On Leonard Control applied to Mine-Hoist.

By

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The present paper undertakes to consider the load diagram, the ampere rating, and the theoretical durations of acceleration and retardation of a D. C. mine-hoist motor controlled by the Leonard system, without entering upon a discussion of the equalizing apparatus.

Mr. Wilfred Sykes' paper[1] has already treated these problems in detail; but, in the rating of the motor, the copper loss only was taken into account, and, to the relation between the time of acceleration and that of retardation, no consideration was given.

The object of this paper is to give decisions on these points, and it is to be regretted that the results, generally, are not of simple form, except in case of the cylindrical drum hoist with tail rope.

Of various classes, the reel hoist may be taken as representative; and any other may be treated as a particular case.

Notation used by the writer:

N = mass of ore to be hoisted.

C = mass of cage, tubs, plus the mass of pulley reduced to its periphery, in one moving side.

m = mass of rope of unit length.

h = depth of the vertical shaft.

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1) "Large Electric Hoisting Plant," A. I. E. E. Vol. XXIX, part I; p. 291 (1910).

r_1 = radius of drum at the beginning of hoisting.

r_2 = radius of drum at the end of hoisting.

r_x = radius of winding at the instant of consideration, in the ascending side.

R_x = radius of winding at the instant of consideration, in the descending side.

δ = increment of winding radius per turn of drum.

T = number of revolutions of the drum.

T_e = number of revolutions of the motor.

n_m' = maximum revolutions of the drum per unit time.

n_m = maximum revolutions of the motor per unit time.

η = efficiency of the drum, gear inclusive.

i = ratio of gear.

M_s' = statical moment at the axis of the drum.

M_a' = accelerating moment at the axis of the drum.

M_v' = retarding moment at the axis of the drum.

M_s = statical moment at the axis of the motor.

M_a = accelerating moment at the axis of the motor.

M_v = retarding moment at the axis of the motor.

M'_{sT} = resultant moment during full speed running, at the axis of the drum.

M'_{aT} = resultant moment during acceleration, at the axis of the drum.

M'_{vT} = resultant moment during retardation, at the axis of the drum.

M_{sT} = resultant moment during full speed running, at the axis of the motor.

M_{aT} = resultant moment during acceleration, at the axis of the motor.

M_{vT} = resultant moment during retardation, at the axis of the motor.

W_a = instantaneous output of the motor during acceleration.

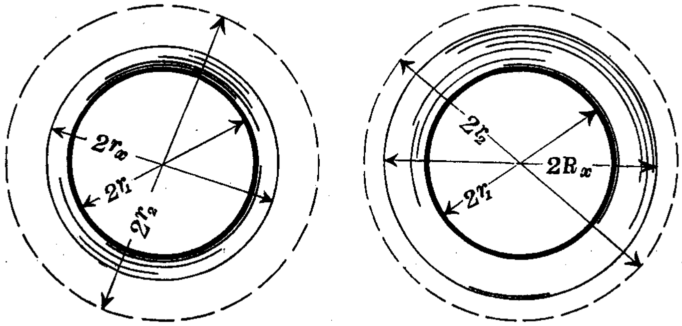
W_s = instantaneous output of the motor during full speed running.

W_v = instantaneous output of the motor during retardation.

G = total input energy to the motor per winding.

As units, kilogram, meter and second are used.

REEL HOIST.

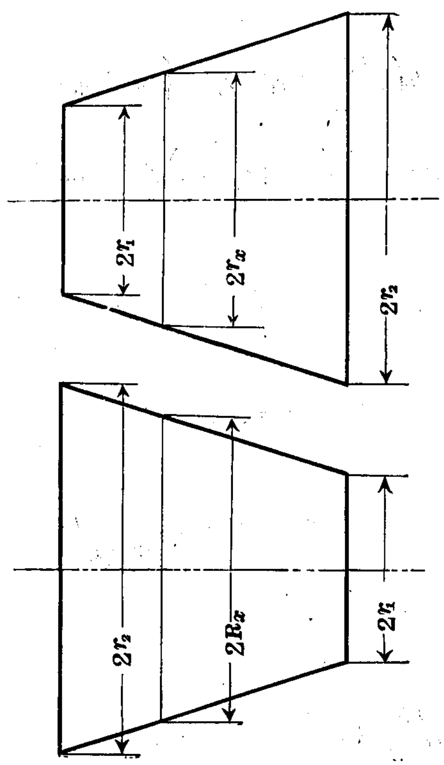


Winding drum. Relieving drum.

Fig. 1.

CONICAL HOIST.

Fig. 2.



Winding drum.

Relieving drum.

I. Resisting Moment.

Resultant resisting moment is composed of:

(1) statical moment,

(2) accelerating or retarding moment.

(1) Statical Moment.

$$M_s' = \{N + C + mh - \pi(r_1 + r_x)Tm\}gr_x - \{C + \pi(r_2 + R_x)Tm\}gR_x,$$

and

$$r_x = r_1 + \delta T,$$

$$R_x = r_2 - \delta T,$$

$$\pi(r_2^2 - r_1^2) = \delta h,$$

then

$$\left.\begin{aligned} M'_s = {} & (Ng + mhg)r_1 - Cg\left(\sqrt{\frac{\delta h}{\pi} + r_1^2} - r_1\right) - \left\{4\pi r_1^2 mg - \frac{mg(2-\sigma h)\delta}{\sigma}\right\}T \\ & + 3\pi mg\delta\left(\sqrt{\frac{\delta h}{\pi} + r_1^2} - r_1\right)T^2 - 2\pi mg\delta^2 T^3 \\ \text{where} \quad & \sigma = 2\frac{m}{N+2C} \end{aligned}\right\} \quad (1).$$

Now, investigating the curve expressed by equation (1), the values of T satisfying $\frac{dM_s'}{dT} = 0$, give a maximum and a minimum to the statical moment.

$$\frac{dM's}{dT} = -4\pi mgr_1^2 + \frac{mg(2-\sigma h)\delta}{\sigma} + 6\pi mg\delta\left(\sqrt{\frac{\delta h}{\pi} + r_1^2} - r_1\right)T$$
$$- 6\pi mg\delta^2 T^2.$$

Put

$$6\pi\delta^2 T_m^2 - 6\pi\delta\left(\sqrt{\frac{\delta h}{\pi} + r_1^2} - r_1\right)T_m + 4\pi r_1^2 - \frac{(2-\sigma h)\delta}{\sigma} = 0.$$

Solving this,

$$T_m = \frac{1}{2\delta}\left\{\left(\sqrt{\frac{\delta h}{\pi}+r_1^2}-r_1\right)\pm\sqrt{\left(\sqrt{\frac{\delta h}{\pi}+r_1^2}-r_1\right)^2-\tfrac{2}{3}\left(4r_1^2-\frac{(2-\sigma h)\delta}{\sigma\pi}\right)}\right\}.$$

Let T_t be the total number of turn of rope per winding,

$$T_m = \tfrac{1}{2}T_t\left\{1\pm\sqrt{1-\tfrac{2}{3}\frac{4r_1^2-\frac{2-\sigma h}{\pi\sigma}\delta}{\left(\sqrt{\frac{\delta h}{\pi}+r_1^2}-r_1\right)^2}}\right\} \qquad (2).$$

These values of T give the maximum and the minimum value to the statical moment.

Now,

$$\frac{d^2M'_s}{dT^2} = 12\pi mg\delta^2 T - 6\pi mg\delta\left(\sqrt{\frac{\delta h}{\pi}+r_1^2}-r_1\right).$$

Putting

$$2\delta T_w-\left(\sqrt{\frac{\delta h}{\pi}+r_1^2}-r_1\right) = 0,$$

we have

$$T_w = \tfrac{1}{2}\frac{\sqrt{\frac{\delta h}{\pi}+r_1^2}-r_1}{\delta}$$

$$= \tfrac{1}{2}T_t \qquad (3).$$

At the point $T = \frac{1}{2}T_t$, the curve has a point of inflexion. Fig. 3 shows the statical moment curve.

The statical moment at T = 0,

$$M'_{ss} = (Ng+mhg)r_1 - Cg\left(\sqrt{\frac{\delta h}{\pi}+r_1^2}-r_1\right).$$

At $T = T_t$,

$$M'_{se} = Ng\sqrt{\frac{\delta h}{\pi}+r_1^2}-mhgr_1+Cg\left(\sqrt{\frac{\delta h}{\pi}+r_1^2}-r_1\right).$$

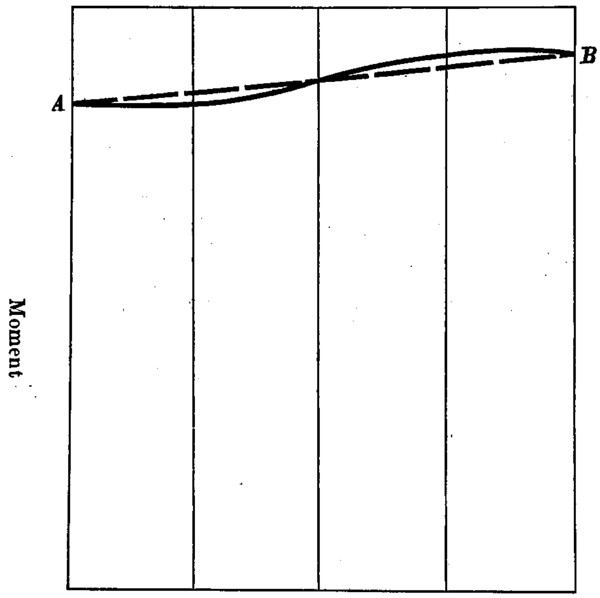


Fig. 3.

The statical moment has a maximum at

$$T = \tfrac{1}{2}T_t\left\{1 + \sqrt{1 - \tfrac{2}{3}\frac{4r_1^2 - \dfrac{2-\sigma h}{\pi\sigma}\delta}{\left(\sqrt{\dfrac{\delta h}{\pi} + r_1^2} - r_1\right)^2}}\right\},$$

and a minimum at

$$T = \tfrac{1}{2}T_t\left\{1 - \sqrt{1 - \tfrac{2}{3}\frac{4r_1^2 - \dfrac{2-\sigma h}{\pi\sigma}\delta}{\left(\sqrt{\dfrac{\delta h}{\pi} + r_1^2} - r_1\right)^2}}\right\},$$

and a point of inflexion at

$$T = \tfrac{1}{2}T_t.$$

But such a curve as contains a higher order of T is not convenient for practical treatment, so that we reform the statical moment curve to a straight line.

Now, take a straight line joining the point A to the point B:

$$M_s' = (Ng+mhg)r_1 - Cg\left(\sqrt{\frac{\delta h}{\pi}+r_1^2}-r_1\right)$$
$$+\frac{(Ng+2Cg)\left(\sqrt{\frac{\delta h}{\pi}+r_1^2}-r_1\right)-2mhgr_1}{T_t}T \qquad (4).$$

The discrepancy between the curve (1) and the curve (4) is very slight compared with the starting statical moment.

From numerous examples, we know that the curve (4) may be substituted for the curve (1) with a error not exceeding a few per cent.

For a conical drum hoist M_{ss}' is to be equal to M_{se}':

$$(Ng+Cg+mhg)r_1 - Cgr_2 = (Ng+Cg)r_2 - (Cg+mhg)r_1.$$

From this,

$$r_2 = \left(1+\frac{2mg}{Ng+2Cg}h\right)r_1$$
$$= (1+\sigma h)r_1 .$$

The statical moment for a conical drum hoist

$$M_s' = (Ng+mhg)r_1 - Cg\sigma hr_1 - \pi mg(\sigma h)^2 r_1^2 T$$
$$+3\pi^2 mg(2+\sigma h)\sigma^2 hr_1^3 T^2 - 2\pi^3 mg(2+\sigma h)^2\sigma^2 r_1^4 T^3 \qquad (1)'.$$

And

$$T_m = \tfrac{1}{2}\left(1\pm\frac{1}{\sqrt{3}}\right)T_t \qquad (2)',$$

$$T_w = \tfrac{1}{2}T_t \qquad (3)'.$$

In this case, the statical moment may be represented by a straight line parallel to the X-axis.

For the cylindrical drum hoist, the value of δ is zero.

Generally the statical moment would be

$$M_s' = P' + Q'T \qquad [6].$$

The value of P':

$$\left.\begin{array}{ll}\text{for reel hoist, } \dots\dots & (Ng+mhg)r_1-Cg\left(\sqrt{\dfrac{\delta h}{\pi}+r_1^2}-r_1\right)\\ \text{for conical hoist, } \dots\dots & (Ng+mhg)r_1-Cg\sigma hr_1\\ \text{for cylindrical hoist, without tail rope, } \dots\dots & (Ng+mhg)r_1\\ \text{for cylindrical hoist, with tail rope, } \dots\dots & Ngr_1\end{array}\right\} \quad [7].$$

The value of Q':

$$\left.\begin{array}{ll}\text{for reel hoist} \dots\dots & \dfrac{(Ng+2Cg)\left(\sqrt{\dfrac{\delta h}{\pi}+r_1^2}-r_1\right)-2mhgr_1}{T_t}\\ \text{for conical hoist, } \dots\dots & 0\\ \text{for cylindrical hoist, without tail rope, } \dots\dots & -4\pi mgr_1^2\\ \text{for cylindrical hoist, with tail rope, } \dots\dots & 0\end{array}\right\} \quad [8].$$

(2) Accelerating or retarding moment.

Total accelerating or retarding moment is composed of:

1 moment to accelerate or retard the rope
- (a) wound on drum,
- (b) suspended from drum.

2 moment to accelerate or retard the ore, cage, tubs, pulley &c.

3 moment to accelerate or retard the drum.

(A) *Accelerating moment.*

1. Moment to accelerate the rope.

When a rope whose total mass is m_d wound on a drum of radius r_1, to a depth d, the moment of inertia of the rope is

$$m_d \frac{r_1^2+(r_1+d)^2}{2}.$$

In our case, the mass of rope wound on the winding drum at any instant is

$$m_w = \pi(2r_1+\delta T)Tm.$$

That on the relieving drum,

$$m'_w = \{h-\pi(2r_2-\delta T)T\}m.$$

Mass of rope suspended on the winding drum,

$$m_s = \{h-\pi(2r_1+\delta T)T\}m.$$

That on the relieving drum,

$$m'_s = \pi(2r_2-\delta T)Tm.$$

From these relations,

$$m_w+m'_w = \{h-2\pi(r_2-r_1)T+2\pi\delta T^2\}m,$$

$$m_s+m'_s = \{h+2\pi(r_2-r_1)T-2\pi\delta T^2\}m,$$

$$m_s+m'_s+m_w+m'_w = 2mh.$$

(a) Accelerating moment due to rope wound on drum is

$$2\pi m_w\frac{r_1^2+r_x^2}{2}\frac{d^2T}{dt^2}+2\pi m_w'\frac{r_1^2+R_x^2}{2}\frac{d^2T}{dt^2}.$$

(b) Accelerating moment due to rope suspended on drum.

The linear acceleration of rope on the winding side is

$$2\pi\left(r_x\frac{d^2T}{dt^2}+\frac{dT}{dt}\frac{dr_x}{dt}\right) = 2\pi\left(r_x\frac{d^2T}{dt^2}+\delta\left(\frac{dT}{dt}\right)^2\right).$$

The linear acceleration of rope on the relieving side is

$$2\pi\left(R_x\frac{d^2T}{dt^2}+\frac{dT}{dt}\frac{dR_x}{dt}\right) = 2\pi\left(R_x\frac{d^2T}{dt^2}-\delta\left(\frac{dT}{dt}\right)^2\right).$$

Therefore the accelerating moment due to suspended rope is

$$2\pi m_s r_x^2\frac{d^2T}{dt^2}+2\pi m_s\delta r_x\left(\frac{dT}{dt}\right)^2+2\pi m'_s R_x^2\frac{d^2T}{dt^2}-2\pi m'_s\delta R_x\left(\frac{dT}{dt}\right)^2.$$

Total accelerating moment due to rope,

$$M'_{ar} = 2\pi m_w \frac{r_1^2+r_x^2}{2}\frac{d^2T}{dt^2} + 2\pi m'_w \frac{r_1^2+R_x^2}{2}\frac{d^2T}{dt^2}$$

$$+ 2\pi m_s r_x^2 \frac{d^2T}{dt^2} + 2\pi m_s \delta r_x \left(\frac{dT}{dt}\right)^2$$

$$+ 2\pi m'_s R_x^2 \frac{d^2T}{dt^2} - 2\pi m_s' \delta R_x \left(\frac{dT}{dt}\right)^2.$$

Substituting values of m_w, m'_w, m_s, m'_s, and $R_x = r_1 + r_2 - r_x$, $\delta h = \pi(r_2^2 - r_1^2)$,

$$M'_{ar} = (4\pi m h r_1^2 + \delta m h^2)\frac{d^2T}{dt^2} + 2\pi\delta m h r_1 \left(\frac{dT}{dt}\right)^2$$

$$+ 4\pi\delta m h r_1 T \frac{d^2T}{dt^2} - (2\pi\delta^2 m h + 8\pi^2\delta m r_1^2)\, T\left(\frac{dT}{dt}\right)^2$$

$$-(2\pi\delta^2 m h + 8\pi^2\delta m r_1^2)\, T^2\frac{d^2T}{dt^2} + 6\pi^2\left(\sqrt{\frac{\delta h}{\pi} + r_1^2} - r_1\right)\delta^2 m T^2\left(\frac{dT}{dt}\right)_2$$

$$+ 4\pi^2\left(\sqrt{\frac{\delta h}{\pi} + r_1^2} - r_1\right)\delta^2 m T^3 \frac{d^2T}{dt^2} - 4\pi^2\delta^3 m T^3\left(\frac{dT}{dt}\right)^2$$

$$-2\pi^3\delta^3 m T^4 \frac{d^2T}{dt^2} \qquad (9).$$

Practically, δ is of small value, so that the last four terms may be neglected for practical purposes; then

$$M'_{ar} = (4\pi m h r_1^2 + \delta m h^2)\frac{d^2T}{dt^2} + 2\pi\delta m h r_1\left(\frac{dT}{dt}\right)^2$$

$$+ 4\pi\delta m h r_1 T\frac{d^2T}{dt^2} - (2\pi\delta^2 m h + 8\pi^2\delta m r_1^2)\, T\left(\frac{dT}{dt}\right)^2$$

$$-(2\pi\delta^2 m h + 8\pi^2\delta m r_1^2)\, T^2\frac{d^2T}{dt^2} \qquad (10).$$

For conical drum,

$$M'_{ar} = \left(4\pi m h r_1^2 + \pi(2+\sigma h)\sigma m h^2 r_1^2\right)\frac{d^2T}{dt^2} + 2\pi^2(2+\sigma h)\sigma m h r_1^3\left(\frac{dT}{dt}\right)^2$$

$$+ 4\pi^2(2+\sigma h)\sigma m h r_1^3 T\frac{d^2T}{dt^2} - 2\pi^3(8 + 8\sigma h + 4\sigma^2 h^2 + \sigma^3 h^3)\sigma m r_1^4 T\left(\frac{dT}{dt}\right)^2$$

$$-2\pi^3(8+8\sigma h+4\sigma^2h^2+\sigma^3h^3)\sigma m r_1^4 T^2\frac{d^2T}{dt^2} \qquad (10)'.$$

Equation (10′) for a conical drum holds more accurately than equation (10) for a reel drum, on account of the smaller value of δ in the former than in the latter case.

2. Accelerating moment due to load, cage, tubs, pulley &c.

$$M'_{ae} = 2\pi(N+C)r_x\left\{r_x\frac{d^2T}{dt^2}+\delta\left(\frac{dT}{dt}\right)^2\right\}+2\pi CR_x\left\{R_x\frac{d^2T}{dt^2}-\delta\left(\frac{dT}{dt}\right)^2\right\}$$

$$= 2\pi\{Nr_1^2+C(r_2^2+r_1^2)\}\frac{d^2T}{dt^2}+2\pi\delta\{Nr_1-C(r_2-r_1)\}\left(\frac{dT}{dt}\right)^2$$

$$+4\pi\delta\{Nr_1-C(r_2-r_1)\}T\frac{d^2T}{dt^2}+2\pi\delta^2(N+2C)T\left(\frac{dT}{dT}\right)^2$$

$$+2\pi\delta^2(N+2C)T^2\frac{d^2T}{dt^2}.$$

And $r_2 = \sqrt{\frac{\delta h}{\pi}+r_1^2}$, therefore,

$$M'_{ae} = 2\pi\left\{(N+2C)r_1^2+\frac{\delta h}{\pi}C\right\}\frac{d^2T}{dt^2}+2\pi\delta\left\{(N+C)r_1-C\sqrt{\frac{\delta h}{\pi}+r_1^2}\right\}\left(\frac{dT}{dt}\right)^2$$

$$+4\pi\delta\left\{(N+C)r_1-C\sqrt{\frac{\delta h}{\pi}+r_1^2}\right\}T\frac{d^2T}{dt^2}+2\pi\delta^2(N+2C)T\left(\frac{dT}{dt}\right)^2$$

$$+2\pi\delta^2(N+2C)T^2\frac{d^2T}{dt^2} \qquad (11).$$

For conical drum,

$$M'_{ae} = 2\pi r_1^2\{(N+2C)+(2+\sigma h)\sigma hC\}\frac{d^2T}{dt^2}$$

$$+2\pi^2r_1^3(2+\sigma h)\sigma(N-\sigma hC)\left(\frac{dT}{dt}\right)^2$$

$$+4\pi^2r_1^3(2+\sigma h)\sigma(N-\sigma hC)T\frac{d^2T}{dt^2}+2\pi^3r_1^4(2+\sigma h)^2\sigma^2(N+2C)T\left(\frac{dT}{dt}\right)^2$$

$$+2\pi^3r_1^4(2+\sigma h)^2\sigma^2(N+2C)T^2\frac{d^2T}{dt^2}.$$

3. Accelerating moment due to drum and motor armature.

Let I_d be the resultant moment of inertia at the axis of the drum,

$$M'_{am} = 2\pi I_d \frac{d^2T}{dt^2} \qquad (12).$$

Total accelerating moment,

$$\begin{aligned} M'_a &= M'_{ar} + M'_{ae} + M'_{am} \\ &= 2\pi\left\{I_d + (N+2C+2mh)r_1^2 + \frac{\delta h}{\pi}\left(C + m\frac{h}{2}\right)\right\}\frac{d^2T}{dt^2} \\ &\quad + 2\pi\delta\left\{(N+C+mh)r_1 - C\sqrt{\frac{\delta h}{\pi} + r_1^2}\right\}\left(\frac{dT}{dt}\right)^2 \\ &\quad + 4\pi\delta\left\{(N+C+mh)r_1 - C\sqrt{\frac{\delta h}{\pi} + r_1^2}\right\}T\frac{d^2T}{dt^2} \\ &\quad + 2\pi\delta\left\{\delta(N+2C-mh) - 4\pi m r_1^2\right\}T\left(\frac{dT}{dt}\right)^2 \\ &\quad + 2\pi\delta\left\{\delta(N+2C-mh) - 4\pi m r_1^2\right\}T^2\frac{d^2T}{dt^2}. \end{aligned}$$

$$M'_a = a_d\frac{d^2T}{dt^2} + 2b_d\left(\frac{dT}{dt}\right)^2 + 4b_d T\frac{d^2T}{dt^2} + 2c_d T\left(\frac{dT}{dt}\right)^2 + 2c_d T^2\frac{d^2T}{dt^2} \qquad [13],$$

where

$$\left.\begin{aligned} a_d &= 2\pi\left\{I_d + (N+2C+2mh)r_1^2 + \frac{\delta h}{\pi}\left(C + m\frac{h}{2}\right)\right\} \\ b_d &= \pi\delta\left\{(N+C+mh)r_1 - C\sqrt{\frac{\delta h}{\pi} + r_1^2}\right\} \\ c_d &= \pi\delta\{\delta(N+2C-mh) - 4\pi m r_1^2\} \end{aligned}\right\} \qquad [14].$$

For conical drum,

$$\left.\begin{aligned} a_d &= 2\pi\{I_d + (N+2C+2mh)r_1^2\} \\ &\qquad + \pi(2+\sigma h)\sigma h(2C+mh)r_1^2 \end{aligned}\right\} \qquad [15].$$

$$\left.\begin{aligned} b_d &= \pi^2(2+\sigma h)^2(m-\sigma C)r_1^3 \\ c_d &= -\pi^3\sigma^3 m h^2(2+\sigma h)r_1^4 \end{aligned}\right\}$$

For cylindrical drum without tail rope,

$$\left.\begin{aligned} a_d &= 2\pi\{I_d+(N+2C+2mh)r_1^2\} \\ b_d &= 0 \\ c_d &= 0 \end{aligned}\right\} \quad [16].$$

For cylindrical drum with tail rope,

$$\left.\begin{aligned} a_d &= 2\pi\{I_d+(N+2C+3mh)r_1^2\} \\ b_d &= 0 \\ c_d &= 0 \end{aligned}\right\} \quad [17].$$

From equations [13], [14], [15], [16], and [17], the accelerating moment for any kind of hoist is easily constructed.

In the case of a cylindrical drum, during full speed running, there is no accelerating moment; but in the cases of reel and conical drums, after the drum reached the full speed of running, the load and a part of the rope are in the state of a slight acceleration due to the increase of winding radius. The moment of this kind is

$$M'_{as} = 2\pi\delta\left(\frac{dT}{dt}\right)^2\left[(N+mh)r_1 - C\left(\sqrt{\frac{\delta h}{\pi}+r_1^2}-r_1\right) - \left\{4\pi m r_1^2 - \frac{m(2-\sigma h)\delta}{\sigma}\right\}T \right.$$
$$\left. + 3\pi\delta m\left(\sqrt{\frac{\delta h}{\pi}+r_1^2}-r_1\right)T^2 - 2\pi\delta^2 m T^3\right].$$

This is of the same form as the statical moment, and can be obtained by substituting $2\pi\delta n'^2_m$ for g in equation (1), and similarly treated; then

$$M'_{as} = \frac{2\pi\delta n'^2_m}{g}\{P'+Q'T\}$$

Putting

$$\theta = \frac{2\pi\delta n'^2_m}{g},$$

we have

$$M'_{as} = \theta(P' + Q'T) \qquad [18].$$

(B) Retarding moment.

The retarding moment can be easily determined by substituting $T_t + T'$ for T in (13), where

$$T' = \int_{t_0}^{t} n'dt,$$

n'......number of revolutions of drum per unit time, at any instant during retardation,

t ......time at the instant of consideration after the beginning of retardation,

t_v ......time of retardation.

In equation (9), put

$$\left.\begin{aligned} d_d &= \pi^2\left(\sqrt{\frac{\delta h}{\pi} + r_1^2} - r_1\right)\delta^2 m \\ e_d &= \pi^2\delta^3 m \end{aligned}\right\} \qquad [19].$$

Then

$$d_d = e_d T_t .$$

For conical drum,

$$\left.\begin{aligned} d_d &= \pi^4 r_1^5(2+\sigma h)^2\sigma^3 mh \\ e_d &= \pi^5 r_1^6(2+\sigma h)^3\sigma^3 m \\ c_d &= -d_d T_t = -e_d T_t^2 \end{aligned}\right\} \qquad [20].$$

Retarding moment

$$M'_v = a_d\frac{d^2T'}{dt^2} + 2b_d\left(\frac{dT'}{dt}\right)^2 + 4b_d(T_t + T')\frac{d^2T'}{dt^2} + 2c_d(T_t + T')\left(\frac{dT'}{dt}\right)^2$$

$$+2c_d(T_t+T')^2\frac{d^2T'}{dt^2}+6d_d(T_t+T')^2\left(\frac{dT'}{dt}\right)^2+4d_d(T_t+T')^3\frac{d^2T'}{dt^2}$$

$$-4e_d(T_t+T')^3\left(\frac{dT'}{dt}\right)^2-2e_d(T_t+T')^4\frac{d^2T'}{dt^2}\ .$$

Neglecting terms containing d_d (not d_dT_t), we have

$$M'_v = \{a_d+4b_dT_t+2c_dT_t^2+2d_dT_t^3\}\frac{d^2T'}{dt^2}$$

$$+2\{b_d+c_dT_t+d_dT_t^2\}\left(\frac{dT'}{dt}\right)^2$$

$$+4(b_d+c_dT_t+d_dT_t^2)T'\frac{d^2T'}{dt^2}$$

$$+2c_dT'\left(\frac{dT'}{dt}\right)^2+2c_dT'^2\frac{d^2T'}{dt^2}\ .$$

$$M'_v = a'_d\frac{d^2T'}{dt^2}+2b'_d\left(\frac{dT'}{dt}\right)^2+4b'_dT'\frac{d^2T'}{dt^2}$$

$$+2c_dT'\left(\frac{dT'}{dt}\right)^2+2c_dT'^2\frac{d^2T'}{dt^2} \qquad [21],$$

$$\left.\begin{aligned} a'_d &= a_d+4b_dT_t+2c_dT_t^2+2d_dT_t^3 \\ b'_d &= b_d+c_dT_t+d_dT_t^2 \end{aligned}\right\} \qquad [22].$$

For conical drum,

$$\left.\begin{aligned} a'_d &= a_d+4b_dT_t\ . \\ b'_d &= b_d \end{aligned}\right\} \qquad [23].$$

For cylindrical drum,

$$\left.\begin{aligned} a'_d &= a_d \\ b'_d &= 0 \end{aligned}\right\} \qquad [24].$$

The retarding moment is determined from eqnations [19], [20], [21], [22], [23], and [24].

Resultant resisting moment.

(a) During acceleration,

$$M'_{aT} = a_a\frac{d^2T}{dt^2} + 2b_a\left(\frac{dT}{dt}\right)^2 + 4b_aT\frac{d^2T}{dt^2} + 2c_aT\left(\frac{dT}{dt}\right)^2 + 2c_aT^2\frac{d^2T}{dt^2}$$
$$+ P' + Q'T \qquad (25).$$

(b) During full speed running,

$$M'_{sT} = P' + Q'T + \theta(P' + Q'T) \qquad (26).$$

(c) During retardation,

$$M'_{vT} = a'_a\frac{d^2T'}{dt^2} + 2b'_a\left(\frac{dT'}{dt}\right)^2 + 4b'_aT'\frac{d^2T'}{dt} + 2c_aT'\left(\frac{dT'}{dt}\right)^2 + 2c_aT'^2\frac{d^2T'}{dt^2}$$
$$+ P' + Q'T \qquad (27).$$

If the mode of acceleration and retardation, that is, the relation between T or T' and time be given, from equations (25), (26), and (27), the resisting moment curve can be readily constructed. The maximum revolution per unit time of the drum, as well as the maximum acceleration or retardation, is restricted from the mechanical point of view.

When the full speed r. p. m. and T or T' curve be fixed, the time of acceleration or retardation can be determined.

Let the ratio of the gear and the efficiency of the drum be i and η respectively, then the moment at the axis of the drum is reduced to that at the axis of the motor by dividing it by the product $i\eta$;

$$M_a = \frac{M_a'}{i\eta}.$$

Substituting $T = T_e/i$, we have

$$M_a = \frac{a_a}{i^2\eta}\frac{d^2T_e}{dt^2} + 2\frac{b_a}{i^3\eta}\left(\frac{dT_e}{dt}\right)^2 + 4\frac{b_a}{i^3\eta}T_e\frac{d^2T_e}{dt^2} + 2\frac{c_a}{i^4\eta}T_e\left(\frac{dT_e}{dt}\right)^2 + 2\frac{c_a}{i^4\eta}T_e^2\frac{d^2T_e}{dt^2}.$$

$$M_a = a\frac{d^2T_e}{dt^2} + 2b\left(\frac{dT_e}{dt}\right)^2 + 4bT_e\frac{d^2T_e}{dt^2} + 2cT_e\left(\frac{dT_e}{dt}\right)^2 + 2cT_e^2\frac{d^2T_e}{dt^2}. \qquad [28],$$

$$\left.\begin{aligned} a &= \frac{a_d}{i^2\eta} \\ b &= \frac{b_d}{i^3\eta} \\ c &= \frac{c_d}{i^4\eta} \end{aligned}\right\} \qquad [29].$$

Similarly,

$$M_s = P + QT_e \qquad [30],$$

$$\left.\begin{aligned} P &= \frac{p'}{i\eta} \\ Q &= \frac{Q'}{i^2\eta} \end{aligned}\right\} \qquad [31].$$

And,

$$M_v = a'\frac{d^2T'_e}{dt^2} + 2b'\left(\frac{dT'_e}{dt}\right)^2 + 4b'T'_e\frac{d^2T'_e}{dt^2} + 2cT'_e\left(\frac{dT'_e}{dt}\right)^2 + 2cT'^2_e\frac{d^2T'_e}{dt^2}$$

$$[32],$$

$$\left.\begin{aligned} a' &= \frac{a'_d}{i^2\eta} \\ b' &= \frac{b'_d}{i^3\eta} \\ c &= \frac{c_d}{i^4\eta} \end{aligned}\right\} \qquad [33].$$

Resultants moments are

$$M_{aT} = a\frac{d^2T_e}{dt^2} + 2b\left(\frac{dT_e}{dt}\right)^2 + 4bT_e\frac{d^2T_e}{dt^2} + 2c\,T_e\left(\frac{dT_e}{dt}\right)^2 + 2cT_e^2\frac{d^2T_e}{dt^2}$$
$$+ P + QT_e \qquad [34],$$

$$M_{sT} = (1+\theta)(P+QT_e) \qquad [35],$$

$$M_{vT} = a'\frac{d^2T'_e}{dt^2} + 2b'\left(\frac{dT'_e}{dt}\right)^2 + 4b'T'_e\frac{d^2T'_e}{dt^2} + 2cT'_e\left(\frac{dT'_e}{dt}\right)^2 + 2cT'^2_e\frac{d^2T'_e}{dt^2}$$

$$+P+QT_e \qquad [36],$$

where
$$T'_e = i\int_{t_v}^{t} n'dt.$$

As usual in the Leonard system, assume that the motor is to be controlled with a constant acceleration and retardation, and let n_m, p, and p_v be the maximum revolution per unit time, acceleration and retardation of the motor respectively, then

$$\frac{dT_e}{dt} = n = pt,$$

$$\left(\frac{dT_e}{dt}\right)_{\max.} = n_m = pt_m\,,$$

$$\frac{d^2T_e}{dt^2} = p,$$

$$\frac{dT'_e}{dt} = n_m - p_v t,$$

$$\frac{d^2T'_e}{dt^2} = -p_v\,,$$

$$T_e = \tfrac{1}{2}pt^2,$$

$$T'_e = (n_m t - \tfrac{1}{2}p_v t^2) - \tfrac{1}{2}n_m t_v\,,$$

where t_m and t_v is time of acceleration and that of retardation, respectively.

Let $T_{te} = iT_t$, then

$$M_{aT} = P + ap + 4bp^2t^2 + \tfrac{1}{2}Qpt^2 + \tfrac{3}{2}cp^3t^4 \qquad (37),$$

$$M_{sT} = (1+\theta)\{P + Q(\tfrac{1}{2}pt_m^2 + n_m t)\} \qquad (38),$$

$$M_{vT} = P + QT_{te} + 4b'n_m^2 - a'p_v - \tfrac{1}{2}(3cn_m^4 + Qn_m^2)\frac{1}{p_v}$$

$$-\{8b'n_m p_v - (6cn_m^3 + Qn_m)\}t$$

$$+\{4b'p_v^2-(9cn_m^2+\tfrac{1}{2}Q)p_v\}t^2$$

$$+6cn_m p_v^2 t^3-\tfrac{3}{2}cp_v^3 t^4 \qquad (39).$$

In (37), (38), and (39), t is measured from the beginning of the acceleration, full speed, and retardation, respectively.

In (37) and (39), it is found that M_{aT} is maximum when $t=t_m$, and M_{vT} is minimum when $t=t_v$.

The starting resisting moment is

$$(M_{aT})_{\text{start.}} = P+ap \qquad (40).$$

$P+ap$ must be smaller than the starting torque of the motor; and p or p_v must be taken at such value that any danger due to mechanical shock may not occur. These conditions give the upper limit to the value of p or p_v.

II. Output of Motor and Times of Acceleration and Retardation.

The power and times of acceleration and retardation are to be considered in regard to the straight line control, above investigated.

The instantaneous output of the motor is $2\pi M\dfrac{dT_e}{dt}$.

During acceleration,

$$W_a = 2\pi pt\left\{P+ap+4bp^2t^2+\tfrac{1}{2}Qpt^2+\tfrac{3}{2}cp^3t^4\right\} \qquad (41).$$

During full speed running,

$$W_s = 2\pi n_m(1+\theta)\{P+Q(\tfrac{1}{2}pt_m^2+n_m t)\} \qquad (42).$$

During retardation,

$$W_v = 2\pi(n_m-p_v t)\Big[a'p_v-(P+QT_{te}+4b'n_m^2)+\tfrac{1}{2}(3cn_m^4+Qn_m^2)\frac{1}{p_v}$$

$$+\{8b'n_m p_v-(6cn_m^3+Qn_m)\}t-\{4b'p_v^2-(9cn_m^2+\tfrac{1}{2}Q)p_v\}t^2$$

$$\left.-6cn_m p_v^2 t^3+\tfrac{3}{2}cp_v^3 t^4\right] \text{ as generator} \qquad (43).$$

In (41), (42), and (43), the zero point of t is at the beginning of the acceleration, full speed running and retardation, respectively. The maximum power as motor and as generator is found to be at the end of acceleration and at the beginning of the retardation, respectively,

$$W_{a\,\max.}=2\pi n_m\left\{P+ap+4bn_m^2+\tfrac{1}{2}(3cn_m^4+Qn_m^2)\frac{1}{p}\right\} \qquad (44),$$

$$W_{v\,\max.}=2\pi n_m\left\{a'p_v-(P+QT_{te}+4b'n_m^2)+\tfrac{1}{2}(3cn_m^4+Qn_m^2)\frac{1}{p_v}\right\} \text{ as generator} \qquad (45).$$

The above mentioned power as generator, which in the case of steam-engine-driven hoist is absorbed by the brakes, is here returned to the electric supply system, thereby improving the economy of operation and reducing the wear on the mechanical brake.

The time required for a winding is fixed from the capacity of cage and the amount of ore to be hoisted a day.

Let t be time of winding, and t' be time for filling the cage, then

$$n_m\left(t-t'-\frac{n_m}{p}-\frac{n_m}{p_v}\right)+\frac{n_m^2}{2p}+\frac{n_m^2}{2p_v}=T_{te}$$

$$\frac{1}{p}+\frac{1}{p_v}=2\left(\frac{t-t'}{n}-\frac{T_{te}}{n_m^2}\right)$$

$$=2S,\quad \text{say} \qquad (46).$$

p and p_v must be determined so as to afford the greatest economy to the winding system under the condition (46).

Now total input to the hoist motor is composed of:

(1) output of the motor,

(2) losses in the motor.

p and p_v must be determined so that the sum of (1) and (2) may be of the minimum value.

(1) Total output energy of the motor per winding,

$$\text{Total output} = \int W_a dt + \int W_s dt + \int W_v dt$$

$$= 2\pi\Big[(1+\theta)(PT_{te} + \tfrac{1}{2}QT_{te}^2) + \frac{n_m^2}{2}(a-a')$$

$$+\Big\{bn_m^4 - \frac{\theta}{2}Pn_m^2 + \frac{S}{4}(2cn_m^6 - \theta Qn_m^4)\Big\}\frac{1}{p}$$

$$+\Big\{b'n_m^4 - \frac{\theta}{2}(P+QT_{te})n_m^2 - \frac{S}{4}(2cn_m^6 - \theta Qn_m^4)\Big\}\frac{1}{p_v}\Big] \qquad [47].$$

Let $\qquad p_v = ap; \quad p = \dfrac{1+a}{2Sa};$

then, total output energy of motor becomes

$$2\pi\Big[(1+\theta)(PT_{te} + \tfrac{1}{2}QT_{te}^2) + \frac{n_m^2}{2}(a-a')$$

$$+\frac{2S}{1+a}\Big\{a\Big(bn_m^4 - \frac{\theta}{2}Pn_m^2 + \frac{S}{4}(2cn_m^6 - \theta Qn_m^4)\Big) + b'n_m^4 - \frac{\theta}{2}(P+QT_{te})$$

$$-\frac{S}{4}(2cn_m^6 - \theta Qn_m^4)\Big\}\Big] \qquad [48].$$

For a cylindrical drum,

$$\text{Total output} = 2\pi\{PT_{te} + \tfrac{1}{2}QT_{te}^2\} \qquad (48)'.$$

As previously mentioned, the motor, during retardation, acts as a generator; and the energy returned to the supply system, may be utilized. Restoring in fly-wheel, the energy, wasted in order to accelerate the moving masses, is again obtained, in retardation.

The second term and succeeding ones in [47] or [48] are all due to the difference between the equivalent moving masses in acceleration and retardation.

(2) Losses in the motor.

In the leonard system, the field of the motor is excited separately;

and it is therefore sufficient to take losses in the armature only into account. The hysteresis loss is proportional to the number of cycle per unit time, and the eddy current loss is proportional to the square of that, so that the total loss at any instant is

$$I^2R + hn + fn^2$$

where n is the instantaneous value of revolution per unit time of the motor, I, current in the armature, R, resistance of the armature coil, h and f, constants.

The torque of the motor is expressed by const. ΦI. Φ is flux from a pole. Therefore the loss at any instant in the armature is

$$w = \text{const.}\,\{M^2 + Hn + Fn^2\} \qquad (49).$$

Total loss per winding,

$$w_t = \text{const.}\sum\int M^2dt + \text{const. } H\sum\int ndt + \text{const. } F\sum\int n^2dt.$$

And,

$$\sum\int M^2dt = \int_0^{t_m} M_{aT}^2 dt + \int_0^{t-t'-t_m-t_v} M_{sT}^2 dt + \int_0^{t_v} M_{vT}^2 dt,$$

$$\sum\int ndt = \int_0^{t_m} ptdt + \int_0^{t-t'-t_m-t_v} n_m dt + \int_0^{t_v}(n_m - p_v t)dt,$$

$$\sum\int n^2dt = \int_0^{t_m} p^2t^2dt + \int_0^{t-t'-t_m-t_v} n_m^2 dt + \int_0^{t_v}(n_m - p_v t)^2dt.$$

$$\int_0^{t_n} M_{aT}^2\,dt = a^2n_m p + 2Pan_m + \tfrac{8}{3}abn_m^3$$

$$+ (P^2n_m + \tfrac{8}{3}bPn_m^3 + \tfrac{1}{3}aQn_m^3 + \tfrac{16}{5}b^2n_m^5 + \tfrac{8}{5}acn_m^5)\frac{1}{p}$$

$$+ (\tfrac{1}{3}PQn_m^3 + \tfrac{3}{5}cPn_m^5 + \tfrac{4}{5}bQn_m^5 + \tfrac{12}{7}bcn_m^7)\frac{1}{p^2}$$

$$+ (\tfrac{1}{20}Q^2n_m^5 + \tfrac{3}{14}cQn_m^7 + \tfrac{1}{4}c^2n_m^9)\frac{1}{p^3}.$$

$$\int_0^{t-t'-t_m-t_v} M_T^2 dt = (1+\theta)^2\Big\{P^2\frac{T_{te}}{n_m}+PQ\frac{T_{te}^2}{n_m}+\tfrac{1}{3}Q^2\frac{T_{te}^3}{n_m}-\tfrac{1}{2}P^2n_m\frac{1}{p}$$

$$-\tfrac{1}{2}(P+QT_{te})^2n_m\frac{1}{p_v}-\tfrac{1}{4}PQn_m^3\frac{1}{p^2}$$

$$+\tfrac{1}{4}Q(P+QT_{te})n_m^3\frac{1}{p_v^2}-\tfrac{1}{24}Q^2n_m^5\frac{1}{p^3}$$

$$-\tfrac{1}{24}Q^2n_m^5\frac{1}{p_v^3}\Big\}.$$

$$\int_0^{t_v} M_{vT}^2\,dt = a'^2n_mp_v-\{2a'(P+QT_{te})n_m+\tfrac{8}{3}a'b'n_m^3\}$$

$$+\{(P+QT_{te})^2n_m+\tfrac{8}{3}b'(P+QT_{te})n_m^3+\tfrac{1}{3}a'Qn_m^3+\tfrac{16}{5}b'^2n_m^5+\tfrac{3}{5}a'cn_m^5\}\frac{1}{p_v}$$

$$-\{\tfrac{1}{3}(P+QT_{te})Qn_m^3+\tfrac{3}{5}c(P+QT_{te})n_m^5+\tfrac{4}{5}b'Qn_m^5+\tfrac{12}{7}b'cn_m^7\}\frac{1}{p_v^2}$$

$$+\{\tfrac{1}{20}Q^2n_m^5+\tfrac{3}{14}cQn_m^7+\tfrac{1}{4}c^2n_m^9\}\frac{1}{p_v^3}.$$

$$\sum\int Hndt+\sum\int Fn^2dt = (H+Fn_m)T_{te}-\tfrac{1}{6}Fn_m^3\frac{1}{p}-\tfrac{1}{6}Fn_m^3\frac{1}{p_v}.$$

Then,

$$w_t = \text{const.}\left\{a^2n_mp+a'^2n_mp_v+A+\frac{B}{p}+\frac{B'}{p_v}+\frac{D}{p^2}-\frac{D'}{p_v^2}+\frac{E}{p^3}+\frac{E}{p_v^3}\right\} \qquad [50],$$

$$\left.\begin{aligned}
A &= (H+Fn_m)T_{te}+2aPn_m-2a'(P+QT_{te})n_m+\tfrac{8}{3}abn_m^3-\tfrac{8}{3}a'b'n_m^3\\
&\qquad+\frac{(1+\theta)^2}{n_m}(P^2T_{te}+PQT_{te}^2+\tfrac{1}{3}Q^2T_{te}^3)\\
B &= \frac{2-(1+\theta)^2}{2}P^2n_m+\tfrac{8}{3}bPn_m^3+\tfrac{1}{3}aQn_m^3-\tfrac{1}{6}Fn_m^3+\tfrac{16}{5}b^2n_m^5+\tfrac{3}{5}acn_m^5\\
B' &= \frac{2-(1+\theta)^2}{2}(P+QT_{te})^2n_m+\tfrac{8}{3}b'(P+QT_{te})n_m^3+\tfrac{1}{3}a'Qn_m^3-\tfrac{1}{6}Fn_m^3\\
&\qquad+\tfrac{16}{5}b'^2n_m^5+\tfrac{3}{5}a'cn_m^5\\
D &= \frac{4-3(1+\theta)^2}{12}PQn_m^3+\tfrac{3}{5}cPn_m^5+\tfrac{4}{5}bQn_m^5+\tfrac{12}{7}bcn_m^4
\end{aligned}\right\} \quad [51].$$

$$\left.\begin{aligned} D' &= \frac{4-3(1+\theta)^2}{12}(P+QT_{te})Qn_m^3+\tfrac{3}{5}c(P+QT_{te})n_m^5+\tfrac{4}{5}b'Qn_m^5+\tfrac{12}{7}b'cn_m^7 \\ E &= \frac{6-5(1+\theta)^2}{120}Q^2n_m^5+\tfrac{3}{14}cQn_m^7+\tfrac{1}{4}c^2n_m^9 \end{aligned}\right\}$$

For conical drum,

$$\left.\begin{aligned} A &= \{Hn_m+Fn_m^2+(1+\theta)^2P^2\}\frac{T_{te}}{n_m}+2(a-a')Pn_m+\tfrac{8}{3}(a-a')bn_m^3 \\ B &= \frac{2-(1+\theta)^2}{2}P^2n_m+\tfrac{8}{3}bPn_m^3-\tfrac{1}{6}Fn_m^3+\tfrac{16}{5}b^2n_m^5+\tfrac{3}{5}acn_m^5 \\ B' &= \frac{2-(1+\theta)^2}{2}P^2n_m+\tfrac{8}{3}bPn_m^3-\tfrac{1}{6}Fn_m^3+\tfrac{16}{5}b^2n_m^5+\tfrac{3}{5}a'cn_m^5 \\ D &= \tfrac{3}{5}cPn_m^5+\tfrac{12}{7}bcn_m^7 \\ D' &= D \\ E &= \tfrac{1}{4}c^2n_m^9 \end{aligned}\right\} [52].$$

For cylindrical drum without tail rope,

$$\left.\begin{aligned} A &= (Hn_m+Fn_m^2+P^2)\frac{T_{te}}{n_m}+PQ\frac{T_{te}^2}{n_m}+\tfrac{1}{3}Q\frac{T_{te}^3}{n_m}-2aQT_{te}n_m \\ B &= \tfrac{1}{2}P^2n_m+\tfrac{1}{3}aQn_m^3-\tfrac{1}{6}Fn_m^3 \\ B' &= \tfrac{1}{2}(P+QT_{te})^2n_m+\tfrac{1}{3}aQn_m^3-\tfrac{1}{6}Fn_m^3 \\ D &= \tfrac{1}{12}PQn_m^3 \\ D' &= \tfrac{1}{12}Q(P+QT_{te})n_m^3 \\ E &= \tfrac{1}{120}Q^2n_m^5 \end{aligned}\right\} [53].$$

For cylindrical drum with tail rope,

$$\left.\begin{aligned} A &= (Hn_m+Fn_m^2+P^2)\frac{T_{te}}{n_m} \\ B &= \tfrac{1}{2}P^2n_m-\tfrac{1}{6}Pn_m^3 \\ B' &= B \end{aligned}\right\} [54].$$

$$D = 0$$
$$D' = 0$$
$$E = 0$$

Inserting $\frac{1}{p}+\frac{1}{p_v} = 2S$, and converting into terms of a,

$$w_t = \frac{\text{const.}}{a(1+a)^2}\left\{\frac{n_m}{2S}(a^2+a'^2a)(1+a)^3+Aa(1+a)^2+2S(Ba+B')a(1+a)\right.$$
$$\left.+4S^2(D+2SE)a^3-4S^2(D'-2SE)a-8S^3Ea^2\right\} \qquad [55].$$

Total input energy to the hoist motor per winding,

$$G = \frac{\text{const.}}{a(1+a)^2}\left\{\frac{n_m}{2S}(a^2+a'^2a)(1+a)^3+Aa(1+a)^2+2S(Ba+B')a(1+a)\right.$$
$$\left.+4S^2(D+2SE)a^3-4S^2(D'-2SE)a-8S^3Ea^2\right\}$$
$$+2\pi\left\{(1+\theta)(PT_{te}+\tfrac{1}{2}QT_{te}^2)+\frac{n_m^2}{2}(a-a')\right\}$$
$$+\frac{4\pi S}{1+a}\left\{a\left(bn_m^4-\frac{\theta}{2}Pn_m^2+\frac{S}{4}(2cn_m^6-\theta Qn_m^4)\right)+b'n_m^4-\frac{\theta}{2}(P+QT_{te})n_m^2\right.$$
$$\left.-\frac{S}{4}(2cn_m^6-\theta Qn_m^4)\right\}$$
$$[56].$$

For the sake of the greatest economy,

$$\frac{n_m}{2S}(a'^2a^2-a^2)(1+a)^3+2S\left\{B-B'+\frac{2\pi}{\text{const.}}\left((b-b')n_m^4+\frac{\theta}{2}QT_{te}n_m^2\right.\right.$$
$$\left.\left.+\frac{S}{2}(2cn_m^6-\theta Qn_m^4)\right)\right\}a^2(1+a)+8S^2(D+3SE)a^3$$
$$+8S^2(D'-3SE)a^2 = 0 \qquad (57).$$

Equation (57) does not contain H and F; but, having a term containing particular constants for a motor (number of poles, armature resistance &c.), it is applicable only for a given motor; for the first design, it is inconvenient. Fortunately, the amount, in the output of the motor, depending on a is comparatively small (for a cylindrical drum, the input to the hoist is independent on a).

For the sake of simplicity, take the loss only into account in discussion of durations of acceleration and retardation. a, obtained from the condition of the minimum value of the loss, makes the motor rating minimum, or, in the case of a given motor, gives the minimum temperature-rise, and differs from that given by (57) by only small amount (for a cylindrical drum, is the same).

The dispersion of heat generated by the losses is not uniform over the cycle of winding on account of the reduced speed at both ends. For a general approximation, take the cooling during acceleration and retardation to be 75 per cent, and that during standstill to be 50 per cent of that at full speed, then the effective time for cooling per winding would be

$$\begin{aligned} &0{\cdot}75(t_m+t_v)+0{\cdot}5\,t'+t-t'-t_m-t_v \\ &= t-0{\cdot}5\,t'-\frac{n_m}{2}\,S. \end{aligned}$$

Mean loss per unit time,

$$w_m = \frac{\text{const.}}{\left(t-0{\cdot}5\,t'-\frac{n_m}{2}S\right)a(1+a)^2}\left\{\frac{n_m}{2S}(a^2+a'^2a)(1+a)^3+Aa(1+a)^2\right.$$
$$+2S(Ba+B')a(1+a)+4S^2(D+2SE)a^3-4S^2(D'-2SE)a$$
$$\left.-8S^3Ea^2\right\} \qquad (58).$$

The temperature-rise of the motor depends on w_m; the value of a such as to make the motor rating minimum, or, to a given motor, to give the minimum temperature-rise, must satisfy the condition

$$\frac{dw_m}{da}=0$$

or

$$\frac{n_m}{2S}(a'^2\alpha^2-a^2)(1+\alpha)^3+2S(B-B')\alpha^2(1+\alpha)+8S^2(D+3SE)\alpha^3$$
$$+8S^2(D'-3SE)\alpha^2=0 \qquad [59].$$

Equation [59] gives the greatest economical value of α, and as readily understood, [59] has no term containing H or F, but only constants connecting with the hoist.

For a cylindrical drum with tail rope [59] becomes

$$\frac{n_m}{2S}a^2(\alpha^2-1)(1+\alpha)^3=0,$$

and α must not be negative, therefore

$$\left.\begin{aligned} \alpha &= 1 \\ p &= p_v \end{aligned}\right\} \qquad [60],$$

i.e., for a cylindrical drum with tail rope, the acceleration is to be taken at equal value to the retardation; and this result is that we expected.

For a conical drum, comparing with a and b, c in (29), the value of which is $-\frac{\pi^3\sigma^3(2+\sigma h)mh^2r_1^4}{i^4\eta}$, may be neglected approximately, thereby [59] becomes

$$\frac{n_m}{2S}(a'^2\alpha^2-a^2)(1+\alpha)^3=0,$$

$$\left.\begin{aligned} \alpha &= \frac{a}{a'} \\ a'p_v &= ap \end{aligned}\right\} \qquad [61].$$

For a reel drum and a cylindrical drum without tailrope, [59] can not be reduced to a simple form, but for the former, [61] may be taken in approximate estimation, and for the latter

$$\frac{n_m}{2S}a^2(\alpha^2-1)(1+\alpha)^3+8\pi Smgr_1^2n_m\{PT_{te}-2\pi mgr_1^2T_{te}^2-\tfrac{1}{3}PSn_m^2$$
$$+\tfrac{2}{3}\pi S^2mgr_1^2n_m^4\}\alpha^2(1+\alpha)+\tfrac{32}{3}\pi^2S^2m^2g^2r_1^4n_m^3T_{te}\alpha^2=0 \qquad [62].$$

In (62), the second and the third terms are all positive, hence the first term must be negative, i.e., α is to be smaller than unity.

If the capacity of the motor or the total loss is considered to be determined by the maximum value of the load, (though this is obviously an error), p and p_v are taken as follows:

$$ap+P+4bn_m^2+\tfrac{1}{2}(3cn_m^4+Qn_m^2)\frac{1}{p}=a'p_v-(P+QT_{te}+4b'n_m^2)+\tfrac{1}{2}(3cn_m^4+Qn_m^2)\frac{1}{p_v},$$

$$2P+QT_{te}+4(b+b')n_m^2+ap-a'p_v+\tfrac{1}{2}(3cn_m^4+Qn_m^2)\Big(\frac{1}{p}+\frac{1}{p_v}\Big)=0.$$

Put $$2P+4(b+b')n_m^2+QT_{te}=\beta,$$

and $$\tfrac{1}{2}(3cn_m^4+Qn_m^2)=\gamma,$$

then

$$(a-a'\alpha)(1+\alpha)^2+2S\beta\alpha(1+\alpha)+4S^2\gamma\alpha(\alpha-1)=0.$$

When $\gamma=0$ (in case of cylindrical drum with tail rope, and approximately in cases of conical and reel drums.),

$$\alpha=\frac{2S\beta-a(\mu-1)+\sqrt{\{a(\mu-1)-2S\beta\}^2+4a^2\mu}}{2a\mu}$$

$$=\frac{1-\mu+4S\dfrac{4bn_m^2+P}{a}+\sqrt{\Big\{1+\mu+4S\dfrac{4bn_m^2+P}{a}\Big\}^2-16\mu S\dfrac{4bn_m^2+P}{a}}}{2\mu}$$

where $$\mu=\frac{a'}{a}.$$

For a cylindrical drum with tail rope,

$$\alpha=\frac{4S\dfrac{P}{a}+\sqrt{4+16S^2\dfrac{P^2}{a^2}}}{2}.$$

For a cylindrical drum without tail rope,

$$a(1-\alpha)(1+\alpha)^2+2S(2P+QT_{te})\alpha(1+\alpha)+2S^2Q\alpha(\alpha-1)n_m^2=0.$$

These results are of comparatively simple form, and this method is sometimes emploied in determination of p and p_v; but they are not strictly rational. [60], [61] and [62] must be emploied in theoretical calculation.

III. Capacity of Motor.

The rating of the motor depends upon the temperature-rise.

The root-mean-square value of the statical moment over the period of winding is

$$M_r = \left[\left\{a^2 n_m p + a'^2 n_m p_v + K + \frac{L}{p} + \frac{L'}{p_v} + \frac{D}{p^2} - \frac{D'}{p_v^2} + 2SE\left(\frac{1}{p^2} + \frac{1}{p_v^2} - \frac{1}{pp_v}\right)\right\}\frac{1}{t}\right]^{\frac{1}{2}}.$$

Substituting a for p and p_v,

$$\left.\begin{aligned}
M_r &= \left[\frac{1}{t\cdot a(1+a)^2}\left\{\frac{n_m}{2S}(a^2+a'^2 a)(1+a)^3 + Ka(1+a)^2\right.\right.\\
&\quad + 2S(La+L')a(1+a) + 4S^2(D+2SE)a^3 - 4S^2(D'-2SE)a\\
&\quad \left.\left. - 8S^3Ea^2\right\}\right]^{\frac{1}{2}}\\
K &= \frac{(1+\theta)^2}{n_m}\left(P^2T_{te} + PQT_{te}^2 + \tfrac{1}{3}Q^2T_{te}^3\right) + 2aPn_m - 2a'(P+QT_{te})n_m\\
&\quad + \tfrac{8}{3}abn_m^3 - \tfrac{8}{3}a'b'n_m^3\\
L &= \frac{2-(1+\theta)^2}{2}P^2n_m + \tfrac{8}{3}bPn_m^3 + \tfrac{1}{3}aQn_m^3 + \tfrac{16}{5}b^2n_m^5 + \tfrac{3}{5}acn_m^5\\
L' &= \frac{2-(1+\theta)^2}{2}(P+QT_{te})^2n_m + \tfrac{8}{3}b'(P+QT_{te})n_m^3 + \tfrac{1}{3}a'Qn_m^3\\
&\quad + \tfrac{16}{5}b'^2n_m^5 + \tfrac{3}{5}a'cn_m^5
\end{aligned}\right\} \quad [63].$$

D, D' and E are defined in [48].

Now, consider a motor specified as:

normal voltage $= e$,

normal speed $= n_m$,

normal capacity $= \dfrac{2\pi}{75}M_r n_m$ HP,

efficiency $= \eta_m$.

This motor is designed to work at a certain safety temperature-rise with the resisting moment M_r, under a constant speed n_m (not cyclic intermittent working). The loss per unit time in the armature of this motor is

$$\text{const.}\,(M_r^2 + Hn_m + Fn_m^2).$$

When this motor is emploied to drive a hoist whose statical moment is $P+QTe$, the ratio of temperature-rise in this case to that in the normal constant speed working, is

$$\frac{1}{M_r^2+Hn_m+Fn_m^2}\,\frac{1}{\left(t-0{\cdot}5t'-\frac{n_m}{2}S\right)a(1+a)^2}\left\{\frac{n_m}{2S}(a^2+a'^2a)(1+a)^3+Aa(1+a)^2\right.$$

$$\left.+2S(Ba+B')a(1+a)+4S^2(D+2SE)a^3-4S^2(D'-2SE)a-8S^3Ea^2\right\}$$

$$=\rho^2,\ \text{say} \qquad [64].$$

Then the capacity of the winding motor would be taken

$$W=\frac{2\pi}{75}\rho M_r n_m\ \text{HP} \qquad [65].$$

Values of H and F.

The exact evaluations of H and F can not be expected; but the error due to the inaccuracy of their values may be comparatively small on account of their existence in both the numerator and the denominator of equation [64]. First, the copper and iron losses of the motor, the capacity of which is $2\pi M_r n_m$, are estimated from practical data, and from the copper loss, the armature resistance is to be calculated. And

hysteresis loss $= hn_m = \xi B_a^{1.6} V n_m\, 10^{-7}$ watts,

eddy curret loss $= fn_m^2 = \varsigma B_a^2\, V n_m^2\, 10^{-11}$ watts.

where

B_a = Mean induction in armature in lines per $(\text{cm})^2$,

V = Volume of armature, teeth inclusive, in $(\text{cm})^3$,

ξ = (Number of pair of pole) $\times$ (hysteresis constant),

ς = (Number of pair of pole)$^2 \times 1{\cdot}645$(thickness of iron sheet in (cm))^2.

The number of pair of pole is to be determined from the actual design. Thickness of iron sheet is from 0.35 to 0.5 mm.; B_a is from 10000 to 15000 c.g.s. according to the capacity of the motor.

$$\frac{f}{h} = \frac{\varsigma}{\xi} 10^{-4} B_a^{0.4},$$

$$f = \frac{\varsigma}{\xi} 10^{-4} B_a^{0.4} h \qquad (66).$$

$$\text{Iron loss} = h n_m + \frac{\varsigma}{\xi} 10^{-4} B_a^{0.4} h n_m^2 \qquad (67).$$

From (66) and (67), h and f are to be determined. Then

$$\left.\begin{aligned} H &= \frac{1}{R}\left(\frac{\eta_m e}{2 \times 9{,}81 \pi n_m}\right)^2 h \\ F &= \frac{1}{R}\left(\frac{\eta_m e}{2 \times 9{,}81 \pi n_m}\right)^2 f \end{aligned}\right\} \qquad [68].$$

The relative values of the copper and iron losses of armature may be assumed as (S. P. Thompson, Dynamo Electric Machinery):

Ccapacity in H.P.	Efficiency in %	% Losses			
		Armature		Field	Friction
		Copper	Iron		
15– 80	91	3.5	3.0	2.1	0.4
50– 150	92	3.2	2.8	1.6	0.4
100– 400	93	2.8	2.3	1.55	0.3
300– 700	94	2.4	1.8	1.5	0.3
600–1500	95	1 9	1.5	1.35	0.25

N. B. In this study, the constancy of speed of the motor during the full voltage running is assumed; but in cases of cylindrical drum without tail rope, and reel drum, the speed of the motor varies, though inconsiderably, according to the increase or decrease of the statical moment during this period.

Assume the ampere-speed curve of the motor to be a straight line

$$M_{so}-M_{sf} = y(n_{mf}-n_{mo}).$$

M_{so} = moment at the beginning of full voltage running,

M_{sf} = moment at the instant of consideration in full voltage running,

n_{mo} = number of revolution per unit time of the motor at the beginning of full voltage running,

n_{mf} = number of revolution per unit time of the motor at the instant of consideration in full voltage running,

y = constant.

For practical purposes, this assumption is acknowledged to be satisfactory.

Let T_s be the number of revolution after the beginning of the full voltage running, then

$$T_e = \frac{n_{mo}^2}{2p}+T_s,$$

$$M_{sf} = P+Q\frac{n_{mo}^2}{2p}+QT_s+\frac{a+a'}{2}\frac{d^2T_s}{dt^2},$$

$$M_{so} = P+Q\frac{n_{mo}^2}{2p}.$$

$$\therefore\quad P+Q\frac{n_{mo}^2}{2p}-\left\{P+Q\frac{n_{mo}^2}{2p}+QT_s+\frac{a+a'}{2}\frac{d^2T_s}{dt^2}\right\} = y\left(\frac{dT_s}{dt}-n_{mo}\right),$$

$$\frac{d^2T_s}{dt^2}+2\frac{y}{a+a'}\frac{dT_s}{dt}+2\frac{Q}{a+a'}T_s-2\frac{y}{a+a'}n_{mo} = 0.$$

Solving this,

$$T_s = C_1e^{m_1t}+C_2e^{m_2t}+\frac{yn_{mo}}{Q}$$

where

$$\left.\begin{aligned}
m_1 &= \frac{-y+\sqrt{y^2-2Q(a+a')}}{a+a'} \\
m_2 &= \frac{-y-\sqrt{y^2-2Q(a+a')}}{a+a'} \\
C_1 &= \frac{(a+a')Q-y^2-y\sqrt{y^2-2Q(a+a')}}{2Q\sqrt{y^2-2Q(a+a')}}n_{mo} \\
C_2 &= \frac{-(a+a')Q+y^2-y\sqrt{y^2-2Q(a+a')}}{2Q\sqrt{y^2-2Q(a+a')}}n_{mo}
\end{aligned}\right\} \quad (69).$$

From these,

$$n_{mf} = C_1 m_1 e^{m_1 t} + C_2 m_2 e^{m_2 t} \quad (70).$$

Speed at the end of the full voltage running,

$$n_{me} = C_1 m_1 e^{m_1(t-t'-t_m-t_v)} + C_2 m_2 e^{m_2(t-t'-t_m-t_v)} \quad (71).$$

For a cylindrical drum without tail rope,

$$\left.\begin{aligned}
m_1 &= \frac{-y+\sqrt{y^2+16\pi mgr_1^2 a}}{2a} \\
m^2 &= \frac{-y-\sqrt{y^2+16\pi mgr_1^2 a}}{2a} \\
C_1 &= \frac{8\pi amgr_1^2+y^2+y\sqrt{y^2+16\pi mgr_1^2 a}}{8\pi mgr_1^2\sqrt{y^2+16\pi mgr_1^2 a}}n_{mo} \\
C_2 &= \frac{-8\pi amgr_1^2-y^2+y\sqrt{y^2+16\pi mgr_1^2 a}}{8\pi mgr_1^2\sqrt{y^2+16\pi mgr_1^2 a}}n_{mo}
\end{aligned}\right\} \quad (72).$$

From these equations, we find the maximum speed of the moving mass, and check if the speed is beyond the limit of mechanical danger.

SUMMARY.

In this study, the general equations of resisting moment of hoist are given; the economical relation between acceleration and retardation may be found from equations [60], [61] and [62]. Taking iron losses into account, the capacity of the hoist motor may be determined, though with some difficulty, from equation [65]. In order perfectly to avoid the mechanical danger, equations (69) and (70) are to be checked.

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