

数理解析研究所講究録 2225

RIMS 共同研究 (公開型)

代数的整数論とその周辺

京都大学数理解析研究所

2022年7月

数理解析研究所講究録は、京都大学数理解析研究所の共同利用研究集会および共同研究の記録として1964年に刊行が開始されました。当研究所が全国共同利用研究所として発足した翌年のことでしたが、以来半世紀、毎年数十巻を刊行し、2016年には第2000巻が刊行されるに至りました。第1巻から第2000巻までに収録された論文数は29,265編、総頁数は342,960頁という膨大なものであり、最先端の数学・数理科学分野の研究状況を伝えるのみならず、我が国の数学・数理科学の発展の歴史を留める文献として、他に類例を見ない論文集となっています。

講究録の内容は当研究所のウェブサイトおよび京都大学の学術情報リポジトリにおいても公開され、年間の総アクセス数は1,380,032回（2017年度）を数えるなど、多数の方にご利用いただいています。

講究録の使用言語は論文著者の判断に任されていますが、結果的に日本語が多用されていることが特徴の一つとなっています。その結果、講究録は、数学・数理科学の広い領域における最先端の専門知識に母国語でアクセスできるものとして、近年の英語化の流れの中で、重要な文献となりつつあります。

当研究所の共同利用事業に参加し講究録の論文を執筆していただいた多数の方々に対し、講究録を大きく成長させていただいたことを深く感謝いたしますとともに、これからも、当研究所の国際共同利用・共同研究拠点(*)としての活動にご参加いただき、講究録の発展にご協力いただけますよう心よりお願い申し上げます。

*数理解析研究所は2018年11月13日、共同利用・共同研究拠点の認定が廃止され、新しく国際共同利用・共同研究拠点に認定されました。

講究録

Kôkyûroku

RIMS Kôkyûroku was started in 1964 as the proceedings of symposia, colloquia and workshops supported by RIMS, the Research Institute for Mathematical Sciences, Kyoto University. It was the next year of the establishment of RIMS as one of the Nationwide Cooperative Research Centers. For half a century since then, several dozen volumes have been issued each year, and the 2,000th volume was issued in 2016. The volumes of Kôkyûroku from the 1st through the 2,000th, containing enormous 29,265 articles and 342,960 pages, not only deliver the latest research activities in mathematics and mathematical sciences but also constitute valuable and incomparable collections of articles that pass down history of progress of mathematics and mathematical science in Japan.

Articles in Kôkyûroku are available on the websites of RIMS and Kyoto University Research Information Repository. They are very frequently accessed on the internet, with a total of as many as 1,380,032 accesses in 2017.

The authors choose the languages to write articles, and many are written in Japanese, which is one of the characteristics of Kôkyûroku. As a result, Kôkyûroku is regarded as a significant and important literature which allows easy access to the latest specialized knowledge in the large fields of mathematics and mathematical sciences written in native language for Japanese readers, while more and more research papers are being written in English in recent years.

We are deeply grateful to many of those who have participated in cooperative research activities of RIMS and greatly developed Kôkyûroku. We heartily ask for your continuous participation in research activities at RIMS as an International Joint Usage/Research Center(*) and your warm support and cooperation for the fruitful development of Kôkyûroku.

* RIMS was certified as an International Joint Usage/Research Center on Nov. 13, 2018.

RIMS Kôkyûroku 2225

Algebraic Number Theory and Related Topics

December 13 ~ 17, 2021

edited by Shinichi Kobayashi

July, 2022

Research Institute for Mathematical Sciences

Kyoto University, Kyoto, Japan

This is a report of research done at the Research Institute for Mathematical Sciences,
an International Joint Usage/Research Center located in Kyoto University.
The papers contained herein are in final form and will not be submitted for publication elsewhere.

Preface

This is the proceedings of the conference “Algebraic Number Theory and Related Topics” held at RIMS, Kyoto University from December 13th (Mon) through 17th (Fri), 2021. The conference was held as one of the workshops supported by the joint research program of RIMS, and was intended to cover a wide range of topics in number theory such as algebraic number theory, arithmetic geometry, algebraic geometry, analytic number theory, automorphic representations, anabelian geometry, Iwasawa theory, and L -functions.

We would like to thank all who kindly helped us to organize the conference. In particular, the graduate students who helped us during the conference, and all the staff of the joint research office at RIMS for their constant support. Finally, we would like to express our sincere gratitude to all who recommended speakers, and to all the speakers of the conference for giving interesting talks and writing articles in this proceeding.

We hope that the articles in this proceeding will lead to further developments in number theory and related subjects.

June, 2022

Shinichi Kobayashi
Tomokazu Kashio
Akinari Hoshi

RIMS 共同研究（公開型）「代数的整数論とその周辺」

京都大学数理解析研究所の共同研究事業の一環として、下記のように研究集会を催しますのでご案内申し上げます。

研究代表者 小林 真一 (九州大学)
研究副代表者 加塩 朋和 (東京理科大学)

記

期間 令和3年12月13日(月)～12月17日(金)
場所 京都大学数理解析研究所420号室 + Zoom のハイブリッド型

12月13日(月)

- 9:50～10:00 はじめに
- 10:00～10:50 山内 卓也^z (東北大学)
5次Dwork族に付随する法2ガロア表現の保型性とある5次3項方程式の相互法則について
- 11:10～12:00 竹平 航平^z (東北大学)
射影直線上の力学系に付随するゼータ関数について
- 13:30～14:20 高松 哲平 (東京大学)
既約正則シンプレクティック多様体の Shafarevich 予想について
- 14:35～15:35 望月 新一^{*,z} (京都大学数理解析研究所)
宇宙際タイヒミュラー理論の論理構造 I
- 15:50～16:50 望月 新一^{*,z} (京都大学数理解析研究所)
宇宙際タイヒミュラー理論の論理構造 II

12月14日(火)

- 9:20～10:10 坂本 龍太郎^z (理化学研究所 革新知能統合研究センター)
 p -Selmer groups and modular symbols
- 10:25～11:15 大下 達也^z (群馬大学)
イデアル類群の漸近挙動と楕円曲線の精セルマー群の岩澤加群について
- 11:30～12:20 山名 俊介 (大阪市立大学)
捻り三重積 p 進 L 関数の例外零点
- 13:40～14:30 室谷 岳寛^z (京都大学数理解析研究所)
混標数完備離散付値体の遠アーベル幾何学と分岐フィルトレーション
- 14:40～15:40 望月 新一^{*,z} (京都大学数理解析研究所)
宇宙際タイヒミュラー理論の論理構造 III
- 15:50～16:50 南出 新^{*,z} (京都大学数理解析研究所)
宇宙際タイヒミュラー理論における明示的評価

12月15日(水)

- 9:20 ~ 10:10 Yen-Tsung Chen^z (National Tsing Hua University)
Linear equations on Drinfeld modules
- 10:25 ~ 11:15 山崎 愛一 (京都大学)
Norm one tori and Hasse norm principle
- 11:30 ~ 12:30 安田 健彦* (大阪大学)
2次元対数的端末特異点の局所エタール基本群
- 12:40 ~ 「代数的整数論とその周辺」運営委員会 (主に委員の方)
自由討論

12月16日(木)

- 10:00 ~ 10:50 李 公彦^z (東京大学)
高レベルブリズマティックおよび q -クリスタリンサイトについて
- 11:10 ~ 12:00 Alex Youcis (東京大学)
A specialization map from the de Jong fundamental group to the pro-étale fundamental group
- 13:30 ~ 14:20 南 範彦^z (名古屋工業大学)
レトラクト $(-i)$ 有理性とそのための, あるコホモロジー論たちによって表される
必要条件 - 有限群 G の閉体上 Noether 問題を例に
- 14:40 ~ 15:30 小山 信也^z (東洋大学)
深リーマン予想を用いた「チェビシェフの偏り」の解明と一般化
- 15:50 ~ 16:50 甲斐 亘* (東北大学)
数体における Green-Tao の定理

12月17日(金)

- 10:00 ~ 11:00 佐野 昂迪* (大阪市立大学)
Hilbert の第 12 問題の最近の進展について
- 11:20 ~ 12:10 熱田 真大 (慶應義塾大学)
 \mathbb{G}_m の同変玉河数予想のマイナス成分について
- 14:00 ~ 14:50 板東 克之^z (東京大学)
混標数の幾何学的佐武対応とシュプリンガー対応
- 15:10 ~ 16:00 三枝 洋一 (東京大学)
 $\mathrm{GSp}(4)$ の Rapoport-Zink 空間の ℓ 進コホモロジーの超尖点部分について
- 16:10 ~ 16:15 おわりに

敬称略, * 印は依頼講演, ^z はオンライン講演

プログラム作成委員

小林 真一 (九州大学), 加塩 朋和 (東京理科大学), 星 明考 (新潟大学)

RIMS Workshop

Algebraic Number Theory and Related Topics

Organizers:

Shinichi Kobayashi (Kyushu University)

Tomokazu Kashio (Tokyo University of Science)

Date: December 13 (Mon)– December 17 (Fri), 2021

Place: Room 420, Research Institute for Mathematical Sciences (RIMS),
Kyoto University, Kyoto 606-8502, JAPAN +Zoom (hybrid)

December 13 (Mon)

09:50 – 10:00 Introduction

10:00 – 10:50 Takuya Yamauchi^z (Tohoku University)

Automorphy of mod 2 Galois representations associated to the quintic Dwork family and reciprocity of some quintic trinomials

11:10 – 12:00 Kohei Takehira^z (Tohoku University)

On the dynamical zeta function associated with a dynamical system on the projective line

13:30 – 14:20 Teppei Takamatsu (The University of Tokyo)

On the Shafarevich conjecture for irreducible holomorphic symplectic varieties

14:35 – 15:35 Shinichi Mochizuki^{*,z} (RIMS)

On the logical structure of inter-universal Teichmüller theory I

15:50 – 16:50 Shinichi Mochizuki^{*,z} (RIMS)

On the logical structure of inter-universal Teichmüller theory II

December 14 (Tue)

09:20 – 10:10 Ryotaro Sakamoto^z (RIKEN Center for Advanced Intelligence Project)
 p -Selmer groups and modular symbols

10:25 – 11:15 Tatsuya Ohshita^z (Gunma University)

Asymptotic behavior of ideal class groups and Iwasawa module of the fine Selmer group of elliptic curves

11:30 – 12:20 Shunsuke Yamana (Osaka City University)

Exceptional zeros of twisted triple product p -adic L-functions

13:40 – 14:30 Takahiro Murotani^z (RIMS)

Anabelian geometry of mixed-characteristic complete discrete valuation fields and ramification filtrations

14:40 – 15:40 Shinichi Mochizuki^{*,z} (RIMS)

On the logical structure of inter-universal Teichmüller theory III

15:50 – 16:50 Arata Minamide^{*,z} (RIMS)

Explicit estimates in inter-universal Teichmüller theory

December 15 (Wed)

- 09:20 – 10:10 Yen-Tsung Chen^z (National Tsing Hua University)
Linear equations on Drinfeld modules
- 10:25 – 11:15 Aiichi Yamasaki (Kyoto University)
Norm one tori and Hasse norm principle
- 11:30 – 12:30 Takehiko Yasuda* (Osaka University)
Local étale fundamental groups of log terminal surface singularities
- 12:40 – Meeting of the steering committee (mainly for the committee members)

December 16 (Thu)

- 10:00 – 10:50 Kimihiko Li^z (The University of Tokyo)
Prismatic and q -crystalline sites of higher level
- 11:10 – 12:00 Alex Youcis (The University of Tokyo)
A specialization map from the de Jong fundamental group to the pro-étale fundamental group
- 13:30 – 14:20 Norihiko Minami^z (Nagoya Institute of Technology)
Retract $(-i)$ rationality and its necessary conditions expressed by certain cohomology theories - Noether's problem of a finite group G over an algebraic closed field, as an example
- 14:40 – 15:30 Shin-ya Koyama^z (Toyo University)
Chebyshev's bias with its generalizations
- 15:50 – 16:50 Wataru Kai* (Tohoku University)
The Green-Tao theorem for number fields

December 17 (Fri)

- 10:00 – 11:00 Takamichi Sano* (Osaka City University)
Recent progress on Hilbert's 12th problem
- 11:20 – 12:10 Mahiro Atsuta (Keio University)
The minus component of the equivariant Tamagawa number conjecture for \mathbb{G}_m
- 14:00 – 14:50 Katsuyuki Bando^z (The University of Tokyo)
Geometric Satake equivalence in mixed characteristic and Springer correspondence
- 15:10 – 16:00 Yoichi Mieda (The University of Tokyo)
On supercuspidal part of the ℓ -adic cohomology of the Rapoport-Zink space for $\mathrm{GSp}(4)$
- 16:10 – 16:15 Closing

* Invited speakers, ^z Online

Program Committee:

Shinichi Kobayashi (Kyushu University),
Tomokazu Kashio (Tokyo University of Science),
Akinari Hoshi (Niigata University)

RIMS Workshop

Algebraic Number Theory and Related Topics

Organizers:

Shinichi Kobayashi (Kyushu University)

Tomokazu Kashio (Tokyo University of Science)

Date: December 13 (Mon)– December 17 (Fri), 2021

Place: Room 420, Research Institute for Mathematical Sciences (RIMS),
Kyoto University, Kyoto 606-8502, JAPAN +Zoom (hybrid)

December 13 (Mon)

10:00 – 10:50 **Takuya Yamauchi**^z (Tohoku University)

Automorphy of mod 2 Galois representations associated to the quintic Dwork family and reciprocity of some quintic trinomials

In this talk, I will explain my recent work on computing the mod 2 Galois representations associated to the mirror motives of rank 4 and of weight 3 coming from the Dwork quintic family defined over a number field. In the course of the computation, we observe that the image of such a mod representation is governed by reciprocity of some quintic trinomial. When the base field is a totally real field, we apply the modularity of 2-dimensional, totally odd Artin representations due to Shu Sasaki to obtain automorphy of mod 2 representations in question after a suitable (at most) quadratic base extension. This is a joint work with Tsuzuki Nobuo.

11:10 – 12:00 **Kohei Takehira**^z (Tohoku University)

On the dynamical zeta function associated with a dynamical system on the projective line

A discrete dynamical system is defined by viewing a univariate rational function ϕ on a field as a self-map on a projective line. In 1995, Hatjispyros and Vivaldi defined the zeta function associated with this dynamical system using a quantity called the multiplier of periodic points, which describes the local behavior of the dynamical system. In this talk, we will give a cohomological interpretation of this zeta function and use it to discuss the rationality of this zeta function, explicit computation of the zeta function, and other issues.

13:30 – 14:20 **Teppei Takamatsu** (The University of Tokyo)

On the Shafarevich conjecture for irreducible holomorphic symplectic varieties

The Shafarevich conjecture, which was proved by Faltings and Zarhin, states the finiteness of isomorphism classes of abelian varieties of a fixed dimension over a fixed number field admitting good reduction away from a fixed finite set of finite places. In this talk, we prove the Shafarevich conjecture for irreducible holomorphic symplectic varieties whose second Betti numbers are greater than 3. As byproducts, we also discuss the cone conjecture over non-algebraically closed field of characteristic 0 and finiteness of twists for irreducible holomorphic symplectic varieties.

14:35 – 15:35 **Shinichi Mochizuki**^{*,z} (RIMS)

On the logical structure of inter-universal Teichmüller theory I, II, III

Introductory lectures and expositions on inter-universal Teichmüller theory — such as, for instance, [Alien] — have a tendency to concentrate on exposing the technical details surrounding the various mathematical objects that appear in the theory. To a certain extent, of course, this is unavoidable. On the other hand, concentrating on such technical details can lead to a situation where one is overwhelmed with seemingly meaningless technicalities to such an extent that one loses sight of the essential logical structure of the theory. The purpose of this series of talks is to discuss this essential logical structure of the theory, as exposed in the preprint [EssLgc] (cf. also the slides [OnEssLgc]).

[Alien] <https://www.kurims.kyoto-u.ac.jp/~motizuki/Alien%20Copies,%20Gaussians,%20and%20Inter-universal%20Teichmuller%20Theory.pdf>

[EssLgc] <https://www.kurims.kyoto-u.ac.jp/~motizuki/Essential%20Logical%20Structure%20of%20Inter-universal%20Teichmuller%20Theory.pdf>

[OnEssLgc] <https://www.kurims.kyoto-u.ac.jp/~motizuki/On%20the%20Essential%20Logical%20Structure%20of%20IUT%20I,%20II,%20III,%20IV,%20V.pdf>

15:50 – 16:50 **Shinichi Mochizuki**^{*,z} (RIMS)

On the logical structure of inter-universal Teichmüller theory I, II, III

December 14 (Tue)

09:20 – 10:10 **Ryotaro Sakamoto**^z (RIKEN Center for Advanced Intelligence Project)
 p -Selmer groups and modular symbols

It is conjectured by Kurihara that the dimension of the p -Selmer group of an elliptic curve over the field of rationals is controlled by analytic quantities associated with modular symbols. In this talk, I will explain the proof of Kurihara's conjecture using the theory of the Kolyvagin systems of rank 0.

10:25 – 11:15 **Tatsuya Ohshita**^z (Gunma University)

Asymptotic behavior of ideal class groups and Iwasawa module of the fine Selmer group of elliptic curves

Sairaiji-Yamauchi and Hiranouchi studied asymptotic behavior of the order of the p -part A_n of the ideal class group of the Galois extension field K_n of \mathbb{Q} generated by coordinates of all p^n -torsion points of an elliptic curve E defined over \mathbb{Q} . Under certain conditions, they give a lower bound of the order of A_n by using the rank of the Mordell-Weil group $E(\mathbb{Q})$. In this talk, by using the Λ -module structure of the cyclotomic Iwasawa module of the fine Selmer group of E , we give finer description, including the group structure, on the asymptotic behavior of certain quotient of A_n cut out by the Galois representation $E[p^n]$. This is joint work with Toshiro Hiranouchi.

11:30 – 12:20 **Shunsuke Yamana** (Osaka City University)

Exceptional zeros of twisted triple product p -adic L -functions

p -adic L -functions involve modified p -factors which measure the discrepancy between the p -adic and complex L -values in the interpolation formula. It is a puzzling fact that this factor can vanish at the central point. Then the p -adic L -function trivially vanishes at the point, and such a zero is called an exceptional zero. The p -adic L -function of an elliptic curve has an exceptional zero if and only if it has split multiplicative reduction at p , and the precise relation between derivative of the p -adic L -function and the algebraic part of the complex L -value was conjectured by Mazur-Tate-Teitelbaum and proved by Greenberg-Stevens. There have been many attempts to extend this result of Greenberg-Stevens to more general automorphic forms.

In this talk I will consider the exceptional zeros of the cyclotomic twisted triple product p -adic L -function associated to elliptic curves over rationals and a real quadratic field, and prove an identity between derivatives of the p -adic L -function and complex L -values. I will also consider exceptional zeros of a certain p -adic L -function of degree 6 associated with two rational elliptic curves. This is a joint work with Ming-Lun Hsieh.

13:40 – 14:30 **Takahiro Murotani**^z (RIMS)

Anabelian geometry of mixed-characteristic complete discrete valuation fields and ramification filtrations

It is known that an analogue of the theorem of Neukirch-Uchida for p -adic local fields fails to hold. However, in 1990s, Mochizuki proved that the absolute Galois groups with ramification filtrations of p -adic local fields determine the isomorphism classes of the fields. So, it is natural to consider to what extent ramification filtrations (of the absolute Galois groups) determine

the isomorphism classes (or other invariants) of general mixed-characteristic complete discrete valuation fields. In this talk, we establish (filtered) group-theoretic algorithms reconstructing various invariants from the absolute Galois groups with ramification filtrations, and prove that the isomorphism classes of the fields are determined by these (filtered) group-theoretic data under certain conditions.

14:40 – 15:40 **Shinichi Mochizuki**^{*,z} (RIMS)

On the logical structure of inter-universal Teichmüller theory I, II, III

15:50 – 16:50 **Arata Minamide**^{*,z} (RIMS)

Explicit estimates in inter-universal Teichmüller theory

In the final paper of a series of papers concerning inter-universal Teichmüller theory, Mochizuki verified various numerically non-effective versions of the Vojta, ABC, and Szpiro Conjectures over number fields. In this talk, we will give various numerically effective versions of Mochizuki's results. This is joint work with Shinichi Mochizuki, Ivan Fesenko, Yuichiro Hoshi, and Wojciech Porowski.

December 15 (Wed)

09:20 – 10:10 **Yen-Tsung Chen**^z (National Tsing Hua University)

Linear equations on Drinfeld modules

Let E be an elliptic curve defined over a number field K . Given finitely many K -rational points on E , Masser proved that there is an explicit upper bound for the size of the generators of linear relations among those points. This upper bound depends on the number of points, Neron-Tate height on $E(K)$, and the size of the torsion subgroup of $E(K)$. Let L be a finite extension of the rational function field over a finite field. We aim to study an analogue of Masser's result for finitely many L -rational points on a given Drinfeld module defined over L .

10:25 – 11:15 **Aiichi Yamasaki** (Kyoto University)

Norm one tori and Hasse norm principle

Let k be a field and T be an algebraic k -torus. In 1969, over a global field k , Voskresenskii proved that there exists an exact sequence $0 \rightarrow A(T) \rightarrow H^1(k, \text{Pic } \overline{X})^\vee \rightarrow \text{III}(T) \rightarrow 0$ where $A(T)$ is the kernel of the weak approximation of T , $\text{III}(T)$ is the Shafarevich-Tate group of T , X is a smooth compactification of T , $\overline{X} = X \times_k \overline{k}$, $\text{Pic } \overline{X}$ is the Picard group of \overline{X} and \vee stands for the Pontryagin dual. In 1984, Kunyavskii showed that, among 73 cases of 3-dimensional k -tori T , there exist exactly 2 cases satisfy $H^1(k, \text{Pic } \overline{X}) \neq 0$. On the other hand, in 1963, Ono proved that $\text{III}(T) = 0$ if and only if the Hasse norm principle holds for K/k where $T = R_{K/k}^{(1)}(\mathbb{G}_m)$ is the norm one torus of K/k . First, we show that, among 710 cases of 4-dimensional algebraic k -tori T , there exist exactly 2 (resp. 20, 688) cases with $H^1(k, \text{Pic } \overline{X}) \simeq (\mathbb{Z}/2\mathbb{Z})^{\oplus 2}$ (resp. $H^1(k, \text{Pic } \overline{X}) \simeq \mathbb{Z}/2\mathbb{Z}$, $H^1(k, \text{Pic } \overline{X}) = 0$). Among 6079 cases of 5-dimensional algebraic k -tori T , there exist exactly 11 (resp. 263, 5805) cases with $H^1(k, \text{Pic } \overline{X}) \simeq (\mathbb{Z}/2\mathbb{Z})^{\oplus 2}$ (resp. $H^1(k, \text{Pic } \overline{X}) \simeq \mathbb{Z}/2\mathbb{Z}$, $H^1(k, \text{Pic } \overline{X}) = 0$). Second, we determine $H^1(k, \text{Pic } \overline{X})$ for norm one tori $T = R_{K/k}^{(1)}(\mathbb{G}_m)$ with $[K : k] = n \leq 15$. We also show that $H^1(k, \text{Pic } \overline{X}) = 0$ for the 5 Mathieu groups $M_n \leq S_n$. Third, we give a necessary and sufficient condition for the Hasse norm principle for K/k with $[K : k] = n \leq 15$. As applications of the results, we get the group $T(k)/R$ of R -equivalence classes over a local field k and the Tamagawa number $\tau(T)$ over a number field k . This is a joint work with Akinari Hoshi and Kazuki Kanai (arXiv:1910.01469).

11:30 – 12:30 **Takehiko Yasuda**^{*} (Osaka University)

Local étale fundamental groups of log terminal surface singularities

The local étale fundamental group of a singular point of an algebraic variety controls local Galois covers that are quasi-étale (equivalently, étale in codimension one). The main result of this talk is that the local étale fundamental group of a log terminal surface singularity is finite in

arbitrary characteristic. Log terminal singularities form an important class of singularities that naturally appears in various contexts, especially in the minimal model program. The main result has been known to hold in characteristic greater than 3. In characteristics 2 and 3, the problem becomes considerably more difficult, because of wild ramification. The strategy of our proof is to study the behavior of stringy motives under quasi-étale Galois covers. This is a joint work with Javier Carvajal-Rojas.

December 16 (Thu)

10:00 – 10:50 **Kimihiko Li**^z (The University of Tokyo)
Prismatic and q -crystalline sites of higher level

Two new p -adic cohomology theories, called prismatic cohomology and q -crystalline cohomology, were defined for generalizing crystalline cohomology and they recover most known integral p -adic cohomology theories. On the other hand, higher level crystalline cohomology was defined for constructing p -adic cohomology theory over a ramified base. In this talk, for a positive integer m , we will give a construction of the level m prismatic and q -crystalline sites and prove a certain equivalence between the category of crystals on the m -prismatic site or the m - q -crystalline site and that on the usual prismatic site or the usual q -crystalline site, which can be regarded as the prismatic analogue of the Frobenius descent. We will also prove the equivalence between the category of crystals on the m -prismatic site and that on the $(m - 1)$ - q -crystalline site.

11:10 – 12:00 **Alex Youcis** (The University of Tokyo)
A specialization map from the de Jong fundamental group to the pro-étale fundamental group

For a normal, locally topologically Noetherian scheme X essentially all interesting algebro-geometric covering spaces are captured by the classical étale fundamental group $\pi_1^{\text{ét}}(X)$. In contrast, even simple smooth rigid analytic varieties \mathcal{X} (e.g. the unit disk) admit many connected infinite-dimensional ‘covering spaces’ (e.g. the period maps of Rapoport–Zink spaces, as well as Tate’s uniformization of elliptic curves). For this reason de Jong introduced an enlarged version $\pi_1^{\text{dJ}}(\mathcal{X})$ of the étale fundamental group $\pi_1^{\text{ét}}(\mathcal{X})$ to account for these spaces. On the other hand, for a non-normal, locally topologically Noetherian scheme X one often has a plethora of covering spaces also not captured by $\pi_1^{\text{ét}}(X)$. To remedy this, Bhatt and Scholze also introduce an enlargement $\pi_1^{\text{proét}}(X)$ of $\pi_1^{\text{ét}}(X)$. In this talk I will discuss recent work of Achinger, Lara, and myself which connects these two ostensibly disparate groups by showing the existence of a *specialization map* $\pi_1^{\text{dJ}}(\mathfrak{X}_\eta) \rightarrow \pi_1^{\text{proét}}(\mathfrak{X}_s)$ for a formal scheme $\mathfrak{X}/\mathcal{O}_K$ where K is a non-archimedean field.

13:30 – 14:20 **Norihiko Minami**^z (Nagoya Institute of Technology)
Retract $(-i)$ rationality and its necessary conditions expressed by certain cohomology theories - Noether’s problem of a finite group G over an algebraic closed field, as an example

First, I shall introduce various algebro-geometric hierarchies in the spirit of lower rationality = higher ruledness, and pay a special attention to the following orthogonal hierarchical relation (where the case $i = 0$ corresponds to the usual hierarchical relation):

$(-i)$ -rational \implies stable $(-i)$ -rational $\implies \implies$ retract $(-i)$ -rational

The main technical purpose of this talk is to two types of necessary conditions for retract $(-i)$ -rationality, represented by appropriate cohomology theory.

The first type is a condition expressed by the unramified subgroup of Rost’s cycle module, Its advantage is that we can avoid the usual “smooth proper” assumption. In particular, if we take the Galois cohomology as Rost’s cycle module, and suppose the base field has finite cohomological dimension, we obtain a user-friendly necessary condition for retract $(-i)$ -rationality.

For instance, if we apply this necessary condition to the “classifying space” BG of a finite group G (which was defined as an ind-variety consisting of non proper varieties) assuming the base field

is algebraic closed, we can upgrade various counter-examples of the Noether problem of finite groups obtained ever since Saltman.

The motivation of the second type is to provide such a necessary condition as in the first type to much wider class of cohomology theories, including at least the unramified subgroup of Rost's cycle module treated in the first type. A clue for this is supplied by Déglise's equivalence between the category of Rost's cycle modules and the category of homotopy invariant Nisnevich sheaves with transfer, in the sense of Voevodsky. Now the second type condition is applicable to the class of \mathbb{P}^1 -invariant Nisnevich sheaves with transfers, which is much wider than the class of the homotopy invariant Nisnevich sheaves with transfers.

As a trade-off, I must impose the usual "smooth proper" assumption (though this condition is not essential in the characteristic 0 case, thanks to Professor Hironaka). Yet, when the second type is restricted to the homotopy invariant Nisnevich sheaves with transfers as in the first type (of course under the smooth proper assumption), the same necessary condition as in the first type is returned. Furthermore, the second type allows us to incorporate the decomposition of the diagonal technique of Bloch-Srinivas.

However, the principal message I would like to deliver in this talk is the fundamental change of our attitude to the (retract) rationality problem prompted by the hierarchical point of view rather than such technical results I alluded above.

In the past research, if the base field is \mathbb{C} then as soon as $H_{nr}^i(X, A) \neq 0$ was found for some $i > 0$, the retract rationality problem of X was finished. However, our necessary condition implies it had only shown the non retract $-(i - 1)$ -rationality of X , and it is nothing but a prelude of the next problem: Is X retract $(-i)$ -rational?

14:40 – 15:30 **Shin-ya Koyama**^z (Toyo University)
Chebyshev's bias with its generalizations

We unravel the mystery of Chebyshev's bias in terms of zeta-parametrization of the counting function of primes. As applications we obtain newly discovered biases. For example, we find the biased distributions of 1) non-principal prime ideals in number fields of class number two, 2) non-splitting primes in certain abelian extensions. (Joint work with Miho Aoki)

15:50 – 16:50 **Wataru Kai**^{*} (Tohoku University)
The Green-Tao theorem for number fields

Based on joint work with Masato Mimura, Akihiro Munemasa, Shin-ichiro Seki and Kiyoto Yoshino. 5, 11, 17, 23 and 29 are prime numbers that form an arithmetic progression of length 5. A famous theorem of Green and Tao says there are arbitrarily long arithmetic progressions of prime numbers. Less widely known, Tao proved something similar for prime elements of the ring of Gaussian integers $\mathbb{Z}[i]$. I discuss our generalization of their results to arbitrary number fields.

December 17 (Fri)

10:00 – 11:00 **Takamichi Sano**^{*} (Osaka City University)
Recent progress on Hilbert's 12th problem

Dasgupta and Kakde have recently obtained an explicit construction of the maximal abelian extension of a totally real field. I will explain their result.

11:20 – 12:10 **Mahiro Atsuta** (Keio University)
The minus component of the equivariant Tamagawa number conjecture for \mathbb{G}_m

In number theory, there are a lot of conjectures which describe the relationship between algebraic objects like class groups and the special values of L -functions. Among those conjectures is the equivariant Tamagawa number conjecture (eTNC), which states a strong relationship involving Galois actions. In this talk, we explain that the minus component of the eTNC for \mathbb{G}_m and for CM abelian extensions holds under mild technical hypotheses. The key idea of the proof

is to extend the method of group ring valued modular forms which was established by Dasgupta and Kakde to prove the Brumer-Stark conjecture. This is joint work with Takenori Kataoka.

14:00 – 14:50 **Katsuyuki Bando**^z (The University of Tokyo)

Geometric Satake equivalence in mixed characteristic and Springer correspondence

The geometric Satake correspondence is an equivalence between the category of equivariant perverse sheaves on the affine Grassmannian and the category of representations of the Langlands dual group. It is known that there is a mixed characteristic version of the geometric Satake correspondence. The Springer correspondence is a correspondence between the category of equivariant perverse sheaves on the nilpotent cone and the category of representation of the Weyl group. In this talk, we will explain some relation between these two correspondences, including the mixed characteristic case.

15:10 – 16:00 **Yoichi Mieda** (The University of Tokyo)

On supercuspidal part of the ℓ -adic cohomology of the Rapoport-Zink space for $\mathrm{GSp}(4)$

Rapoport-Zink spaces are p -adic local counterparts of Shimura varieties. Their ℓ -adic cohomology are expected to be strongly related with the local Langlands correspondence. In this talk, I will explain how the supercuspidal part of the ℓ -adic cohomology of the Rapoport-Zink space for $\mathrm{GSp}(4)$ is described by using the local Langlands correspondence and the local Arthur classification. The recent result of Scholze and Fargues is crucial to this work. If time permits, I will also give some perspective to the case of $\mathrm{GSp}(2n)$ with $n \geq 3$.

* Invited speakers, ^z Online

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代数的整数論とその周辺
Algebraic Number Theory and Related Topics
RIMS 共同研究 (公開型) 報告集

2021 年 12 月 13 日～12 月 17 日
研究代表者 小林 真一 (Shinichi Kobayashi)

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