Journal of Risk **24**(5), 51–73
DOI: 10.21314/JOR.2022.032

Research Paper

A new approach to detecting change in credit quality

Rusudan Kevkhishvili

Graduate School of Economics, Kyoto University, Yoshida-honmachi, Sakyo-ku, Kyoto 606-8501, Japan; email: kevkhishvili.rusudan.2x@kyoto-u.ac.jp

(Received January 13, 2021; revised February 21, 2022; accepted April 19, 2022)

ABSTRACT

This paper provides a new framework for the detection and quantification of change in the corporate credit quality of established companies. These companies are important players in the economy, and analysis of their creditworthiness is of great interest to investors. Our approach is based on the observation that the deterioration of credit quality in established companies is a long-term process, because these firms use their resources as well as strong customer and supplier relationships to respond to market and technological changes. Based solely on the realized path of the stochastic process that represents creditworthiness, we propose an economically plausible endogenous mechanism that governs the shift in the long-term mean of the process. Specifically, we construct a model that detects a change in the mean-reversion level of an Ornstein–Uhlenbeck process representing the company's leverage. Our model captures this change based on a number of realized upcrossings of a certain level (estimated from data) by the process itself, without introducing an additional source of uncertainty. Our approach is computationally simple and provides an efficient

Print ISSN 1465-1211 | Online ISSN 1755-2842

tool for monitoring changes in quantities related to credit quality, such as default probability.

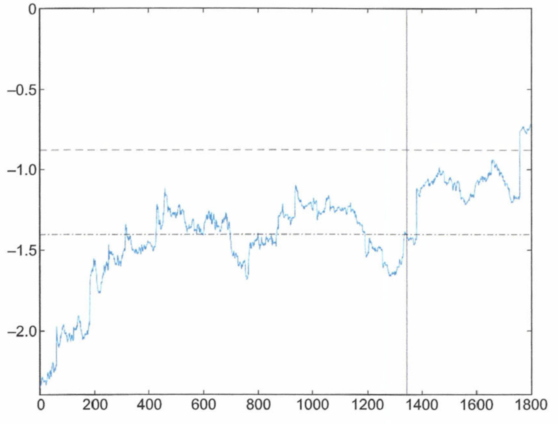
Keywords: credit quality; credit risk; detection; Ornstein–Uhlenbeck; mean reversion; long-term mean.

1 INTRODUCTION

In this paper we focus on the credit quality deterioration of established companies and present a new, explicit and computationally simple approach for the detection and quantification of change in credit quality. Well-known examples of such companies are constituents of the Dow Jones Industrial Average index. Such firms are important players in the economy, and analysis of their credit quality is of great interest to investors. Our approach is based on the observation that credit quality deterioration of established companies is a long-term process. This is because such firms use their resources as well as strong customer and supplier relationships to respond to market and technological changes. Established companies will fail when they use up all their resources and cannot meet market and technological demands anymore. This is the main reasoning behind our model. A possible cause for such failure is the inability to respond to disruptive technological innovations. In contrast to sustaining innovations (improvements of established products), disruptive innovations underperform incumbent products but they offer novel features and may eventually take away the customers of well-established businesses (Christensen 1997).

In our approach we analyze creditworthiness based on a firm's leverage ratio, defined as the ratio of financial debt to capital. We call the process representing the dynamics of the logarithm of this ratio the "leverage process", and we model it using an Ornstein–Uhlenbeck (OU) diffusion. In this way the leverage process is mean-reverting, and it possesses a restoring force directed toward some constant long-term mean level α. By introducing a switching mechanism in this mean level, we analyze the long-term process of a company's response to market and technological changes. Unlike the position of the process, which changes every day, the long-term mean is a stable parameter, and for this reason we focus on this parameter when discussing credit quality. In our model the shift in the long-term mean represents a change in credit quality and this change occurs endogenously. In contrast to models where a switch in the parameters occurs based on an unobservable process (such as Markov-switching models), we do not introduce an additional source of uncertainty. Specifically, the switch in our model is triggered when the leverage process upcrosses α from below a certain number of times. The long-term mean level α and the threshold for the upcrossing number that triggers a switch in the parameter are both unknown and are to be estimated from the time series of the leverage ratio.

FIGURE 1 Estimation result for the Eastman Kodak Company (Kodak) based on the daily data (January 3, 2000–March 6, 2007) of the leverage process.



The horizontal axis shows the trading days from January 3, 2000 to March 6, 2007, the vertical axis shows the values of the leverage process. The fluctuating blue line plots the daily data points. Other lines are explained in the text.

Using Figure 1 we emphasize three estimates that are obtained from our model: long-term mean levels (dashed lines) before and after the switch and the detected switching point in the data (vertical line). In addition, using the data after the switch we can check the mean-reversion speed of the data toward the pre-switch mean level and verify the significance of the switch. We next illustrate the reasoning behind our endogenous switching mechanism.

An increase in leverage raises the probability of insolvency. Therefore, the upcrossing of some sufficiently high-level α can be interpreted as the entrance into a relatively higher credit risk area. This means that the existing operation style of the company is not efficient anymore. The company will then adopt different strategies to improve its business conditions. For example, the company may try to use existing resources more productively to improve business operations or it may start exploring new opportunities in new markets by reallocating its resources. The increase in the number of upcrossings of the level α indicates that there is increasingly less room for improvement and the probability of comeback decreases. This is the reason why we use the number of upcrossings as the triggering threshold for a change in the dynamics of the leverage process.

Using the endogenous switching mechanism described above, our model enables us to detect and quantify the change in credit quality. This quantification is achieved by comparing the long-term mean levels before and after the switch. Our model is computationally simple, and we demonstrate its effectiveness by analyzing both defaulted and surviving companies that have a long business history.

1.1 Literature review

To the best of our knowledge this study is the first to analyze the change in credit quality with an endogenous switching mechanism for the long-term mean of the leverage process. We point out that our model is different from the mean-reverting model proposed in Collin-Dufresne and Goldstein (2001). Their study assumes that firms adjust debt levels in response to changes in the firm value, therefore adjusting the leverage ratio toward a target level. The focus of our study, in contrast, is to detect deteriorated credit quality, and it is natural to think that firms in financial distress will not be able to adjust debt as desired. We use the mean-reverting process to detect the shift in the long-term mean, and we do not impose the assumption of firms having target leverage ratios. For studies regarding the stability of the capital structure we refer the reader to Hovakimian *et al* (2001), DeAngelo and Roll (2015) and Bontempi *et al* (2020).

Regime-switching models are standard tools for detecting changes in the dynamics of financial time series. The switch in the model parameters can be modeled by using an unobservable process, as in Markov-switching models, or by using an observable process, as in threshold models with an unknown constant threshold. We refer the reader to Franses and van Dijk (2000, Chapter 3) for an overview of these approaches. The former method is different from our approach because it does not provide an explicit mechanism for the switch based on the observable time series. Our model belongs to threshold models of the latter type, which are explained in detail in Tong (1983). Common examples of such models are threshold autoregressive (TAR), self-exciting TAR and threshold generalized autoregressive conditional heteroscedasticity models, which are discussed in Tong (2011) and Tsay (2010). To the best of our knowledge there are no threshold models that analyze the shift in the mean-reversion level of the leverage process. Examples of credit-risk-related studies using threshold models are Nunes and Rodrigues (2011), which uses a TAR model to study the sensitivity of credit risk to macroeconomic conditions, and Marcucci and Quagliariello (2009), which uses a threshold regression model to analyze the relationship between credit risk and business cycles. In contrast to our approach, the switch in the threshold models used in the literature is usually triggered according to the position of the process at some predetermined point in time. By using the number of upcrossings as a trigger for the change in the mean-reversion level, our model accounts for the long-term trend of the leverage process and therefore incorporates more information in the switching mechanism.

The rest of the paper is organized as follows. Section 2 presents the methodology. Specifically, Section 2.1 introduces the new model and Section 2.2 shows how to estimate the model parameters from data. In Section 2.3 we present a method for verifying the detected switch and in Section 2.4 we show a method for measuring the change in quantities related to credit quality. Section 3 explains the type of data used for parameter estimation and Section 4 discusses our estimation results. Finally, Section 5 states our conclusions.

2 METHODOLOGY

2.1 Model

Let $(\Omega, \mathcal{H}, \mathbb{P})$ denote the usual probability space. All the stochastic processes in the following are defined on this space. We assume that $B = \{B_t;\ 0 \leqslant t < \infty\}$ is a standard Brownian motion with respect to the filtration $\mathcal{F} = \{\mathcal{F}_t;\ 0 \leqslant t < \infty\}$. We let $X = \{X_t;\ 0 \leqslant t < \infty\}$ denote the leverage process describing the dynamics of the natural logarithm of the leverage ratio. We model X using an OU diffusion with endogenously switching mean-reversion level, which represents a long-term mean of the process.

When considering an OU process in continuous time, we need to deal with rapid oscillations, as shown in Cox and Miller (2001, Example 5.4). Specifically, if an OU process Z hits some level z at some time t, then it will hit the level z infinitely often in the time interval $(t, t+\varepsilon)$ for any $\varepsilon > 0$. The proof in this example deals with the case when the mean-reversion level is 0; however, it is easy to see that the result applies to a general case of Z with the mean-reversion level $\alpha \in \mathbb{R}$ by considering the process $\alpha - Z$. To define the number of upcrossings of some fixed level $\alpha \in \mathbb{R}$ by the process X until time $t \in \mathbb{R}_+$ (denoted by $U_t(\alpha)$) in a way that is relevant from the perspective of credit risk analysis, we base our definition on a finite subset of $[0, t]$. This removes the problem of rapid oscillations and makes $U_t(\alpha)$ an integer-valued random variable. We give the formal definition of $U_t(\alpha)$ after the presentation of the model. Our model for the leverage process X is given by

$$\mathrm{d}X_t = \kappa(\beta_t - X_t)\,\mathrm{d}t + \sigma\,\mathrm{d}B_t, \quad \beta_t = \begin{cases} \alpha & \text{if } U_t(\alpha) < N, \\ \alpha' & \text{if } U_t(\alpha) \geqslant N, \end{cases} \quad X_0 = x_0. \tag{2.1}$$

It is possible to assume that the parameter κ (the speed of mean reversion) also changes when there is a shift in α; however, to keep the model simple and to not increase the number of parameters, we do not include a switch in κ, focusing instead on the switch in the mean-reversion level.

The parameters $\kappa > 0, \alpha, \alpha', \sigma > 0$ and $N > 0$ are unknown constants and are to be estimated from data. With the exception that the mean-reversion level β_t changes from α to α' after the number of upcrossings of α exceeds N, (2.1) represents the stochastic differential equation of the OU process. We note that, even though the number of upcrossings can only be an integer, we regard the parameter N as a real number, as this does not make any difference.

The only source of uncertainty in (2.1) is the Brownian motion B, and, as will be shown below, the number of upcrossings $U_t(\alpha)$ is $\mathcal{F}_t$-measurable. The shift in the mean-reversion level takes place at a stopping time $S^\alpha := \inf\{t \geqslant 0 \colon U_t(\alpha) \geqslant N\}$. The mechanism in (2.1) enables us to detect change in credit quality by observing the shift in the mean-reversion level of the leverage process. This is accomplished by analyzing only the path of the process (that is, without introducing an additional source of uncertainty). As described in Section 1, detection of any change in credit quality of established companies requires an analysis of the long-term trend. This is because such companies are engaged in the long-term process of responding to market and technological changes. When the leverage process upcrosses some sufficiently high α, the probability of insolvency increases. As the number of upcrossings of the level α increases, it is natural to think that the possibility for improvement has decreased, and the probability of the leverage ratio returning to a lower level goes down. This will trigger a shift in the long-term mean as proposed in (2.1).

Here we formally define the number of upcrossings $U_t(\alpha)$ in the model (2.1). Our formulation is partly based on the definition of the number of upcrossings of a nonzero interval in Karatzas and Shreve (1998, Section 1.3). To deal with the rapid oscillations discussed at the beginning of this section we consider finite sets. This removes the problem of the infinite number of upcrossings and allows us to define upcrossings in a way that is relevant from the perspective of credit risk analysis. First, we let F_t denote a finite subset of $[0, t]$ such that $F_s \subset F_t$ whenever $s \leqslant t$. Then, we define random times τ_t^j for $j = 1, 2, \dots$ recursively as follows:

$$\tau_t^1(\omega) = \min\{u \in F_t : X_u(\omega) < \alpha\}$$

$$\tau_t^j(\omega) = \begin{cases} \min\{u \in F_t \colon u > \tau_t^{j-1},\ X_u(\omega) \geqslant \alpha\} & \text{if } j(> 1) \text{ is even,} \\ \min\{u \in F_t \colon u > \tau_t^{j-1},\ X_u(\omega) < \alpha\} & \text{if } j(> 1) \text{ is odd.} \end{cases}$$

We adopt the convention that $\min \emptyset = +\infty$. The random times τ_t^j for even j represent upcrossing times of the level α by the restricted sample path $\{X_u; u \in F_t\}$. Using this observation, the number of upcrossings of the level α by $\{X_u \colon u \in F_t\}$ is given by

$$U_{F_t}(\alpha) = \frac{\text{largest even integer } j \text{ such that } \tau_t^j < +\infty}{2}.$$

If there is no even integer j such that $\tau_t^j < +\infty$, we set $U_{F_t}(\alpha) = 0$. Finally, we define

$$U_t(\alpha) := U_{F_t}(\alpha). \tag{2.2}$$

It is obvious that $U_t(\alpha)$ is $\mathcal{F}_t$-measurable. As F_t includes only a finite number of elements, $U_t(\alpha) < +\infty$. Moreover, $U_s(\alpha) \leqslant U_t(\alpha)$ whenever $s \leqslant t$.

REMARK 2.1 As the finite set F_t is arbitrary, the definition (2.2) is general and can easily be applied to real-world data by specifying the set F_t appropriately. This is illustrated in the next section.

2.2 Estimation procedure

We use discrete time series data of the leverage ratio to estimate five unknown parameters in (2.1). Let the constant $\Delta_t > 0$ denote the time interval between the consecutive data points. For each time $s \in \mathbb{R}_+$, let n_s denote the largest integer such that $n_s \Delta_t \leqslant s$. To apply definition (2.2) to financial time series, we set $F_s = \{0, \Delta_t, \dots, n_s \Delta_t\}$ for each time $s \in \mathbb{R}_+$.

Set $T = \{0, \Delta_t, 2\Delta_t, \dots\}$. Consider the process $\{X_t; t \in T\}$, which is a discretized version of X. By setting $T_t = \{0, \Delta_t, 2\Delta_t, \dots, t\}$ for $t \in T$, we see that the number of upcrossings of α by $\{X_t; t \in T\}$ until time $t \in T$ is given by $U_{T_t}(\alpha)$ (defined in the same way as U_{F_t} for finite F_t). We consider the discretized version of (2.1):

$$\left.\begin{aligned} X_{t+\Delta_t} - X_t &= \kappa(\beta_t - X_t)\Delta_t + \sigma(B_{t+\Delta_t} - B_t), \\ \beta_t &= \alpha \mathbf{1}_{\{U_{T_t}(\alpha) < N\}} + \alpha' \mathbf{1}_{\{U_{T_t}(\alpha) \geqslant N\}}, \quad t \in T, \\ X_0 &= x_0. \end{aligned}\right\} \tag{2.3}$$

The random variable $X_{t+\Delta_t}$ conditioned on $\mathcal{F}_t$ follows a normal distribution, and its density function is given by

$$\begin{aligned} f(x_{t+\Delta_t} \mid \mathcal{F}_t) &= \frac{1}{\sqrt{2\pi\sigma^2\Delta_t}} \\ &\quad \times \exp\left(-\frac{(x_{t+\Delta_t} - x_t - \kappa(\alpha \mathbf{1}_{\{U_{T_t}(\alpha) < N\}} + \alpha' \mathbf{1}_{\{U_{T_t}(\alpha) \geqslant N\}} - x_t)\Delta_t)^2}{2\sigma^2\Delta_t}\right), \end{aligned}$$

where $(x_0, x_{\Delta_t}, \dots, x_t)$ are known values of the process based on the information accumulated up to time t. Note that the set $\{U_t(\alpha) < N\}$ is measurable with respect to $\mathcal{F}_t$ and is determined by the path of X until time t. We obtain the loglikelihood function $\mathcal{L}$ as the product of conditional densities. Given that $(x_0, x_{\Delta_t}, \dots, x_{n\Delta_t})$ is the realization of $(X_0, X_{\Delta_t}, \dots, X_{n\Delta_t})$, we have

$$\begin{aligned}
\mathcal{L}(\kappa, \alpha, \alpha', \sigma, N \mid x_0, x_{\Delta_t}, \dots, x_{n\Delta_t}) \\
&= \sum_{i=1}^{n} \ln f(x_{i\Delta_t} \mid \mathcal{F}_{(i-1)\Delta_t}) \\
&= -\frac{n}{2} \ln(2\pi\sigma^2 \Delta_t) \\
&\quad - \frac{1}{2\sigma^2 \Delta_t} \Bigg(\sum_{i=1}^{\bar{I}^\alpha} (x_{i\Delta_t} - x_{(i-1)\Delta_t} - \kappa(\alpha - x_{(i-1)\Delta_t})\Delta_t)^2 \\
&\qquad + \sum_{i=\bar{I}^\alpha+1}^{n} (x_{i\Delta_t} - x_{(i-1)\Delta_t} - \kappa(\alpha' - x_{(i-1)\Delta_t})\Delta_t)^2 \Bigg), \qquad (2.4)
\end{aligned}$$

where $\bar{I}^\alpha$ denotes the realized value of I^α given by

$$I^\alpha = \min\{i : U_{i\Delta_t}(\alpha) \geqslant N\}. \qquad (2.5)$$

The unknown parameters in (2.1) can be estimated by numerically maximizing $\mathcal{L}$ in (2.4). Note that for any α the set $D^\alpha = \{X_{j\Delta_t} = \alpha \text{ for some } j = 1, \dots, n\}$ has a probability of 0.

It is obvious that the function $\mathcal{L}$ is twice differentiable with respect to κ, α' and σ. Note also that, for each $\omega \notin D^\alpha$, $I^\alpha(\omega) = I^{\alpha+\varepsilon}(\omega)$ for sufficiently small $\varepsilon \in \mathbb{R}$ when $X_0 \neq \alpha$. Take any sequence $(\alpha_m)_{m\geqslant 1}$ converging to α. For $\alpha \notin \{x_{i\Delta_t}; 0 \leqslant i \leqslant n\}$, $\lim_{m\to\infty} \bar{I}^{\alpha_m} = \bar{I}^\alpha$ and $\mathcal{L}$ is twice differentiable with respect to α at such points. For $\alpha \in \{x_{i\Delta_t}; 0 \leqslant i \leqslant n\}$, $\lim_{m\to\infty} \bar{I}^{\alpha_m}$ is not necessarily equal to $\bar{I}^\alpha$; therefore, the possible set of points at which $\mathcal{L}$ is not differentiable with respect to α is the set $\{x_{i\Delta_t}; 0 \leqslant i \leqslant n\}$, which has a zero Lebesgue measure. Similarly, we check the differentiability of $\mathcal{L}$ with respect to N. We point out that the function $\mathcal{L}$ is constant between any two integer values of N (which is a positive real number) given the realization of the data, with all other parameters fixed. Therefore, the derivative of $\mathcal{L}$ with respect to N is 0 outside the set $A = \{1, \dots, n\}$. The set of possible points at which $\mathcal{L}$ is not differentiable with respect to N is the set A, which has a Lebesgue measure of 0.

We use the gradient-based MATLAB function `fmincon` for optimization. Therefore, the estimated value of α satisfies $\alpha \notin \{x_{i\Delta_t}; 0 \leqslant i \leqslant n\}$. Although we cannot exclude the possibility that the true value of α is equal to one of the realized data points, we believe that this is highly unlikely as we are dealing with continuously distributed random variables. The estimated value of N is not an integer, but this does not make any difference because the function $\mathcal{L}$ is left-continuous with respect to N at any integer $N > 0$ and is constant between any two integer values of N.

2.3 Verification of the shift in the long-term mean level

Using the estimated parameters we can compute the realized value of I^{α} in (2.5), which will tell us when the shift in the long-term mean took place in the data. Then, we can split the data into two parts: before and after the switch. We use the data after the switching point and estimate the following model with the long-term mean fixed at the estimated value of $\hat{\alpha}$:

$$\mathrm{d}X'_t = \kappa'(\hat{\alpha} - X'_t)\,\mathrm{d}t + \sigma'\mathrm{d}B_t, \quad t \geqslant 0. \tag{2.6}$$

We consider the discretized process $X'_{t+\Delta_t} - X'_t$ and estimate the unknown parameters $\kappa' > 0$ and $\sigma' > 0$ with the maximum likelihood estimation. We focus on the mean-reversion speed κ'. If the estimated value of κ' is close to 0, we see that the data after the switch does not revert to the pre-switch level $\hat{\alpha}$. This provides strong evidence that the detected switch indeed occurred in the data.

2.4 Occupation time and default probability

The shift in the long-term mean of the leverage process affects both the time the process spends in a relatively higher credit risk region (occupation time) and the distribution of the hitting time of level 0. Level 0 corresponds to the case when the ratio of financial debt to capital equals 1; therefore, we may interpret default as a hitting time of 0. Once we have estimated the model parameters, we will compare the change in the occupation time and the probability of hitting 0 using the mean-reversion levels before and after the switch. We set the parameters to estimated values and consider two processes:

$$\left.\begin{aligned} \mathrm{d}Y_t = \hat{k}(\hat{\alpha} - Y_t)\,\mathrm{d}t + \hat{\sigma}\mathrm{d}B_t, \quad \mathrm{d}Y'_t = \hat{k}(\hat{\alpha}' - Y'_t)\,\mathrm{d}t + \hat{\sigma}\mathrm{d}B_t, \quad t \geqslant 0, \\ Y_0 = Y'_0 = y. \end{aligned}\right\} \tag{2.7}$$

The two processes in (2.7) have the same parameters except for the mean-reversion level: $\hat{\alpha}$ is the level before the switch and $\hat{\alpha}'$ is the level after the switch. As the initial value we use the final data point y of the leverage process used for the parameter estimation. We fix the time horizon as T_m (which we set to five years in our analysis) and calculate the following probabilities by simulation:

$$\left.\begin{aligned} &\mathbb{E}_y\biggl[\int_0^{T_m} \mathbf{1}_{\{Y_t \geqslant \hat{\alpha}'\}}\mathrm{d}t\biggr], \qquad \mathbb{P}_y(\inf\{t : Y_t \geqslant 0\} \leqslant T_m), \\ &\mathbb{E}_y\biggl[\int_0^{T_m} \mathbf{1}_{\{Y'_t \geqslant \hat{\alpha}'\}}\mathrm{d}t\biggr], \qquad \mathbb{P}_y(\inf\{t : Y'_t \geqslant 0\} \leqslant T_m). \end{aligned}\right\} \tag{2.8}$$

In this way, we find the expected occupation time of the process above $\hat{\alpha}'$ until time T_m and the probability that the process hits 0 before time T_m. By comparing these quantities we can see the effect of the shift (in the long-term mean) on the company's credit quality.

3 DATA

The data used in this paper were obtained from Thomson Reuters Eikon. We use the daily time series of the leverage ratio to estimate the unknown parameters in (2.3). We define the leverage ratio as the ratio of the financial debt to the sum of financial debt and market capitalization. We estimate daily debt values using quarterly balance sheets. Specifically, we keep the debt level constant between the quarters, and we use the value of the previous quarter until the new quarter value is available. We use the standardized balance sheets provided by Eikon and compute the financial debt as the sum of notes payable, short-term debt, long-term debt and capital leases.

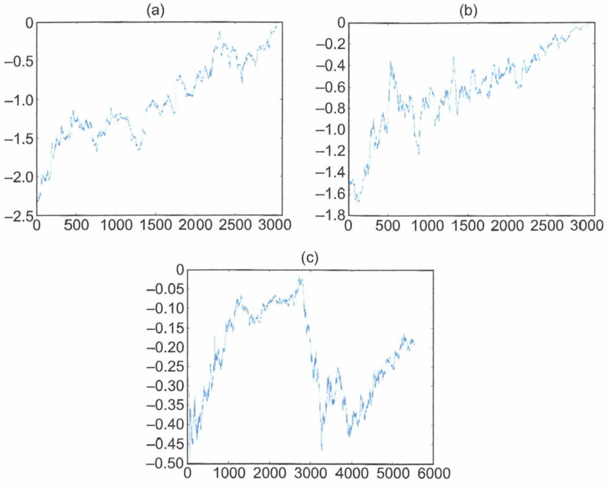
We demonstrate the application of our model for three companies: the Eastman Kodak Company, Sears Holdings Corporation and Ford Motor Company (Kodak, Sears and Ford for short). The daily data (ie, the logarithms of the leverage ratios) are depicted in Figure 2. For each company we analyze around 12 years of data preceding bankruptcy (Kodak and Sears) or the climax of the distress period (Ford).

We refer the reader to Spector *et al* (2012) regarding Kodak's bankruptcy. Kodak filed for bankruptcy protection on January 19, 2012, and we analyze the data from the period from January 3, 2000 to January 17, 2012 for this company. We refer the reader to Rizzo (2018) regarding the bankruptcy of Sears, which was filed on October 15, 2018. For this company we analyze the data from the period from October 2, 2006 to October 12, 2018. In the case of Ford we focus on the increase in the leverage ratio and analyze the data from December 31, 1997 to February 10, 2009, which is the period prior to data point 2790 in Figure 2. Regarding Ford's distress and recovery, we refer the reader to Dolan (2010).

4 ESTIMATION, RESULTS AND DISCUSSION

For each company we conduct 12 estimations. First, we estimate the parameters using data points from 1 to 1800. Then, we move forward in time by 90 days and run the estimation using the data points from 1 to 1890. We continue in this manner until we have used the data points 1–2790. As we are analyzing daily data, we set $\Delta_t = 1/360$ and estimate the parameters in (2.3) and (2.6). We show the estimated values up to four decimal places.

FIGURE 2 The logarithm of the leverage ratios (daily data of Kodak, Sears and Ford).



The horizontal axis denotes the trading days in the following periods: (a) Kodak (January 3, 2000–January 17, 2012); (b) Sears (October 2, 2006–October 12, 2018); (c) Ford (December 31, 1997–December 31, 2019).

For brevity, we provide 12 estimation results for each company in an online appendix to this paper,[1] and we report every other estimation result here. That is, we report the estimation results by moving forward in time by 180 days. In each table we specify the data used in the estimation. In addition to the estimated parameters we report the realized number of upcrossings of the estimated level $\hat{\alpha}$, denoted by $U_{n\Delta_t}(\hat{\alpha})$, based on the sample used for each estimation. We also indicate the point in time (the data point) at which the switch was detected. Finally, we set the time horizon T_m to five years and display the calculated values of the quantities in (2.8). When conducting the simulation we simulate 10 000 paths. As we minimize the negative loglikelihood using the `fmincon` function, the reported standard errors are the square root of the diagonal elements of the inverted Hessian matrix produced by `fmincon`. For each estimation we provide figures that display the data,

[1] Note that ranges in the online appendix are indicated by Japanese "wave dash" notation.

the switching point and the two long-term means. In addition, we display the average simulated path (with the first observation used as the starting point) of the leverage process in (2.3) using the estimated parameters.

4.1 Kodak

The estimation results for Kodak are given in Figure 3 and Table 1. We point out that in only one case when we used the data points 1–1890 (see the online appendix), the estimated value of κ' is 0.27. In all other estimation results the parameter κ' is practically zero. This parameter measures the mean-reversion speed of the post-switch data toward the pre-switch mean-reversion level $\hat{\alpha}$ (see Section 2.3). When κ' is close to 0 there is strong evidence that the switch indeed occurred in the data.

The long-term mean level after the switch $\hat{\alpha}'$ is -0.88 in the first estimation result, and it gradually changes as more data is used in the estimation. As the fourth reported result in Table 1 demonstrates, the shift in the long-term mean level results in a significant difference in five-year default probabilities. This indicates an increased risk of insolvency. This difference is detected starting with the estimation based on 1–2250 data points. As expected, the occupation time above the new level $\hat{\alpha}'$ is significantly lower for the process with the pre-switch mean-reversion level $\hat{\alpha}$. As demonstrated in this example, by observing changes in the quantities related to credit risk it is possible to monitor the company's credit quality using our model.

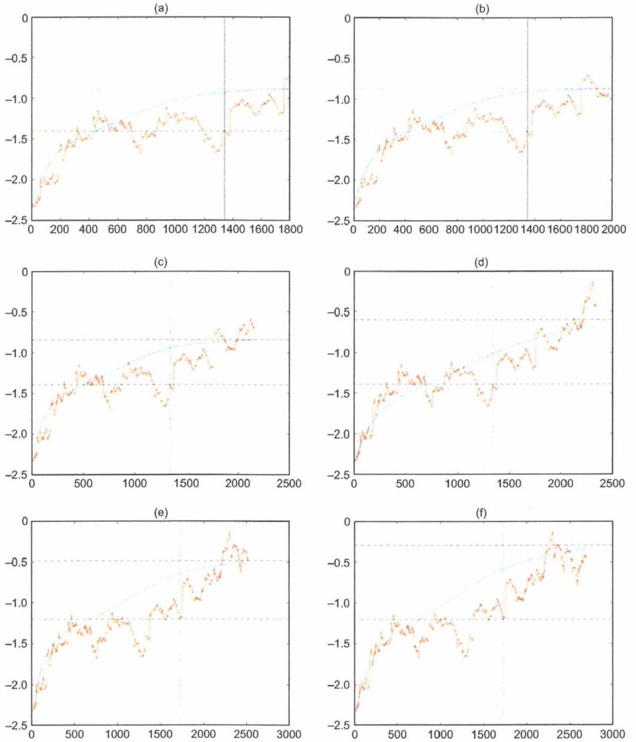
4.2 Sears

The estimation results for Sears are displayed in Figure 4 and Table 2. We point out that in all the estimations (including the online appendix), the mean-reversion speed $\hat{\kappa}'$ of the post-switch data toward the pre-switch mean-reversion level $\hat{\alpha}$ is practically zero. Note that the estimate of the long-term mean after the switch $\hat{\alpha}'$ gradually changes as we use more data in the estimation. In the second estimation result in Table 2 we already see that the shift in the long-term mean affects five-year default probability estimates. The difference in default probabilities is already detected in the estimation based on the data 1–1890 (see the online appendix). In the final result in Table 2 the difference in default probabilities is more than 50%. As in the case of Kodak, the process with the pre-switch mean-reversion level $\hat{\alpha}$ spends significantly less time above the level $\hat{\alpha}'$.

4.3 Ford

The estimation results for Ford are displayed in Figure 5 and Table 3. As in the case of Sears, the mean-reversion speed $\hat{\kappa}'$ is practically zero in all the estimations (including the online appendix). This provides strong evidence that the data after the switch does not revert to the pre-switch mean-reversion level $\hat{\alpha}$. In the first estimation

FIGURE 3 Estimation results for Kodak.



Pre-switch and post-switch mean-reversion levels (horizontal dashed lines), detected switching point (vertical line), the data used in the estimation (fluctuating orange line), and the average simulated path from the model (2.3) based on the estimated parameters (smooth blue line), with the horizontal axis denoting trading days. Data used: (a) 1–1800, (b) 1–1980, (c) 1–2160, (d) 1–2340, (e) 1–2520, (f) 1–2700.

the long-term mean after the switch $\hat{\alpha}'$ is -0.15, and this value is higher than the results of the other two companies. Also, in all the estimations the shift in the long-term mean level affects five-year default probabilities significantly. As in the case

TABLE 1 Estimation results for Kodak. [Table continues on next page.]

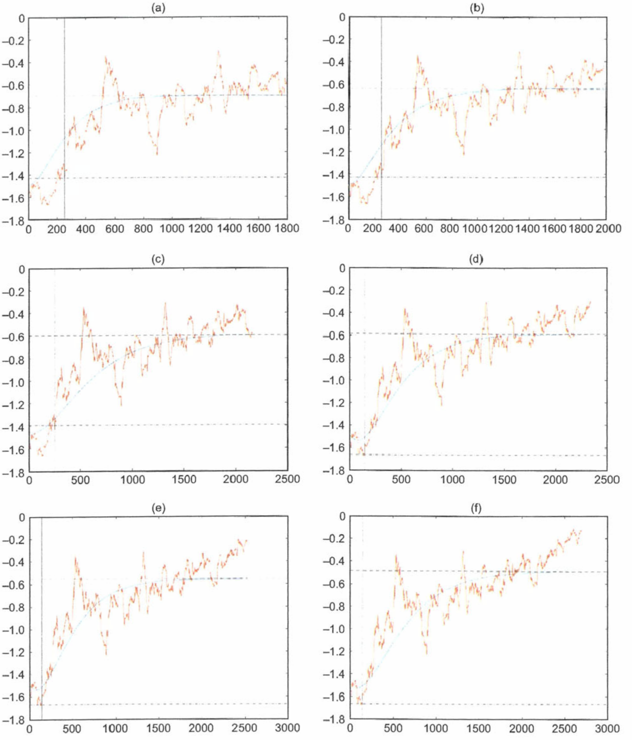
	Data used					
	1–1800		1–1980		1–2160	
$\hat{\kappa}$	2.5031	(0.6249)	2.5920	(0.3201)	2.4504	(0.1517)
$\hat{\alpha}$	−1.4025	(0.1282)	−1.4046	(0.1058)	−1.3945	(0.1146)
$\hat{\alpha}'$	−0.8794	(0.1432)	−0.8730	(0.1788)	−0.8389	(0.0776)
$\hat{\sigma}$	0.4267	(0.0682)	0.4157	(0.0096)	0.4072	(0.0440)
$\hat{N}$	7.5127	(0.0093)	8.0000	(8.23E−07)	10.2044	(2.0465)
Realized $U_{n\Delta_t}(\hat{\alpha})$	9		9		12	
Switching point in "Data used"	1343		1343		1345	
$\hat{\kappa}'$	2.83E−05	(0.0976)	0.0724	(0.0328)	0.0024	(0.1028)
$\hat{\sigma}'$	0.5099	(0.0169)	0.4577	(0.0924)	0.4284	(0.0214)
$\mathbb{E}[\int_0^5 \mathbf{1}_{\{Y_t \geqslant \hat{\alpha}'\}} \mathrm{d}t]$	0.1708		0.0805		0.1096	
$\mathbb{E}[\int_0^5 \mathbf{1}_{\{Y_t' \geqslant \hat{\alpha}'\}} \mathrm{d}t]$	2.6617		2.5434		2.6095	
$\mathbb{P}(\inf\{t : Y_t \geqslant 0\} \leqslant 5)$	0.0000		0.0000		0.0000	
$\mathbb{P}(\inf\{t : Y_t' \geqslant 0\} \leqslant 5)$	0.0003		0.0002		0.0005	

TABLE 1 Continued.

	Data used					
	1–2340		1–2520		1–2700	
$\hat{\kappa}$	1.4180	(0.1065)	1.8934	(0.4090)	1.3373	(0.0397)
$\hat{\alpha}$	−1.3875	(0.3043)	−1.2011	(3.39E−08)	−1.2050	(6.97E−08)
$\hat{\alpha}'$	−0.5978	(0.1073)	−0.4803	(0.0931)	−0.2999	(0.0729)
$\hat{\sigma}$	0.4115	(0.0399)	0.4069	(0.0269)	0.4061	(0.0118)
$\hat{N}$	10.1361	(1.0004)	12.0291	(1.0006)	9.0192	(1.1120)
Realized $U_{n\Delta_t}(\hat{\alpha})$	12		13		10	
Switching point in "Data used"	1338		1733		1728	
$\hat{\kappa}'$	6.13E−06	(0.0255)	0.0002	(0.0503)	3.47E−05	(0.0048)
$\hat{\sigma}'$	0.4327	(0.0674)	0.4388	(0.0386)	0.4286	(0.0246)
$\mathbb{E}[\int_0^5 1_{\{Y_t \geqslant \hat{\alpha}'\}} \mathrm{d}t]$	0.2307		0.0372		0.0102	
$\mathbb{E}[\int_0^5 1_{\{Y'_t \geqslant \hat{\alpha}'\}} \mathrm{d}t]$	2.7779		2.4814		2.2811	
$\mathbb{P}(\inf\{t : Y_t \geqslant 0\} \leqslant 5)$	0.0006		0.0000		0.0010	
$\mathbb{P}(\inf\{t : Y'_t \geqslant 0\} \leqslant 5)$	0.2766		0.3662		0.8445	

The estimated parameters are given by (2.3) and (2.6); the last four quantities are based on (2.8); the realized number of upcrossings of $\hat{\alpha}$ (denoted by $U_{n\Delta_t}(\hat{\alpha})$) and the detected switching point are based on the data used for the estimation; the standard errors are displayed in parentheses.

FIGURE 4 Estimation results for Sears.



Pre-switch and post-switch mean-reversion levels (horizontal dashed lines), detected switching point (vertical line), the data used in the estimation (fluctuating orange line), and the average simulated path from the model (2.3) based on the estimated parameters (smooth blue line), with the horizontal axis denoting trading days. Data used: (a) 1–1800, (b) 1–1980, (c) 1–2160, (d) 1–2340, (e) 1–2520, (f) 1–2700.

of the other two companies, the process with the mean-reversion level $\hat{\alpha}'$ spends significantly more time above the level $\hat{\alpha}'$ compared with the process with the mean-reversion level $\hat{\alpha}$.

TABLE 2 Estimation results for Sears. [Table continues on next page.]

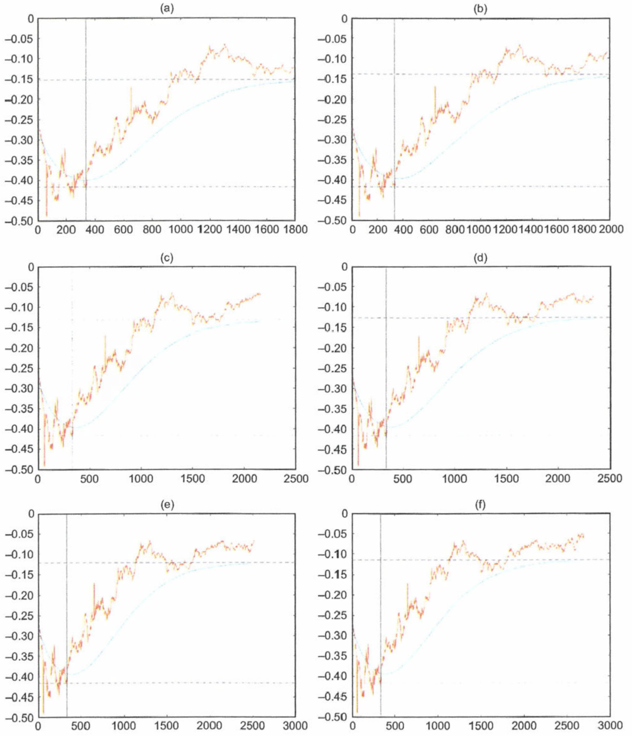
	Data used					
	1–1800		1–1980		1–2160	
$\hat{\kappa}$	3.4411	(0.6949)	2.3766	(0.3862)	1.8355	(0.5838)
$\hat{\alpha}$	−1.4279	(3.09E−08)	−1.4279	(2.45E−08)	−1.3946	(6.23E−08)
$\hat{\alpha}'$	−0.7036	(0.0862)	−0.6421	(0.0860)	−0.5993	(0.0990)
$\hat{\sigma}$	0.4094	(0.0474)	0.4002	(0.0538)	0.3941	(0.0462)
$\hat{N}$	3.5251	(1.0298)	3.4065	(1.0068)	6.7346	(1.0005)
Realized $U_{n\Delta_t}(\hat{\alpha})$	4		4		7	
Switching point in "Data used"	254		254		256	
$\hat{\kappa}'$	0.0218	(0.0088)	0.0001	(0.0036)	0.0390	(0.0129)
$\hat{\sigma}'$	0.4254	(0.0090)	0.4137	(0.0292)	0.4056	(0.0094)
$\mathbb{E}[\int_0^5 \mathbf{1}_{\{Y_t \geqslant \hat{\alpha}'\}} \mathrm{d}t]$	0.0490		0.1135		0.0445	
$\mathbb{E}[\int_0^5 \mathbf{1}_{\{Y_t' \geqslant \hat{\alpha}'\}} \mathrm{d}t]$	2.5966		2.6882		2.5299	
$\mathbb{P}(\inf\{t : Y_t \geqslant 0\} \leqslant 5)$	0.0000		0.0000		0.0000	
$\mathbb{P}(\inf\{t : Y_t' \geqslant 0\} \leqslant 5)$	0.0006		0.0333		0.1046	

TABLE 2 Continued.

	Data used					
	1–2340		1–2520		1–2700	
$\hat{\kappa}$	1.7908	(0.3091)	1.7508	(0.0828)	1.3430	(0.2832)
$\hat{\alpha}$	−1.6613	(2.09E−07)	−1.6613	(6.39E−08)	−1.6613	(8.48E−09)
$\hat{\alpha}'$	−0.5876	(0.0816)	−0.5551	(0.0732)	−0.4897	(0.0928)
$\hat{\sigma}$	0.3843	(0.0729)	0.3738	(0.0349)	0.3635	(0.0386)
$\hat{N}$	2.4363	(1.0083)	2.9359	(1.0249)	3.0000	(1.75E−07)
Realized $U_{n\Delta_t}(\hat{\alpha})$	3		3		3	
Switching point in "Data used"	143		143		143	
$\hat{\kappa}'$	6.70E−07	(0.0149)	1.02E−05	(0.0131)	8.24E−06	(0.0284)
$\hat{\sigma}'$	0.3907	(0.0041)	0.3793	(0.0075)	0.3680	(0.0102)
$\mathbb{E}[\int_0^5 \mathbf{1}_{\{Y_t \geqslant \hat{\alpha}'\}} \mathrm{d}t]$	0.1476		0.1645		0.2095	
$\mathbb{E}[\int_0^5 \mathbf{1}_{\{Y_t' \geqslant \hat{\alpha}'\}} \mathrm{d}t]$	2.8259		2.8610		2.9412	
$\mathbb{P}(\inf\{t : Y_t \geqslant 0\} \leqslant 5)$	0.0000		0.0002		0.0046	
$\mathbb{P}(\inf\{t : Y_t' \geqslant 0\} \leqslant 5)$	0.1606		0.2258		0.5173	

The estimated parameters are based on (2.3) and (2.6); the last four quantities are given by (2.8); the realized number of upcrossings of $\hat{\alpha}$ (denoted by $U_{n\Delta_t}(\hat{\alpha})$) and the detected switching point are based on the data used for the estimation; the standard errors are displayed in parentheses.

FIGURE 5 Estimation results for Ford.



Pre-switch and post-switch mean-reversion levels (horizontal dashed lines), detected switching point (vertical line), the data used in the estimation (fluctuating orange line), and the average simulated path from the model (2.3) based on the estimated parameters (smooth blue line), with the horizontal axis denoting trading days. Data used: (a) 1–1800, (b) 1–1980, (c) 1–2160, (d) 1–2340, (e) 1–2520, (f) 1–2700.

REMARK 4.1 It is possible to extend the model to include more than one switch in the mean-reversion level. In our model the mean-reversion level after the first switch is α'. The mechanism for the second switch can be based on the number of

TABLE 3 Estimation results for Ford. [Table continues on next page.]

	Data used					
	1–1800		1–1980		1–2160	
$\hat{\kappa}$	2.6994	(0.7823)	2.3772	(0.6740)	2.2705	(0.6759)
$\hat{\alpha}$	−0.4171	(2.01E−06)	−0.4171	(1.00E−07)	−0.4171	(3.49E−05)
$\hat{\alpha}'$	−0.1534	(0.0891)	−0.1395	(0.0320)	−0.1322	(0.0270)
$\hat{\sigma}$	0.1407	(0.0630)	0.1347	(0.0134)	0.1292	(0.0020)
$\hat{N}$	11.0000	(2.52E−06)	10.7183	(1.0003)	10.9997	(1.1340)
Realized $U_{n\Delta_t}(\hat{\alpha})$	11		11		11	
Switching point in "Data used"	337		337		337	
$\hat{\kappa}'$	0.0001	(0.0023)	1.07E−05	(0.0086)	1.74E−06	(0.0046)
$\hat{\sigma}'$	0.1009	(0.0023)	0.0960	(0.0024)	0.0917	(0.0012)
$\mathbb{E}[\int_0^5 \mathbf{1}_{\{Y_t \geqslant \hat{\alpha}'\}} \mathrm{d}t]$	0.0629		0.0938		0.1011	
$\mathbb{E}[\int_0^5 \mathbf{1}_{\{Y_t' \geqslant \hat{\alpha}'\}} \mathrm{d}t]$	2.6053		2.6725		2.6917	
$\mathbb{P}(\inf\{t : Y_t \geqslant 0\} \leqslant 5)$	0.0000		0.0004		0.0003	
$\mathbb{P}(\inf\{t : Y_t' \geqslant 0\} \leqslant 5)$	0.3423		0.5182		0.5685	

TABLE 3 Continued.

	Data used					
	1–2340		**1–2520**		**1–2700**	
$\hat{\kappa}$	2.1897	(0.5967)	2.1107	(0.5799)	1.9973	(0.6783)
$\hat{\alpha}$	−0.4171	(1.43E−06)	−0.4171	(9.57E−08)	−0.4171	(6.29E−07)
$\hat{\alpha}'$	−0.1273	(0.0251)	−0.1215	(0.0259)	−0.1155	(0.0285)
$\hat{\sigma}$	0.1245	(0.0018)	0.1204	(0.0018)	0.1172	(0.0340)
$\hat{N}$	11.0000	(1.1384)	10.9997	(2.8145)	11.0000	(5.22E−06)
Realized $U_{n\Delta_t}(\hat{\alpha})$	11		11		11	
Switching point in "Data used"	337		337		337	
$\hat{\kappa}'$	1.73E−06	(0.0036)	8.17E−08	(0.0062)	6.54E−06	(0.0099)
$\hat{\sigma}'$	0.0880	(0.0013)	0.0850	(0.0038)	0.0831	(0.0012)
$\mathbb{E}[\int_0^5 \mathbf{1}_{\{Y_t \geqslant \hat{\alpha}'\}} \mathrm{d}t]$	0.0878		0.0987		0.1044	
$\mathbb{E}[\int_0^5 \mathbf{1}_{\{Y_t' \geqslant \hat{\alpha}'\}} \mathrm{d}t]$	2.6714		2.7268		2.7241	
$\mathbb{P}(\inf\{t : Y_t \geqslant 0\} \leqslant 5)$	0.0002		0.0007		0.0014	
$\mathbb{P}(\inf\{t : Y_t' \geqslant 0\} \leqslant 5)$	0.5726		0.6190		0.6775	

The estimated parameters are based on (2.3) and (2.6); the last four quantities are given by (2.8); the realized number of upcrossings of $\hat{\alpha}$ (denoted by $U_{n\Delta_t}(\hat{\alpha})$) and the detected switching point are based on the data used for the estimation; the standard errors are displayed in parentheses.

upcrossings of the level α' after the first switching time. When this upcrossing number exceeds a certain threshold, the second switch will occur and the mean-reversion level will change. This mechanism can be extended to more than two switches similarly. All parameters (including mean-reversion levels and thresholds for switches) should be estimated from the data. To determine an appropriate number of switches, we need to check the mean-reversion speed of the post-switch data toward the pre-switch mean-reversion level using the method in Section 2.3. For example, in the case of two switches we need to check the mean-reversion speed of the data after the second switching point toward the mean-reversion level resulting from the first switch. We also need to check the mean-reversion speed of the data between the first and second switching points toward the mean-reversion level before the first switch. If both of these mean-reversion speeds are close to 0, there is strong evidence that the switch indeed occurred twice in the data. Otherwise, we should consider decreasing the number of switches in the model. A similar method can be applied to the model with more than two switches.

5 CONCLUSION

In this paper we proposed a new model with an endogenous switching mechanism that governs the shift in the long-term mean of the leverage process. We constructed this model based on the observation that the credit quality deterioration of established companies is a long-term process. Such firms will fail when they use up all their resources and cannot meet market and technological demands anymore. The switching mechanism in the model depends only on the number of upcrossings of the leverage process itself. Our model is computationally simple and its parameters are interpretable from a credit risk perspective. As our empirical analysis demonstrates, the shift in the long-term mean affects quantities related to credit quality. We show that occupation time and default probability estimates change with the shift, and this triggers an alarm indicating that the company's credit quality has changed. We also demonstrated how to test the significance of the change by checking the mean-reversion speed of the post-switch data toward the pre-switch long-term mean. Our model is designed for established companies and may not be an appropriate tool for those firms that may experience a sudden decline in credit quality. For such companies we might need to consider processes that are not mean-reverting.

DECLARATION OF INTEREST

The author reports no conflicts of interest. The author alone is responsible for the content and writing of the paper.

ACKNOWLEDGEMENTS

This work is supported by the Japan Society for the Promotion of Science KAKENHI Grant-in-Aid for Early-Career Scientists grant 21K13324.

REFERENCES

Bontempi, M. E., Bottazzi, L., and Golinelli, R. (2020). A multilevel index of heterogeneous short-term and long-term debt dynamics. *Journal of Corporate Finance* **64**, paper 101666 (https://doi.org/10.1016/j.jcorpfin.2020.101666).

Christensen, C. M. (1997). *The Innovator's Dilemma: When New Technologies Cause Great Firms to Fail*. Harvard Business School Press, Boston, MA.

Collin-Dufresne, P., and Goldstein, R. S. (2001). Do credit spreads reflect stationary leverage ratios? *Journal of Finance* **56**(5), 1929–1957 (https://doi.org/10.1111/0022-1082.00395).

Cox, D. R., and Miller, H. D. (2001). *The Theory of Stochastic Processes*. Chapman & Hall/CRC, Boca Raton, FL.

DeAngelo, H., and Roll, R. (2015). How stable are corporate capital structures? *Journal of Finance* **70**(1), 373–418 (https://doi.org/10.1111/jofi.12163).

Dolan, M. (2010). Governments help Ford cut debt. *Wall Street Journal*, August 8. URL: https://on.wsj.com/3OrOwSP.

Franses, P. H., and van Dijk, D. (2000). *Non-Linear Time Series Models in Empirical Finance*. Cambridge University Press (https://doi.org/10.1017/CBO9780511754067).

Hovakimian, A., Opler, T., and Titman, S. (2001). The debt–equity choice. *Journal of Financial and Quantitative Analysis* **36**(1), 1–24 (https://doi.org/10.2307/2676195).

Karatzas, I., and Shreve, S. E. (1998). *Brownian Motion and Stochastic Calculus*, 2nd edn. Springer (https://doi.org/10.1007/978-1-4612-0949-2).

Marcucci, J., and Quagliariello, M. (2009). Asymmetric effects of the business cycle on bank credit risk. *Journal of Banking and Finance* **33**(9), 1624–1635 (https://doi.org/10.1016/j.jbankfin.2009.03.010).

Nunes, T. M. T., and Rodrigues, P. M. M. (2011). Threshold effects in credit risk and stress scenarios. *International Journal of Finance and Economics* **16**(4), 393–407 (https://doi.org/10.1002/ijfe.436).

Rizzo, L. (2018). Sears, a onetime retail giant, now banks on bankruptcy. *Wall Street Journal*, October 15. URL: https://on.wsj.com/2yFhMQz.

Spector, M., Mattioli, D., and Brickley, P. (2012). Can bankruptcy filing save Kodak? *Wall Street Journal*, January 20. URL: https://on.wsj.com/2Cf9UdG.

Tong, H. (1983). *Threshold Models in Non-linear Time Series Analysis*. Springer (https://doi.org/10.1007/978-1-4684-7888-4).

Tong, H. (2011). Threshold models in time series analysis: 30 years on. *Statistics and Its Interface* **4**(2), 107–118 (https://doi.org/10.4310/SII.2011.v4.n2.a1).

Tsay, R. S. (2010). *Analysis of Financial Time Series*, 3rd edn. Wiley (https://doi.org/10.1002/9780470644560).