input energy and the instantaneous input energy are easily maximized in the same time for elastic-plastic models throughout the design procedure. Although these characteristics will be useful for the optimal damper design, a rational method to adjust the input velocity amplitudes of DI has not been investigated sufficiently.

In this paper, an input velocity adjustment method of the critical double impulse (DI) is newly presented for efficient design of viscous dampers for elastic-plastic moment frames. The input velocity is adjusted so that the input energy to the lowest eigenmode under the critical DI is equal to those under the selected recorded ground motions. This adjustment makes the critical DI work as the active earthquake. The response bounding property of the critical DI is supported by the fact that (1), while the critical DI possesses a multi-modal property, recorded ground motions usually have not, and (2) the lowest-mode response is often excited under recorded ground motions, (3) the proportion of the instantaneous input energy to the total input energy is large (tend to cause larger deformation). The proposed method can treat not only pulse-like near-fault ground motions but also ground motions of random nature. In order to validate the proposed adjustment method, the input energies and the maximum interstory drifts under the selected recorded ground motions and the critical DI with the adjusted input velocity are investigated for elastic SDOF models, elastic proportionally damped MDOF models, and elastic-plastic proportionally damped MDOF models. The optimization method by Akehashi and Takewaki is extended so that the critical DI can be treated for the design of viscous dampers for elastic-plastic moment frames. Finally, it is demonstrated through numerical examples that the use of the critical DI efficiently provides effective designs for elastic-plastic moment resisting frames with viscous dampers.

2. Critical DI and its Use for Design of Viscous Dampers for Elastic-plastic Moment Frames

In this section, DI is introduced as a substitute for fling-step motions. In Sections 2.1, 2.2, DI and its critical timing are explained briefly. In Section 2.3, the way of the time-history response analysis under the critical DI is explained. In Section 2.4, the merits of using the critical DI for the design of viscous dampers for elastic-plastic moment frames are summarized.

2.1 Double impulse (DI)

It is widely known that a one-cycle sine wave represents the main characteristics of a fling-step near-fault motion. Kojima and Takewaki simplified the fling-step motion to DI for treating the critical responses of elastic-plastic structures (Figure 1). The ground acceleration of DI with the input velocity amplitude \( V \) and the time interval \( t_0 \) is given by

\[
\ddot{u}_g(t) = V\delta(t) - V\delta(t-t_0),
\]

where \( \delta(t) \) is the Dirac delta function. Each impulse input causes the instantaneous change of the velocity response and only the free vibration occurs after each impulse input. Since the critical condition on the time interval is easily realized as explained in Sections 2.2, 2.3, the critical maximum responses are derived straightforwardly for elastic-plastic SDOF models based on the energy balance law.

2.2 Critical condition on time interval of DI

Akehashi and Takewaki derived the impulse timing which maximizes the total input energy by the second impulse to elastic-plastic MDOF models. This timing is called the ‘critical timing’, and the critical timing is briefly explained here since the maximization of the total input energy by the second impulse plays a key role for the optimal design of viscous dampers for elastic-plastic moment frames.

Consider a \( N \)-story shear mass model. Let \( \mathbf{u} = (u_1, \ldots, u_N)^T \), \( \mathbf{M} = \text{diag}(m_1, \ldots, m_N) \), \( \mathbf{c} \), \( f_i \) denote the horizontal displacements of all the floors, the mass matrix with respect to \( \mathbf{u} \), the damping coefficient, and the restoring force in the \( i \)-th story. The input energies \( E_1, E_2 \) by the first and second impulses are expressed by

![FIGURE 1. Double impulse and corresponding one-cycle sine wave](image-url)
A self-archived copy in Kyoto University Research Information Repository
https://repository.kulib.kyoto-u.ac.jp

FIGURE 2. Change of velocity response by impulse inputs

\[
\begin{align*}
E_1 &= \frac{1}{2} (m_1 + \ldots + m_{N_f}) V^2 \\
E_2 &= \frac{1}{2} (\mathbf{u} + V \mathbf{l})^T \mathbf{M} (\mathbf{u} + V \mathbf{l}) - \frac{1}{2} \mathbf{u}^T \mathbf{M} \mathbf{u} \\
&= V (m_1, \ldots, m_{N_f}) \mathbf{u} + \frac{1}{2} (m_1 + \ldots + m_{N_f}) V^2
\end{align*}
\]

where \( \mathbf{1} = (1, \ldots, 1)^T \). The maximization of the total input energy by the second impulse is equivalent to the maximization of \( E_2 \). It is clear that \( E_2 \) is maximized when \((m_1, \ldots, m_{N_f}) \mathbf{u}\) is maximized. Therefore, the critical condition is expressed by \((m_1, \ldots, m_{N_f}) \mathbf{u} = -(f_1 + c_1 u_1) = 0 \) (Figure 2). It is noted that the critical condition is common in the case of moment frames because each impulse input causes the instantaneous change of the horizontal velocity responses, but not the change of the vertical and rotational velocity responses. In other words, since the input direction of DI is horizontal, the vertical and rotational velocity responses of the nodes do not change just after the impulse input.

2.3 Time-history response analysis under critical DI

Consider first the time-history response analysis under a single impulse input \( u_i = V \delta(t) \). In the analysis, the impulse input is not treated as the ground acceleration. Instead, the change of the state just before and after the impulse input is treated, but not the moment the impulse act. The impulse input causes the instantaneous change \(-V \mathbf{l}\) of the horizontal velocity responses, and this leads to the change \(-\mathbf{M}^{-1} \mathbf{C}(V \mathbf{l})\) of the relative acceleration responses, where \( \mathbf{C} \) denotes the damping matrix. It is noted that \( \mathbf{C}(V \mathbf{l})\) is the change of the damping force responses by the impulse input. In other words, the instantaneous change \(-V \mathbf{l}\) of the horizontal velocity responses means the instantaneous change of the interstory velocity response and the instantaneous change of the damping force response only in the first story. In the time-history response analysis, the changes of the responses are given and the free vibration analysis is conducted.

Consider next the time-history response analysis under the critical DI. The computer code for the time-history response analysis under the critical DI requires the judgment of the critical condition on the time interval. Only the free vibration analysis (without any ground acceleration) is conducted here again since each impulse input is treated as the instantaneous change of the velocity and acceleration responses. Moreover, the responses before the second impulse input is independent of \( t_0 \). Therefore, \( t_0 \) is not necessary to be set beforehand in the analysis, and the analysis after the second impulse is continuously conducted. In other words, different from the input of ordinary sinusoidal waves, DI does not require the specification of the input frequency before the input of the second impulse. Then, the critical condition is automatically judged at the time of the second impulse and the changes of the velocity and acceleration responses are also automatically provided.

It is noted that an originally made computer code easily treats the change of the responses by the impulse inputs. However, such treatment is difficult for general purpose structural analysis software. In such cases, a substitute ground acceleration such as a triangular wave with short duration is treated instead of the impulse input. Especially in the case of the critical DI, the analysis under the single impulse input is conducted to find the critical \( t_0 \), then the analysis under the DI with fixed \( t_0 \) is conducted.

It is also noted that, in the case of a substitute ground acceleration for DI, the repetition of the convergence calculation is required, depending on \( t_0 \). This is because the large seismic load acts in a short time, and it may greatly increase the nonlinearity of the responses or the magnitude of unbalanced forces. However, in the case of the critical DI, such problem does not occur because both the first and second impulses act where all the restoring forces are almost zero (elastic response range).

2.4 Merits of using critical DI for design of elastic-plastic moment frames with viscous dampers

The merits of using the critical DI for the design of viscous dampers for elastic-plastic moment frames are summarized as follows: (1) the duration is short and the computational load is small for the time-history response analysis, (2) the sensitivity-based optimization is easily applicable because of the simple expression of the input, (3) the rapid convergence design is effective for the responses including the multi-eigenmodes because DI has multiple frequency component, (4) the critical condition on the impulse timing is easily treated for elastic-plastic models throughout the design procedure.

For the purpose of the calculation of the maximum responses under the critical DI, it may be sufficient to set the
duration of the critical DI as about a few times the fundamental natural period of the model. It is also noted that, in the case of the critical DI, both of the total input energy and the instantaneous input energy are simultaneously maximized since the input timing of the second impulse affects $E_2$ but not $E_1$. It is widely known that the total input energy corresponds well to the total damage of the structure, and the instantaneous input energy corresponds well to the maximum response of the structure. The use of the critical timing of the impulses is reasonable to obtain the damper design with high safety.

3. Input Velocity Adjustment of DI Based on Energy Spectrum

In this section, an input velocity adjustment method of the critical DI is presented based on the energy spectra of recorded ground motions. The input velocity is adjusted so that the input energy to the lowest eigenmode under the critical DI is equal to those under the recorded ground motions. This adjustment makes the critical DI work as the ‘active earthquake’, which bounds or maximizes the structural response under the selected motions. It is noted that the proposed method can treat not only pulse-like near-fault ground motions but also ground motions of random nature, although the method proposed by Kojima and Takewaki is not applicable to ground motions of random nature. The latter aims for the extraction and the simplification of pulse-like components in near-fault ground motions. Figure 3 shows the overviews of the proposed method and the method by Kojima and Takewaki.

In Sections 3.1-3.3, the input energies and the maximum interstory drifts under recorded ground motions and the critical DI are investigated for elastic SDOF models, elastic MDOF models, and elastic-plastic MDOF models. In Section 3.4, an efficient design procedure of viscous dampers is presented for elastic-plastic moment frames.

**FIGURE 3.** Adjustment of input velocity of critical double impulse and amplitude of recorded ground motion, (A) proposed method, (B) method by Kojima and Takewaki.
3.1 Input energies to elastic SDOF model under recorded ground motions and critical DI

The critical impulse timing for a damped elastic SDOF model is almost equal to a half of the damped natural period since the relative acceleration response and the displacement response attain zero after the first impulse almost at the same time. The total input energy \( E_{DI} \) by the critical DI to an elastic SDOF model with the damping ratio \( h \) model is expressed by

\[
E_{DI} = E_1 + E_2 = \frac{1}{2}mV^2 + \frac{1}{2}m(V + v_e)^2 - \frac{1}{2}mv_e^2
\]

\[
= \frac{1}{2}mV^2 \left\{ 2 + 2\exp\left(\frac{-\pi h}{\sqrt{1 - h^2}}\right) \right\},
\]

\( (3) \)

where \( m, v_e \) denote the mass and the velocity response just before the second impulse input. It is noted that \( E_{DI} \) is independent of the natural period of the model since the input timing of the second impulse is adjusted depending on the natural period of the model.

Let us define the input energy \( E_{EQ} \) by the ground acceleration \( \ddot{u}_g \) as

\[
E_{EQ} = \int_0^t (-m\ddot{u}_g)dt,
\]

\( (5) \)

where \( \ddot{u} \) denotes the relative velocity response. It is noted that the input energy is sometimes defined as the work \( \int_0^t m(\ddot{u} + \ddot{u}_g)u_g dt \) of the ground on the structural system. When the end time \( t_e \) of the ground acceleration is applied to \( t \), both expressions coincide. The total input energy and the maximum input energy are given by

\[
E_{EQ} = \int_0^{t_e} (-m\ddot{u}_g)dt = \frac{1}{2}mV_e^2,
\]

\( (6) \)

\[
\max t \int_0^t (-m\ddot{u}_g)dt = \frac{1}{2}mV_{E,max}^2.
\]

\( (7) \)

\( V_e \) is the equivalent velocity to the input energy. \(^{27}\) Regarding to the maximum input energy, the similar expression \( V_{E,max} \) can be defined. By equating \( E_{DI} \) and \( E_{EQ} \), the following relation can be obtained.

\[
\frac{V}{V_e} = \left\{ 2 + 2\exp\left(\frac{-\pi h}{\sqrt{1 - h^2}}\right) \right\}^{-\frac{1}{2}}.
\]

\( (8) \)

Equation (8) indicates that \( V/V_e \) depends only on \( h \), not on the natural period. It is noted that the change of the value of the right-hand side in Equation (8) is small compared to the change of \( h \) (for example, the values are 0.5116, 0.5192, 0.5377, 0.5725 when \( h = 0.03, 0.05, 0.1, 0.2 \)).

Figure 4A shows \( V_e, V_{E,max} \) and the velocity response spectrums \( S_V \) under the 5 recorded ground motions in the case of \( h = 0.1 \). \( V \) calculated by Equation (8) for \( V_e \) (evaluated in Equation (6)) is shown in Figure 4B. Figure 4C shows the displacement response spectrums \( S_D \) under each ground motion and the maximum displacement \( d_{max} \) under DI with the adjusted \( V \). The peak ground velocities of the recorded ground motions are adjusted to \( PGV = 0.5[m/s] \). It can be observed that \( d_{max} \) is always larger than the corresponding \( S_D \). This is because the proportion of the instantaneous input energy to the total input energy is large in the case of DI (tend to cause larger deformation). In other words, the total input energy is given to the model in a short time. In the cases of El Centro NS component, Taft EW component and Hachinohe NS component, \( d_{max} \) is much larger than \( S_D \). On the other hand, in the cases of Rinaldi Sta. FN component and Kobe Univ. NS component, \( d_{max} \) is a little larger than \( S_D \). This is because El Centro NS component, Taft EW component, and Hachinohe NS component are the ground motions of random nature, and Rinaldi Sta. FN component and Kobe Univ. NS component are the pulse-like ground motions. The case of \( h = 0.03 \) is treated in Appendix 1.

3.2 Input energies to elastic proportionally damped MDOF model under recorded ground motions and critical DI

Consider next an elastic proportionally damped MDOF model under recorded ground motions and DI. The displacement responses \( u_{DI} \) under DI are expressed by

\[
u_{DI}(t) = \begin{cases} 
\sum_{i=1}^{N} - (\beta_i \phi_i) \frac{V}{\omega_i} e^{-h_0t} \sin \omega_i t 
& (t < t_0) \\
\sum_{i=1}^{N} - (\beta_i \phi_i) \frac{V}{\omega_i} \left( e^{-h_0t} \sin \omega_i t - e^{-h_0(t-t_0)} \sin \omega_i (t-t_0) \right) 
& (t > t_0)
\end{cases}
\]

\( (9) \)

where \( \phi_i, \beta_i, h_i, \omega_i \) denote the \( i \)-th eigenmode, the \( i \)-th participation vector, the \( i \)-th damping ratio, and the \( i \)-th undamped natural circular frequency, and \( \omega_i = \sqrt{1 - h_i^2} \omega_0 \).

The velocity response after the first impulse input can be represented by

\[
u_{DI}(t) = \sum_{i=1}^{N} - (\beta_i \phi_i) \frac{V}{\sqrt{1 - h_i^2}} e^{-h_0t} \cos (\omega_i t + \phi_i)
\]

\( (10) \)

The total input energy \( E_{DI} \) by DI is expressed as

\[
E_{DI} = \frac{1}{2}V^2 I^T M I + \frac{1}{2} (u_{DI}(t_0) + V I) \overline{T} M (\ddot{u}_{DI}(t_0) + V I) - \frac{1}{2} \ddot{u}_{DI}(t_0) M u_{DI}(t_0)
\]

\( (11) \)

From the expansion theorem,

\[
1 = \beta_1 \phi_1 + \ldots + \beta_N \phi_N
\]

\( (12) \)
FIGURE 4. Comparison of responses under recorded ground motions and DI \( (h = 0.1) \), (A) \( V_E, V_{E,max}, S_V \), (B) \( V \) by Equation (8), (C) \( S_D, d_{max} \)
By substituting Equations (10), (12) into Equation (11), Equation (13) can be obtained

$$E_{Di} = \sum_{i=1}^{N} \left\{ \frac{-V^2(\beta_i \varphi_i)^T \mathbf{M}(\beta_i \varphi_i)}{\sqrt{1 - h_i^2}} e^{-h_i \alpha_{10} \cos(\alpha_i t_0 + \phi_i)} + V^2(\beta_i \varphi_i)^T \mathbf{M}(\beta_i \varphi_i) \right\} = \sum_{i=1}^{N} \sum_{j=1}^{N} \mathbf{F}_{D_{ij}}^c,$$

(13)

where $E_{Di}$ denotes the input energy to the $i$-th mode. On the other hand, the total input energy $E_{EQ}$ by the recorded ground acceleration is expressed by

$$E_{EQ} = \int_0^t \left\{ -\mathbf{u}_{EQ}^T \mathbf{M} \mathbf{u}_{EQ} \right\} dt,$$

(14)

where $\mathbf{u}_{EQ}$ denotes the velocity response under the recorded ground acceleration. The decomposition of $\mathbf{u}_{EQ}$ as Equation (15) leads to Equation (16).

$$\mathbf{u}_{EQ} = \mathbf{u}_{EQ,1}(t) \beta_1 \varphi_1 + \ldots + \mathbf{u}_{EQ,N}(t) \beta_N \varphi_N,$$

(15)

$$E_{EQ} = \sum_{i=1}^{N} \left\{ -\int_0^t (\beta_i \varphi_i)^T \mathbf{M}(\beta_i \varphi_i) \mathbf{u}_{EQ,i} \mathbf{u}_{EQ,i} dt \right\} = \sum_{i=1}^{N} \sum_{j=1}^{N} \frac{1}{2} (\beta_i \varphi_i)^T \mathbf{M}(\beta_i \varphi_i) V_E^2(T_i, h_i),$$

(16)

where $E_{EQ,i}$ denotes the input energy to the $i$-th mode. By equating $E_{Di,1}$ and $E_{EQ,i}$, the following relation is obtained.

$$\frac{1}{2} (\beta_i \varphi_i)^T \mathbf{M}(\beta_i \varphi_i) V_E^2(T_i, h_i) = -\frac{V^2(\beta_i \varphi_i)^T \mathbf{M}(\beta_i \varphi_i)}{\sqrt{1 - h_i^2}} e^{-h_i \alpha_{10} \cos(\alpha_i t_0 + \phi_i)} + V^2(\beta_i \varphi_i)^T \mathbf{M}(\beta_i \varphi_i).$$

(17a)

Further simplification of Equation (17a) leads to

$$\frac{V}{V_E(T_i, h_i)} = \left\{ 2 - \frac{2}{\sqrt{1 - h_i^2}} e^{-h_i \alpha_{10} \cos(\alpha_i t_0 + \phi_i)} \right\}^{-\frac{1}{2}}. \quad (17b)$$

Since the elastic proportionally damped MDOF model is treated, the critical timing only for the lowest mode is common to the case of SDOF models. By substituting $t_0 = \pi/\alpha_1$, the following relation can be obtained.

$$\frac{V}{V_E(T_i, h_i)} = \left\{ 2 + 2 \exp \left( \frac{-\pi h_i}{\sqrt{1 - h_i^2}} \right) \right\}^{-\frac{1}{2}}. \quad (18)$$

It is understood that Equation (18) for MDOF models is equivalent to Equation (8) for SDOF models.

Figures 5, 6 show the time histories of the input energies and the distributions of the maximum interstory drifts under El Centro NS component, Rinaldi Station FN component, and the critical DI. The dotted lines in the figures correspond to the lowest mode responses. A proportionally damped 12-story shear mass model is used ($h_i = 0.01, 0.05, 0.1$). All the floor masses have the same value ($m_i = 400 \times 10^3$ [kg]). The undamped fundamental natural period of these models is 1.2[s]. The common story height is 4 [m]. The peak ground velocities of these recorded motions are adjusted to $PGV = 0.5$ [m/s], and $V$ is calculated by Equation (18), where $V_E(T_i, h_i)$ is evaluated by Equation (16), for each case. It is noted that the critical timing only for the lowest mode slightly differs from that for all the modes, which is defined in Section 2.2. The latter is adopted here. It can be observed from Figures 5, 6 that the input energy to the lowest mode under the critical DI almost corresponds to that under each motion.

In the cases of El Centro NS component, the increase of $h_i$ decreases the total input energy. On the other hand, in the cases of the Rinaldi Station FN component, the increase of $h_i$ increases the total input energy. It is noted that the total input energy becomes much smaller than the maximum input energy as in the case of the Rinaldi Station FN component with a small $h_i$. In such cases, the mechanical energy of the model is kept high for relatively long time due to the small damping. Moreover, the energy transfer function $F(\omega)$ gives another explanation of the difference between these motions. The $F(\omega)$ is equivalent to the real part of the velocity response transfer function $H_v$, and the input energy per unit mass can be expressed as $\int_0^\infty F(\omega) |U_v(\omega)|^2 d\omega$. Figure 7A,B show the Fourier amplitudes of the ground motions and $F(\omega)$ for the lowest mode of the model. The increase of $h_i$ decreases the peak value of $F(\omega)$. However, the increase of $h_i$ enlarges the non-peak values around the peak. When the peak of $|U_v(\omega)|$ does not correspond to that of $F(\omega)$, the input energy may increase due to the non-peak values of $F(\omega)$.

It is noted that, in the cases of $h_i = 0.01, 0.05$, the higher mode response greatly affects the maximum interstory drifts under El Centro NS component and the critical DI. This is because El Centro NS component and DI have relatively large components in the high-frequency range (see Figure 7A,C). Especially, the critical DI possesses a multi-modal property (every eigenmode is equally excited) due to its impulsive nature. Since recorded ground motions usually do not have such multi-modal property, the critical DI excites the higher mode response more greatly than El Centro NS component (see Figures 5A, 7A,C). The contribution of higher mode responses can be checked by subtracting the fundamental-mode response (dotted lines in Figure 5A) from the total response (solid lines in Figure 5A). In the cases of $h_i = 0.1$, the lowest mode almost contributes to the maximum interstory drifts under El Centro NS component and the critical DI because the higher mode responses are effectively reduced by the sufficient damping. On the other hand, all in the cases of $h_i = 0.01, 0.05, 0.1$, the lowest mode almost contributes to the maximum interstory drifts under the Rinaldi Station FN component. This is because the Rinaldi Station FN component has relatively small components in the high-frequency range. It is pointed out that the maximum interstory drifts under El Centro NS component are almost twice as small as those under the corresponding DI. However, the maximum interstory drifts under the Rinaldi Station FN component are slightly smaller than those under the corresponding DI. In the cases of DI and...
FIGURE 5. Comparison of responses under El Centro NS component and DI with $V$ calculated by Equation (18), (A) distribution of maximum interstory drifts, (B) time history of energy response

FIGURE 6. Comparison of responses under Rinaldi Sta. FN component and DI with $V$ calculated by Equation (18), (A) distribution of maximum interstory drifts, (B) time history of energy response
Rinaldi Station FN component, the total input energy is given in a short time. On the other hand, in the case of El Centro NS component, the total input energy is given in a relatively long time because it is the long duration ground motion of random nature. In other words, when the proportion of the instantaneous input energy to the total input energy is high, the maximum interstory drifts become large.30–32

3.3 Input energies to elastic-plastic MDOF model under recorded ground motions and critical DI

The input energy and the maximum interstory drifts are investigated for an elastic-plastic MDOF model under recorded ground motions and DI. As in the case of Section 3.2, the input velocity $V$ of DI is calculated by Equation (18), where $V_s(T, h)$ is evaluated by Equation (16), and the critical condition of DI is judged according to Section 2.2. The model is common to that in the case of Section 3.2. The common yield interstory drift throughout the stories is $4/150 \, [m]$ and the story shear-interstory drift relation obeys the elastic perfectly plastic rule.

Figures 8, 9 show the time histories of the input energies and the distributions of the maximum interstory drifts under El Centro NS component, Rinaldi Station FN component, and the critical DI. The peak ground velocities of these recorded motions are adjusted to $PGV = 1.0\, [m/s]$ for investigating the safety for ground motions with a rather larger amplitude. The dotted lines in Figures 8B, 9B indicate the time histories of the hysteretic energies in the mainframe. When $h_1$ is small, the maximum interstory drifts under DI do not always surpass those under the recorded motions, and the plastic deformation concentrates to the different stories for each input. This is because each input has different characteristics and Equation (18) does not consider the change of the equivalent fundamental natural period due to the elastic-plastic responses. However, when $h_1$ becomes large, the maximum interstory drifts decrease and the equivalent fundamental natural period gets close to the elastic fundamental natural period. Therefore, the maximum interstory drifts under DI surpass those under the recorded motions. Moreover, the relation between the input energy by DI and that by the recorded motions also gets close to the case of the elastic model in Section 3.2. In other words, when $h_1$ becomes large, the input energy by El Centro NS component almost corresponds to that by DI, and the input energy by the Rinaldi Station FN component is smaller than that by DI.

3.4 Efficient design procedure of viscous dampers for elastic-plastic moment frames by using critical DI as active earthquake

It is easily understood that the use of multiple ground motions for optimal damper design leads to the design with high reliability. However, it greatly increases the total computational load. The use of the ‘active earthquake’, which bounds or maximizes the structural response under the selected motions, saves the computational load.16,20 Although the energy spectrum may help to choose one of the motions as the active earthquake,16 the chosen motion will not always maximize the response. For example, in the cases of Figures 5, 6, the input energy to the model with $h_1 = 0.1$ under the Rinaldi Station FN component is smaller than that under El Centro NS component. However, the maximum interstory drifts under the former surpass those under the latter. The proportion of the instantaneous input energy to the total input energy plays another significant role for the maximization of the responses. To overcome these difficulties, an efficient design procedure of viscous dampers for elastic-plastic moment frames is proposed in the following.
FIGURE 8. Comparison of responses under El Centro NS component and DI with \( V \) calculated by Equation (18), (A) distribution of maximum interstory drifts, (B) time history of energy response

FIGURE 9. Comparison of responses under Rinaldi Sta. FN component and DI with \( V \) calculated by Equation (18), (A) distribution of maximum interstory drifts, (B) time history of energy response
[Efficient design procedure of viscous dampers for elastic-plastic moment frames by using critical DI as active earthquake]

Step 1 Set the ground motions for the design.
Step 2 Calculate $V_c(T_i, h)$ for each motion ($T_i$: the undamped elastic fundamental natural period). $h$ is determined with reference to the targeted value of the sum of the added damping coefficients by the dampers.
Step 3 Calculate the input velocity for each motion by Equation (18). Find the maximum value among the obtained input velocities.
Step 4 Optimize the damper placement under the critical DI.

The proposed design procedure uses the critical DI as the active earthquake to obtain the damper placement. Since only the critical DI is used through the optimization and the duration of the critical DI is short, the optimization is conducted with low computational load. The adjustment of the input velocity of the critical DI makes the input energy by the critical DI largest in those by all the ground motions. As explained in Sections 3.1-3.3, since the proportion of the instantaneous input energy to the total energy is large in the case of DI, the critical DI will be the active earthquake at the end of the optimization. Therefore, the proposed design procedure is expected to efficiently provide the damper design with high accuracy. It is noted that the proposed design procedure cannot treat the cases of much larger input levels accurately. In such cases, the plastic deformation ductility becomes large and the adjustment of the input velocity by Equation (18) is not valid. The detail of the optimization in Step 4 is explained in Sections 4.1, 4.2, and the validity of the procedure is demonstrated through the numerical examples in Section 4.3.

4. Damper Optimization

In Section 4.1, an optimization problem of viscous damper placement is stated for elastic-plastic moment frames under the critical DI. The input velocity of the critical DI is adjusted through the procedure proposed in Section 3.4. In Section 4.2, the solution algorithm is presented. The method by Akehashi and Takewaki is extended so that the critical DI can be treated for the optimization. In Section 4.3, the effectiveness of the proposed method is investigated through numerical examples. The comparison of the maximum interstory drifts under the selected recorded ground motions and those under the critical DI with the adjusted input velocity is also conducted.

4.1 Optimization problem

Consider the following problem of optimal damper placement for $N_F$-story $N_B$-span elastic-plastic moment frames.

[Problem]

Find $c_{add} = (c_{1,1}, \ldots, c_{1,N_B}, \ldots, c_{N_F,1}, \ldots, c_{N_F,N_B})$ so as to minimize $d_{max} = \max_i \{d_{max,i}\} \ (i = 1, \ldots, N_F)$ subject to

$$c_{add}^T \cdot 1 = W_c \ (\text{const.})$$
$$0 \leq c_{ij} \leq c_{ij}^U \ (for \ i = 1, \ldots, N_F, j = 1, \ldots, N_B)$$

where $c_{ij}^U, W_c, d_{max,i}$ denote the upper bound of the damping coefficient added at the $j$-th bay in the $i$-th story, the sum of added damping coefficients, and the maximum deformation of the $i$-th story.

It is noted that the constraint on the sum of added damping coefficients is almost equivalent to the constraint on the total cost of added dampers since the former is almost proportional to the latter (in the case of the use of multiple dampers with fixed size).

In addition, the sum of added damping coefficients is referred to the value of $h$ (damping ratio) and the equivalent input velocity of DI. It is also noted that the constraints on member stress are not included in this optimization problem, although they can be easily included in the cases of small input velocities. In the cases of large input velocities, the nonlinear responses of members are automatically considered in the optimization procedure since plastic-elastic moment frames are treated.

4.2 Algorithm

The optimization method by Akehashi and Takewaki is extended to the case where the critical DI is used for the damper design. The solution algorithm may be described as follows.

[Algorithm]

Step 1 Transform the linear elastic bare frame into the lowest mode equivalent shear mass system.
Step 2 Obtain the optimal damper placement for the elastic shear mass system using an optimization method.
Step 3 Inversely transform the shear mass system with dampers into the elastic frame model with added dampers so as to maximize the approximate value of the lowest mode damping ratio.
Step 4 Input the single triangular wave (STW) into the elastic-plastic moment frame to obtain the critical time interval $t_{cL}$ of DI. Then generate the ground acceleration of the double triangular wave (DTW) with $t_{cL}$.
Step 5 Solve a linear programming problem (as stated below) and update $c_{add} \rightarrow c_{add} + \Delta c_{add}$. If this newly obtained design totally coincides with the other obtained designs, go to Step 7.
Step 6 If $i_{LS} < i_{LS}$, go to Step 7. Otherwise, put $i_{LS} = i_{LS} + 1$ and return to Step 4.
Step 7 Select the design which exhibits the minimum value of $d_{max,i}$ from all the obtained designs and finalize the process.

[Linear programming problem]

$$\text{Find} \ \ \Delta c_{add} = (\Delta c_{1,1}, \ldots, \Delta c_{1,N_B}, \ldots, \Delta c_{N_F,1}, \ldots, \Delta c_{N_F,N_B})$$

so as to minimize $f = d_{max} + \sum_{i=1}^{N_F} \sum_{j=1}^{N_B} \Delta c_{ij}$ subject to

$$\Delta c_{add}^T \cdot 1 = 0$$
$$\Delta c_{ij}^L \leq \Delta c_{ij} \leq \Delta c_{ij}^U \ (for \ i = 1, \ldots, N_F, j = 1, \ldots, N_B)$$
$$c_{ij} + \Delta c_{ij}^L \geq 0 \ (for \ i = 1, \ldots, N_F, j = 1, \ldots, N_B)$$
$$c_{ij} + \Delta c_{ij}^U \leq c_{ij} \ (for \ i = 1, \ldots, N_F, j = 1, \ldots, N_B)$$

where $\Delta c_{ij}^L, \Delta c_{ij}^U$ denote the lower and upper bounds of $\Delta c_{ij}$, $s_{ij}$ denotes the finite difference approximation of the first-order sensitivity $(\partial d_{max,i}/\partial c_{ij})$, and $k = \arg \max_{i} d_{max,i}$. After solving the above-stated linear programming problem, update $c_{add} \rightarrow c_{add} + \Delta c_{add}$. 
Figure 10 shows the flowchart of the proposed solution algorithm. The procedures in Steps 1-3 search an initial design of the damper placement. Then, the local search-based optimization is conducted in Steps 4-7. Since the optimal damper placement obtained for a linear elastic moment frame can be a good initial design for an elastic-plastic moment frame, only linear elastic responses are treated in Steps 1-3. The sensitivity-based optimization for the equivalent shear mass system under the critical DI in Step 2 corresponds to the search of the optimal damper distribution along the building height. The damper inverse transformation in Step 3 determines an efficient distribution of dampers into the different bay so that the approximate value of the lowest mode damping ratio is maximized. This procedure is simply and easily conducted because the lowest mode damping ratio can be evaluated in closed form by using the undamped lowest eigenmode. It is noted that the maximization of the lowest mode damping ratio plays a key role in the minimization of the interstory drifts. Although the numerical sensitivity analysis can search a better distribution of dampers into the different bay more precisely, it requires much computational load, since it requires the repetition of the time-history response analysis. The procedures in Steps 4-7 correspond to the modification of the damper design obtained in Step 3, and elastic-plastic moment frames are treated. The value of $n_{LS}$ should be determined considering the convergence and the total computational load. It is noted that the calculation of $s_{ij}$ for the linear programming problem requires the time-history response analysis for the model with slightly added damping at the $j$-th bay in the $i$-th story. Although all the $s_{ij}$'s are calculated in the numerical examples in Section 4.3, the exclusion of some stories from the calculation of $s_{ij}$ greatly reduces the computational load. For example, the added damping in the upper stories will not be effective for the reduction of the interstory drifts in the lower stories. Such treatment will work well, especially in the case of large-scale models.

It is noted that the proposed algorithm assumes the use of general purpose structural analysis software to implement the time-history response analysis for elastic-plastic moment frames, and DTW is substituted for DI. As explained in Section 2.3, the time-history response analysis under STW is required for finding the critical time interval before the time-history response analysis under DTW. The time interval is kept constant in the calculation of $s_{ij}$ to save the computational load, and after updating the damper distribution, the time-history response analysis under STW is conducted again to obtain the critical time interval. The validity of using STW and DTW in place of the single impulse and DI is demonstrated in Appendix 2.

While most of the existing damper optimization methods (for example, the Ref. [18]) employ two-directional sensitivity-
based optimization procedures (story direction and span direction), the proposed method principally uses a one-directional (only span directional) sensitivity-based optimization procedure with the aid of a shear-mass model for the story-directional optimization without the time-history response analysis for frame models. Although the proposed method also uses a two-directional sensitivity-based optimization procedure at the final stage, the computational load is quite small because of the capturing of a good initial design. This fact indicates that, while the existing method has to conduct the sensitivity analysis for $N_F \times N_B$ parameters, the proposed method principally needs the sensitivity analysis for $N_B$ parameters. This clearly demonstrates the computational efficiency of the proposed method.

4.3 Numerical example

4.3.1 Model
A 10-story, 3-span elastic-plastic moment frame used in the previous paper\textsuperscript{21} is adopted in this paper. The details of the model are shown in the previous paper.\textsuperscript{21} The participation vectors and the natural periods are presented in Figure 11A. It is noted that the horizontal displacements of the modes in Figure 11B are scaled up. The model parameters are briefly shown in Figure 11C. In addition, the structural damping ratio is set to 0.02 (stiffness proportional type). The linear viscous dampers are treated and the K-type dampers can be installed at all bays in all stories. The structural analysis software OpenSees\textsuperscript{34} is used to conduct the time-history response analysis for the elastic-plastic frame.

4.3.2 Optimization results
The input velocities $V = 0.8, 1.15\, [m/s]$ of DI are used for the optimization. These amplitudes were selected for demonstrating the elastic-plastic properties of the moment frame in detail. Table 1 shows the corresponding peak ground velocities of the 5 selected recorded motions. In other words, they are adjusted so that the energy spectra $V_E$ satisfy Equation (18). It is noted that $h_1 = 0.1$ is used for the calculation of $V_E$ because the sum of the added damping coefficients is set so that the lowest mode damping ratio almost becomes 0.1, as stated below. It is noted that the input velocities $V = 0.8, 1.15\, [m/s]$ of DI were determined so that the peak ground velocities of El Centro NS component, Taft EW component, and Hachinohe NS component become larger than 0.5, 0.75 $[m/s]$.

The sensitivity-based algorithm\textsuperscript{33} is used to optimize the damper placement for the equivalent shear mass model. In the application of the sensitivity-based algorithm, the added damping coefficient $3 \times 10^7$ [Ns/m] is given to each story at first. Then, a small damping coefficient $1 \times 10^6$ [Ns/m] (: decrement of damping coefficient in each step) is sequentially removed.

![Figure 11. Participation vectors (horizontal displacements), natural periods and model parameters](image)

1. Participation vectors and natural periods, (A) participation vectors and natural periods,
2. modes, (C) model parameters

- $T_1 = 1.29\, [s]$
- $T_2 = 0.447\, [s]$
- $T_3 = 0.248\, [s]$
- $T_4 = 0.172\, [s]$

**floor mass: $100 \times 10^3$ [kg]**

- $W21 \times 201, \sigma_y = 320$ [N/mm$^2$]
- $W21 \times 182, \sigma_y = 320$ [N/mm$^2$]
- $W33 \times 130, \sigma_y = 240$ [N/mm$^2$]
- $W30 \times 99, \sigma_y = 240$ [N/mm$^2$]
until the lowest mode damping ratio becomes almost 0.1. The obtained damper placement for the shear mass model is inversely transformed into the damper placement of the moment frame. It is noted here that \( W_c = \frac{c_{\text{add}}}{C_1} \).

Figure 12A illustrates the distributions of the maximum interstory drifts of the shear mass system, the moment frame with the inversely transformed added dampers, and the bare frame under the critical DI with \( V = 0.2 \,[\text{m/s}] \). \( V = 0.2 \,[\text{m/s}] \) is adopted to compare the linear elastic responses of these models. The distribution of inversely transformed added dampers for the moment frame is also shown. All the added dampers are allocated to the center bays through the inverse transformation because the constraints on the upper bound of \( c_{ij} \) are not considered and the elongation of the columns adjacent to the center bay is smaller than that of the outer columns (higher horizontal resistance). The distribution of the maximum interstory drifts of the shear mass system with dampers almost coincides with that of the inversely transformed frame with dampers. It can also be observed that the proposed method effectively reduces the maximum interstory drifts. Moreover, all the interstory drifts are almost uniform except for the stories without dampers, and those stories’ interstory drifts are smaller than the interstory drifts of the stories with dampers. It is pointed out that the stiffness of the 1st story is large, since the column

### TABLE 1. Peak ground velocities of recorded ground motions

<table>
<thead>
<tr>
<th>Name of ground motion</th>
<th>( V = 0.8 ,[\text{m/s}] )</th>
<th>( V = 1.15 ,[\text{m/s}] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Imperial Valley 1940 El Centro NS</td>
<td>PGV = 0.564 ,[\text{m/s}]</td>
<td>PGV = 0.810 ,[\text{m/s}]</td>
</tr>
<tr>
<td>Kern County 1952 Taft EW NS</td>
<td>PGV = 0.522 ,[\text{m/s}]</td>
<td>PGV = 0.750 ,[\text{m/s}]</td>
</tr>
<tr>
<td>Tokachi-oki 1968 Hachinohe NS</td>
<td>PGV = 0.612 ,[\text{m/s}]</td>
<td>PGV = 0.880 ,[\text{m/s}]</td>
</tr>
<tr>
<td>Northridge 1994 Rinaldi Sta FN</td>
<td>PGV = 0.709 ,[\text{m/s}]</td>
<td>PGV = 1.02 ,[\text{m/s}]</td>
</tr>
<tr>
<td>Hyogoken-Nanbu 1995 Kobe Univ NS</td>
<td>PGV = 0.368 ,[\text{m/s}]</td>
<td>PGV = 0.529 ,[\text{m/s}]</td>
</tr>
</tbody>
</table>

**FIGURE 12.** Distribution of maximum interstory drift ratios and distribution of added damping coefficients, (A) moment frame with inversely transformed added dampers (initial design), (B) design for \( V = 0.8 \,[\text{m/s}] \), (C) design for \( V = 1.15 \,[\text{m/s}] \).
In the case of \( V = 0.8 [\text{m/s}] \), plastic hinges occur at the beams and the column bases in the 1st story.

The authors have tackled the viscous damper optimization under El Centro NS component for the same model. The settings of the structural parameters and the damper parameters are common in both cases, and only the input ground motions are different. Although the values of the maximum interstory drifts under El Centro NS component are close to those under the critical DI, the critical DI provides more damping to the side bays in the lower stories than El Centro NS component. The duration of DI is short and the interstory drifts are rapidly enlarged under DI. In such case, the allocation of the dampers to a single bay in the same floor may increase the member force responses and the maximum interstory drifts.

As a comparison with an existing damper optimization method, the method using the transfer function amplitude was used. Under the same total damper amount condition, the method using the transfer function amplitude leads to a similar story-direction distribution of dampers. However, the dampers were concentrated to the mid-span while the proposed method provided some damper allocations to the outer spans. This may result from the fact that, while the method using the transfer function amplitude deals with only the lowest mode and the influence of outer-column elongation is remarkable, the proposed method using DI treats also higher mode effects with a slight influence of outer-column elongation.

Figure 14 shows the distributions of the maximum interstory drifts under the recorded ground motions whose peak ground velocities are adjusted according to Table 1. It can be observed that the maximum interstory drifts under the critical DI surpass those under the ground motions.

Finally, it can be concluded that the use of the critical DI efficiently provides effective designs for elastic-plastic moment frames with viscous dampers.

5. Conclusions

An input velocity adjustment method of the critical double impulse (DI) was presented for the efficient design of viscous
dampers for elastic-plastic moment frames. The main conclusions can be summarized as follows.

1. An input velocity adjustment method of the critical DI was presented based on the energy spectra of recorded ground motions. The input velocity is adjusted so that the input energy to the lowest eigenmode under the critical DI is equal to those under the recorded ground motions. This adjustment makes the critical DI work as the active earthquake, which bounds the structural responses under the selected ground motions. The response bounding property of the critical DI can be explained by (1) the multi-modal property of the critical DI, (2) the lowest mode response excitation of recorded ground motions, (3) the larger proportion of the instantaneous input energy to the total input energy. The proposed method can treat not only pulse-like near-fault ground motions but also ground motions of random nature.

2. The input energies and the maximum interstory drifts under the recorded ground motions and the critical DI with the adjusted input velocity were investigated for elastic SDOF models, elastic proportionally damped MDOF models, and elastic-proportionally damped MDOF models. In the cases of elastic SDOF models and elastic proportionally damped MDOF models, the maximum interstory drifts under the critical DI are much larger than those under the ground motions of random nature, and the maximum interstory drifts under the critical DI are slightly larger than those under the pulse-like motions. This tendency is common for elastic-proportionally damped MDOF models for the large damping ratio. Both of the instantaneous input energy and the total input energy affect the maximum responses.

3. The optimal damper design method by Akehashi and Takewaki\(^2\) for a recorded ground motion was extended to the case under the critical DI. The use of the critical DI efficiently provides effective designs for elastic-plastic moment frames with viscous dampers. The merits of using the critical DI for the design of viscous dampers for elastic-plastic moment frames were summarized as follows: (1) the short duration enables the small computational load for the time-history response analysis, (2) the simple input motion enables the smart sensitivity-based optimization, (3) the use of multi-modal DI leads to a damper design effective for the responses including the multi-modes, (4) the total input energy and the instantaneous input energy are easily maximized at the same time for elastic-plastic models.

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### Disclosure

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Appendix 1. Comparison of responses of elastic SDOF model with small damping under recorded ground motions and DI

Figure A1 shows $V_E$, $V_{E,max}$, $S_V$, and $S_D$ under the 5 recorded ground motions in the case of $h = 0.03$. The maximum displacement $d_{max}$ under DI with the adjusted $V$ is also plotted. It can be observed that $d_{max}$ is almost always larger than the corresponding $S_D$. It can be also observed that $S_D$ occasionally becomes larger than $d_{max}$ when $V_{E,max}$ is larger than $V_E$. It is widely known that the correspondence of $V_E$ and $V_{E,max}$ becomes better with the increase of $h$. In the case of small $h$,
the use of $V_{E,max}$ for calculating $V$ in place of $V_E$ provides the bounds of $S_D$.

Appendix 2. Validity of using STW and DTW in place of single impulse and DI

Figure A2 shows the time histories of the displacement responses for an elastic SDOF model with the natural period $T$.
and the damping ratio $h = 0.01$ under a single impulse $\ddot{u}_e = V\delta(t)$ and the corresponding STW. $t_{STW}$ in Figure A2 denotes the duration of STW, and the peak acceleration of STW is set to $2V/t_{STW}$. It can be observed that the displacement response under the single impulse corresponds well to those under STW with a small $t_{STW}/T$ as shown in the figure. It is noted that STW and DTW with $t_{STW}/T = 0.02/T_1 = 0.0155$ are adopted in Section 4.