Transmission Losses in Telephone Lines.

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I. General Discussion.

As is well known to all telephone engineers, the transmission loss in any nonloaded telephone line is usually represented by a length of standard cable (88 ohms resistance per loop mile and 0.054 microfarad wire-to-wire capacity per mile) having an attenuation length equal to that of the line under consideration, assuming the same terminal apparatus to be connected with both the line and standard cable. This convenient method of expressing roughly the transmission efficiency of a telephone line is based on the assumptions that the current at any point on an uniform line is represented by the formula (1), and the currents at the sending ends of the line and standard cable are equal.

$$C = Ie^{-\alpha l'}\sqrt{\beta l'} \quad \text{.............................(1)}$$

where C is the current at any point on the line distant l' miles from the sending end;

I is the current at the sending end;

α is the attenuation constant of the line; and

β is the wave length constant of the line.

If the above assumptions hold good even at the receiving end of a line of finite length, then the formula becomes:

$$C_r = Ie^{-\alpha l}\,|\!\overline{\beta l}\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

where C_r is the current at the receiving end, and

l is the total length of the line.

If the phase difference between the currents at the sending and receiving ends is left out of consideration formula (2) becomes

$$C_r = Ie^{-\alpha l}\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3).$$

Since the loudness of receiving sound naturally does not depend upon the phase difference, we may use formula (3) as one showing the loudness of receiving sound.

Therefore, to get an equivalent length, in terms of standard cable, of a telephone line of length l_1, we have to equate the receiving current of the line and that of standard cable, that is,

$$Ie^{-\alpha_0 l_0} = Ie^{-\alpha_1 l_1}$$

or

$$\alpha_0 l_0 = \alpha_1 l_1\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

where α_0 and α_1 are the attenuation constants of standard cable and the line under consideration respectively; and

l_0 and l_1 are the respective lengths.

Formula (4) readily gives the equivalent length required.

In the method given above of obtaining the equivalent length, the loudness of receiving sound only is taken in consideration, its clearness being disregarded. This is necessary in the present state of technical development. In the following, therefore, if nothing is said of clearness, loudness only will be taken as the measure of the transmission efficiency of the telephone line.

Formula (1), however, holds good only when the length of a line is infinite in which case

$$I = \frac{E}{Z_0 + Z_s}$$

where E is the impressed E.M.F.;

Z_s is the impedance of the apparatus at the sending end; and

Z_0 is the initial sending end impedance of the line.

Hence formula (1) becomes

$$C = \frac{E}{Z_0 + Z_s} e^{-\alpha l'} \underline{|\beta l'} \quad \text{.......................(5).}$$

Formula (5) shows that the current I at the sending end differs in accordance with the initial sending end impedance of the line. Therefore, some error must exist if formula (4) is used in comparing the transmission efficiencies of lines having different initial sending end impedances.

In addition, if the line is of finite length, as is the case with the actual line, the formula expressing the current at any point on the line is

$$C = \frac{E\{Z_0 \cosh a(l-l') + Z_r \sinh a(l-l')\}}{(Z_0^2 + Z_s Z_r) \sinh al + Z_0(Z_s + Z_r) \cosh al} \quad \text{...........(6)}$$

where C, E, Z_s, l' and Z_0 are the same as given before;

a is the vector attenuation constant and is equal to $\alpha + j\beta$;

l is the total length of the line; and

Z_r is the impedance of the apparatus at the receiving end.

Therefore, the transmission equivalent of any telephone line in terms of standard cable can not be obtained so simply as is stated above, if some degree of accuracy is required.

Since the table of hyperbolic functions of complex quantities was completed by the indefatigable diligence of Dr. A. E. Kennelly, it has become possible to facilitate, in some degree, the calculation of the formula containing them. Still, it is not an easy task to calculate such a formula as (6). It will perhaps take some hours to calculate; and such laborious calculations are necessary in each case of finding the transmission equivalent of a telephone line.

In addition to such laborious work, the equivalent length can only be worked out from a curve drawn by taking the receiving currents and the line lengths of standard cable as co-ordinates. That is, we have, at first, to calculate the receiving end current of the line from formula (6) by putting $l'=l$. Then the length of standard cable which gives equal receiving current is to be found from the curve previously drawn. This is perhaps the only way of finding the equivalent length if the full formula is to be used, or, it is almost impossible to find the equivalent length from the formula directly.

In practical telephone engineering, therefore, it is not convenient to use the complete formula; and some convenient approximate formula is needed. It is for this reason that formula (1) is generally used as simplest to calculate the equivalents. But the error is sometimes so large that we can not be content with such a rough formula.

Before going further, we have here to describe the telephone lines and the terminal apparatus used throughout the experiments in this paper.

Standard cable: The standard cable has electrical constants as previously described; but as we did not have one with exactly such constants in our laboratory, an artificial cable having electrical constants as given in Table 1 was used.

400 and 200 lb. copper aerial lines: These were artificial lines having electrical constants as given in Table 1.

100 and 50 lb. copper aerial lines: These were formed of 200 lb. copper artificial aerial line and resistances connected in series at each section so as to obtain the conductor resistances of 18 and 36 ohms per loop mile respectively.

No. 19 gauge testing cable: This was 180 pair lead covered paper cable, 0·261 mile long with conductors connected to and fro so as to form a cable circuit of the length of 46·98 miles.

No. 22 gauge special cable: This was formed of No. 19 gauge testing cable and a resistance connected in series at each section so as to obtain the conductor resistance of 176 ohms per loop mile.

No. 19 gauge special cable: This was formed of two artificial standard cables so connected as to double the electrical capacity.

Loaded No. 19 gauge cable: This was formed of No. 19 gauge testing cable and loading coils of the following description:—

Effective resistance of each loading coil ...	6 ohms
Effective inductance of each loading coil ...	0·165 henry
Space of loading coils	1·566 miles.

Terminal apparatus: These were the subscribers' sets for the magneto system and the No. 25–A repeating coils for the common battery system. The impedance of the subscriber's set at the sending end is $387{\cdot}4+j\,243{\cdot}0=456{\cdot}21\,\underline{|32°6'}$ ohms when measured by a current of 5 miliamperes and of 800 cycles per second. The impedance of the subscriber's set at the receiving end is $402+j\,290=495{\cdot}68\,\underline{|38°48'}$ ohms when measured by a current of 0·5 miliampere and of 800 cycles per second. The impedance of the No. 25–A repeating coil is given in Table 20, Section 4.

In this paper, it shall hereafter, be understood that the lines and apparatus thus named shall, if not otherwise specified, be as described above.

Table 1. Electrical constants of several telephone lines used in the experiments.

Kinds of Lines	R	L	S	K	Initial sending end impedance		Vector attenuation constant.	
					$Z_0 \lvert \theta$	$A-jB$	$P \lvert \psi$	$\alpha+j\beta$
Standard cable	86·75	0·00008	3·97	0·0607	534·35 | 44°29′	381·2 $-j$374·42	0·162 | 44°45′	0·1152 $+j$0·114
400 lb. copper aerial line	4·08	0·00344	1·57	0·0086	642·8 | 5°37′	639·7 $-j$ 62·9	0·02752 | 82°21′	0·00369 $+j$0·0273
200 lb. copper aerial line	8·15	0·00345	1·90	0·0084	673·06 | 11°20′	659·93 $-j$132·26	0·02838 | 76°3′	0·00683 $+j$0·0275
100 lb. copper aerial line	15·299	0·003433	2·33	0·00872	725·68 | 19°19′	684·82 $-j$240·05	0·03169 | 67°37′	0·01207 $+j$0·0293
50 lb. copper aerial line	33·54	0·00382	3·22	0·00842	956·77 | 27°55′	845·4 $-j$447·96	0·04040 | 57°43′	0·02157 $+j$0·03415
No. 19 gauge testing cable	90·50	0·00132	0·017	0·0507	598·44 | 42°55′	438·26 $-j$407·51	0·151 | 47°8′	0·103 $+j$0·111
No. 22 gauge special cable	169·88	0·001168	0·234	0·0522	806·92 | 44°20′	577·19 $-j$563·87	0·2105 | 45°37′	0·1473 $+j$0·1505
No. 19 gauge special cable	87·61	0·00029	18·700	0·1204	381·27 | 43°38′	275·96 $-j$263·08	0·2298 | 44°35′	0·1637 $+j$0·1613
Loaded No. 19 gauge cable	94·33	0·1067	0·017	0·0507	1461·9 | 5°1′	1456·05 $-j$127·77	0·3706 | 84°59′	0·0324 $+j$0·3692

Notes:—R is the conductor resistance per loop mile in ohms.

L is the inductance per loop mile in henry.

K is the wire-to-wire electrostatic capacity per mile in microfarad.

S is the wire-to-wire leakance per mile in micromhos.

The figures are the results measured with a high frequency alternating current of 800 cycles per second.

II. The Case of Uniform Line in Magneto System.

First let us see to what extent there exists error by using the method of attenuation lengths in obtaining the transmission equivalent of an uniform line.

Tables 2–7 show the differences between the observed and calculated results of the transmission equivalents of 400 lb., 200 lb., 100 lb. and 50 lb. copper aerial lines, No. 19 gauge testing cable and loaded No. 19 gauge cable, the calculated results being those obtained by equating attenuation lengths as is always done by telephone engineers, at present, though some reflection losses obtained by experiments are usually added for loaded lines.

Table 2. Comparison of the observed and calculated results of the transmission equivalents of 400 lb. copper aerial line in terms of standard cable.

Lengths of 400 lb. copper aerial line (miles)	Observed equivalents (1) (miles of S.C.)	Calculated equivalents by att. lengths (2) (miles of S.C.)	Differences between (1) & (2)
20	2·00	0·641	1·359
40	2·00	1·281	0·719
60	1·50	1·922	−0·422
80	3·00	2·562	0·438
100	5·80	3·203	2·597
110	6·75	3·523	3·227
120	7·50	3·854	3·646
140	8·15	4·484	3·666
160	8·60	5.125	3·475
180	9·00	5·765	3·235
200	9·75	6·406	3·344
220	10·75	7·047	3·703
250	11·85	8·008	3·842
270	12·40	8·648	3·752
300	13·10	9·609	3·491
330	14·20	10·570	3·630
350	14·90	11·211	3·689
370	15·62	11·851	3·769
400	16·33	12·812	3·518
420	17·00	13·453	3·547
450	17·87	14·439	3·431
470	18·62	15·054	3·566
500	19·67	16·015	3·655
550	21·20	17·617	3·583
600	23·00	19·219	3·731

(Differences for 110–600 miles: Average of these values 3·578.)

The reflection equivalent calculated by the second term of formula (12)...................................... 3.5426.

Table 3. Comparison of the observed and calculated results of the transmission equivalents of 200 lb. copper aerial line in terms of standard cable.

Lengths of 200 lb. copper aerial line (miles)	Observed equivalents (1) (miles of S.C.)	Calculated equivalents by att. lengths (2) (miles of S.C.)	Differences between (1) & (2)
20	3·50	1·188	2·312
40	4·50	2·371	2·129
60	5·25	3·557	1·693
80	7·00	4·743	2·257
90	8·25	5·336	2·914
100	9·12	5·929	3·191
110	10·00	6·522	3·478
120	10·70	7·115	3·585
140	11·75	8·301	3·449
160	12·75	9·486	3·264
180	13·70	10·672	3·028
200	15·20	11·858	3·342
220	16·50	13·044	3·456
250	18·25	14·822	3·428
270	19·43	16·008	3·422
300	21·20	17·786	3·414
330	23·15	19·564	3·586
350	24·10	20·751	3·349
370	25·12	21·936	3·184
400	27·07	23·715	3·335
420	28·00	24·901	3·099
450	29·80	26·680	3·120
470	31·10	27·865	3·235
500	33·10	29·644	3·456
550	36·00	32·608	3·392
600	38·50	35·573	2·923

Average of these values 3·302. (differences for 90 to 600 miles)

The reflection equivalent calculated by the second term of formula (12)................................. 3·2915.

Table 4. Comparison of the observed and calculated results of the transmission equivalents of 100 lb. copper aerial line in terms of standard cable.

Lengths of 100 lb. copper aerial line (miles)	Observed equivalents (1) (miles of S.C.)	Calculated equivalents by att. lengths (2) (miles of S.C.)	Differences between (1) & (2)
10	3·75	1·048	2·702
20	5·60	2·095	3·505
30	6·70	3·143	3·557
40	7·25	4·191	3·059
50	8·00	5·238	2·762
60	9·00	6·286	2·714
70	10·25	7·334	2·916
80	11·50	8·382	3·118
90	12·40	9·429	2·971
100	13·50	10·477	3·023

Average of these values 3.007.

The reflection equivalent calculated by the second term of formula (12)................................... 2.9607.

Table 5. Comparison of the observed and calculated results of the transmission equivalents of 50 lb. copper aerial line in terms of standard cable.

Lengths of 50 lb. copper aerial line (miles)	Observed equivalents (1) (miles of S.C.)	Calculated equivalents by att. lengths (2) (miles of S.C.)	Differences between (1) & (2)
10	5·80	1·872	3·928
20	8·00	3·745	4·255
30	9·50	5·617	3·883
40	10·83	7·489	3·341
50	12·55	9·361	3·189
60	14·20	11·234	2·966
70	16·20	13·106	3·094
80	18·12	14·978	3·142
90	20·00	16·851	3·149
100	21·85	18·723	3·127

Average of these values 3.144.

The reflection equivalent calculated by the second term of ormula (12)................................... 3.1595.

Table 6. Comparison of the observed and calculated results of the transmission equivalents of No. 19 gauge testing cable in terms of standard cable.

Lengths of No. 19 gauge testing cable (miles)	Observed equivalents (1) (miles of S.C.)	Calculated equivalents by att. lengths (2) (miles of S.C.)	Differences between (1). & (2).	
1·044	1·50	0·935	0·565	
2·088	2·50	1·870	0·630	
4·176	4·30	3·741	0·559	
6·264	6·20	5·611	0·589	
8·352	7·90	7·482	0·418	
10·440	9·73	9·352	0·378	
12·528	11·55	11·223	0·327	Average of these values 0·3504.
14·616	13·40	13·093	0·307	
16·704	15·27	14·964	0·306	
18·792	17·16	16·834	0·326	
20·880	19·05	18·705	0·345	
22·968	20·95	20·575	0·375	
26·100	23·75	23·381	0·369	
31·320	28·50	28·057	0·443	
36·540	33·10	32·734	0·366	
41·760	37·75	37·410	0·340	

The reflection equivalent calculated by the second term of formula (12)................................. 0·3226.

Table 7. Comparison of the observed and calculated results of the transmission equivalents of loaded No. 19 gauge cable in terms of standard cable.

Lengths of loaded No. 19 gauge cable (miles)	Observed equivalents (1) (miles of S.C.)	Calculated equivalents by att. lengths (2) (miles of S.C.)	Differences between (1) & (2)
4·698	8·75	1·321	7·429
7·830	6·50	2·202	4·298
9·396	9·83	2·643	7·187
12·528	11·00	3·523	7·477
15.660	10·10	4·404	5·696
18·792	12·92	5·285	7·635
20·358	13·00	5·726	7·274
23·490	12·75	6·607	6·143
25·056	13·80	7·047	6·753
29·754	14·70	8·368	6·332
32·886	15·80	9·249	6·551
37·584	17·15	10·570	6·580
42·282	18·60	11·892	6·708

Average of these values 6·5427.

The reflection equivalent calculated by the second term of formula (12).................................. 6·2109.

In the tables 2—7, the observed equivalents are obtained by the following method:—

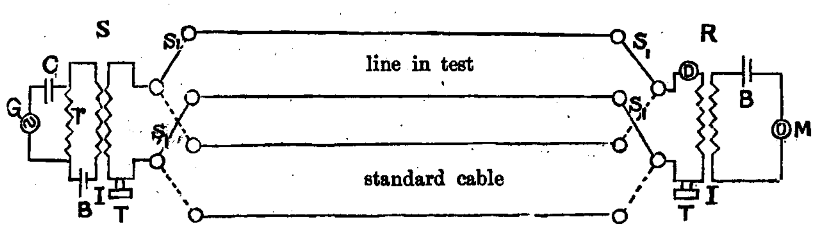


Fig. 1. Connection for measuring the transmission loss in a line.

In Fig. 1, S and R are the sending and receiving sets respectively, G a high frequency generator, r a resistance replacing the transmitter at the sending end, S_1 four switches which are capable of cutting over from the line in test to standard cable or vise versa, T receivers, D Duddell's thermo-galvanometer, I induction coils, M microphone, B batteries, C condenser.

A high frequency alternating electromotive-force is applied by G to the line in test or to the standard cable, the switches being used to cut over from the one to the other, and the length of the standard cable is so adjusted as to make the deflections on D the same at any position of the switches. But, as the smallest section of standard cable used for the experiments is 0·5 mile, we can not, in general, obtain exactly equal deflection. So that the equivalents are calculated by proportion from two lengths of standard cable which give the nearest deflections on both sides of the deflection for each length of the line in test.

From the preceding tables we see that the differences between the observed and calculated equivalents are nearly constant for any except a short length of line, and that we are neglecting some miles in the calculation of the transmission equivalents which are due to reflection at both ends of the line. We will call this part of the equivalent the "reflection equivalent".

The author, therefore, proposes the following formula for the transmission equivalent of any line:—

$$l_0 = \frac{a_1 l_1}{a_0} + \frac{2{\cdot}303}{a_0} \log_{10} \frac{Z_0(Z_1+Z_s)(Z_1+Z_r)}{Z_1(Z_0+Z_s)(Z_0+Z_r)} \quad \text{.........(7)}$$

where l_0 is the transmission equivalent of l_1 miles of any line under consideration, in miles of standard cable;

a_0 and a_1 are the attenuation constants of standard cable and the line under consideration respectively;

Z_0 and Z_1 are their initial sending end impedances; and

Z_s and Z_r are the impedances of the apparatus at the sending and receiving ends respectively.

Let us now proceed to discuss how formula (7) is obtained.

From formula (6) we get the following formula for the current at the receiving end.

$$C_r = \frac{EZ_0}{(Z_0^2+Z_sZ_r)\sinh al+(Z_0Z+Z_0Z_r)\cosh al} \quad \text{............(8)}$$

or

$$C_r = \frac{2EZ_0}{(Z_0^2+Z_sZ_r)(e^{al}-e^{-al})+(Z_0Z_s+Z_0Z_r)(e^{al}+e^{-al})} \quad \text{......(9).}$$

In formula (9), if al is large or the length of line is not too short, e^{-al} becomes negligibly small compared with e^{al}, and we get

$$C_r = \frac{2EZ_0}{(Z_0+Z_s)(Z_0+Z_r)}\, e^{-al} \quad \text{..................(10).}$$

From formula (8), it can be shown that the current at the receiving end of a line decreases with the increase of its length, forming a wavy curve of a period equal to one-half of the natural period of the line. But the amplitude of the wave decreases as the length increases and at length a smooth logarithmically decreasing curve is formed which may be shown by formula (10).

Now, to obtain the transmission equivalent of a line in terms of standard cable, it is only necessary to know the miles of standard cable, the receiving end current of which is equal to that of the line under consideration. But as the calculation of formula (8) is very troublesome, the author proposes to use formula (10).

Using the symbols given above, if the receiving currents for both standard cable and the line under consideration are equal, we have

$$\frac{2EZ_0}{(Z_0+Z_s)(Z_0+Z_r)}\, e^{-a_0l_0} = \frac{2EZ_1}{(Z_1+Z_s)(Z_1+Z_r)}\, e^{-a_1l_1} \quad \text{...(11).}$$

But as the loudness of the receiving sound does not depend upon the phase of the current, we can leave the angle of both sides of equation (11) out of consideration and obtain

$$l_0 = \frac{\alpha_1 l_1}{\alpha_0} + \frac{2{\cdot}303}{\alpha_0} \log_{10} \frac{Z_0(Z_1+Z_s)(Z_1+Z_r)}{Z_1(Z_0+Z_s)(Z_0+Z_r)} \quad \text{.........(12)}$$

where α_0 and α_1 are the real parts of a_0 and a_1 respectively.

Formula (12) is identical with formula (7); and its second term corresponds to the reflection equivalent.

In the next place, let us examine the degree of accuracy of formula (12).

The following theory is the application of that given by Dr. A. E. Kennelly in his "The application of hyperbolic functions to electrical engineering problem".

If the receiving end of a line is closed through an impedance Z_r, any current on arriving at that end is split into two parts, namely, a transmitted part which is absorbed without further reflection, and a reflected part which goes back as though from an open end. Let m be the fraction of the wave that is transmitted or the transmission coefficient, and $1-m$ the fraction that is reflected or the reflection coefficient. It was shown by Heaviside that in the symbols here used

$$m = \frac{2Z_0}{Z_0+Z_r} \quad \text{..........................(13)}$$

$$1-m = \frac{Z_r-Z_0}{Z_0+Z_r} \quad \text{.......................(14)}$$

The reflected current-wave retreats with its sign reversed, or as with a coefficient $m-1$.

The current wave that is absorbed at the receiving end on first arrival is mI_0e^{-al}, and $(m-1)I_0e^{-al}$ goes back to the sending end, where I_0 is the initial sending end current. It reaches the sending end in the condition $(m-1)I_0e^{-2al}$. Here, the condition is just about the same as it is at the receiving end. If, therefore, m' is the transmission coefficient at the sending end, we have

$$m' = \frac{2Z_0}{Z_0+Z_s} \quad \text{..........................(15)}$$

$$1-m' = \frac{Z_s - Z_0}{Z_0 + Z_s} \quad \text{.......................(16)}.$$

Then the reflected current wave is

$$(m'-1)(m-1)I_0 e^{-2al} \quad \text{.......................(17)}$$

which arrives at the receiving end for the second time in the condition

$$(m'-1)(m-1)I_0 e^{-3al} \quad \text{.......................(18)}.$$

Of this

$$m(m'-1)(m-1)I_0 e^{-3al} \quad \text{.......................(19)}$$

is absorbed and the remainder

$$(m'-1)(m-1)^2 I_0 e^{-3al} \quad \text{.......................(20)}$$

retreats to the sending end. The final summation absorbed at the receiving end is :—

$$C_r = mI_0 e^{-al} + m(m'-1)(m-1)I_0 e^{-3al} + m(m'-1)^2(m-1)^2 I_0 e^{-5al}$$
$$+ \text{....................} \quad \text{..........................(21)}$$

$$= \frac{mI_0 e^{-al}}{1-(m'-1)(m-1)e^{-2al}} = \frac{mI_0}{e^{al}-(m'-1)(m-1)e^{-al}} \text{......(22)}.$$

If we put $I_0 = \dfrac{E}{Z_0 + Z_s}$ and give m and m' their proper values in (22), we obtain

$$C_r = \frac{EZ_0}{(Z_0^2 + Z_s Z_r)\sinh al + Z_0(Z_r + Z_s)\cosh al} \text{.........(23)}$$

which is identical with formula (8).

Suppose, now, that the absorbed part of the current wave which arrives at the receiving end for the second time is so small that it can be negligible compared with that which arrives at that end the first time, then the second and the following terms may be neglected and we obtain

$$C_r = mI_0 e^{-al} = \frac{2Z_0}{Z_0 + Z_r} \cdot \frac{E}{Z_0 + Z_s} e^{-al} \quad \text{...........(24)}.$$

The above reasoning holds good, of course, on any uniform telephone line. If, therefore, the receiving end currents for standard cable and the line under consideration are equal, we get

$$\frac{2Z_0}{Z_0+Z_r}\,\frac{E}{Z_0+Z_s}\,e^{-a_0l_0} = \frac{2Z_1}{Z_1+Z_r}\cdot\frac{E}{Z_1+Z}\,e^{-a_1l_1} \quad\ldots\ldots(25)$$

which is identical with equation (11). This shows us that formula (12) is the result obtained by neglecting the currents arriving at the receiving end for the second and the following times and being there absorbed by the receiver.

Now let us calculate the value of the reflection equivalent

$$\frac{2{\cdot}303}{a_0}\cdot\log_{10}\frac{Z_0(Z_1+Z_s)(Z_1+Z_r)}{Z_1(Z_0+Z_s)(Z_0+Z_r)} \quad\ldots\ldots\ldots\ldots\ldots(26)$$

in formula (12) for 400 lb. and 200 lb. copper aerial lines, No. 19 gauge testing cable and loaded No. 19 gauge cable.

Here we may leave the angle of each factor out of consideration as has been shown when equation (12) was obtained. Therefore, for standard cable (see Table 8)

$$\log_{10} Z_0 = \log_{10} 534{\cdot}35 = 2{\cdot}72783$$
$$\log_{10}(Z_0+Z_s) = \log_{10} 780{\cdot}14 = 2{\cdot}89217$$
$$\log_{10}(Z_0+Z_r) = \log_{10} 787{\cdot}74 = 2{\cdot}89638$$
$$a_0 = 0{\cdot}1152.$$

For 400 lb. copper aerial line

$$\log_{10} Z_1 = \log_{10} 642{\cdot}80 = 2{\cdot}80808$$
$$\log_{10}(Z_1+Z_s) = \log_{10} 1042{\cdot}77 = 3{\cdot}01819$$
$$\log_{10}(Z_1+Z_r) = \log_{10} 1066{\cdot}16 = 3{\cdot}02782$$

$$\therefore\quad \frac{2{\cdot}303}{a_0}\log_{10}\frac{Z_0(Z_1+Z_s)(Z_1+Z_r)}{Z_1(Z_0+Z_s)(Z_0+Z_r)} = \frac{2{\cdot}303\times 0.17721}{0{\cdot}1152} = 3{\cdot}5426.$$

For 200 lb. copper aerial line

$$\log_{10} Z_1 = \log_{10}\ 673{\cdot}06 = 2{\cdot}82805$$

$$\log_{10}(Z_1+Z_s) = \log_{10} 1053{\cdot}16 = 3{\cdot}02250$$

$$\log_{10}(Z_1+Z_r) = \log_{10} 1073{\cdot}58 = 3{\cdot}03083$$

$$\therefore\quad \frac{2{\cdot}303}{a_0}\log_{10}\frac{Z_0(Z_1+Z_s)(Z_1+Z_r)}{Z_1(Z_0+Z_s)(Z_0+Z_r)} = \frac{2{\cdot}303\times 0{\cdot}16456}{0{\cdot}1152} = 3{\cdot}2915.$$

For No. 19 gauge testing cable

$$\log_{10} Z_1 = \log_{10} 598{\cdot}44 = 2{\cdot}77702$$

$$\log_{10}(Z_1+Z_s) = \log_{10} 841{\cdot}89 = 2{\cdot}92526$$

$$\log_{10}(Z_1+Z_r) = \log_{10} 848{\cdot}44 = 2{\cdot}92862$$

$$\therefore\quad \frac{2{\cdot}303}{a_0}\log_{10}\frac{Z_0(Z_1+Z_s)(Z_1+Z_r)}{Z_1(Z_0+Z_s)(Z_0+Z_r)} = \frac{2{\cdot}303\times 0{\cdot}01614}{0{\cdot}1152} = 0{\cdot}3226.$$

For loaded No. 19 gauge cable

$$\log_{10} Z_1 = \log_{10} 1461{\cdot}9 = 3{\cdot}16592$$

$$\log_{10}(Z_1+Z_s) = \log_{10} 1847{\cdot}33 = 3{\cdot}26655$$

$$\log_{10}(Z_1+Z_r) = \log_{10} 1865{\cdot}4 = 3{\cdot}27077$$

$$\therefore\quad \frac{2{\cdot}303}{a_0}\log_{10}\frac{Z_0(Z_1+Z_s)(Z_1+Z_r)}{Z_1(Z_0+Z_s)(Z_0+Z_r)} = \frac{2{\cdot}303\times 0{\cdot}31068}{0{\cdot}1152} = 6{\cdot}2109.$$

These four examples show that the average values of the differences between (1) and (2) in tables 2, 3, 6 and 7 are very nearly equal to the four values obtained above, the differences being only a few per cent.

As Z_0, Z_s, Z_r, etc. are vector quantities it is somewhat laborious to calculate the value shown in formula (26). But as it is independent of the length of the line, it may previously be calculated as we do with the attenuation constants of several different lines in case the equivalents are calculated by comparing the attenuation lengths.

In table 8 all such constants as are necessary to calculate the formulas of the reflection equivalents are given.

Table 8. Constants necessary to calculate the reflection equivalents of the various lines used for these experiments.

(Magneto system.)

Kinds of lines	Z_0	$\log_{10} Z_0$	Z_0+Z_s	$\log_{10}(Z_0+Z_s)$	Z_0+Z_r	$\log_{10}(Z_0+Z_r)$	Z_0-Z_s	$\log_{10}(Z_0-Z_s)$	Z_0-Z_r	$\log_{10}(Z_0-Z_r)$
Standard cable	534·35	2·72783	780·14	2·89217	787·74	2·89638	617·45	2·79060	664·75	2·82266
400 lb. copper aerial line	642·80	2·80808	1042·77	3·01819	1066·16	3·02782	396·52	2·59827	425·49	2·69889
200 lb. copper aerial line	673·06	2·82805	1053·16	3·02250	1073·58	3·03083	463·78	2·66631	494·80	2·69443
100 lb. copper aerial line	725·68	2·86098	1072·22	3·02979	1087·97	3·03662	567·27	2·75379	600·79	2·77872
50 lb. copper aerial line	956·77	2·98081	1249·75	3·09682	1257·39	3·09947	828·97	2·91854	860·92	2·93496
No. 19 gauge testing cable	598·44	2·77702	841·89	2·92526	848·44	2·92862	652·49	2·81458	698·46	2·84414
No. 22 gauge special cable	806·92	2·90683	1016·56	3·00713	1016·77	3·00722	828·89	2·91850	871·67	2·94035
No. 19 gauge special cable	275·96	2·58123	663·66	2·82195	678·49	2·83154	518·18	2·71450	567·26	2·75378
Loaded No. 19 gauge cable	1461·90	3·16592	1847·33	3·26655	1865·40	3·27077	1131·19	3·05352	1133·85	3·05455

Figs. 2, 3 and 4 show the differences between (1) and (2) in Tables 2, 3, 6 and 7 and the values calculated from formula (26). The full line curves show the differences between (1) and (2) in those tables, i.e., the reflection equivalents obtained by the experiments and the dotted lines show the values calculated from formula (26).

These curves and straight lines show us clearly that formula (7), given by the author, is practically accurate for securing the equivalent miles of a telephone line in terms of standard cable, so long as the length of the line is not very short. In the instances given above, it may be seen from the curves or tables that the formula is practically correct for lines longer than the following:—

for 400 lb. copper aerial line longer than			...	110 miles
" 200 lb. copper aerial line	"	"	...	90 miles
" 100 lb. copper aerial line	"	"	...	65 miles
" 50 lb. copper aerial line	"	"	...	45 miles
" No. 19 gauge testing cable	"	"	...	11 miles
" Loaded No. 19 gauge cable	"	"	...	27 miles.

Formula (7), proposed by the author, must be more exact, if the ratio of the sum of the second and the following terms to the first term of formula (21) is smaller. But, as the sum of the third and the following terms is very small, the condition will be met if the ratio of the second term to the first is very small.

Now, let us calculate the ratios for those six lines as exemplified above.

$$\text{The ratio is } \frac{m(m'-1)(m-1)I_0e^{-3al}}{m\,I_0e^{-al}}=(m'-1)(m-1)e^{-2al}$$

$$=\frac{Z_0-Z_s}{Z_0+Z_s}\cdot\frac{Z_0-Z_r}{Z_0+Z_r}e^{-2al}=\frac{(Z_0-Z_s)(Z_0-Z_r)}{(Z_0+Z_s)(Z_0+Z_r)}\,e^{-2\alpha l}\,\underline{|2\beta l}\quad\ldots(27a)$$

400. lb. copper aerial line:—

Leaving the angle of formula (27*a*) out of consideration, we get

$$\text{the ratio }=\frac{396{\cdot}52\times425{\cdot}49}{1042{\cdot}77\times1066{\cdot}16}\,e^{-2\times0{\cdot}00369\times l}.$$

Putting $l = 110$ and taking common logarithms, we get

$$2{\cdot}59827 + 2{\cdot}62889 - 3{\cdot}01819 - 3{\cdot}02782$$
$$- 2 \times 0{\cdot}00369 \times 110 \times 0{\cdot}4343 = -1{\cdot}17141$$

$$\therefore \text{ the ratio } = \frac{1}{14{\cdot}81} = 0.067.$$

200 lb. copper aerial line :—

Putting $l = 90$, we get

$$\text{the ratio } = \frac{463{\cdot}78 \times 494{\cdot}80}{1053{\cdot}16 \times 1073{\cdot}58} e^{-2\times 0{\cdot}00683 \times 90} = \frac{1}{16{\cdot}847} = 0{\cdot}059.$$

100 lb. copper aerial line :—

Putting $l = 65$, we get

$$\text{the ratio } = \frac{567{\cdot}27 \times 600{\cdot}79}{1072{\cdot}22 \times 1087{\cdot}97} e^{-2\times 0{\cdot}01207 \times 65} = \frac{1}{16{\cdot}42} = 0{\cdot}061.$$

50 lb. copper aerial line :—

Putting $l = 45$, we get

$$\text{the ratio } = \frac{828{\cdot}97 \times 860{\cdot}92}{1249{\cdot}75 \times 1257{\cdot}39} e^{-2\times 0{\cdot}02157 \times 45} = \frac{1}{15{\cdot}332} = 0{\cdot}065.$$

No. 19 gauge testing cable :—

Putting $l = 11$, we get

$$\text{the ratio } = \frac{652{\cdot}49 \times 698{\cdot}46}{841{\cdot}89 \times 848{\cdot}44} e^{-2\times 0{\cdot}103 \times 11} = \frac{1}{15{\cdot}178} = 0.066.$$

Loaded No. 19 gauge cable :—

Putting $l = 27$, we get

$$\text{the ratio } = \frac{1131{\cdot}19 \times 1133{\cdot}85}{1847{\cdot}33 \times 1865{\cdot}40} e^{-2\times 0{\cdot}0324 \times 27} = \frac{1}{15{\cdot}453} = 0{\cdot}065.$$

The above examples show us that the ratios are all nearly equal and their average is 0·064. It is easily seen, therefore, that formula (7) is practically applicable to any line having such length as to make the value of formula (27*a*) less than 0·064. This limit of length of a line is, thus, found from the following formula and will, hereafter, be referred to as the "limiting length of a line".

$$l=\frac{2{\cdot}303}{2a}\log_{10}\frac{(Z_0-Z_s)(Z_0-Z_r)}{(Z_0+Z_s)(Z_0+Z_r)\times 0{\cdot}064}\;\ldots\ldots\ldots\ldots(27b).$$

III. The Case When the Telephone Line Consists of More Than One Uniform Line in Magneto System.

The author proposes formula (28) for the transmission equivalent of a line consisting of more than one uniform line

$$l_0=\frac{a_1l_1+a_2l_2+a_3l_3+\ldots\ldots\ldots\ldots}{a_0}$$

$$+\frac{2{\cdot}303}{a_0}\log_{10}\frac{Z_0(Z_1+Z_s)(Z_1+Z_2)(Z_2+Z_3)\ldots\ldots\ldots(Z_n+Z_r)}{2^{n-1}(Z_0+Z_s)Z_1.Z_2.Z_3\ldots\ldots\ldots Z_n(Z_0+Z_r)}\;\ldots(28).$$

This can easily be obtained from formula (25).

For the transmission equivalent of the first section of the line, we have the equation

$$\frac{E}{Z_0+Z_s}e^{-a_0l_0'}=\frac{E}{Z_1+Z_s}\cdot\frac{2Z_1}{Z_1+Z_2}e^{-a_1l_1}\;\ldots\ldots\ldots\ldots\ldots(29)$$

as standard cable is continued still further and the impedance at the first junction point of the line is Z_2 instead of Z_r if the second line is not short. In the same way for the first and second sections, we have

$$\frac{E}{Z_0+Z_s}e^{-a_0(l_0'+l_0'')}=\frac{E}{Z_1+Z_s}\cdot\frac{2Z_1}{Z_1+Z_2}\cdot\frac{2Z_2}{Z_2+Z_3}e^{-a_1l_1-a_2l_2}\;\ldots\ldots(30)$$

For up-to nth section, we have therefore,

$$\frac{2Z_0}{Z_0+Z_r}\cdot\frac{E}{Z_0+Z_s}\,e^{-\alpha_0 l_0}=\frac{E}{Z_1+Z_s}\cdot\frac{2Z_1}{Z_1+Z_2}\cdot\frac{2Z_2}{Z_2+Z_3}\cdots\cdots\frac{2Z_n}{Z_n+Z_r}\,e^{-\alpha_1 l_1-\alpha_2 l_2\cdots\cdots-\alpha_n l_n} \qquad \cdots\cdots\cdots\cdots\cdots\cdots(31)$$

where $l_0=l_0'+l_0''+l_0'''+\cdots\cdots\cdots\cdots$up-to nth term.

Leaving the angles of both sides of equation (31) out of consideration, we have an equation the same as (28).

Formula (28) is applicable to a line consisting of several different lines each of which is not shorter than a certain length. This limiting length is found by the ratio of the first and second terms of the formula for the current at the receiving end of that section.

Let us, first, consider a line consisting of two different lines, one of which is long enough to make the initial and final sending end impedances practically equal.

In case the long line is at the sending end and the other line under consideration is at the receiving end, the limiting length is found in the following way:—

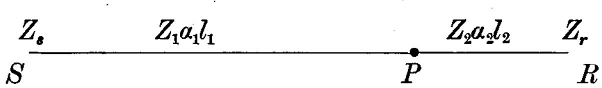


Fig. 5.

Let the line under consideration be as shown in Fig. 5, Z_1, Z_2, α_1, α_2, l_1, l_2, being the initial sending end impedances, the attenuation constants and the lengths of two lines, and Z_s and Z_r the impedances of the apparatus at the sending and receiving ends respectively.

Then the incident current at the junction point P is

$$\frac{E}{Z_1+Z_s}\,e^{-\alpha_1 l_1}$$

E being the electromotive-force applied to the sending end apparatus.

The transmission and reflection coefficients are respectively

$$m_1 = \frac{2Z_1}{Z_1+Z_f}, \qquad 1-m_1 = \frac{Z_f-Z_1}{Z_1+Z_f} \quad \text{............(32)}$$

where Z_f is the final sending end impedance of the second line.

Hence, the current transmitted to the second line is

$$m_1 \frac{E}{Z_1+Z_s} e^{-\alpha_1 l_1}.$$

The incident current at the receiving end is, therefore,

$$m_1 \frac{E}{Z_1+Z_s} e^{-\alpha_1 l_1-\alpha_2 l_2}.$$

Here as the transmission and reflection coefficients are

$$m_2 = \frac{2Z_2}{Z_2+Z_r} \quad \text{and} \quad 1-m_2 = \frac{Z_r-Z_2}{Z_2+Z_r} \quad \text{............(33)}$$

the currents transmitted and reflected are respectively

$$m_1 m_2 \frac{E}{Z_1+Z_s} e^{-\alpha_1 l_1-\alpha_2 l_2} \quad \text{and} \quad (1-m_2)m_1 \frac{E}{Z_1+Z_s} e^{-\alpha_1 l_1-\alpha_2 l_2} \quad \text{...(34).}$$

But the reflected current changes its sign and retreats to the junction point, where it is again reflected and comes back to the receiving end with a current value of

$$(m_3-1)(m_2-1)m_1 \frac{E}{Z_1+Z_s} e^{-\alpha_1 l_1-3\alpha_2 l_2} \quad \text{..................(35)}$$

$$\text{where} \quad m_3-1 = \frac{Z_2-Z_1}{Z_1+Z_2}.$$

The last current is partly absorbed by the receiver coil and partly reflected. The absorbed part is

$$(m_3-1)(m_2-1)m_1 m_2 \frac{E}{Z_1+Z_s} e^{-\alpha_1 l_1-3\alpha_2 l_2}.$$

These processes are repeated until the first reflected current at the receiving end becomes, at last, zero.

The current at the receiving end is, therefore, the sum of the currents absorbed by the receiver, that is

$$C_r = m_1 m_2 \frac{E}{Z_1+Z_s} e^{-\alpha_1 l_1 - \alpha_2 l_2} + (m_3-1)(m_2-1) m_1 m_2 \frac{E}{Z_1+Z_s} e^{-\alpha_1 l_1 - 3\alpha_2 l_2}$$
$$+ (m_3-1)^2 (m_2-1)^2 m_1 m_2 \frac{E}{Z_1+Z_s} e^{-\alpha_1 l_1 - 5\alpha_2 l_2} + \text{etc.} \quad \ldots\ldots\ldots\ldots(36).$$

The ratio of the second term to the first is

$$(m_3-1)(m_2-1)e^{-2\alpha_2 l_2} = \frac{Z_2-Z_1}{Z_1+Z_2} \frac{Z_2-Z_r}{Z_2+Z_r} e^{-2\alpha_2 l_2} \quad \ldots\ldots(37).$$

Similarly, in case the long line is at the receiving end and the other line under consideration is at the sending end, the ratio of the second term to the first is easily found as follows:—

$$\frac{(Z_1-Z_s)(Z_1-Z_2)}{(Z_1+Z_s)(Z_1+Z_2)} e^{-2\alpha_1 l_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(38)$$

where Z_s is the impedance of the sending end apparatus and the other symbols are the same as before.

From the results of experiments shown in Tables 9–16, it is seen that the approximate limiting length of several lines connected to other lines longer than the limiting lengths are as follows:—

The long line at the sending end and short line at the receiving end.

Long 200 lb. copper aerial line
+ short No. 19 gauge testing cable ...... 6·5 miles

Long No. 19 gauge testing cable
+ short 200 lb. copper aerial line......... 55 miles

Long 400 lb. copper aerial line
+ short No. 19 gauge testing cable ...... 7 miles

Long No. 19 gauge testing cable
+ short 400 lb. copper aerial line......... 90 miles.

If the above lengths of lines are put into formula (37), we get

0·062, 0·063, 0·066, 0·070 respectively.

The average of these values is 0·065 which is nearly equal to that of the uniform lines.

The short line at the sending end and the long line at the receiving end.

Short 200 lb. copper aerial line
+ long No. 19 gauge testing cable ...... 50 miles

Short No. 19 gauge testing cable
+ long 200 lb. copper aerial line .... 6·2 miles

Short 400 lb. copper aerial line
+ long No. 19 gauge testing cable ...... 90 miles

Short No. 19 gauge testing cable
+ long 400 lb. copper aerial line .... 7 miles.

If the above lengths of lines are put into formula (38), we get

0·064, 0·062, 0·066, 0·062 respectively.

The average of these values is 0·064 which is just equal to that of the uniform lines.

It is to be noticed here that, in Tables 9–16, the limiting lengths of the shorter lines do not coincide with those mentioned above when the longer lines are 100 miles of 200 lb. aerial line or 10·44 miles of No. 19 gauge testing cable. This is because the lengths of the longer lines were not long enough to make the initial and final sending end impedances practically equal.

In Table 18, all such constants as are necessary to calculate the formulas of the reflection equivalents and the limiting lengths of the several lines used for these experiments are given, excepting those constants which are shown in Table 8.

Table 9. Comparison of the observed and calculated results of the transmission equivalents of 100 and 300 miles of 200 lb. copper aerial line connected to various lengths of No. 19 gauge testing cable in terms of standard cable.

Lengths of lines (miles) 200 lb. line + Testing cable	Observed equivalents (1) (miles of S.C.)	Calculated equivalents by att. lengths (2) (miles of S.C.)	Differences between (1) & (2)	
100 + 0	9·12	5·929	3·191	
„ + 2·088	10·42	7·799	2·621	
„ + 4·176	11·60	9·670	1·930	
„ + 6·264	13·05	11·540	1·510	Average of these values 1·214.
„ + 8·352	14·70	13·411	1·289	
„ + 10·440	16·50	15·281	1·219	
„ + 12·528	18·33	17·152	1·278	
„ + 14·616	20·20	19·022	1·178	
„ + 16·704	22·05	20·893	1·167	
„ + 18·792	23·90	22·763	1·137	
„ + 20·880	25·75	24·634	1·116	
„ + 22·968	27·84	26·504	1·336	
„ + 26·100	30·50	29·310	1·190	
„ + 31·320	35·00	33·986	1·014	
„ + 36·540	39·80	38·663	1·137	
300 + 0	21·20	17·786	3·414	
„ + 2·088	22·00	19·656	2·344	
„ + 4·176	23·25	21·527	1·723	
„ + 6·264	24·68	23·397	1·283	Average of these values 1·203.
„ + 8·352	26·60	25·268	1·332	
„ + 12·528	30·25	29·009	1·241	
„ + 16·704	33·90	32·750	1·150	
„ + 20·880	37·50	36·491	1·009	

The reflection equivalent calculated by the second term of formula (28).................................. 1·4434.

Table 10. Comparison of the observed and calculated results of the transmission equivalents of various lengths of No. 19 gauge testing cable connected to 100 and 300 miles of 200 lb. copper aerial line in terms of standard cable.

Lengths of lines (miles) Testing cable + 200 lb. line	Observed equivalents (1) (miles of S.C.)	Calculated equivalents by att. lengths (2) (miles of S.C.)	Differences between (1) & (2)
0 +100	9·12	5·929	3·191
2·088 + „	10·40	7·799	2·601
4·176 + „	11·83	9·670	2·160
6·264 + „	13·30	11·540	1·760
8·352 + „	14·90	13·411	1·489
10·440 + „	16·65	15·281	1·369
12·528 + „	18·50	17·152	1·348
14·616 + „	20·35	19·022	1·328
16·704 + „	22·16	20·893	1·267
18·792 + „	24·20	22·763	1·437
20·880 + „	26·10	24·634	1·466
22·968 + „	28·00	26·504	1·496
26·100 + „	30·75	29·310	1·440
31·320 + „	35·30	33·986	1·314
36·540 + „	40·00	38·663	1·337
0 +300	21·20	17·786	3·414
2·088 + „	22·00	19·656	2·344
4·176 + „	23·25	21·527	1·723
6·264 + „	24·90	23·397	1·457
8·352 + „	26·70	25·268	1·432
12·528 + „	30·37	29·009	1·361
16·704 + „	34·10	32·750	1·350
20·880 + „	37·80	36·491	1·309

(Rows 6·264 to 36·540 + 100:) Average of these values 1·421.

(Rows 6·264 to 20·880 + 300:) Average of these values 1·382.

The reflection equivalent calculated by the second term of formula (28)................................. 1·5426.

Table 11. Comparison of the observed and calculated results of the transmission equivalents of 10·44 and 20·88 miles of No. 19 gauge testing cable connected to various lengths of 200 lb. copper aerial line in terms of standard cable.

Lengths of lines (miles) Testing cable + 200 lb. line	Observed equivalents (1) (miles of S.C.)	Calculated equivalents by att. lengths (2) (miles of S.C.)	Differences between (1) & (2)	
10·44 + 0	9·73	9·352	0·378	
„ + 10	10.40	9·946	0·454	
„ + 20	11·50	10·540	0·962	
„ + 30	12·50	11·131	1·369	
„ + 40	13·60	11·723	1·877	
„ + 60	15·00	12·909	2·091	
„ + 80	15·75	14·095	1·655	Average of these values 1·470.
„ + 100	16·65	15·281	1·369	
„ + 120	17·75	16·467	1·283	
„ + 150	19·65	18·245	1·405	
„ + 180	21·50	20·024	1·476	
„ + 200	22·65	21·210	1·440	
„ + 220	23·85	22·396	1·454	
„ + 250	25·65	24·174	1·476	
„ + 270	26·85	25·360	1·490	
„ + 300	28·70	27·138	1·562	
„ + 320	29·90	28·324	1·576	
„ + 350	31·60	30·103	1·497	
„ + 400	34·50	33·067	1·433	
20·88 + 0	19·05	18·705	0·345	
„ + 20	21·20	19·893	1·307	
„ + 40	22·90	21·076	1·824	
„ + 60	23·90	22·262	1·638	
„ + 80	25·10	23·448	1·652	Average of these values 1·492.
„ + 120	27·25	25·820	1·430	
„ + 150	29·10	27·598	1·502	
„ + 200	32·00	30·563	1·437	
„ + 250	35·00	33·527	1·473	
„ + 300	37·80	36·491	1·309	

The reflection equivalent calculated by the second term of formula (28).................................... 1·5426.

Table 12. Comparison of the observed and calculated results of the transmission equivalents of various lengths of 200 lb. copper aerial line connected to 10·44 and 20·88 miles of No. 19 gauge testing cable in terms of standard cable.

Lengths of lines (miles) 200 lb. line + Testing cable	Observed equivalents (1) (miles of S.C.)	Calculated equivalents by att. lengths (2) (miles of S.C.)	Differences between (1) & (2)	
0+ 10·44	9·73	9·352	0·378	
10+ ,,	10·15	9·946	0·204	
20+ ,,	11·10	10·540	0·560	
30+ ,,	12·10	11·131	0·969	
40+ ,,	13·10	11·723	1·377	
60+ ,,	14·55	12·909	1·691	Average of these values 1·260.
80+ ,,	15·50	14·095	1·405	
100+ ,,	16·50	15·281	1·219	
120+ ,,	17·65	16·467	1·183	
150+ ,,	19·50	18·245	1·255	
180+ ,,	21·25	20·024	1·226	
200+ ,,	22·50	21·210	1·290	
220+ ,,	23·58	22·396	1·184	
250+ ,,	25·25	24·174	1·076	
270+ ,,	26·50	25·360	1·140	
300+ ,,	28·40	27·138	1·262	
320+ ,,	29·65	28·324	1·326	
350+ ,,	31·35	30·103	1·247	
400+ ,,	34·20	33·067	1·133	
0+ 20·88	19·05	18·705	0·345	
20+ ,,	20·80	19·893	0·907	
40+ ,,	22·65	21·076	1·574	
60+ ,,	23·76	22·262	1·498	Average of these values 1·289.
80+ ,,	24·87	23·448	1·422	
120+ ,,	27·00	25·820	1·180	
150+ ,,	29·00	27·598	1·402	
200+ ,,	31·90	30·563	1·337	
250+ ,,	34·70	33·527	1·173	
300+ ,,	37·50	36·491	1·009	

The reflection equivalent calculated by the second term of formula (28).................................... 1·4434.

Table 13. Comparison of the observed and calculated results of the transmission equivalents of 300 miles of 400 lb. copper aerial line connected to various lengths of No. 19 gauge testing cable in terms of standard cable.

Lengths of lines (miles) 400 lb. line + Testing cable	Observed equivalents (1) (miles of S.C.)	Calculated equivalents by att. lengths (2) (miles of S.C.)	Differences between (1) & (2)
300 + 0	13·10	9·609	3·491
,, + 2·088	14·20	11·479	2·721
,, + 4·176	15·50	13·350	2·150
,, + 6·264	16·93	15·220	1·710
,, + 8·352	18·65	17·091	1·559
,, +12·528	22·16	20·832	1·328
,, +16·704	26·00	24·573	1·427
,, +20·880	29·80	28·314	1·486
,, +26·100	34·50	32·990	1·510

Average of these values 1·462. (last five values, 1·559 to 1·510)

The reflection equivalent calculated by the second term of formula (28)................................... 1·4124.

Table 14. Comparison of the observed and calculated results of the transmission equivalents of various lengths of No. 19 gauge testing cable connected to 300 miles of 400 lb. copper aerial line in terms of standard cable.

Lengths of lines (miles) Testing cable + 400 lb. line	Observed equivalents (1) (miles of S.C.)	Calculated equivalents by att. lengths (2) (miles of S.C.)	Differences between (1) & (2)	
0 + 300	13·10	9·609	3·491	
1·044 + ,,	13·67	10·544	3·126	
2·088 + ,,	14·25	11·479	2·771	
4·176 + ,,	15·70	13·350	2·350	
6·264 + ,,	17·20	15·221	1·979	
8·352 + ,,	18·75	17·091	1·659	Average of these values 1·640.
10·440 + ,,	20·50	18·962	1·538	
12·528 + ,,	22·35	20·832	1·518	
16·704 + ,,	26·20	24·573	1·627	
20·880 + ,,	30·00	28·314	1·686	
26·100 + ,,	34·80	32·990	1·810	

The reflection equivalent calculated by the second term of formula (28)................................... 1·5376.

Table 15. Comparison of the observed and calculated results of the transmission equivalents of 15·66 miles of No. 19 gauge testing cable connected to various lengths of 400 lb. copper aerial line in terms of standard cable.

Lengths of lines (miles) Testing cable + 400 lb. line	Observed equivalents (1) (miles of S.C.)	Calculated equivalents by att. lengths (2) (miles of S.C.)	Differences between (1) & (2)	
15·66 + 0	14·35	14·029	0·321	
„ + 20	15·70	14·670	1·030	
„ + 40	17·50	15·310	2·190	
„ + 60	18·30	15·951	2·349	
„ + 80	18·50	16·591	1·909	
„ + 100	18·65	17·232	1·418	Average of these values 1·563.
„ + 120	19·15	17·883	1·267	
„ + 150	20·40	18·834	1·566	
„ + 200	22·20	20·435	1·765	
„ + 250	23·50	22·037	1·463	
„ + 300	25·30	23·638	1·662	
„ + 350	26·85	25·240	1·610	
„ + 400	28·50	26·841	1·659	
„ + 450	30·10	28·468	1·632	
„ + 500	31·75	30·044	1·706	
„ + 550	33·20	31·646	1·554	
„ + 600	34·70	33·248	1·452	

The reflection equivalent calculated by the second term of formula (28).................................. 1·5376.

Table 16. Comparison of the observed and calculated results of the transmission equivalents of various lengths of 400 lb. copper aerial line connected to 15·66 miles of No. 19 gauge testing cable in terms of standard cable.

Lengths of lines (miles) 400 lb. line + Testing cable	Observed equivalents (1) (miles of S.C.)	Calculated equivalents by att. lengths (2) (miles of S.C.)	Differences between (1) & (2)	
0 + 15·66	14·35	14·029	0·321	
10 + ,,	14·76	14·349	0·411	
20 + ,,	15·40	14·670	0·730	
30 + ,,	16·00	14·989	1·011	
40 + ,,	16·95	15·310	1·640	
60 + ,,	17·87	15·951	1·919	
80 + ,,	18·20	16·591	1·609	
100 + ,,	18·30	17·232	1·068	Average of these values 1·257.
120 + ,,	18·88	17·883	0·997	
150 + ,,	20·20	18·834	1·366	
200 + ,,	21·80	20·435	1·365	
250 + ,,	23·20	22·037	1·163	
300 + ,,	24·80	23·638	1·162	
350 + ,,	26·55	25·240	1·310	
400 + ,,	28·20	26·841	1·359	
450 + ,,	29·80	28·468	1·332	
500 + ,,	31·40	30·044	1·356	
550 + ,,	32·90	31·646	1·254	
600 + ,,	34·60	33.248	1·352	

The reflection equivalent calculated by the second term of formula (28)................................... 1·4124.

The above examples relate only to lines consisting of two uniform lines. In the following, therefore, some examples are given of lines consisting of three uniform lines.

Table 17. Comparison of the observed and calculated results of the transmission equivalents of various lines consisting of three uniform lines.

Kinds of lines	Observed results (1) (miles of S.C.)	Calculated results by att. lengths (2) (miles of S.C.)	Differences between (1) & (2)	Calculated reflection equivalents
200^m of 400 lb. a.l. + 100^m of 200 lb. a.l. + $10{\cdot}44^m$ of No. 19 g.t.c.	23·10	21·687	1·513	1·528
200^m of 400 lb. a.l. + $10{\cdot}44^m$ of No. 19 g.t.c. + 100^m of 200 lb. a.l.	24·20	21·687	2·513	2·624
200^m of 400 lb. a.l. + $5{\cdot}22^m$ of No. 19 g.t.c. + 100^m of 200 lb. a.l.	19·80	17·011	2·789	2·624
$10{\cdot}44^m$ of No. 19 g.t.c. + 200^m of 400 lb. a.l. + 100^m of 200 lb. a.l.	23·15	21·687	1·463	1·3894
$10{\cdot}44^m$ of No. 19 g.t.c. + 100^m of 400 lb. a.l. + 200^m of 200 lb. a.l.	25·90	24·413	1·487	1·3894

Notes:—"200^m of 400 lb. a.l." means 200 miles of 400 lb. copper aerial line.
"$10{\cdot}44^m$ of No. 19 g.t.c." means 10·44 miles of No. 19 gauge testing cable.
The others have the same meaning as above.

Table 18. $\log_{10}(Z_1 + Z_2)$ **and** $\log_{10}(Z_1 - Z_2)$.

Kinds of lines	Standard cable	400 lb. copper aerial line	200 lb. copper aerial line	100 lb. copper aerial line	50 lb. copper aerial line	No. 19 gauge testing cable	No. 22 gauge special cable	No. 19 gauge special cable	Loaded No. 19 gauge cable
Standard cable	— —	3·04556 2·60723	3·06366 2·56723	3·09004 2·52116	3·16932 2 67210	3·05410 1·81954	3·12751 2·43546	2·96170 2·18511	3·27980 3·04251
400 lb. copper aerial line	3·04556 2·60723	— —	3·11866 1·85884	3·13313 2·26199	3·19604 2·64004	3·07045 2·60116	3·13655 2·70317	2·98767 2·61821	3·32313 2·91324
200 lb. copper aerial line	3·06366 2·56723	3·11866 1·85884	— —	3·14468 2·04386	3·20771 2·56366	3.08766 2·54828	3·15214 2·64293	3·00688 2·60810	3·32877 2·90100
100 lb. copper aerial line	3·09004 2·52116	3·13313 2·26199	3·14468 2·04386	— —	3·22473 2·41946	3·11274 2·47428	3·17502 2·53306	3·03525 2·61220	3·33691 2·89174
50 lb. copper aerial line	3·16932 2·67210	3·19604 2·64004	3·20771 2·56366	3·22473 2·41946	— —	3·18826 2·61184	3·24199 2·46564	3·12313 2·77721	3·37519 2·83851
No. 19. gauge testing cable	3·05410 1·81954	3·07045 2·60116	3·08766 2·54828	3·11274 2·47428	3·18826 2·61184	— —	3·14777 2·32051	2·99109 2·33694	3·29413 3·02393
No. 22. gauge special cable	3·12751 2·43546	3·13635 2·70317	3·15214 2·64293	3·17502 2·53306	3·24199 2·46564	3·14777 2·32051	— —	3·07537 2·62909	3·33196 2·99169
No. 19. gauge special cable	2·96170 2·18511	2·98767 2·61821	3·00688 2·60810	3·03525 2·61220	3·12313 2·77721	2·99109 2·33694	3·07537 2·62908	— —	3·24934 3·07475
Loaded No. 19 gauge cable	3·27980 3·04251	3·32313 2·91324	3·32877 2·90100	3·33691 2·89174	3·37519 2·83851	3·29413 3·02393	3·33196 2·99169	3·24934 3·07475	— —

Notes:—The upper figures correspond to $\log_{10}(Z_1 + Z_2)$.

The lower „ „ „ $\log_{10}(Z_1 - Z_2)$.

Formula (27) is applicable to a line inserted between two long lines, too.

In this case, Z_s and Z_r are replaced by the initial sending end impedances of the lines at both ends. For example, suppose that No. 19 gauge testing cable is inserted between 200 lb. copper aerial lines. Then the formula is reduced to

$$\frac{-(Z_1-Z_2)^2}{(Z_1+Z_2)^2}\, e^{-2\alpha_2 l_2} \left|\overline{2\beta_2 l_2}\right. \quad \text{.....................(39)}$$

where Z_1 and Z_2 are the initial sending end impedances of 200 lb. copper aerial line and No. 19 gauge testing cable respectively and α_2, β_2 and l_2 are the attenuation constant, wave length constant and length of No. 19 gauge testing cable.

Now, leaving the angle of expression (39) out of consideration and putting in the corresponding values, we get

$$\frac{\overline{353{\cdot}41}^{\,2}}{\overline{1223{\cdot}67}^{\,2}}\, e^{-2\times 0{\cdot}1032\times l_2}\,.$$

The results of experiments are shown in Table 19 and it is seen that formula (28) is applicable, if No. 19 gauge testing cable is longer than 1·5 miles.

Putting $l_2=1{\cdot}5$ in the above expression and taking logarithms, we get

$$2{\cdot}54828\times 2-3{\cdot}08766\times 2-2\times 0{\cdot}1032\times 1{\cdot}5\times 0{\cdot}4343=-1{\cdot}21322.$$

Hence the above expression is equal to $\dfrac{1}{16{\cdot}338}=0{\cdot}061$ which is nearly equal to the values of the preceding cases.

In the same way, we can determine the limit of the length of any line to which formula (28) is applicable, the line being inserted between two lines whose final sending end impedances are known.

Table 19. Comparison of the observed and calculated results of the transmission equivalents of various lengths of No. 19 gauge testing cable inserted between two 150 miles of 200 lb. copper aerial lines.

Lengths of lines (miles) 200 lb. line+testing cable +200 lb. line	Observed equivalents (1) (miles of S.C.)	Calculated equivalents by att. lengths (2) (miles of S.C.)	Differences between (1) & (2)	
150+ 0·00 +150	21·20	17·786	3·414	
„ + 0·261+ „	21·20	18·020	3·180	
„ + 0·522+ „	21·40	18·254	3·146	
„ + 0·783+ „	21·50	18·488	3·012	
„ + 1·044+ „	21·60	18·715	2·885	
„ + 1·305+ „	21·70	18·955	2·745	
„ + 1·566+ „	21·85	19·189	2·661	Average of these values 2·531.
„ + 1·783+ „	22·00	19·384	2·616	
„ + 2·088+ „	22·25	19·657	2·593	
„ + 2·610+ „	22·70	20·125	2·575	
„ + 3·132+ „	23·05	20·592	2·456	
„ + 3·654+ „	23·50	21·060	2·440	
„ + 4·176+ „	24·10	21·528	2·572	
„ + 5·220+ „	25·00	22·463	2·537	
„ + 6·264+ „	25.90	23·399	2·501	
„ + 7·308+ „	26·80	24·334	2·466	
„ + 8·352+ „	27·75	25·269	2·481	
„ + 9·396+ „	28·70	26·205	2·495	
„ +10·440+ „	29·65	27·140	2·510	

The reflection equivalent calculated by the second term of formula (28) .... 2·6592.

IV. Transmission Losses in the Toll and Trunk Lines Connected to Common Battery Subscribers' Loops.

In the preceding sections, we considered the transmission losses in telephone lines without making any distinction between subscribers' loops and toll or trunk lines. This was because they are usually connected directly in case of magneto system, though they are sometimes connected through repeating coils. But, in case of a common battery system, there are some conditions under which it is necessary to consider separately the transmission losses in subscribers' loops and toll or trunk lines. The transmission losses in common battery subscribers' loops, however, have been already discussed by the author in Report No. 16 of the Second Section of Electro-technical Laboratory. In this section, therefore, it remains to consider those in toll and trunk lines only.

It was shown in the preceding section that the transmission loss in any telephone line could be calculated by formula (28). This must, of course, hold good also in case of toll and trunk lines connected to common battery subscribers' loops. In this case, however, those lines are not directly connected to subscribers' loops but through No. 25 repeating coils. Therefore the impedance of the terminal apparatus is not that of the subscriber's set but of the secondary of the repeating coil. The value of the impedance of the secondary coil is not, of course, constant for the different lengths of subscriber's loop connected to the primary coil and also for the different measuring currents. In these experiments, 8 and 0·5 miliamperes are taken as the currents for measuring the impedance, 8 miliamperes expressing the sending end current and 0·5 miliampere the receiving end current. The sending and receiving end currents are often less. But we used the above values for convenience of measurement. The results of measurements for the various lengths of subscribers' loops connected to the primary are shown in Tables 20 and 21, and in Figs. 6 and 7.

Table 20. The effective resistances and inductances of the secondary of No. 25-A repeating coil, the primary of which is connected to various lengths of No. 19 gauge testing cable.

Lengths of subscriber's loop (miles)	Values at the transmitting end or with the measuring current of 8 miliamperes		Values at the receiving end or with the measuring current of 0·5 miliampere	
	Effective resistances (ohms)	Effective inductances (mili-henries)	Effective resistances (ohms)	Effective inductances (mili-henries)
0	219·0	60·7	221·0	52·0
1·044	357·0	69·6	345·0	58·2
2·088	486·0	75·0	463·0	61·8
3·132	635·0	73·0	590·0	66·0
4·176	783·0	66·0	727·0	63·0
5·220	923·0	51·0	850·0	55·8

Table 21. The effective resistances and inductances of the secondary of No. 25-A repeating coil, the primary of which is connected to various lengths of No. 22 gauge special cable.

Lengths of subscriber's loop (miles)	Values at the transmitting end or with the measuring current of 8 miliamperes		Values at the receiving end or with the measuring current of 0·5 miliampere	
	Effective resistances (ohms)	Effective inductances (mili-henries)	Effective resistances (ohms)	Effective inductances (mili-henries)
0	219·0	60·7	221·0	52·0
1·044	418·1	75·0	391·2	66·0
2·088	617·2	85·0	574·0	77·0
3·132	843·6	90·0	775·0	96·0
4·176	1100·0	81·3	1000·0	100·0
5·220	1301·2	48·2	1203·0	86·0

Now, let us calculate the transmission losses in various lines by the method of attenuation lengths and compare the results with the observed ones. Tables 22, 23 and 24 show the comparison and we see that there are some differences and that those differences are not constant for the different lengths of subscribers' loops, even for the same trunk or toll line. The differences are due to the reflections at both ends of the lines and they are not constant as the impedance of the terminal apparatus is not constant for the various lengths of subscribers' loops.

In the last column of each of Tables 22, 23 and 24, the reflection equivalents calculated by the second term of formula (28) are shown and it is seen that they are very like the differences shown in the last column but one. The curves in Figs. 8, 9, 10 and 11 show the values of the reflection equivalents obtained by taking differences between the losses observed and those calculated by the method of attenuation lengths. By examining these curves, it is seen that the curve for 400 lb. aerial line in Fig. 8 coincides almost with that in Fig. 9 but not with that in Fig. 10, that is, the reflection equivalents do not depend so much upon the capacities of subscribers' loops as upon the resistances, so long as the cables in present use are concerned.

These results show that formula (28) is applicable also for the practical calculations of the transmission losses in the toll or trunk lines connected to common battery subscribers' loops.

But as the impedance of the terminal apparatus is not constant for the different kinds and lengths of subscribers' loops, it is somewhat laborious to employ the formula. This drawback may, however, be remedied to some degree by previously drawing curves for several toll or trunk lines for each of the subscribers' loops of No. 19 and No. 22 B.S. gauge cables.

In Table 25, there are shown the constants necessary to calculate the reflection equivalents and the limiting lengths of the various lines used for these experiments.

Table 22. Comparison of the observed and calculated results of the transmission equivalents of 400 miles of 400 lb. copper aerial line connected to various lengths of subscribers' loops in terms of standard cable.

	Lengths of subscribers' loops (miles)	Observed equivalents (1) (miles of S.C.)	Calculated equivalent by att. length (2) (miles of S.C.)	Differences between (1) & (2)	Calculated reflection equivalents (miles of S.C.)
No. 19 gauge testing cable (90ω 0·053 *mcf.*)	0	17·80	12·847	4·953	4·942
	1·044	16.90	,,	4·053	4·154
	2·088	16·20	,,	3·353	3·484
	3·132	15·50	,,	2·653	2·841
	4·176	14·78	,,	1·933	2·237
	5·220	14·22	,,	1·373	1·714
	7·308	13·34	,,	0·493	—
No. 19 gauge special cable (88ω 0·07 *mcf.*)	0	17·80	12·847	4·953	—
	1·044	17·00	,,	4·153	—
	2·088	16·20	,,	3·353	—
	3·132	15·35	,,	2·503	—
	4·176	14·70	,,	1·853	—
	5·220	14·10	,,	1·253	—
No. 22 gauge special cable (176ω 0·053 *mcf.*)	0	17·80	12·847	4.953	4·942
	1·044	16·60	,,	3·753	3·967
	2·088	15·75	,,	2·903	3·117
	3·132	15·00	,,	2·153	2·406
	4·176	14·40	,,	1·553	1·720
	5·220	13·70	,,	0·853	1·122

Table 23. Comparison of the observed and calculated results of the transmission equivalents of 200 miles of 200 lb. copper aerial line connected to various lengths of subscribers' loops in terms of standard cable.

	Lengths of subscribers' loops (miles)	Observed equivalents (1) (miles of S.C.)	Calculated equivalent by att. length (2) (miles of S.C.)	Differences between (1) & (2)	Calculated reflection equivalents (miles of S.C.)
No. 19. gauge testing cable (90ω 0·053 mcf.)	0	16·30	11·858	4·442	4·652
	1·044	15·50	,,	3·642	3·835
	2·088	14·75	,,	2·892	3·161
	3·132	14·15	,,	2·292	2·528
	4·176	13·58	,,	1·722	1·949
	5·220	12·85	,,	0·993	1·455
	7·308	11·90	,,	0·042	—

Table 24. Comparison of the observed and calculated results of the transmission equivalents of 31·32 miles of loaded No. 19 gauge testing cable connected to various lengths of subscribers' loops in terms of standard cable.

	Lengths of subscribers' loops (miles)	Observed equivalents (1) (miles of S.C.)	Calculated equivalent by att. length (2) (miles of S.C.)	Differences between (1) & (2)	Calculated reflection equivalents (miles of S.C.)
No. 19 gauge testing cable (90ω 0·053 mcf.)	0	17·30	8·809	8·491	8·931
	1·044	15·62	,,	6·811	6·977
	2·088	14·20	,,	5·391	5·475
	3·132	12·87	,,	4·061	4·066
	4·176	11·70	,,	2·891	2·881
	5·220	10·80	,,	2·061	1·779
	6·264	10·00	,,	1·191	—

Table 25. Constants necessary to calculate the reflection equivalents of the various lines used for these experiments. (Common battery system).

Kinds of lines	Z_0	$\log_{10} Z_0$	lengths of sub. loops (miles)	Z_0+Z_s	$\log_{10}(Z_0+Z_s)$	Z_0+Z_r	$\log_{10}(Z_0+Z_r)$	Z_0-Z_s	$\log_{10}(Z_0-Z_s)$	Z_0-Z_r	$\log_{10}(Z_0-Z_r)$
Standard cable	534·35	2·72783	0	604·5	2·78138	613·0	2·78744	697·0	2·84325	654·3	2·81580
			5.22	1309·7	3·11715	1234·9	3·09163	830·5	2·91933	804·2	2·90536
400 lb. copper aerial line	642·80	2·80808	0	891·9	2·95030	882·9	2·94589	557·8	2·74649	528·7	2·72325
			5·22	1574·5	3·19713	1505·3	3·17762	425·8	2·62922	401·4	2·60358
200 lb. copper aerial line	673·06	2·82805	0	895·6	2·95209	890·1	2·94946	619·9	2·79229	588·7	2·76987
			5·22	1587·7	3·20076	1517·0	3·18100	468·2	2·67040	453·1	2·65615
100 lb. copper aerial line	725·68	2·86098	0	906·1	2·95720	906·0	2·95715	715·8	2·85478	682·0	2·83381
			5·22	1607·8	3·20626	1535·3	3·18620	549·3	2·73986	544·7	2·73616
50 lb. copper aerial line	956·77	2·98081	0	1074·2	3·01111	1082·8	3·03456	978·2	2·99044	944·0	2·97496
			5·22	1778·8	3·25015	1703·8	3·23142	707·2	2·84956	726·9	2·86152
No. 19 gauge testing cable	598·44	2·77702	0	665·5	2·82317	675·6	2·82966	744·0	2·87158	702·0	2·84632
			5·22	1369·8	3·13665	1294·7	3·11215	820·9	2·91429	800·5	2·90337
Loaded No. 19 gauge cable	1461·90	3·16592	0	1684·6	3·22650	1682·5	3·22596	1310·0	3·11726	1294·5	3·11210
			5·22	2382·4	3·37702	2311·0	3·36380	656·2	2·81706	729·9	2·86327

Notes:—1. Average impedances of the secondaries of No. 25-A repeating coils.

When the length of subscriber's loop is zero

$$Z_s = 219·1 + j303·5$$

$$Z_r = 221·0 + j260·0$$

When the length of subscriber's loop is 5·22 miles

$$Z_s = 923·0 + j255·0$$

$$Z_r = 850·0 + j279·0$$

2. Subscribers' loops are of No. 19 gauge testing cable.

V. Application of the Above Theories to the Lines Chiefly used in Japan at Present.

In the preceding sections we referred only to the lines specially used for these experiments. In this section we will give the Tables which contain the constants necessary to calculate the transmission equivalents and the limiting lengths by the author's formulas, of the lines chiefly used in Japan at present and also the calculated results of the reflection equivalents and the limiting lengths of those lines.

Table 26. Electrical constants of the telephone lines chiefly used in Japan at present.

Kinds of lines	Resistance per loop mile (ohms)	Inductance per loop mile (henry)	Wire-to-wire capacity per mile (mcf.)	Vector attenuation constants		Initial sending end impedances	
				$P \underline{\|\psi}$	$\alpha+j\beta$	$Z_0 \overline{\|\theta}$	$A-jB$
No. 22 B.S.G. cable	176	0·001	0·066	0·214 $\underline{\|45°48'}$	0·168 $+j$0·1728	736·02 $\overline{\|44°11'}$	527·9 $-j$513·1
No. 20 B.S.G. cable	111	0·001	0·053	0·1715 $\underline{\|46°18'}$	0·1158 $+j$0·124	647·52 $\overline{\|43°43'}$	467·9 $-j$447·4
No. 19 B.S.G. cable	88	0·001	0·057	0·1584 $\underline{\|46°38'}$	0·1088 $+j$0·1152	556·11 $\overline{\|43°22'}$	404·3 $-j$381·5
No. 16 B.S.G. cable	44	0·001	0·072	0·1263 $\underline{\|48°15'}$	0·0841 $+j$0·0942	350·71 $\overline{\|41°46'}$	261·6 $-j$233.6
No. 13 B.S.G. cable	22	0·001	0·072	0·0901 $\underline{\|51°24'}$	0·0562 $+j$0·0704	250·33 $\overline{\|38°36'}$	195·63 $-j$156·18
50 lb. copper aerial line	36	0·0039	0·00738	0·0389 $\underline{\|58°27'}$	0·0203 $+j$0·0331	1053·1 $\overline{\|30°}$	912·0 $-j$526·5
100 lb. copper aerial line	18	0·00385	0·00782	0·0321 $\underline{\|67°44'}$	0·0122 $+j$0·0297	820·8 $\overline{\|20°48'}$	767·3 $-j$291·4
150 lb. copper aerial line	12	0·0037	0·00812	0·0299 $\underline{\|72°49'}$	0·0088 $+j$0·0286	736·8 $\overline{\|15°46'}$	709·1 $-j$200·2
200 lb. copper aerial line	9	0·0036	0·00833	0·029 $\underline{\|76°\ 2'}$	0·007 $+j$0·0281	695·0 $\overline{\|12°36'}$	678·2 $-j$151·6
300 lb. copper aerial line	6	0·0035	0·0086	0·0282 $\underline{\|79°52'}$	0·005 $+j$0·0278	655·8 $\overline{\|8°48'}$	648·1 $-j$100·3
400 lb. copper aerial line	4·5	0·0034	0·0089	0·028 $\underline{\|81°57'}$	0·0039 $+j$0·0277	628·5 $\overline{\|6°47'}$	624·1 $-j$ 74·2
50 lb. silicium bronze aerial line	64	0·0039	0·00738	0·0497 $\underline{\|52°42'}$	0·0301 $+j$0·0395	1346·3 $\overline{\|35°45'}$	1092·5 $-j$786·5
Artificial cable	88	0	0·060	0·1625 $\underline{\|45°}$	0·1149 $+j$0·1149	541·6 $\overline{\|45°}$	382·8 $-j$382·8
Standard cable	88	0	0.054	0·1541 $\underline{\|45°}$	0·109 $+j$0·109	570·9 $\overline{\|45°}$	403·6 $-j$403·6

Notes:—(1) Axial distance between the wires of aerial lines .... 12 inches.
(2) Insulation resistance between the wires of aerial lines .... 1 megohm.
(3) Insulation resistance between the wires of cables.................... infinite.
(4) Angular velocity ($2\pi n$).................... 5000.

Table 27. Constants necessary to calculate the reflection equivalents and the limiting lengths of the various lines chiefly used in Japan at present. (Magneto system.)

Kinds of lines	Z_0	$\log_{10} Z_0$	Z_0+Z_s	$\log_{10}(Z_0+Z_s)$	Z_0+Z_r	$\log_{10}(Z_0+Z_r)$	Z_0-Z_s	$\log_{10}(Z_0-Z_s)$	Z_0-Z_r	$\log_{10}(Z_0-Z_r)$
No. 22 B.S.G. cable	736·02	2·86689	922·04	2·96475	913·50	2·96071	744·63	2·87194	739·11	2·86871
No. 20 B.S.G. cable	647·52	2·81125	844·73	2·92672	835·58	2·92199	668·81	2·82530	660·97	2·82018
No. 19 B.S.G. cable	556·11	2·74516	766·67	2·88461	756·84	2·87900	523·16	2·71863	585·65	2·76764
No. 16 B.S.G. cable	350·71	2·54495	604·34	2·78128	592·52	2·77270	448·90	2·65215	437·96	2·64143
No. 13 B.S.G. cable	250·33	2·39851	540·32	2·73265	527·27	2·72203	392·57	2·59392	379·71	2·57945
50 lb. copper aerial line	1053·10	3·02247	1293·99	3·11193	1284·40	3·10870	929·60	2·96830	930·16	2·96856
100 lb. copper aerial line	820·80	2·91424	1112·62	3·04634	1101·10	3·04183	655·83	2·81679	657·12	2·81764
150 lb. copper aerial line	736·80	2·86735	1051·31	3·02173	1039·01	3·01662	548·86	2·73946	550·59	2·74083
200 lb. copper aerial line	695·00	2·84198	1022·00	3·00945	1009·21	3·00398	492·15	2·69210	494·20	2·69390
300 lb. copper aerial line	655·80	2·81677	996·14	2·99832	982·97	2·99254	434·31	2·63780	436·84	2·64032
400 lb. copper aerial line	628·50	2·79831	975·53	2·98924	962·12	2·98323	398·88	2·60084	401·48	2·60366
50 lb. silicium bronze aerial line	1346·30	3·12914	1546·89	3·18946	1539·01	3·18724	1241·76	3·09404	1246·10	3·09555
Artificial cable	541·60	2·73368	745·76	2·87260	736·02	2·86689	592·19	2·77246	584·20	2·76656
Standard cable	570·90	2·75656	771·03	2·88707	761·50	2·88167	614·67	2·78864	607·09	2·78325

Note:—Average impedances of the magneto subscriber's set

$Z_s = 342{\cdot}2 + j208{\cdot}0 \qquad Z_r = 329{\cdot}9 + j199{\cdot}0$

Table 28. Constants necessary to calculate the reflection equivalents and the limiting lengths of the various lines chiefly used in Japan at present. (Common battery system.)

Sheet No. 1. When the length of subscriber's loop is zero.

Kinds of lines	Z_0	$\log_{10} Z_0$	Z_0+Z_s	$\log_{10}(Z_0+Z_s)$	Z_0+Z_r	$\log_{10}(Z_0+Z_r)$	Z_0-Z_s	$\log_{10}(Z_0-Z_s)$	Z_0-Z_r	$\log_{10}(Z_0-Z_r)$
No. 22 B.S.G. cable	736·02	2·86689	775·86	2·88978	790·52	2·89791	873·03	2·94103	831·78	2·92001
No. 20 B.S.G. cable	647·52	2·81125	701·91	2·84623	713·94	2·85366	791·04	2·89820	749·25	2·87463
No. 19 B.S.G. cable	556·11	2·74516	628·56	2·79835	637·30	2·80434	709·59	2·85101	667·17	2·82424
No. 16 B.S.G. cable	350·71	2·54495	485·76	2·68642	483·33	2·68421	538·78	2·73141	495·27	2·69484
No. 13 B.S.G. cable	250·33	2·39851	440·12	2·64357	429·37	2·63283	460·30	2·66304	416·97	2·62010
50 lb. copper aerial line	1053·10	3·02247	1152·87	3·06178	1163·91	3·06592	1081·21	3·03391	1046·93	3·02092
100 lb. copper aerial line	820·80	2·91424	986·48	2·99409	988·80	2·99511	808·97	2·90793	776·19	2·88997
150 lb. copper aerial line	736·80	2·86735	933·92	2·97031	932·03	2·96943	702·72	2·84678	670·84	2·82662
200 lb. copper aerial line	695·00	2·84198	910·06	2·95907	905·71	2·95699	646·44	2·81053	615·18	2·78900
300 lb. copper aerial line	655·80	2·81677	890·70	2·94973	883·54	2·94623	583·15	2·77023	558·77	2·74723
400 lb. copper aerial line	628·50	2·79831	873·82	2·94142	865·29	2·93716	553·79	2·74335	523·62	2·71902
50 lb. silicium bronze aerial line	1346·30	3·12914	1397·72	3·14542	1415·08	3·15078	1396·76	3·14512	1861·36	3·13413
Artificial cable	541·60	2·73368	607·28	2·78339	616·35	2·78983	705·55	2·84853	662·85	2·82142
Standard cable	570·90	2·75656	630·81	2·79990	641·02	2·80687	730·77	2·86378	688·26	2·83775

Notes:—1. Average impedances of the secondary of No. 25-A. repeating coil

$Z_s = 219{\cdot}1 + j\,303{\cdot}5 \qquad Z_r = 221{\cdot}0 + j\,260{\cdot}0$

2. subscribers' loops are of No 19 gauge testing cable.

Table 28. Constants necessary to calculate the reflection equivalents and the limiting lengths of the various lines chiefly used in Japan at present. (Common battery system.)

Sheet No. 2. When the length of subscriber's loop is 2·61 miles.

Kinds of lines.	Z_0+Z_s	$\log_{10}(Z_0+Z_s)$	Z_0+Z_r	$\log_{10}(Z_0+Z_r)$	Z_0-Z_s	$\log_{10}(Z_0-Z_s)$	Z_0-Z_r	$\log_{10}(Z_0-Z_r)$
No. 22 B.S.G. cable....................	1101·59	3·04202	1078·97	3·03301	888·89	2·94884	840·63	2·92460
No. 20 B.S.G. cable....................	1035·43	3·01512	1010·04	3·00434	828·11	2·91809	777·80	2·89087
No. 19 B.S.G. cable....................	969·63	2·98660	941·16	2·97366	773·38	2·88839	720·95	2·85791
No. 16 B.S.G. cable....................	838·61	2·92356	802·12	2·90424	680·03	2·83253	624·16	2·79530
No. 13 B.S.G. cable....................	791·48	2·89844	750·40	2·87532	647·02	2·81092	590·90	2·77151
50 lb. copper aerial line..............	1484·06	3·17145	1394·89	3·14454	965·98	2·98497	933·51	2·97012
100 lb. copper aerial line	1334·87	3·12544	1302·84	3·11488	696·43	2·84288	661·06	2·82024
150 lb. copper aerial line	1287·51	3·10975	1250·60	3·09712	592·98	2·77304	555·68	2·74482
200 lb. copper aerial line	1263·10	3·10144	1225·88	3·08845	538·63	2·73129	500·04	2·69900
300 lb. copper aerial line	1243·83	3·09476	1204·73	3·08089	482·52	2·68351	442·50	2·64591
400 lb. copper aerial line	1226·00	3·08849	1186·45	3·07425	453·07	2·65617	411·46	2·61433
50 lb. silicium bronze aerial line...	1707·81	3·23244	1678·73	3·22498	1275·67	3·10574	1245·71	3·09542
Artificial cable	947·83	2·97673	919·46	2·96353	779·40	2·89176	726·42	2·86119
Standard cable	969·13	2·98638	941·78	2·97395	795·15	2·90045	742·81	2·87088

Notes:—1. Average impedances of the secondary of No. 25–A repeating coil.

$Z_s = 565{\cdot}0 + j\,375{\cdot}0$ $\qquad Z_r = 535 + j\,327{\cdot}5$

2. Subscribers' loops are of No. 19 gauge testing cable.

Table 28. Constants necessary to calculate the reflection equivalents and the limiting lengths of the various lines chiefly used in Japan at present. (Common battery system).

Sheet No. 3. When the length of subscriber's loop is 5·22 miles.

Kinds of lines	Z_0+Z_s	$\log_{10}(Z_0+Z_s)$	Z_0+Z_r	$\log_{10}(Z_0+Z_r)$	Z_0-Z_s	$\log_{10}(Z_0-Z_s)$	Z_0-Z_r	$\log_{10}(Z_0-Z_r)$
No. 22 B.S.G. cable....................	1473·7	3·16840	1397·66	3·14540	863·76	2·93639	855·08	2·93201
No. 20 B.S.G. cable....................	1404·14	3·14741	1328·62	3·12340	836·95	2·92270	820·77	2·91422
No. 19 B.S.G. cable....................	1332·10	3·12453	1258·78	3·09995	821·40	2·91455	797·14	2·90153
No. 16 B.S.G. cable....................	1184·79	3·07364	1112·53	3.04631	822·30	2·91503	780·37	2·89230
No. 13 B.S.G. cable....................	1122·97	3·05037	1052·81	3·02235	835·55	2·92197	785·90	2·89537
50 lb. copper aerial line	1854·98	3·26834	1779·33	3·25025	781·57	2·89297	807·88	2·90735
100 lb. copper aerial line	1690·64	3·22806	1617·30	3·20880	568·15	2·75446	576·36	2·76069
150 lb. copper aerial line	1633·01	3·21299	1561·10	3·19343	502·95	2·70152	499·49	2·69853
200 lb. copper aerial line	1604·54	3·20535	1533·49	3·18568	474·61	2·67634	463·61	2·66615
300 lb. copper aerial line	1578·70	3·19830	1508·72	3·17861	449·23	2·65247	426·69	2·63011
400 lb. copper aerial line	1557·62	3·19246	1488·88	3·17268	444·65	2.64802	419·26	2·62248
50 lb. silicium bronze aerial line...	2084·40	3·31898	2007·70	3·30270	1055·20	3·02333	1092·75	3·03852
Artificial cable	1312·20	3·11800	1237·34	3·09249	835·83	2·92212	810·10	2·90854
Standard cable	1335·00	3·12548	1259·88	3·10033	838·77	2·92364	815·61	2·91148

Notes:—1. Average impedances of the secondary of No. 25-A repeating coil

$$Z_s = 923·0 + j255·0 \qquad Z_r = 850·0 + j279·0$$

2. Subscribers' loops are of No. 19 gauge testing cable.

Table 29. $\log_{10}(Z_1+Z_2)$ and $\log_{10}(Z_1-Z_2)$ of the lines chiefly used in Japan at present.

Kinds of lines	No. 22 B.S.G. cable	No. 20 B.S.G. cable	No. 19 B.S.G. cable	No. 16 B.S.G. cable	No. 13 B.S.G. cable	50 lb. copper aerial line	100 lb. copper aerial line	150 lb. copper aerial line	200 lb. copper aerial line	300 lb. copper aerial line	400 lb. copper aerial line	50 lb. silicium bronze aerial line
No. 22 B.S.G. cable ...	— —	3·14099 1·94924	3·11134 3·25588	3·03610 2·55664	2·99370 2·68810	3·24944 2·58470	3·18319 2·51359	3·15470 2·55820	3·13898 2·59273	3·12266 2·63341	3·11162 2·54322	3·31747 2·79748
No. 20 B.S.G. cable ...	3·14099 1·94924	— —	3·08043 1·96042	2·99911 2·47290	2·95278 2·60062	3·22762 2·65426	3·15815 2·52840	3·12820 2·53828	3·11166 2·55982	3·09452 2·59228	3·08285 2·60699	3·29870 2·85169
No. 19 B.S.G. cable ...	3·11134 2·25588	3·08043 1·96042	— —	2·95748 2·31347	2·90627 2·48770	3·20394 2·72254	3·13077 2·57300	3·09915 2·55004	3·08168 2·55370	3·06360 2·57109	3·05120 2·57765	3·27848 2·90218
No. 16 B.S.G. cable ...	3·03610 2·55664	2·99911 2·47290	2·95748 2·31347	— —	2·77874 2·00736	3·14558 2·85328	3·06262 2·70671	3·02662 2·65199	3·00676 2·62797	2·98634 2·61155	2·97205 2·59768	3·22926 2·99915
No. 13 B.S.G. cable ...	2·99370 2·68810	2·95278 2·60062	2·90627 2·48770	2·77874 2·00736	— —	3·11431 2·90657	3·02606 2·76899	2·98783 2·71213	2·96681 2·68361	2·94538 2·65891	2·93017 2·63977	3·20309 3·03990
50 lb. copper aerial line	3·24944 2·58470	3·22762 2·65426	3·20394 2·72254	3·14558 2·85328	3·11431 2·90657	— —	3·27135 2·44100	3·24957 2·58460	3·23773 2·64526	3·22565 2·70009	3·21732 2·72929	3·37953 2·50039
100 lb. copper aerial line	3·18319 2·51359	3·15815 2·52840	3·13077 2·57300	3·06262 2·70671	3.02606 2·76899	3·27135 2·44100	— —	3·19204 2·03419	3·17951 2·21953	3·16669 2·35263	3·15784 2·41524	3·33236 2·77258
150 lb. copper aerial line	3·15470 2·55820	3·12820 2·53828	3·09915 2·55004	3·02662 2·65199	2·98783 2·71213	3·24957 2·58460	3·19204 2·03419	— —	3·15570 1·76035	3·14304 2·06837	3·13390 2·18182	3·31262 2·84543
200 lb. copper aerial line	3·13898 2·59273	3·11166 2·55982	3·08168 2·55370	3·00676 2·62797	2·96681 2·68361	3·23773 2·64526	3·17951 2·21953	3·15570 1·76035	— —	3·13034 1·77437	3·12114 1·97511	3·30186 2·87974
300 lb. copper aerial line	3·12266 2·63341	3·09452 2·59228	3·06360 2·57109	2·98634 2·61155	2·94538 2·65891	3·22565 2·70009	3·16669 2·35263	3·14304 2·06837	3·13034 1·77437	— —	3·10860 1·54920	3·29081 2·91250
400 lb. copper aerial line	3·11162 2·54322	3·08285 2·60699	3·05120 2·57765	2·97205 2·59768	2·93017 2·63977	3·21732 2·72929	3·15784 2·41524	3·13390 2·18182	3·12114 1·97511	3·10860 1·54920	— —	3·28337 2·93070
50 lb. silicium bronze aerial line	3·31747 2·79748	3·29870 2·85169	3·27848 2·90218	3·22926 2·99915	3·20309 3·03990	3·37953 2·50039	3·33236 2·77258	3·31262 2·84543	3·30186 2·87974	3·29081 2·91250	3·28337 2·93070	— —

Notes:—The upper figures correspond to $\log_{10}(Z_1+Z_2)$. The lower figures correspond to $\log_{10}(Z_1-Z_2)$.

Table 30. The reflection equivalents of the various lines chiefly used in Japan at present. (Magneto system.)

Sheet No. 1.

	Kinds of lines	Reflection equivalents (miles)		Kinds of lines	Reflection equivalents (miles)
	No. 22 B.S.G. cable	0·9798		100 lb. copper aerial line	3·4162
	No. 20 B.S.G. cable	0·5339		150 lb. copper aerial line	3·3543
	No. 19 B.S.G. cable	0·1324		200 lb. copper aerial line	3·3638
	No. 16 B.S.G. cable	−0.0665		300 lb. copper aerial line	3·4195
	No. 13 B.S.G. cable	0·9290		400 lb. copper aerial line	3·4210
	50 lb. copper aerial line	3·9279		50 lb. silicium bronze aerial line	4·9712
No. 22 B.S.G. cable with	50 lb. copper aerial line	2·5412 2·5241	No. 20 B.S.G. cable with	50 lb. copper aerial line	2·4522 2·4206
	100 lb. copper aerial line	2·0152 2·0254		100 lb. copper aerial line	1·8586 1·8539
	200 lb. copper aerial line	1·8085 1·8387		200 lb. copper aerial line	1·6034 1·6191
	300 lb. copper aerial line	1·7546 1·7914		300 lb. copper aerial line	1·5223 1·5544
	400 lb. copper aerial line	1·7147 1·7563		400 lb. copper aerial line	1·4790 1·5061
	50 lb. silicium bronze aerial line	3·3838 3·3454		50 lb. silicium bronze aerial line	3·3593 3·3063

Notes:—In case of joint lines, the upper figures represent the reflection equivalents when the speech is transmitted from the cable side, the lower ones when from the aerial line side.

Table 30. The reflection equivalents of the various lines chiefly used in Japan at present. (Magneto system.)

Sheet No. 2.

Kinds of lines		Reflection equivalents (miles)
No. 19 B.S.G. cable with	50 lb. copper aerial line	2·4586 2·4083
	100 lb. copper aerial line	1·7868 1·7635
	200 lb. copper aerial line	1·4767 1·4738
	300 lb. copper aerial line	1·3857 1·3893
	400 lb. copper aerial line	1·3170 1·3255
	50 lb. silicium bronze aerial line	3·4388 3·3672
No. 16 B.S.G. cable with	50 lb. copper aerial line	3·2721 3·1591
	100 lb. copper aerial line	2·3935 2·3076
	200 lb. copper aerial line	1·9405 1·8748
50 lb. copper aerial line with	No. 22 B.S.G. cable at both ends	1·1373
	No. 20 B.S.G. cable at both ends	0·9449
	No. 19 B.S.G. cable at both ends	0·9390
100 lb. copper aerial line with	No. 22 B.S.G. cable at both ends	0·6670
	No. 20 B.S.G. cable at both ends	0·2963
	No. 19 B.S.G. cable at both ends	0·1341
200 lb. copper aerial line with	No. 22 B.S.G. cable at both ends	0·2834
	No. 20 B.S.G. cable at both ends	−0·1413

Kinds of lines		Reflection equivalents (miles.)
No. 16 B.S.G. cable with	300 lb. copper aerial line	1·8001 1·7409
	400 lb. copper aerial line	1·6915 1·6372
	50 lb. silicium bronze aerial line	4·4453 4·3110
No. 13 B.S.G. cable with	50 lb. copper aerial line	4·6774 4·5273
	100 lb. copper aerial line	3·6871 3·5581
	200 lb. copper aerial line	3·1625 3·0537
	300 lb. copper aerial line	3·0007 2·8985
	400 lb. copper aerial line	2·8727 2·8387
	50 lb. silicium bronze aerial line	5·9584 5·7810
200 lb. copper aerial line with	No. 19 B.S.G. cable at both ends	−0·4133
	No. 16 B.S.G. cable at both ends	0·4515
400 lb. copper aerial line with	No. 22 B.S.G. cable at both ends	0·0501
	No. 20 B.S.G. cable at both ends	−0·4359
	No. 19 B.S.G. cable at both ends	−0·7785
	No. 16 B.S.G. cable at both ends	−0·0923
	No. 13 B.S.G. cable at both ends	2·2277

Table 31. The reflection equivalents of the various lines chiefly used in Japan at present. (Common battery system.)

Sheet No. 1.

Kinds of lines		Reflection equivalents (miles)		
		When sub. loop 0 mile	When sub. loop 2·61 miles	When sub. loop 5·22 miles
No. 22 B.S.G. cable		1·4909	0·0923	−0·4718
No. 20 B.S.G. cable		0·8127	0·0938	−0·2047
No. 19 B.S.G. cable		0·1546	0·2393	0·2127
No. 16 B.S.G. cable		−0·5174	1·6702	2·2334
No. 13 B.S.G. cable		0·5846	3·6217	4·3288
50 lb. copper aerial line		5·3860	2·3221	0·5675
100 lb. copper aerial line		4·7467	2·5836	1·1272
150 lb. copper aerial line		4·6924	2·8670	1·4746
200 lb. copper aerial line		4·7281	3·0442	1·6854
300 lb. copper aerial line		4·8371	3·2759	1·9196
400 lb. copper aerial line		4·8589	3·3974	2·0609
50 lb. silicium bronze aerial line...		6·6919	2·6297	0·4919
No. 22 B.S.G. cable with	50 lb. copper aerial line	3·4751 3·5594	1·3039 1·2659	0·1785 0·0748
	100 lb. copper aerial line	2·8662 3·0164	1·1438 1·1770	0·1897 0·1107
	200 lb. copper aerial line	2·6535 2·8692	1·1781 1·2621	0·2938 0·2234

Note:—In case of joint lines, the upper figures represent the reflection equivalents when the speech is transmitted from the cable side, the lower ones when from the aerial line side.

Table 31. The reflection equivalents of the various lines chiefly used in Japan at present. (Common battery system.)

Sheet No. 2.

Kinds of lines		Reflection equivalents (miles)		
		When sub. loop 0 mile	When sub. loop 2·61 miles	When sub. loop 5·22 miles
No. 22 B.S.G. cable with	300 lb. copper aerial line...	2·6151 2·8596	1·2062 1·3084	0·3322 0·2623
	400 lb. copper aerial line...	2·0956 2·3574	1·2226 1·3331	0·3637 0·2957
	50 lb. silicium bronze aerial line	4·4513 4·5098	1·7665 1·7337	0·4701 0·3282
No. 20 B.S.G. cable with	50 lb. copper aerial line ...	3·2706 3·3533	1·4522 1·3747	0·4494 0·3244
	100 lb. copper aerial line...	2·5937 2·7281	1·2220 1·2174	0·3926 0·2923
	200 lb. copper aerial line...	2·3329 2·5327	1·2081 1·2547	0·4486 0·3569
	300 lb. copper aerial line...	2·2772 2·5059	1·2188 1·2841	0·4697 0·3775
	400 lb. copper aerial line...	2·2279 2·4738	1·1108 1·2951	0·4879 0·3985
	50 lb. silicium bronze aerial line	4·3112 4·3539	1·9770 1·9069	0·8055 0·6423

Note:—The upper figures represent the reflection equivalents when the speech is transmitted from the cable side, the lower ones when from the aerial line side.

Table 31. The reflection equivalents of the various lines chiefly used in Japan at present. (Common battery system.)

Sheet No. 3.

Kinds of lines		Reflection equivalents (miles)		
		When sub. loop 0 mile	When sub. loop 2·61 miles	When sub. loop 5·22 miles
No. 19 B.S.G. cable with	50 lb. copper aerial line ...	3·1541 3·1931	1·7456 1·6224	0·8619 0·7248
	100 lb. copper aerial line...	2·3991 2·5040	1·4376 1·3874	1·5718 1·4594
	200 lb. copper aerial line...	2·0833 2·2537	1·3688 1·3698	0·7280 0·6243
	300 lb. copper aerial line...	2·0077 2·2070	1·3580 1·3789	0·7293 0·6260
	400 lb. copper aerial line...	1·9430 2·0515	1·3470 1·3787	0·7320 0·6306
	50 lb. silicium bronze aerial line	4·2677 4·2810	2·3435 2·2277	1·2911 1·1158
No. 16 B.S.G. cable with	50 lb. copper aerial line ...	3·7860 3·6525	3·4100 3·1522	2·7829 2·5878
	100 lb. copper aerial line...	2·8242 2·7566	2·8949 2·7103	2·4413 2·2708
	200 lb. copper aerial line...	2·3654 2·3633	2·6831 2·5494	2·2993 2·1376

Note:—The upper figures represent the reflection equivalents when the speech is transmitted from the cable side, the lower ones when from the aerial line side.

Table 31. The reflection equivalents of the various lines chiefly used in Japan at present. (Common battery system.)

Sheet No. 4.

Kinds of lines		Reflection equivalents (miles)		
		When sub. loop 0 mile	When sub. loop 2·61 miles	When sub. loop 5·22 miles
No. 16 B.S.G. cable with	300 lb. copper aerial line...	2·2404 2·2672	2·6246 2·5095	2·2512 2·0898
	400 lb. copper aerial line...	2·1359 2·1798	2·5724 2·4694	2·2140 2·0546
	50 lb. silicium bronze aerial line	5·0927 4·9334	4·2010 3·9505	3·4052 3·1718
No. 13 B.S.G. cable with	50 lb. copper aerial line ...	5·3134 4·9991	5·3119 4·9730	4·7239 4·5122
	100 lb. copper aerial line...	4·2398 3·9915	4·6850 4·4198	4·2705 4·0855
	200 lb. copper aerial line...	3·7095 3·5266	4·1904 4·1877	4·0569 3·8806
	300 lb. copper aerial line...	3·5632 3·4092	4·3218 4·2675	3·9875 3·8115
	400 lb. copper aerial line...	3·4392 3·3023	4·2502 4·0669	3·9309 3·7568
	50 lb. silicium bronze aerial line	6·7278 6·3877	6·2105 5·8798	5·4538 5·2059

Note:—The upper figures represent the reflection equivalents when the speech is transmitted from the cable side, the lower ones when from the aerial line side.

Table 31. The reflection equivalents of the various lines chiefly used in Japan at present. (Common battery system.)

Sheet No. 5.

Kinds of lines		Reflection equivalents (miles)		
		When sub. loop 0 mile	When sub. loop 2·61 miles	When sub. loop 5·22 miles
50 lb. copper aerial line with	No. 22 B.S.G. cable at both ends	1·6484	0·2498	−0·3142
	No. 20 B.S.G. cable at both ends	1·2237	0·5448	0·2062
	No. 19 B.S.G. cable at both ends	0·9612	1·0459	1·0193
100 lb. copper aerial line with	No. 22 B.S.G. cable at both ends	1·1357	−0·2627	−0·8268
	No. 20 B.S.G. cable at both ends	0·5751	−0·1438	−0·4423
	No. 19 B.S.G. cable at both ends	0·1584	0·2410	0·2144
200 lb. copper aerial line with	No. 22 B.S.G. cable at both ends	0·7945	−0·6040	−1·1681
	No. 20 B.S.G. cable at both ends	0·1375	−0·5814	−0·8799
	No. 19 B.S.G. cable at both ends	−0·3890	−0·3065	−0·3331
	No. 16 B.S.G. cable at both ends	0·0006	2·1882	2·7515
400 lb. copper aerial line with	No. 22 B.S.G. cable at both ends	0·5612	−0·8374	−1·4015
	No. 20 B.S.G. cable at both ends	−0·1571	−0·8761	−1·1745
	No. 19 B.S.G. cable at both ends	−0·7542	−0·6716	−0·6982
	No. 16 B.S.G. cable at both ends	−0·5432	1·6444	2·2077
	No. 13 B.S.G. cable at both ends	1·8823	4·9195	5·6248

Table 32. Limiting lengths of the various lines chiefly used in Japan at present.

Kinds of lines	Limiting lengths for magneto system (miles)	Limiting lengths for common battery system		
		When sub. loop 0 mile	When sub. loop 2·61 miles	When sub. loop 5·22 miles
No. 22 B.S.G. cable	6·92	8·69	6·73	5·17
No. 20 B.S.G. cable	9·63	12·31	9·56	7·39
No. 19 B.S.G. cable	9·70	13·40	10·37	8·31
No. 16 B.S.G. cable	12·78	17·10	13·61	12·06
No. 13 B.S.G. cable	18·69	24·59	20·53	19·22
50 lb. copper aerial line	51·62	63·58	47·24	26·97
100 lb. copper aerial line	69·82	94·67	58·17	25·67
150 lb. copper aerial line	83·17	121·33	66·04	24·52
200 lb. copper aerial line	93·16	144·29	71·42	23·89
300 lb. copper aerial line	110·77	187·73	80·03	22·91
400 lb. copper aerial line	125·73	229·52	89·01	29·27
50 lb. silicium bronze aerial line	38·51	45·02	35·87	24·25
Artificial cable....................	9·95	13·27	10·09	8·16

Notes:—1. Subscribers' loops in common battery system are of No. 19 gauge testing cable.

2. These figures are the values of l obtained from formula (27) by equating it to 0·064.

From Table 32, it can be seen that the limiting lengths are too long to allow the use of the proposed formula in practical cases. But these are the shortest lengths of which the comparatively accurate reflection equivalents can be expressed by the proposed formula. If, therefore, great accuracy is not required, the formula is applicable to much shorter lines than the limiting lengths and especially so for lines of thin wire.

Thus, taking the examples of 100 lb. and 50 lb. copper aerial lines shown in Tables 4 and 5, we see that the calculated equivalents added to the reflection equivalents approach very near to the observed results, even in case of very short lengths of lines, such as 10 miles.

Again, taking the examples of No. 19 gauge testing cable and loaded No. 19 gauge cable shown in Tables 6 and 7, we see that the same is true.

This holds good also in cases of lines consisting of more than one uniform line as shown in Tables 9–16. This shows that the formula is applicable to any lines in practical use and that the results obtained, though not very accurate, are much nearer to the observed than those obtained by simply comparing the attenuation lengths. Therefore, the reflection equivalents shown in Tables 30 and 31 are applicable, without much error, to almost all practical cases which may possibly arise.

The writer's sincere thanks are due to Mr. Toshio Arakawa for his kind and earnest assistance in experimental work.

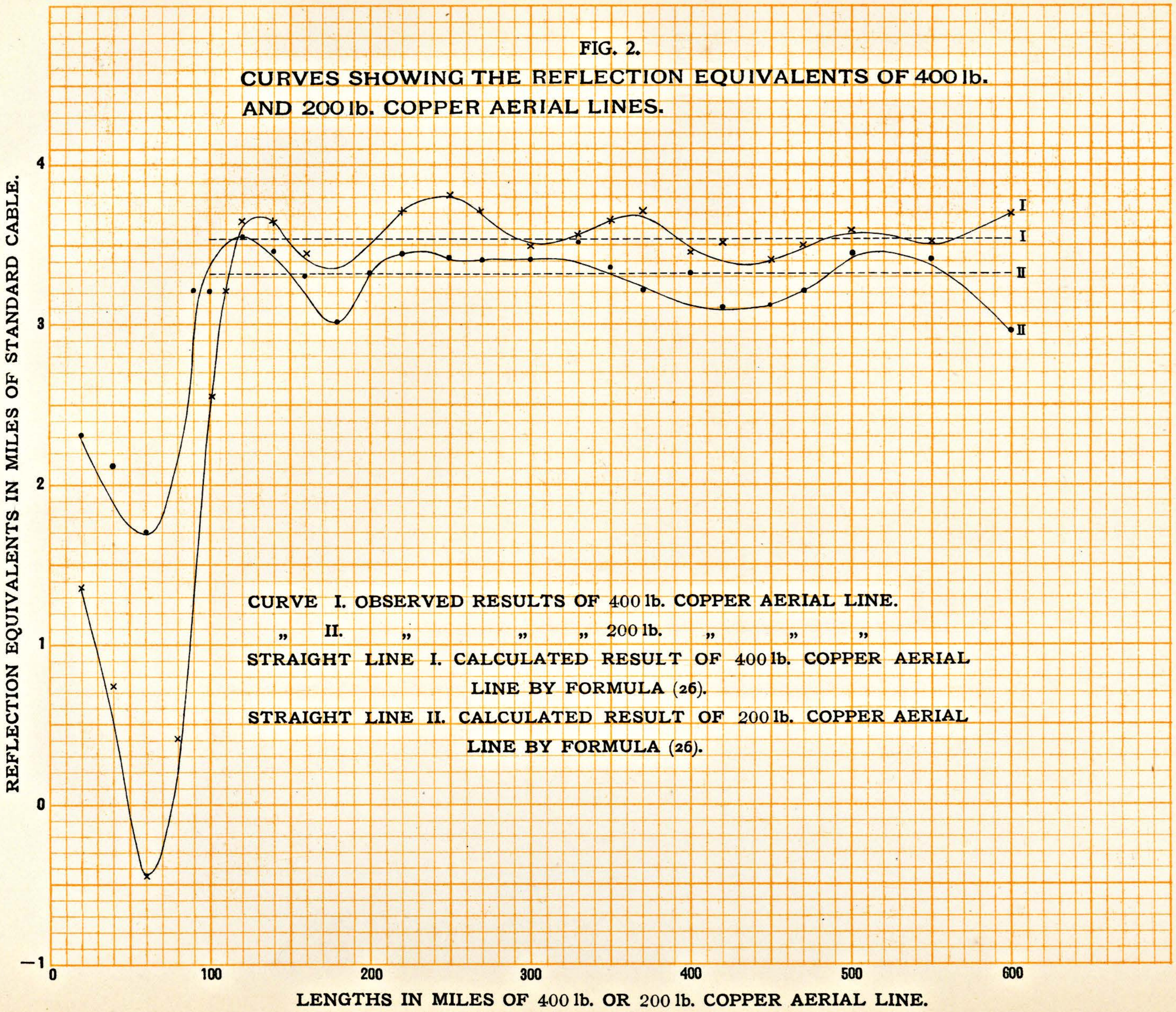


FIG. 2.
CURVES SHOWING THE REFLECTION EQUIVALENTS OF 400 lb. AND 200 lb. COPPER AERIAL LINES.
CURVE I. OBSERVED RESULTS OF 400 lb. COPPER AERIAL LINE.
„ II. „ „ „ 200 lb. „ „ „
STRAIGHT LINE I. CALCULATED RESULT OF 400 lb. COPPER AERIAL LINE BY FORMULA (26).
STRAIGHT LINE II. CALCULATED RESULT OF 200 lb. COPPER AERIAL LINE BY FORMULA (26).
REFLECTION EQUIVALENTS IN MILES OF STANDARD CABLE.
LENGTHS IN MILES OF 400 lb. OR 200 lb. COPPER AERIAL LINE.
I
I
II
II
4
3
2
1
0
−1
0
100
200
300
400
500
600

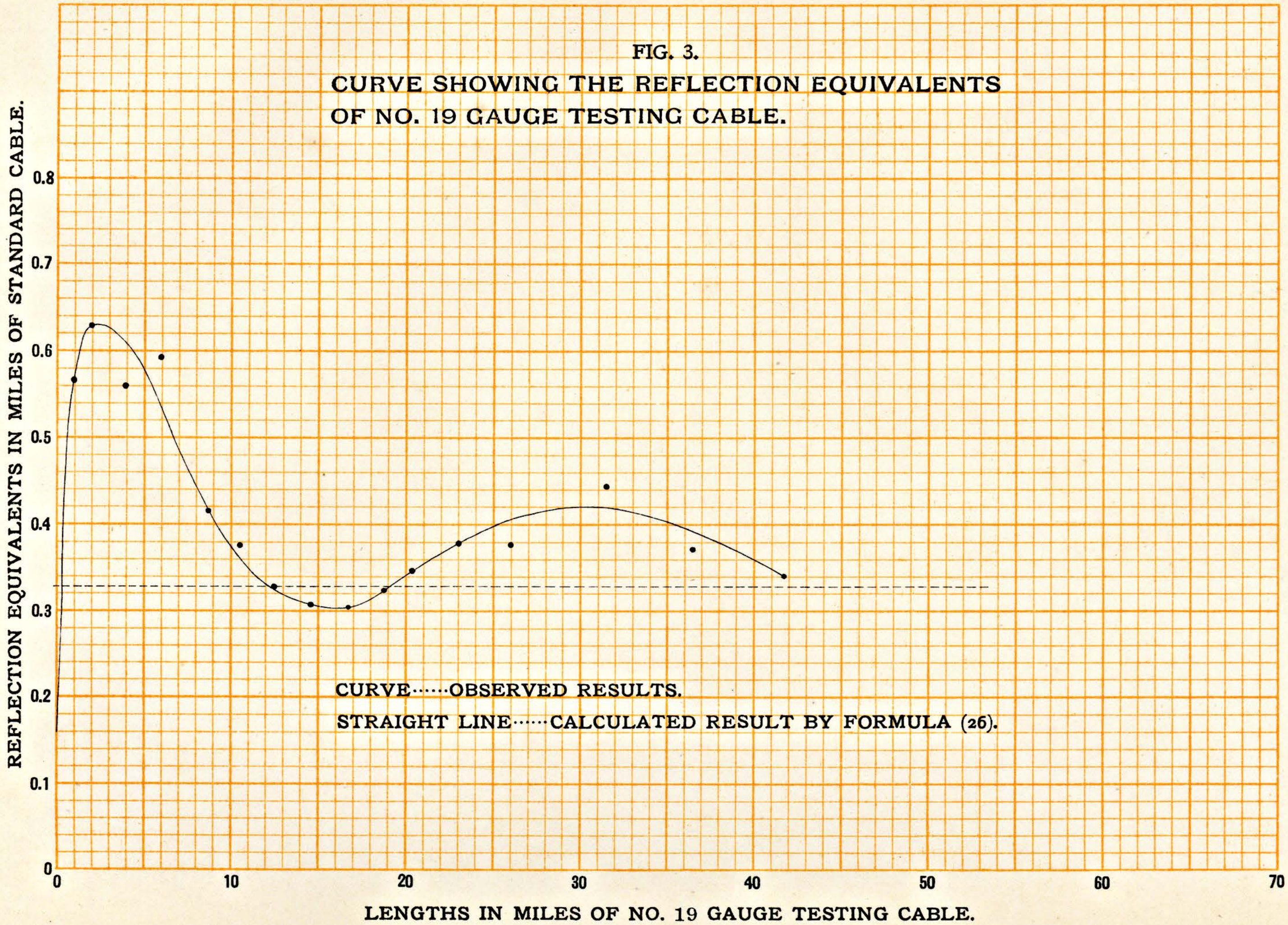
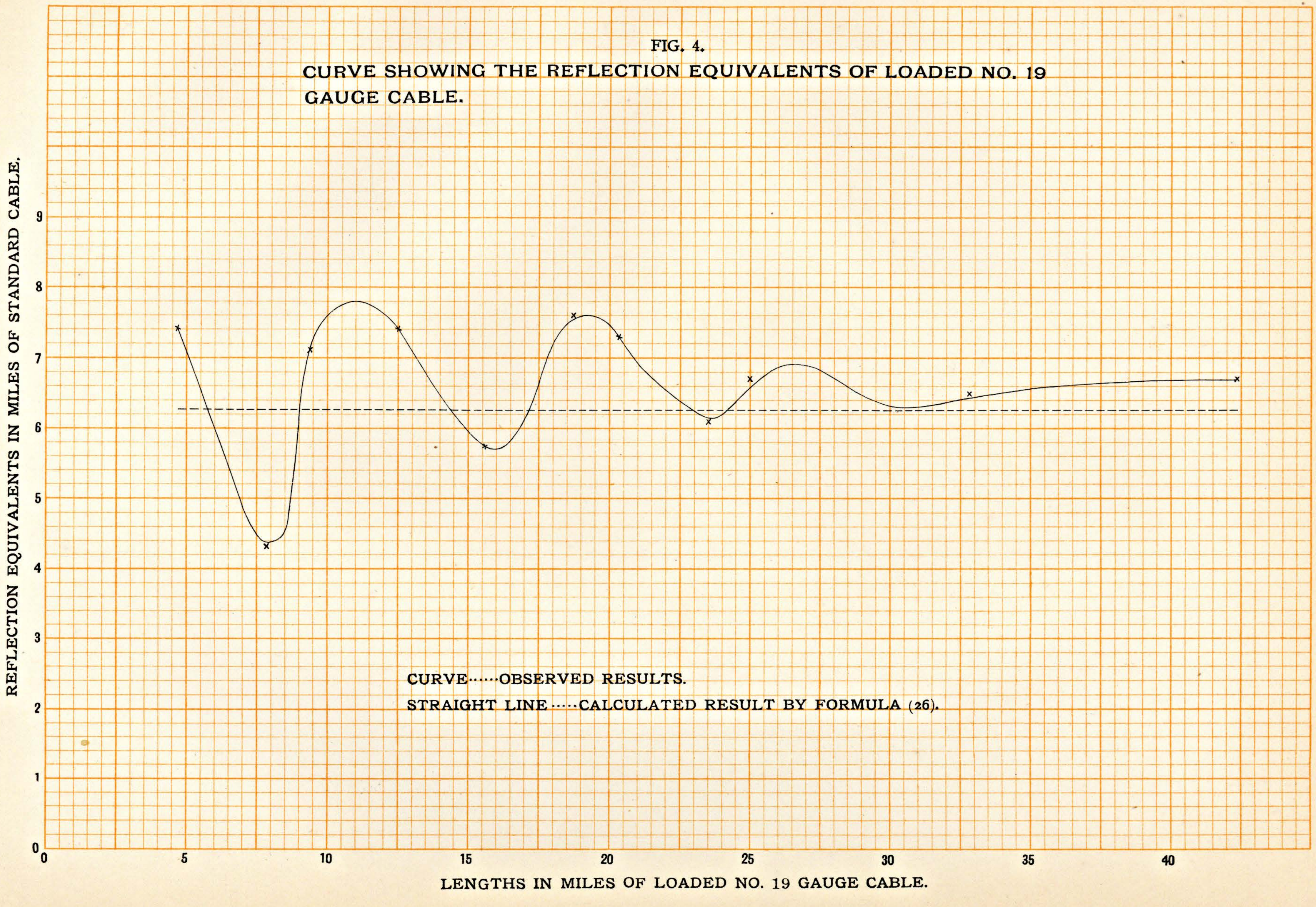


FIG. 3.
CURVE SHOWING THE REFLECTION EQUIVALENTS
OF NO. 19 GAUGE TESTING CABLE.
REFLECTION EQUIVALENTS IN MILES OF STANDARD CABLE.
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
10
20
30
40
50
60
70
CURVE……OBSERVED RESULTS.
STRAIGHT LINE……CALCULATED RESULT BY FORMULA (26).
LENGTHS IN MILES OF NO. 19 GAUGE TESTING CABLE.

FIG. 4.

CURVE SHOWING THE REFLECTION EQUIVALENTS OF LOADED NO. 19 GAUGE CABLE.



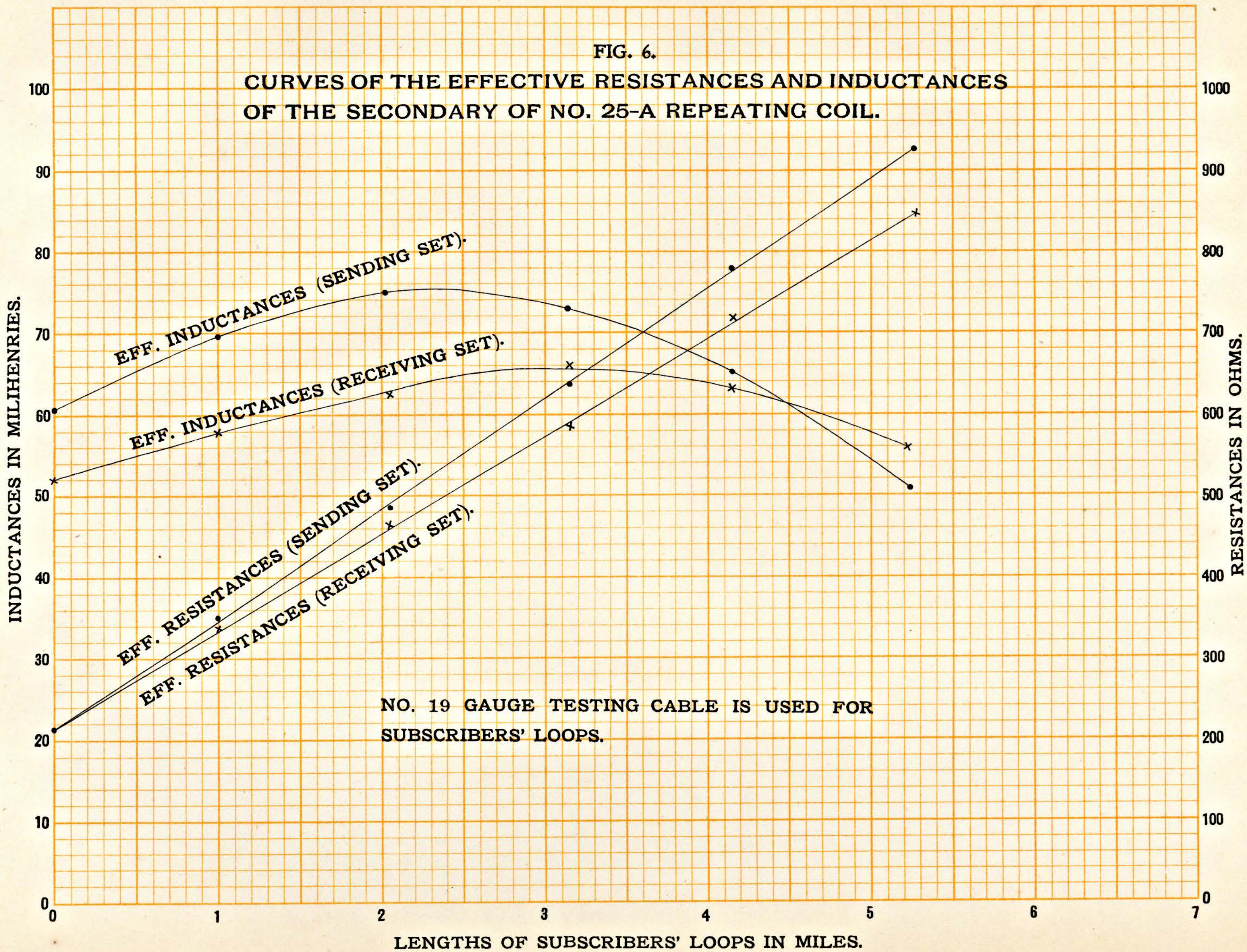


FIG. 6.
CURVES OF THE EFFECTIVE RESISTANCES AND INDUCTANCES
OF THE SECONDARY OF NO. 25-A REPEATING COIL.
EFF. INDUCTANCES (SENDING SET).
EFF. INDUCTANCES (RECEIVING SET).
EFF. RESISTANCES (SENDING SET).
EFF. RESISTANCES (RECEIVING SET).
NO. 19 GAUGE TESTING CABLE IS USED FOR
SUBSCRIBERS' LOOPS.
INDUCTANCES IN MILIHENRIES.
RESISTANCES IN OHMS.
LENGTHS OF SUBSCRIBERS' LOOPS IN MILES.
0
10
20
30
40
50
60
70
80
90
100
0
100
200
300
400
500
600
700
800
900
1000
0
1
2
3
4
5
6
7

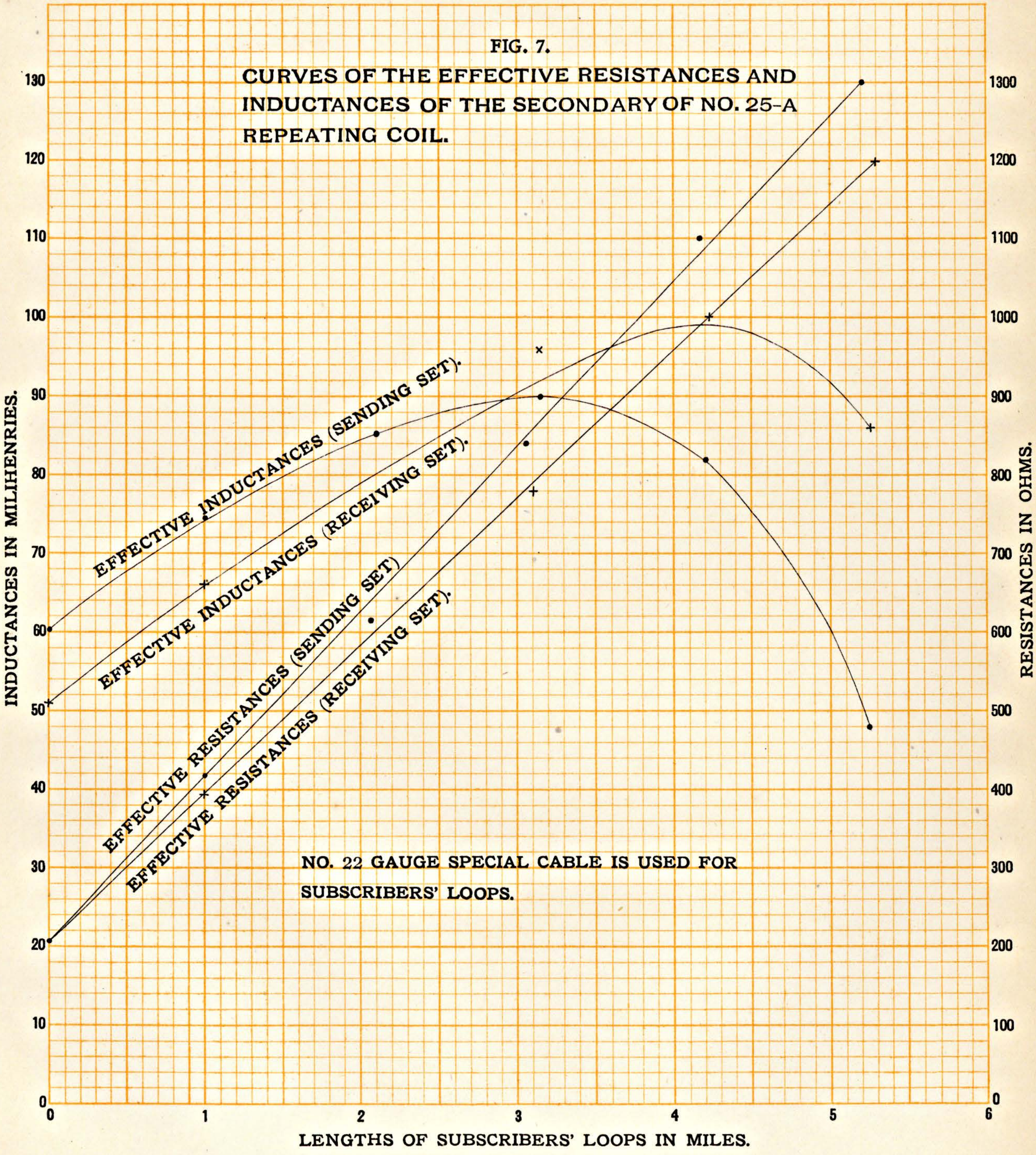


FIG. 7.
CURVES OF THE EFFECTIVE RESISTANCES AND INDUCTANCES OF THE SECONDARY OF NO. 25-A REPEATING COIL.
EFFECTIVE INDUCTANCES (SENDING SET).
EFFECTIVE INDUCTANCES (RECEIVING SET).
EFFECTIVE RESISTANCES (SENDING SET)
EFFECTIVE RESISTANCES (RECEIVING SET).
NO. 22 GAUGE SPECIAL CABLE IS USED FOR SUBSCRIBERS' LOOPS.
INDUCTANCES IN MILIHENRIES.
RESISTANCES IN OHMS.
LENGTHS OF SUBSCRIBERS' LOOPS IN MILES.
0
10
20
30
40
50
60
70
80
90
100
110
120
130
100
200
300
400
500
600
700
800
900
1000
1100
1200
1300
1
2
3
4
5
6

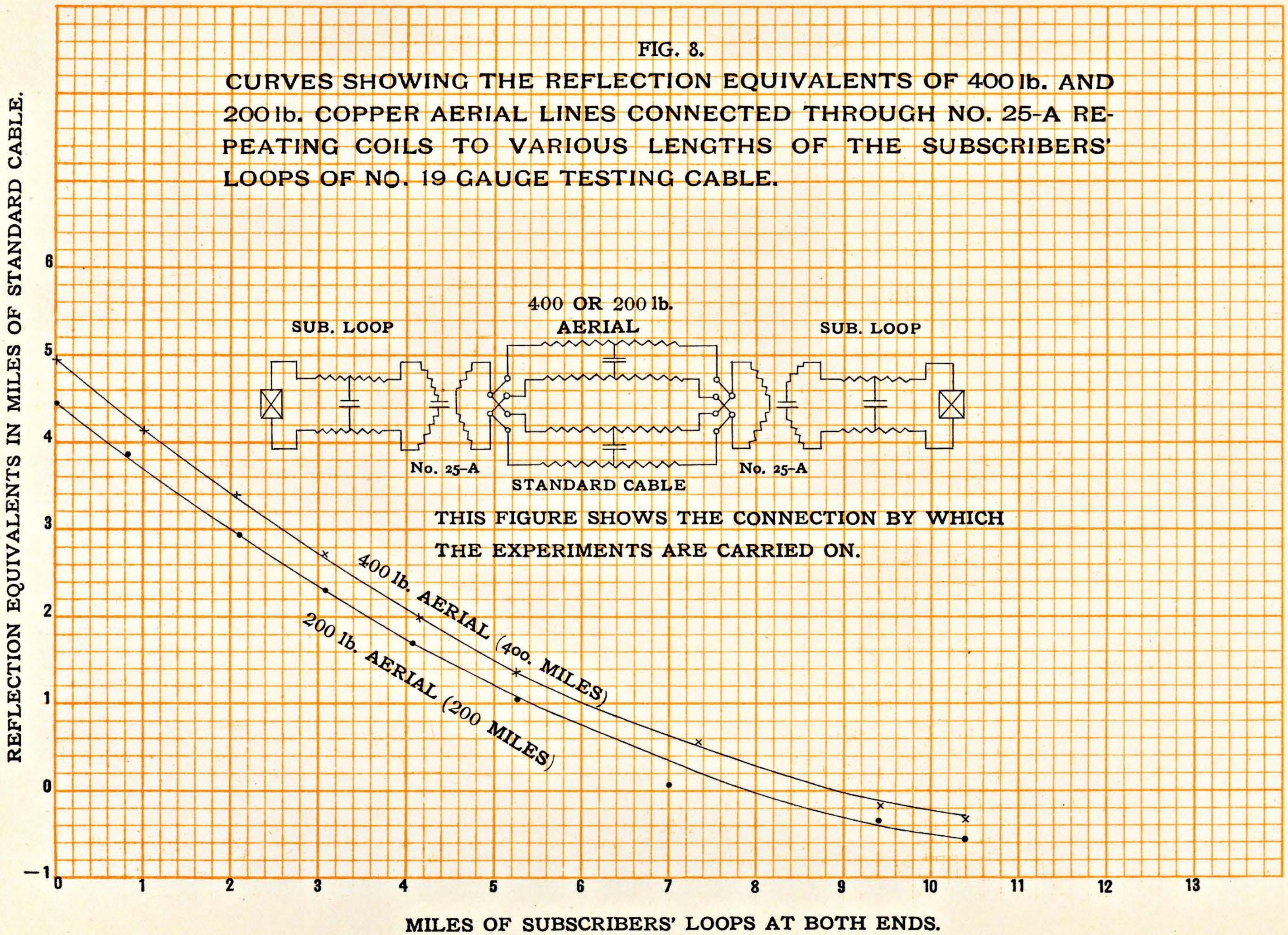


FIG. 8.
CURVES SHOWING THE REFLECTION EQUIVALENTS OF 400 lb. AND 200 lb. COPPER AERIAL LINES CONNECTED THROUGH NO. 25-A REPEATING COILS TO VARIOUS LENGTHS OF THE SUBSCRIBERS' LOOPS OF NO. 19 GAUGE TESTING CABLE.
400 OR 200 lb.
AERIAL
SUB. LOOP
SUB. LOOP
No. 25-A
No. 25-A
STANDARD CABLE
THIS FIGURE SHOWS THE CONNECTION BY WHICH THE EXPERIMENTS ARE CARRIED ON.
400 lb. AERIAL (400. MILES)
200 lb. AERIAL (200 MILES)
REFLECTION EQUIVALENTS IN MILES OF STANDARD CABLE.
6
5
4
3
2
1
0
−1
0
1
2
3
4
5
6
7
8
9
10
11
12
13
MILES OF SUBSCRIBERS' LOOPS AT BOTH ENDS.

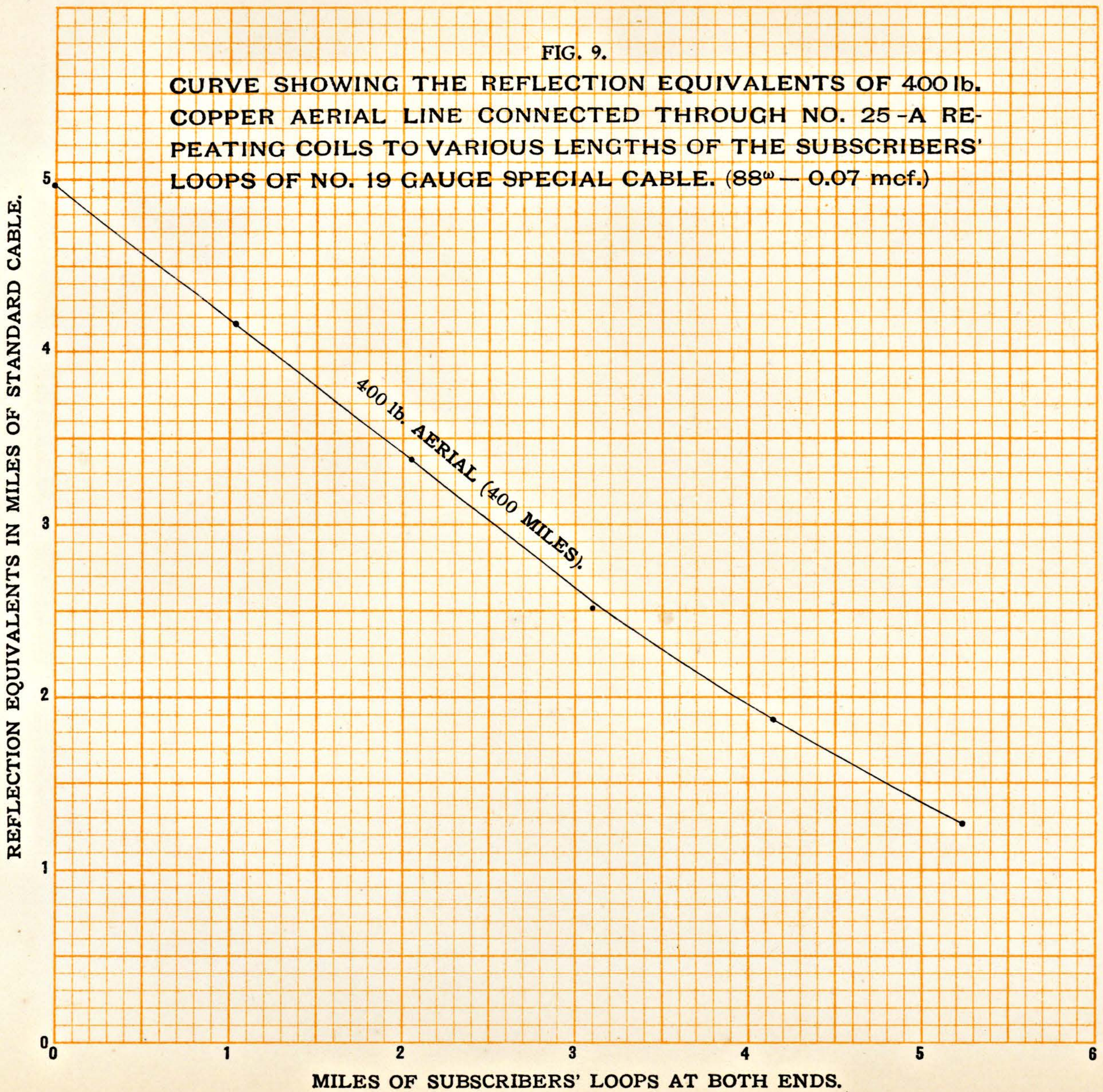


FIG. 9.
CURVE SHOWING THE REFLECTION EQUIVALENTS OF 400 lb. COPPER AERIAL LINE CONNECTED THROUGH NO. 25-A REPEATING COILS TO VARIOUS LENGTHS OF THE SUBSCRIBERS' LOOPS OF NO. 19 GAUGE SPECIAL CABLE. (88ω — 0.07 mcf.)
400 lb. AERIAL (400 MILES).
REFLECTION EQUIVALENTS IN MILES OF STANDARD CABLE.
0
1
2
3
4
5
0
1
2
3
4
5
6
MILES OF SUBSCRIBERS' LOOPS AT BOTH ENDS.

FIG. 10.

CURVE SHOWING THE REFLECTION EQUIVALENTS OF 400 lb. COPPER AERIAL LINE CONNECTED THROUGH NO. 25-A REPEATING COILS TO VARIOUS LENGTHS OF THE SUBSCRIBERS' LOOPS OF NO. 22 GAUGE SPECIAL CABLE.

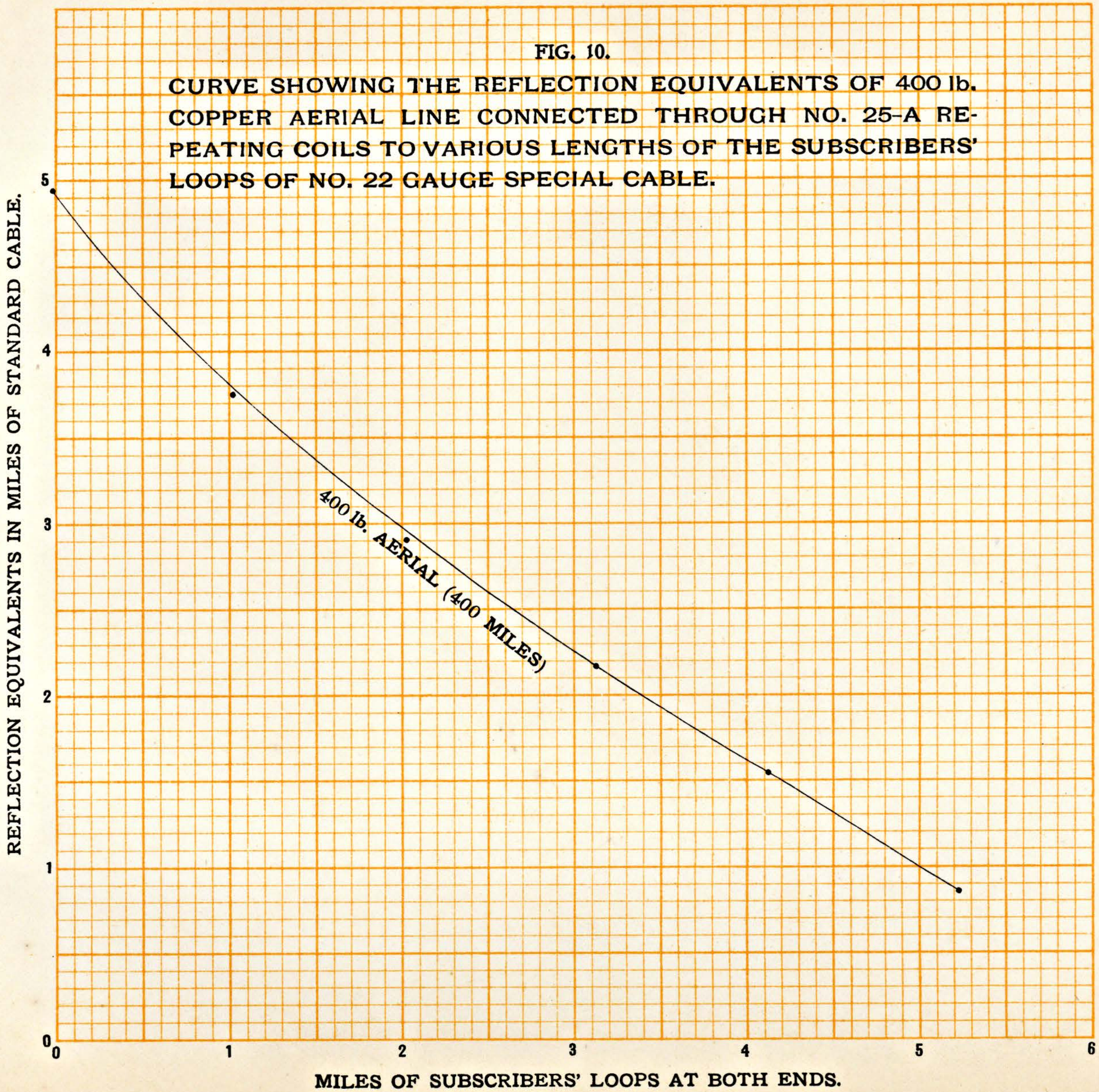


FIG. 11.

CURVE SHOWING THE REFLECTION EQUIVALENTS OF LOADED TESTING CABLE CONNECTED THROUGH NO. 25-A REPEATING COILS TO VARIOUS LENGTHS OF THE SUBSCRIBERS' LOOPS OF NO. 19 GAUGE TESTING CABLE.