Formula for the Strength of Struts.

By

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The strength of a strut subject to compression depends on its length. Only for very short struts it equals to the product of the area of cross section into the compressive yielding stress or strength. Very long struts on the other hand behave according to Euler's formula; hence this formula is commonly used when the slenderness

$$\frac{l}{i}=\frac{\text{length}}{\text{radius of gyration}}$$

is greater than about 100. In practice however cases of medium lengths less than 100 i occur most often and in which either the calculation in regard to pure compressive strength or that in regard to pure buckling strength should give a too weak strut.

For such cases a set of empirical formlae has been established, among which may be mentioned Johnson's and Tetmajer's as are well known.

The former is

$$P=FK\left[1-\alpha\left(\frac{l}{i}\right)^2\right],\quad \text{for } \frac{l}{i}<\sqrt{\frac{1}{2\alpha}}$$

and the latter

$$P=a-b\frac{l}{i}, \quad \text{for } \frac{l}{i}<\text{about } 100,$$

where P = critical load,

K = compressive yielding stress or strength,

F = cross-sectional area,

α = a theoretical constant,

a, b = empirical constants,

The use of discontinuous two functions for the whole range of $\frac{l}{i}$ accompanies a trouble. When in machine design it is required to find the cross-sectional dimension, having given l and P (the actual load × factor of sefety), it will be of question which function of the two is to be taken for use, the value of $\frac{l}{i}$ being unknown at the start. It is therefore very desirable to have a single continuous function holding good for the whole range of $\frac{l}{i}$.

Rankine's formula (with theoretical constant):

$$P=FK\frac{1}{1+\frac{K}{\pi^2 E}\left(\frac{l}{i}\right)^2}$$

is the single continuous function to be used for any value of $\frac{l}{i}$, E being the modulus of elasticity, but it is, as is known, too inaccurate when applied to the struts of meduim lengths.

Natalis proposed a formula[1], which is, for the strut pivoted at each end,

$$P=FK\frac{1+\frac{K}{\pi^2E}\left(\frac{l}{i}\right)^2}{1+\frac{K}{\pi^2E}\left(\frac{l}{i}\right)^2+\left(\frac{K}{\pi^2E}\right)^2\left(\frac{l}{i}\right)^4}$$

He made experiments[2] with air-craft materials and proved the fitness of his formula by the results.

The author wishes to propose in this paper another formula, which may be written, as its general form, as

$$P=FK\frac{1}{\left\{1+\left[\frac{K}{\pi^2E}\left(\frac{l}{i}\right)^2\right]^n\right\}^{\frac{1}{n}}}, \qquad (1)$$

n being a constant.

According as $\frac{l}{i}$ diminishes and approaches zero, the second term in the parenthesis { } becomes negligible against the first and at the limit (1) reduces to

$$P=FK, \ldots\ldots\text{ the compression formula,}$$

and according as $\frac{l}{i}$ increases and approaches infinity the first term becomes negligible against the second and at the limit we have

$$P=\pi^2EF\frac{1}{\left(\frac{l}{i}\right)^2}, \ldots\ldots\text{ Euler's formula.}$$

With n=1 Equ. (1) reduces to Rankine's formula already referred to.

Applying Equ. (1) to experimental results obtained by different investigators we see that n may be taken at 2 as a good average value, although for certain series of test it is to be considerably greater than 2 and for certain other series considerably less. With $n=2$ Equ. (1) becomes

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[1] [2] Dinglers Polytechnisches Journal, 1919, S. 71.

$$P=FK\frac{1}{\sqrt{1+\left(\frac{K}{\pi^2E}\right)^2\left(\frac{l}{i}\right)^4}} \qquad (2)$$

In Fig. 1 points are plotted according to Natalis' results on the struts of solid drawn steel tube, taking $\frac{l}{i}$ as abscissa and $\frac{P}{F}$ as ordinate, and the lines are drawn according to Natalis' and the author's formulae. It will be seen that the author's line fits better. In Fig. 2 Natalis' results on the struts of " Kieferholz " are plotted and the author's lines with respectively $n=2$ and $n=1.7$ are drawn.

Natalis' line almost coincides with the author's line with $n=1.7$ and is here more fitting than the same with $n=2$.

The value of Equ. (2) will furthar be tested by Tetmajer's experimental results. In Figs. 3 to 8 points are plotted according to his report " die Mitteilungen der Materialprüfungsanstalt " published in 1901 and the author's curve is drawn. In all these a sufficient accordance will be observed.

Tetmajer's results on the wooden struts with $\frac{l}{i}$ of about 9 show a considerable fluctuation, the maximum resistance being often almost twice as great as the minimum one. This may be attributed to the degree of homogeneity, whose influence on the resistance becomes striking in very short struts. As a highly homogeneous material is not always to be expected, it will do to exclude high values of resistance of such struts from consideration.

Further the struts with $\frac{l}{i}$ of about 1.8 all show fairly high resistances. This is qualitatively in accordance with Baumann's experimental result[3], namely the strength of timber increases with decreasing length of specimen and in fact at the rate of 1 to 1.11 when $\frac{l}{i}$ decreases from 15.75 to 1.73 (as the mean from 45 specimens).

Such increase of resistance in very short struts, caused probably by the friction on the end surfaces of specimen, may be disregarded in the problem of buckling.

For these reasons, in Figs 6 to 8 the curve is drawn without respect to the points with $\frac{l}{i}$ less than about 10.

Cast Iron Struts.

Cast iron is the material disobedient to ordinary laws of bending strength. Accordingly Equ. (2) is not applicable to cast iron struts.

For them n may be taken at $\frac{5}{4}$, so that (1) becomes

$$P=\alpha FK \quad \text{with} \quad \alpha=\frac{1}{\left\{1+\left[\frac{K}{\pi^2 E}\left(\frac{l}{i}\right)^2\right]^{\frac{5}{4}}\right\}^{\frac{4}{5}}}, \qquad (3)$$

where E may be taken ordinarily at 1,000,000 to 1,050,000 kg/cm².

In Figs 9 to 12 the fitness of this formula is verified by Tetmajer's results on the hollow cyclindrical struts given in the same report already referred to.

The values of α for $\sqrt{\frac{K}{\pi^2 E}}\frac{l}{i}=0.1$ to 3.5 were computed and are given in the following table, which may serve for calculation.

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[3] Forschungsarbeit, Heft 231.

$\sqrt{\frac{K}{\pi^2 E}}\frac{l}{i}$	α	Diff.	$\sqrt{\frac{K}{\pi^2 E}}\frac{l}{i}$	α	Diff.	$\sqrt{\frac{K}{\pi^2 E}}\frac{l}{i}$	α	Diff.
					0.050			0.013
0.0	1.000		1.2	0.469		2.4	0.159	
		0.003			0.045			0.011
0.1	0.997		1.3	0.424		2.5	0.148	
		0.011			0.041			0.010
0.2	0.986		1.4	0.383		2.6	0.138	
		0.024			0.035			0.009
0.3	0.962		1.5	0.347		2.7	0.129	
		0.036			0.032			0.009
0.4	0.926		1.6	0.315		2.8	0.120	
		0.048			0.028			0.007
0.5	0.878		1.7	0.287		2.9	0.113	
		0.057			0.025			0.007
0.6	0.821		1.8	0.262		3.0	0.106	
		0.061			0.023			0.007
0.7	0.760		1.9	0.239		3.1	0.099	
		0.064			0.020			0.006
0.8	0.696		2.0	0.219		3.2	0.063	
		0.062			0.017			0.005
0.9	0.634		2.1	0.202		3.3	0.088	
		0.060			0.016			0.005
1.0	0.574		2.2	0.186		3.4	0.083	
		0.055			0.014			0.004
1.1	0.519		2.3	0,172		3.5	3.079	

On Tetmajer's Formulae.

In 1888 and in 1893 to 1895 Tetmajer performed the most accurate test on more than eight hundred test struts and from the result he deduced the following formulae,

$$\frac{P}{F}=3.03-0.0129\frac{l}{i}$$

for wrought iron struts and for $\frac{l}{i}=$ 10 to 112,

$$\frac{P}{F}=3.10-0.0114\frac{l}{i}$$

for mild steel struts and for $\frac{l}{i}=$ 10 to 105,

$$\frac{P}{F}=0.293-0.00194\frac{l}{i}$$

for wooden struts and for $\frac{l}{i}=$ 1.5 to 100,

$$\frac{P}{F}=0.00053\left(\frac{l}{i}\right)^2-0.120\frac{l}{i}+7.76$$

for cast iron struts and for $\frac{l}{i}=$ 8 to 30, where $\frac{P}{F}$ is in 1000 kg./cm².

These are used by many engineers as the most reliable ones. Tetmajer's achievement can not be erased, but in forming the formulae he plotted, in a sheet of paper, points according to all the results for certain kind of material, including miscellaneous grades, and drew a line passing through or near by the centres of groups of points. The above formulae were built up in this manner and hence they afford no freedom of taking the individual properties of a certain grade of material into account. It will be seen that, for the air-craft materials tested by Natalis Tetmajer's fomulae lose completely their validity.

Fig. 1. Solid Drawn Steel Tube, 3 cm in dia. and 0.1 cm. in Thickness.

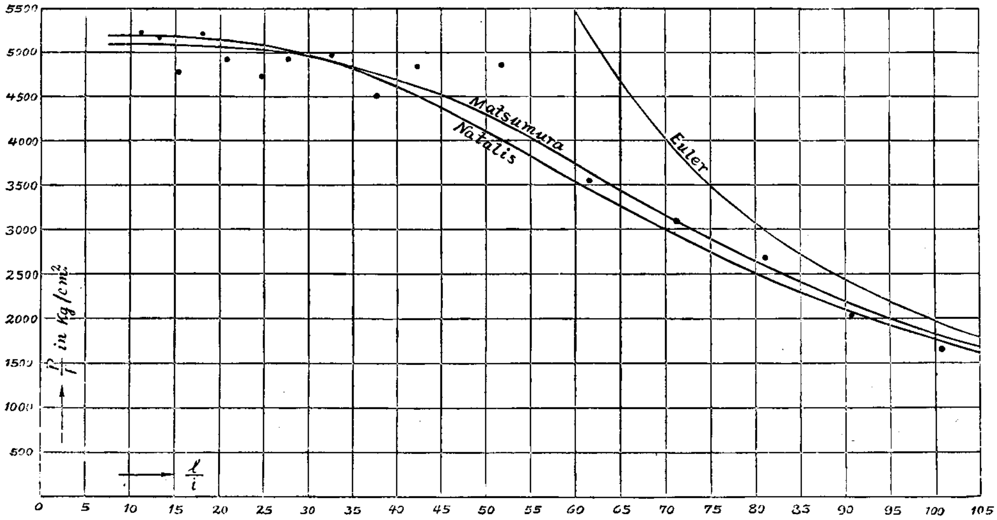


Fig. 2. Strut of "Kieferholz", 4 × 4 cm. square section.

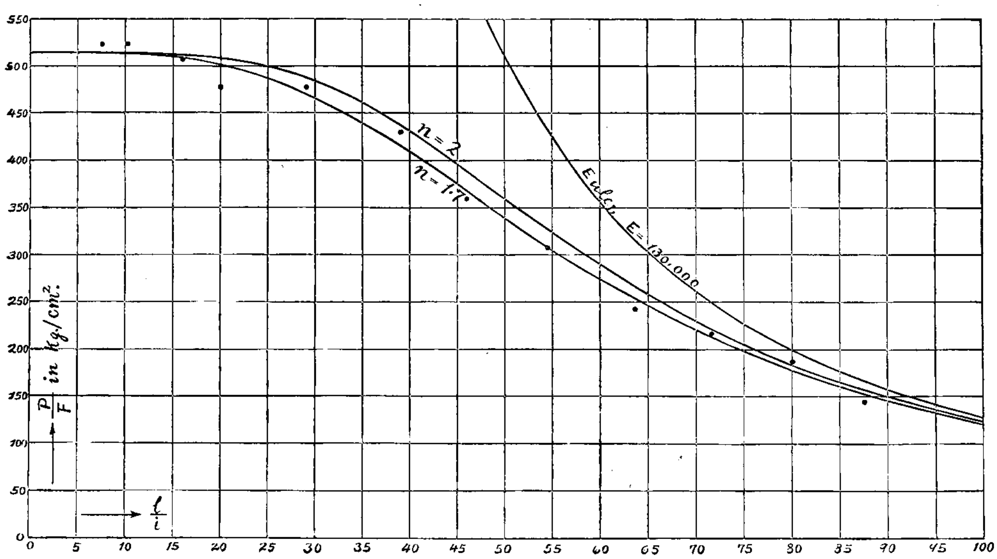


Fig. 3. Wrought Iron Strut.

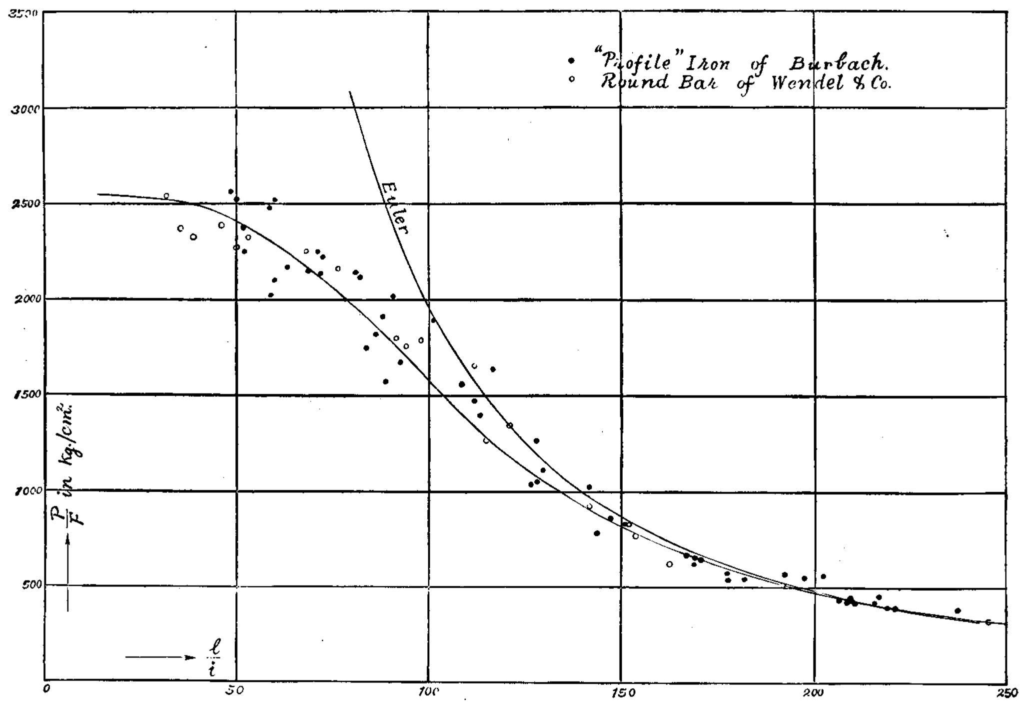


Fig. 4. Wrought Iron Strut, "Profile" Iron of Wendel & Co.

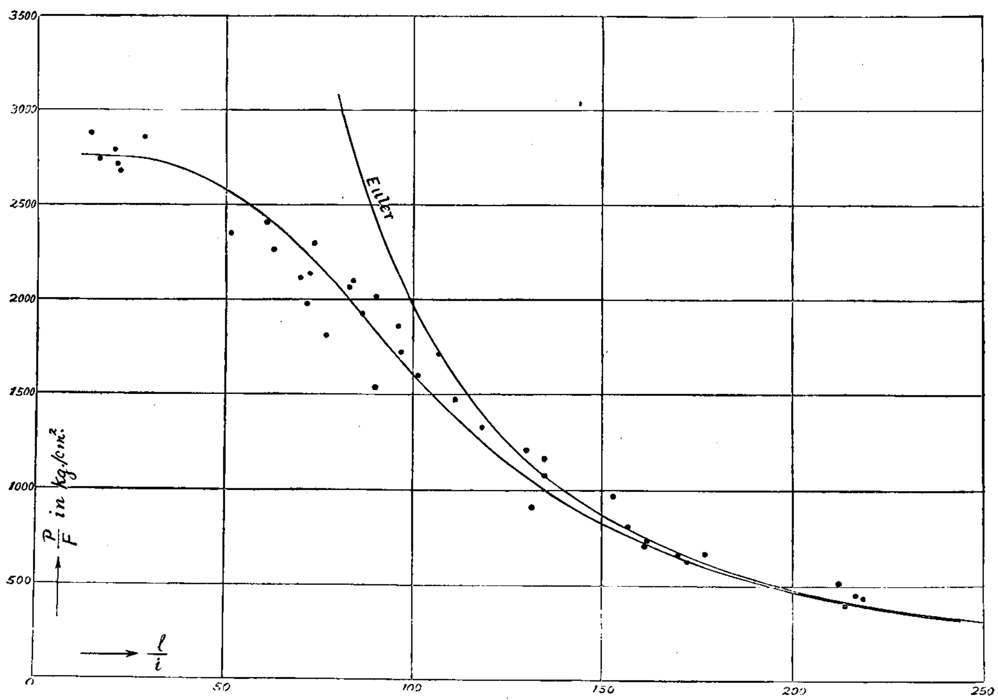


Fig. 5. Mild Steel Strut.

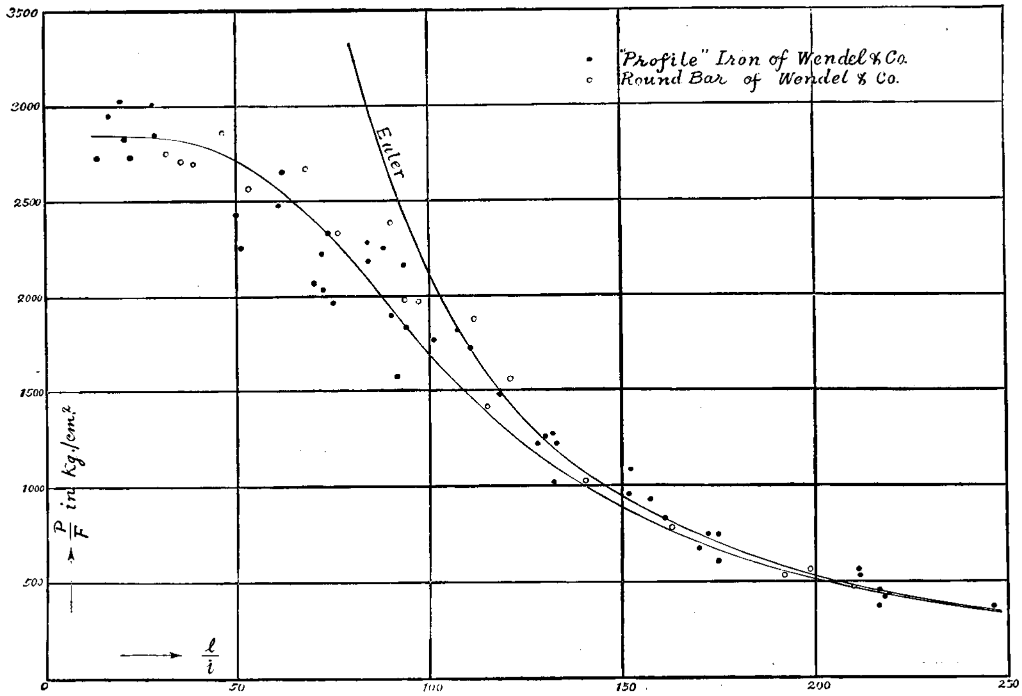
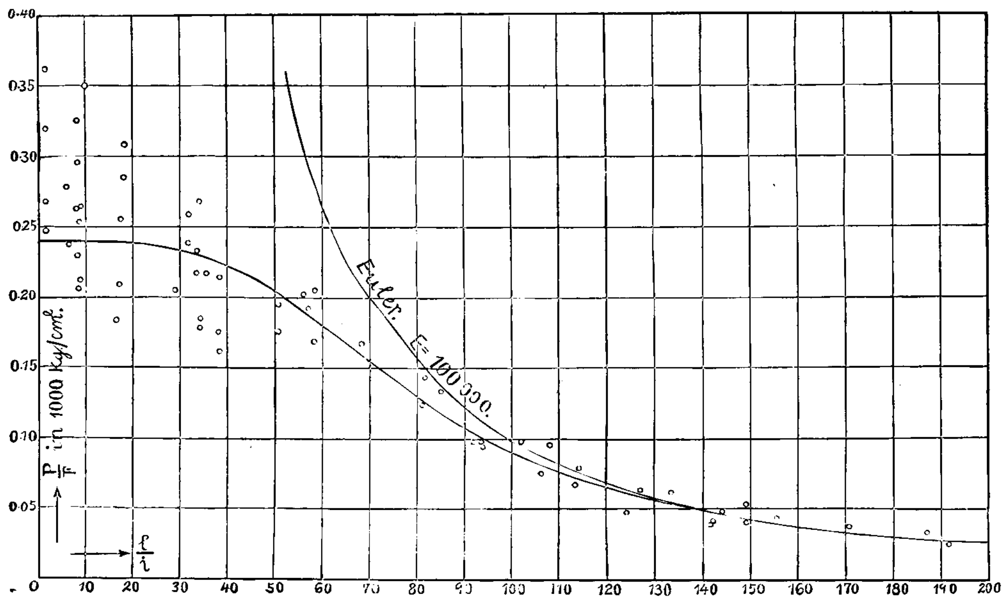
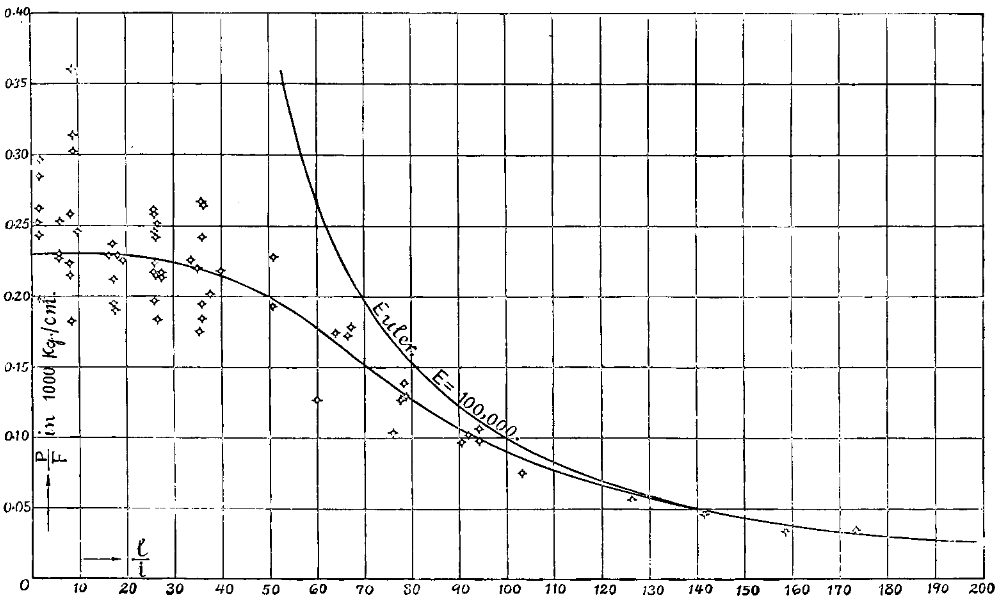


Fig. 6. Strut of "Föhre".



Fig, 7. Strut of "Weisstanne".



Fig, 8. Strut of "Lärche".

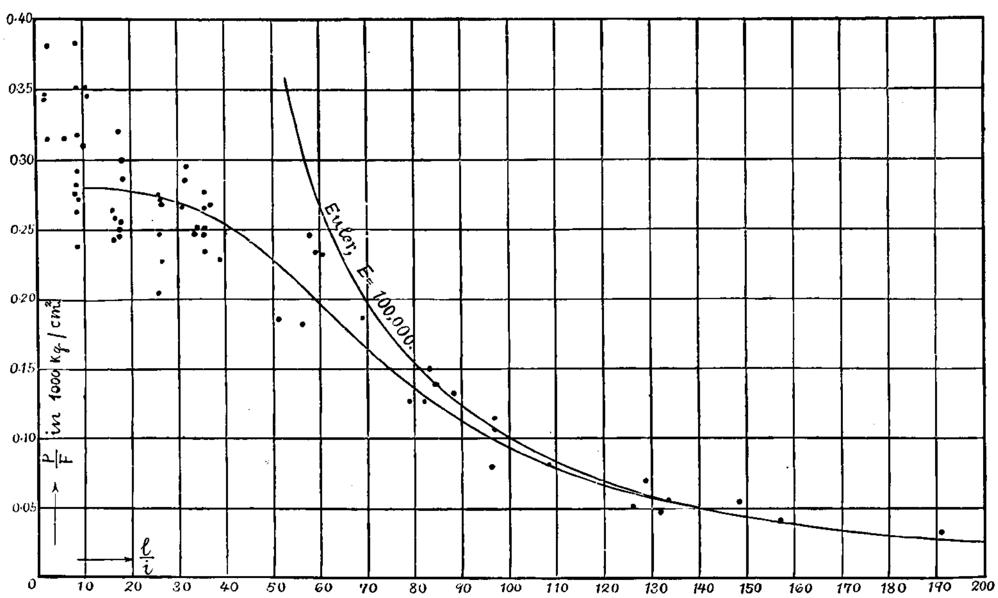


Fig. 9. Cast Iron Pipe of Brebach.

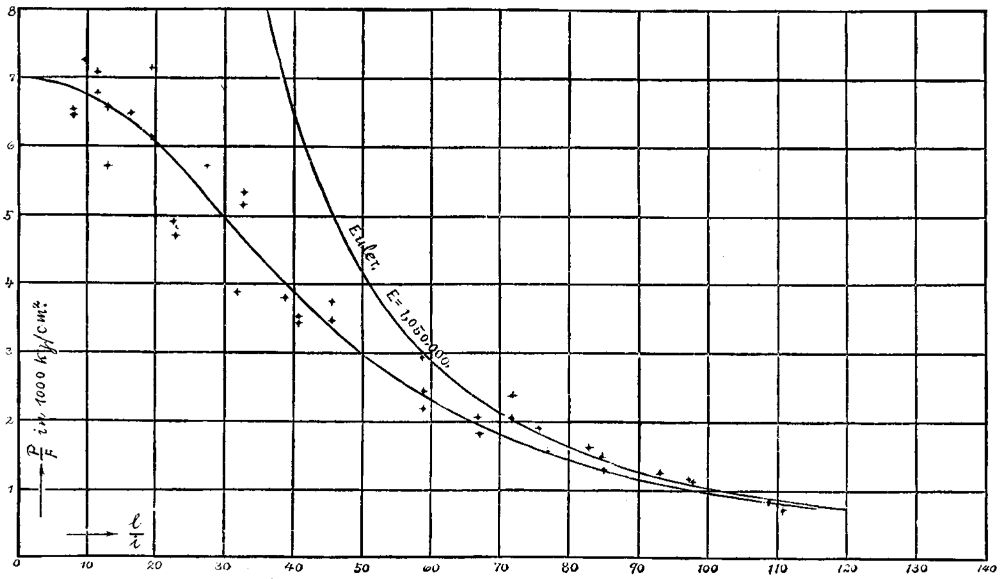


Fig. 10. Cast Iron Column of Choindez.

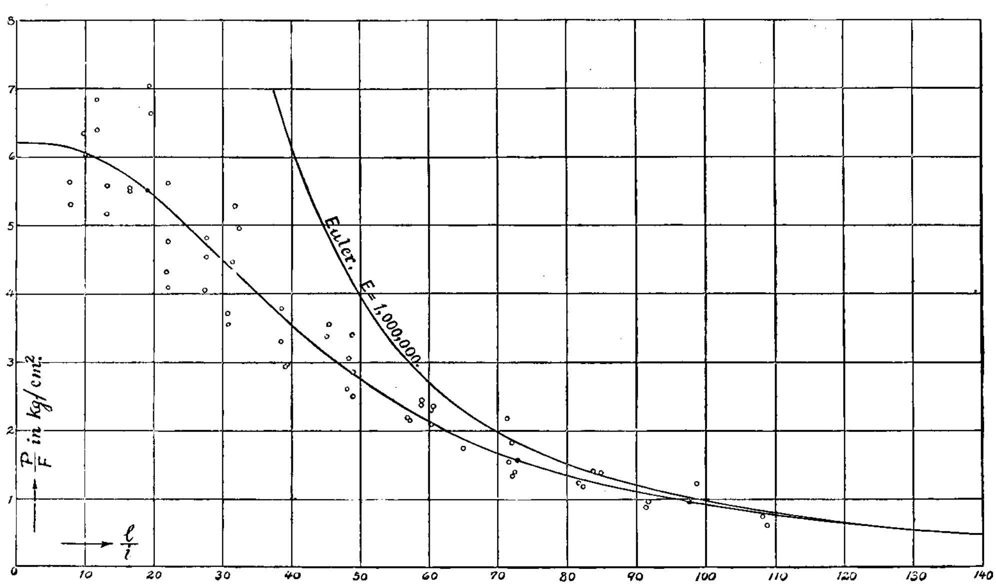


Fig. 11. Cast Iron Column of Brebach

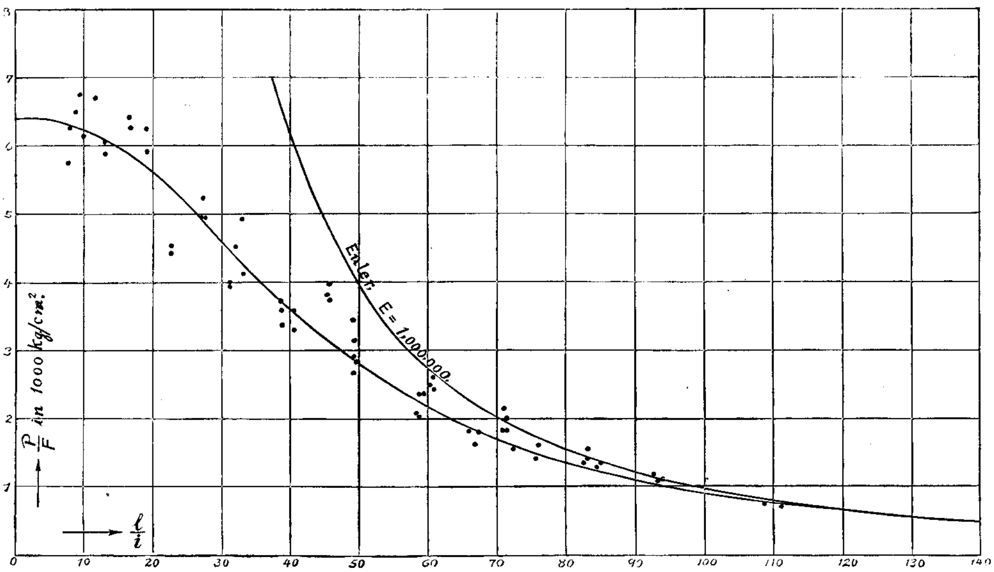


Fig. 12. High Grade Cast Iron of Brebach.