Experiment on the Transmission of Power by Belting.

By

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Since the time of Grashof, various theories and experiments on the transmission of power by belting have been reported by Gehrkens, Kammerer, Skutsch, Stiel, Taylor, Lewis, Okamoto, Sunatani and others. Many points with regard to this method of transmission, however, appear to be still obscure. Lewis stated in one of his conclusions that further experiments are required in order to elucidate more facts in connection with this problem. Using the testing apparatus designed by themselves, the authors have made tests on the transmission by rather narrow leather and cotton belts, the results of which are described in the present paper.

I. TESTING APPARATUS.

The transmission of small power is in most cases accomplished by a vertical or steeply inclined belting. As the present tests were intended to deal with belts of rather narrow widths, a testing apparatus of the vertical belt type as described below was prepared according to the writers' design.

From the line shaft of the college laboratory is driven a counter shaft by two parts of stepped cone pulleys as shown in Figs. 1 & 2. The driving pulley for the experiment is keyed to one end of the counter shaft. The driven pulley is placed directly beneath the driving pulley and is supported on both sides by ball bearings provided in the weighing lever which can swivel in the plane of the two pulleys round an axis or fulcrum.

The sum of the belt tensions on the tight and the slack sides is measured by means of a dead weight W hung on the end of the weighing lever as shown in Fig. 2. The horizontal distances from the fulcrum to the pulley center and to the weight W are in the ratio of 1 : 3. The weight of the driven pulley is counteracted by a balance weight and an oil pot is provided to damp the vibration of the lever during the test. By adjusting the amount of W so as to keep the distance between the driving and driven pulleys constant, we are enabled to measure the variation of the sum of the belt tensions on the tight and the slack sides.

The torque on the driven shaft is transmitted through two universal joints to a brake dynamometer and is measured therein. The brake arm is 70 cms. long.

The vertical shaft A in Figs. 1 and 5 is connected with the counter-shaft by a worm gear and the speed of the driving pulley can be determined by a stop watch. The lag of the driven to the driving pulley due to the slip of the belt is measured with Matsumura's slip meter shown in Fig. 5, a short discription of which is as follows: The toothed wheels C and C′ of equal diameters are fixed to the axes A and B respectively. Between C and E and between C′ and E two equal wheels D and D′ gear respectively. The axis of D′ is fixed to the frame and that of D is fixed to a pointer F, which is movable around the axis A. The motion of F is read by a dial. The numbers of teeth in the wheels C, D and E are 72, 48 and 168 respectively.

When the axes A and B revolve at equal speeds in the same direction, say in the direction of the arrow, F stands still. If B is one revolution behind A, F moves $\frac{72}{168+72}=\frac{3}{10}$ revolutions in the sense of the arrow marked. As the circumference is divided into 1000 equal parts, $\frac{3}{10}$ revolutions correspond to 300 divisions. When F moves x divisions for m rotations of A, the lag of the driven to the driving pulley is $\frac{x}{1000}\,\frac{10}{3}\,\frac{1}{m}=\frac{x}{300\,m}$ per revolution and if $m=10$, a slip of 0.033% can be read.

II. ELASTICITY OF BELT.

The relation between the tension and the elongation of a belt is affected by the condition whether the belt is new or old, by the amount of tension applied before the test and the duration of the application as well as the humidity of the belt. Leather and cotton belts seem to have quite different elastic properties.

The belts used in the present experiments were:

Leather belt: 4 Dia—shirushi, Nitta,

Breadth $b=8.83$ cm. ($3\frac{1}{2}$ inches),

Thickness $t=0.52$ cm.

Balata belt: Chikyu—shirushi, Nitta,

Breadth $b=8.86$ cm. ($3\frac{1}{2}$ inches),

Thickness $t=0.44$ cm.

1. Tension and Elongation of Belt.

In a leather belt stress and strain are not proportional, the stress σ increasing faster than the strain ε, as shown in Fig. 6. The relation between σ and ε may be expressed by

$$\varepsilon = a\,\sigma - a'\,\sigma^n \qquad (1)$$

where $n>1$ and a, a', n are constants. With $n=\frac{3}{2}$ the above expression is exact enough to represent the experimental result. a and a' take different values according to whether the belt is quite new or has been used for some time before the test.

The relation between σ and ε may also be expressed by

$$\varepsilon = \frac{a_1\,\sigma}{1 + a_2\,\sigma} \qquad (2)$$

which is more convenient for further deduction.

The constants a_1 and a_2 were found to be

$a_1=0.00084$, $a_2=0.012$ for new belt (Test No. 1),

$a_1=0.00055$, $a_2=0.0068$ for belt used for some time (Test No. 7),

σ being expressed in kg./cm^2.

The strain ε of the balata belt is either nearly proportional to σ or increases a little faster than σ according to the history. When the belt has been used for some time before the test, ε is proportional to σ as shown in Figs. 7 a and 7 b and when the belt is new or has been let free for several days after having been used, the relation of ε to σ is something like that expressed by

$$\varepsilon = a\,\sigma + a'\,\sigma^{\frac{3}{2}} \qquad (3)$$

or

$$\varepsilon = \frac{a_1\,\sigma}{1 - a_2\,\sigma} \qquad (4)$$

The curves in Figs. 7 c and 7 d were drawn from (3) and (4) respectively with

$$a = 0.000044, \quad a' = 0.0000168$$

$$a_1 = 0.000102, \quad a_2 = 0.00725$$

2. Time and Elongation.

The effect of time on the elongation of a belt is considerable. Fig. 8 a shows the relation of time to elongation when a tension of 32.7 kg./cm. (W=240 lbs.) was applied to a new leather belt of 5 cms. (2 inches) width, initially stressed by 4.1 kg./cm. (W_0=30 lbs.) for several hours. Fig. 8 b shows the effect of time on the contraction of the belt when the tension was released to 4.1 kg./cm. (W=30 lbs.) after it had been stressed to 32.7 kg./cm. (W_0=240 lbs.) for several hours. Fig. 8 c shows the variation of W after it was increased to 240 lbs. from its initial value W_0=30 lbs., the distance between the two pulleys being kept constant. For balata belt we have a similar relation as shown in Figs. 9 a, 9 b and 9 c.

When the belt is in motion, force $T_1' + T_2'$ acting on the pulley axis is the sum of the belt tensions on the tight and the slack sides less the effect of centrifugal force. The variation of $T_1' + T_2'$ during the transmission is shown in Fig. 10 for various rotary speeds of the pulley. The number of revolutions n was varied from 100 to 500 per min. and the force Q measured at the end of the brake arm was 0, 20 and 40 kg. The effect of starting and stopping on the belt tension is perceptible in Figs. 11 a, 11 b and 11 c.

3. Elastic Hysterisis of Belt.

As stated before, the elongation of a belt is influenced by time. From this fact it is to be expected that when it is pulled by increasing load and then released it will show a hysterisis. Figs. 12 a and 12 b show the elastic hysterisis of leather and balata belts respectively.

III. SLIP OF BELT.

The slip s between the belt and pulley may be assumed to consist of, say, the elastic slip s' and, say, the sliding slip s''. The elastic slip is due to the elasticity of the belt and is unavoidable. It is influenced by the relation of ε to σ and may be considered to be proportional(1) to the peripheral force P transmitted even when ε is not proportional to σ, being independent of the coefficient of friction between the belt and pulley. The sliding slip begins to appear when P exceeds a certain limiting value P_1. The magnitude of P_1 depends upon the frictional coefficient of the belt, and is greater according as the coefficient of friction is greater. When the sliding slip begins to occur, the total slip s and the force P deviate from a linear relation, s increasing faster than P until at last the belt runs off the pulley. See Fig. A.

Fig. A.

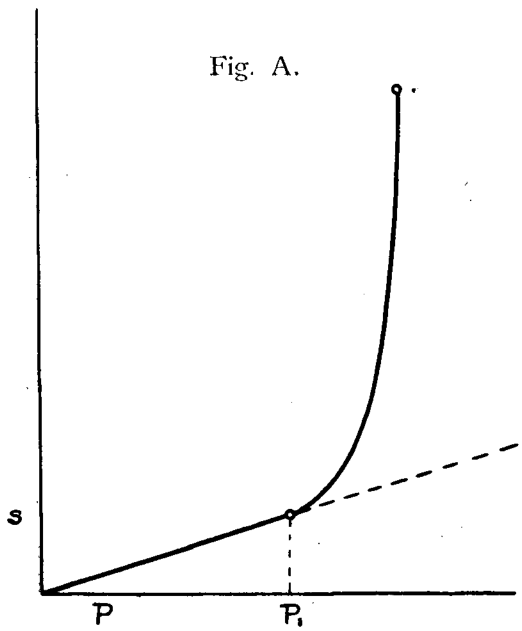
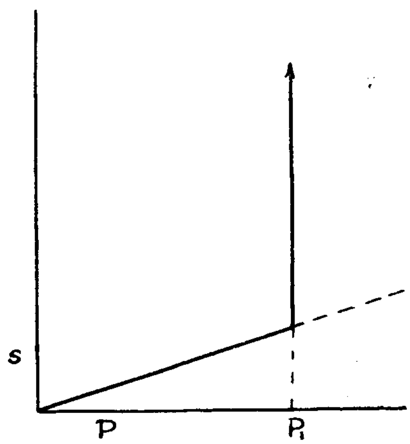


Fig. B.



(1) Journal of the Society of Mechanical Engineering, Japan, No. 138, p. 944.

If the coefficient of friction μ be constant, being independent of the sliding velocity of belt on pulley surface, the frictional resistance would not increase with the sliding velocity until the beginning of the sliding slip. When P passes its limiting value P_1, the slip increases suddenly and the belt runs off at once. See Fig. B.

The slip tests were made under the following conditions:

Pulley diameter	$d=$	15	19	24	30 inches	
	$=$	37.85,	48.11,	61.09,	76.02 cms.	
Revolution of pulley	$n=$	100,	200,	300,	400,	500 /min.
Initial tension weight	$W_0=$	140,		175,		210 lbs.
Initial tension	$T_0=$	60,		75,		90 lb./inch
	$=$	10.8,		13.5,		16.2 kg./cm.

In each test, the distance between the axes of the driving and the driven pulleys was kept constant and the peripheral force P was raised step by step. At every step the slip and the sum of the tensions $T_1'+T_2'$ acting on the pulley axis were measured. When the slip became considerable or the power transmitted reached the maximum available, the peripheral force of the pulley was released step by step and at each step s and $T_1'+T_2'$ were measured. Figs. 13 to 32 show the relations of $\frac{T_1'+T_2'}{2T_0}$ and s in dependence on P as plotted from the test results for the leather belt, the full line being the loading and the chain line the unloading curves.

Figs. 33 to 37 show similar diagrams for the balata belt on 30-inch pulleys. Leather and balata belts behave differently with regard to the relation between s and P. In the case of the leather belt, the sliding slip appears to take place gradually when P passes its limiting value P_1, while in the balata belt the sliding slip grows suddenly and the belt runs off at once, as shown in Fig. 38.

As already seen in Fig. B, this phenomenon occurs when the coefficient of friction μ is constant, being independent of the sliding velocity of belt on pulley surface. If μ is constant the limiting value P_1 is given by

$$P_1=2\left(T_0-\frac{\gamma}{g}v^2\right)\frac{e^{\mu\alpha}-1}{e^{\mu\alpha}+1} \qquad (5)$$

e being the base of natural logarithm and α the angle of contact. As the above expression involves no term of the sliding velocity, the sliding slip for a given peripheral force is indeterminate. When P exceeds P_1, the sliding slip occurs suddenly and the belt can slide to any extent up to running off the pulley immediately. From the result shown in Fig. 38, the coefficient of friction of balata belt may be considered to be nearly constant.

1. Slip and Speed of Pulley.

The influence of the rotary speed of the pulley on the slip of the leather belt is shown in Figs. 39 to 42. As the coefficient of friction of a leather belt on an iron pulley increases with the sliding velocity, the slip for a given peripheral force is less as the speed increases, which is shown in the test results.

The coefficient of friction of balata belt is nearly constant, as mentioned above, and the speed of the pulleys has only a slight influence on the slip. Fig. 43 shows the influence of the number of revolutions on the slip for the balata belt.

2. Slip and Pulley Diameter.

If there exists a part of friction which is proportional to the contact area of the belt and pulley, a smaller pulley diameter must result in a greater slip for given peripheral speed and peripheral force[(1)]. Figs. 44, 45 and 46 show the relation between the slip s and the peripheral speed v of leather belt for the peripheral force $P=40$, 60 and 80 kg. respectively. In a certain range of v, s seems to consist only of the elastic slip s', being independent of v. For smaller values of v, less than about 5 m./sec., the sliding slip is perceptible. The effect of the pulley diameter on the slip is found to be very small.

For a given number of revolutions, a pulley of smaller diameter has

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a smaller peripheral speed and causes a greater slip, as shown in Figs. 47 and 48.

3. Slip and Initial Tension.

In leather belting, stress σ increases faster than strain ε, hence a smaller initial tension T_0 must give a greater elastic slip s' for a given peripheral force. The limiting value P_1 is smaller according as the initial tension is smaller. From these two facts it is clear that a smaller initial tension will result in a greater total slip s. Fig. 49 shows the influence of the initial tension on the slip.

In balata belt, ε is either proportional to or increases faster than σ. If ε is proportional to σ, the initial tension has no influence on the elastic slip and if ε increases faster than σ, an increased initial tension must give a smaller slip. In Fig. 50 the slip for $W_0=175$ lbs. is found to be smaller than those for $W_0=140$ and 210 lbs. This is perhaps because the tests for $W_0=175$ lbs. were made last and the influence of time surpassed that of initial tension.

The limiting value P_1 appears proportional to the initial tension, a greater initial tension giving a greater P_1.

4. Slip and Coefficient of Friction.

If the coefficient of friction μ is a function of the sliding velocity, the relation between the slip and the peripheral force P becomes an expression of a complicated form[1]. The expression (5) which is the result of the common theory of belt transmission, taking μ as constant, gives the value of P at the instant of the beginning of the sliding slip. For a smaller value of P, (5) does not hold even when μ is constant. When a leather belt slides on a cast iron surface, μ increases with the velocity of sliding. So we can not use (5) to find the relation between P and μ. Let us imagine, however, a coefficient of friction μ' which satisfies (5) for any value of P. The relation of such μ' to s can be established from the

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corresponding values of P and s found in the present tests. The relation is shown in Figs. 51 to 54 for various pulley diameters and numbers of revolutions. By the aid of these diagrams we can calculate P from (5) when s is given.

Fig. 55 shows μ' as a function of peripheral speed v for $s=0.5\%$. We see that μ' increases with v.

IV. VARIATION OF BELT TENSION DURING THE TRANSMISSION.

Tests were made in which the distance of the driving and driven pulleys was kept constant and the variation of the sum of the belt tensions $T_1'+T_2'$ acting on the pulley axis was measured. The measured sum of the tensions $T_1'+T_2'$ was less than the sum of actual tensions T_1+T_2 by $2\frac{\gamma}{g}v^2$, where γ is the weight of the belt per unit length. The leather belt used in the tests was 8.33 m. in length and 3.825 kg. in weight, so that $\gamma=0.459$ kg./m.

Tension weight W' in lbs. at the lever end corresponding to $2\frac{\gamma}{g}v^2$ is of the following values:

d \ n	100	200	300	400	500
30″	1.09	4.34	9.76	17.36	27.12
24″	0.72	2.88	6.47	11.53	17.91
19″	0.45	1.79	4.02	7.15	11.72
15″	0.28	1.11	2.50	4.45	6.95

When the belt begins to run, the sum of the belt tensions acting on the pulley axis should become $T_1+T_2-2\frac{\gamma}{g}v^2$. The part of the tension weight W' removed during the test was nearly equal to the value given in the above table.

The results of the test with the leather belt in Fig. 10 show that in the case of constant peripheral force the sum of tensions decreased rapidly by $2\frac{\gamma}{g}v^2$ and then increased gradually.

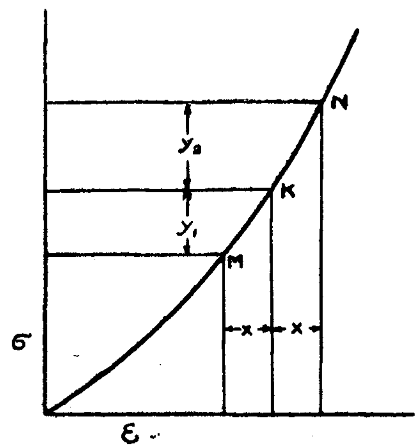
The tensions acting on the pulley axis increase with the peripheral force and decrease on its return path showing a hysterisis, as shown in Figs. 13 to 32. The explanation of this phenomenon may be as follows:

1. The belt tension being affected by time, it approaches asymptotically a new state of equilibrium.

2. The slip work done by the belt is converted into heat, resulting in a drop in humidity and this drop causes leather belting to contract.

3. In leather belt, stress increases faster than strain. In Fig. C, if K be a point representing the initial tension and M and N points representing tensions on the slack and the tight sides, as the distance between pulleys is kept constant during the test, the horizontal distances x are equal while the vertical distance y_1 is greater than that y_2. Hence when y_1+y_2 increases with P the sum of tensions must increase.

Fig. C.



Balata belt contracts, unlike the leather belt, when humidity increases and in it ε increases faster than σ. The tensions acting on the pulley axis decrease as P increases, as is seen in Figs. 33 to 37. Under a constant P the tensions decrease during the transmission because of a drop in humidity, as shown in Figs. 56 and 57.

V. CONCLUSIONS.

1. In leather belt the stress σ increases faster than the strain ε, while in cotton belt ε is either proportional to or increases faster than σ.

2. In leather belt the slip s diminishes within the range of the peripheral speed experimented with when the initial tension, the number of revolutions and the pulley diameter are made greater. The sliding slip s'' steals when the force transmitted reaches a limit value P_1 and then increases gradually. In cotton belt the effect of initial tension and peripheral speed

on s is comparatively small and s'' occurs abruptly at P_1 and the belt runs off the pulley at once.

3. Variation of belt tension during the transmission is affected by the heat of friction, the leather belt having a tendency to contract and the cotton belt to elongate.

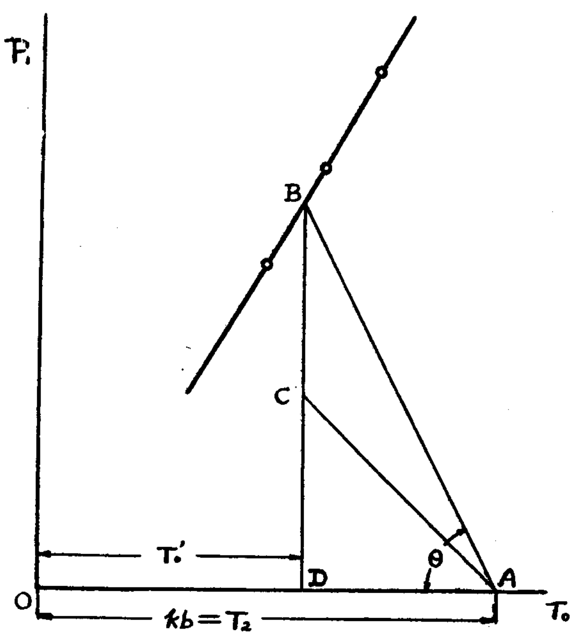
4. As to the calculation of the belt required to transmit a given force, the force should be within the limit value P_1 at which the sliding occurs. P_1 is a function of the initial tension T_0 of the belt and the peripheral speed v, as is seen in Fig. 58. Under a given initial tension, P_1 increases with v and at a given running speed a greater T_0 results in a greater P_1.

The maximum allowable initial tension must depend on the stress allowable for a belt. The allowable stress varies according to the pulley diameter d and the peripheral speed v. For greater values of d and v the allowable stress may be greater. In order to elucidate these relations, experiments on the life of belting will be necessary.

5. When the allowable stress is chosen properly for a given pulley diameter d and a given peripheral speed v, the following method may be employed to determine the initial tension of the belt in order to make P_1 maximum :—

From the relation between P_1 and v found in the present tests shown in Fig. 58, we obtain the relation of P_1 to T_0 for various peripheral speeds v's as will be seen in Fig. 59. In the $P_1 - T_0$ diagram for the

Fig. D.



above value of v, Fig. D, take OA on the horizontal axis equal to $T_2=kb$, where T_2 is the tight side tension, b the breadth of the belt and k the allowable stress. Then draw AB making $\theta=\tan^{-1}2$ and from the intersection B drop a perpendicular BD. Then OD gives the required initial tension T_0', because

$$T_0' + \frac{1}{2} P_1 = T_2 = kb$$

In conclusion the writers wish to express their cordial thanks to Mr. R. Shirai and Mr. T. Mashimo who have zealously assisted them in completing the present experiments.

Fig. 1. Elevation of the Plant.

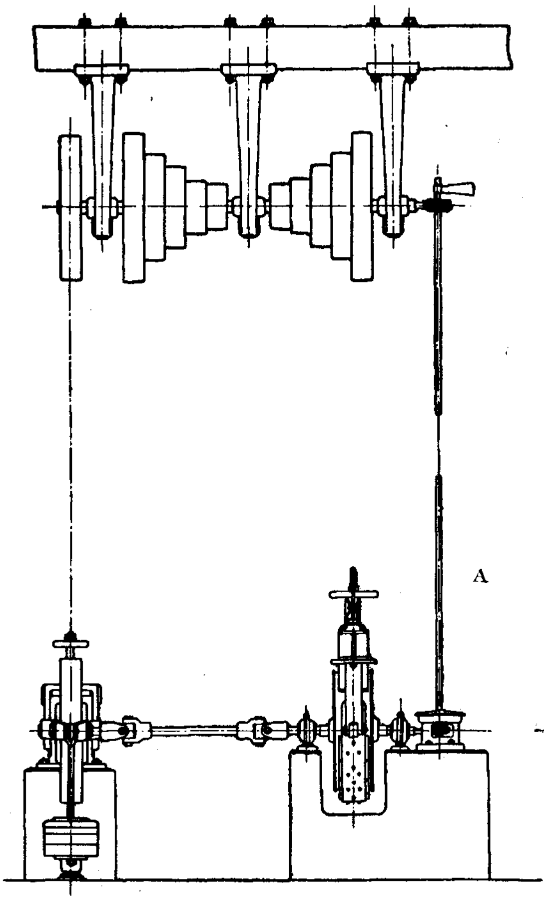


Fig. 2. Side View of the Plant.

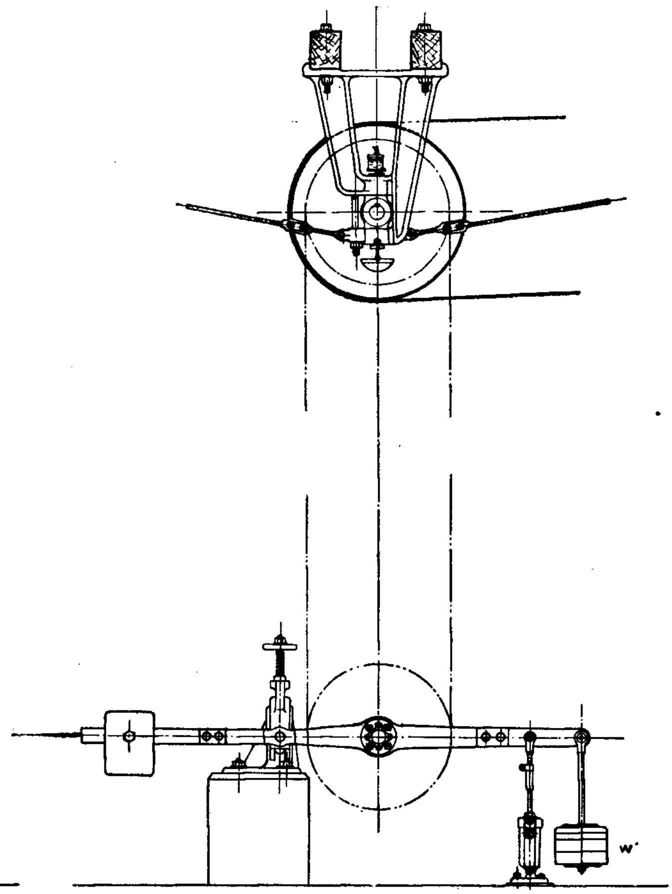


Fig. 3. Plan of the Plant.

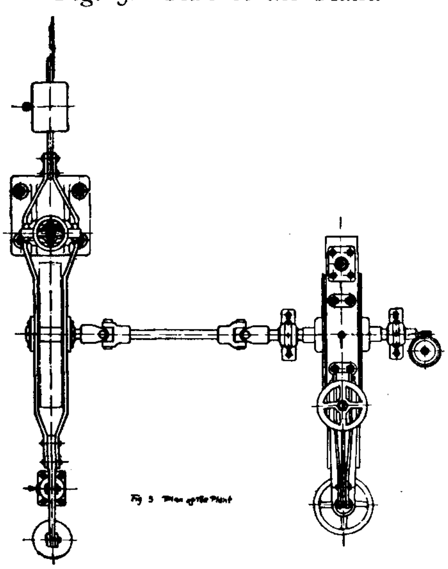


Fig. 4. Prony Brake.

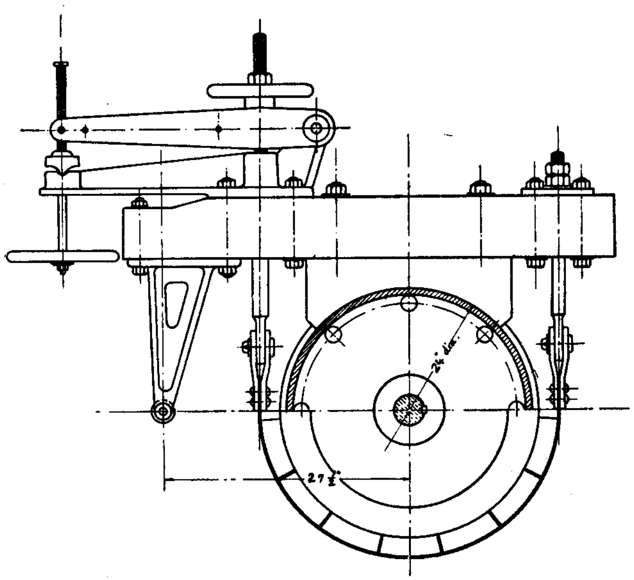


Fig. 5. Matsumura's Slip Meter.

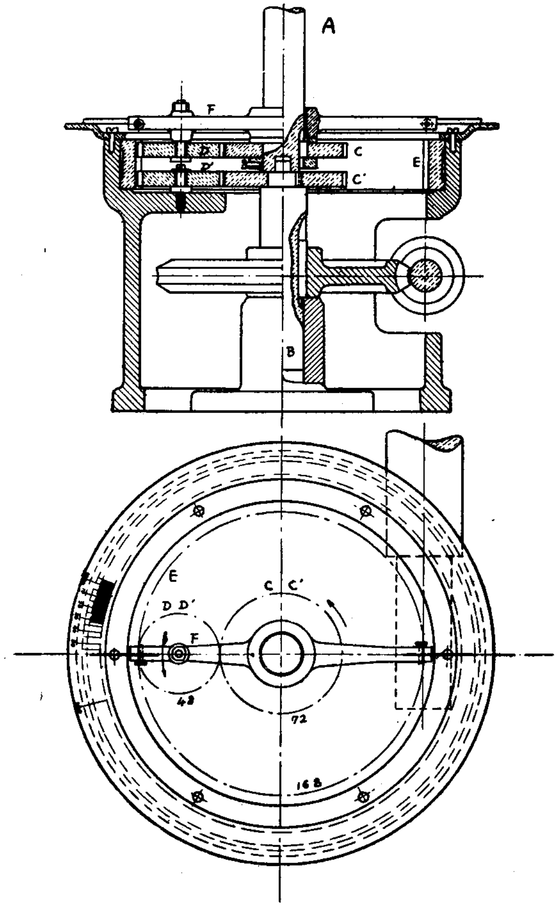
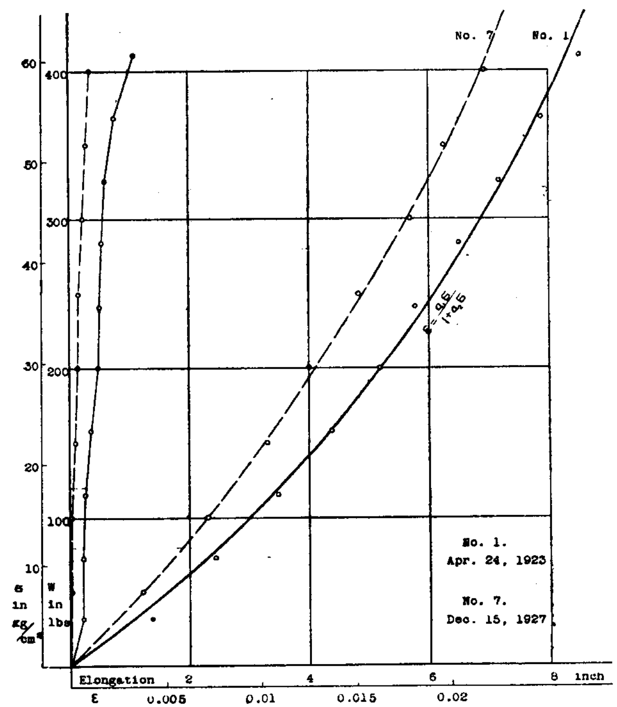


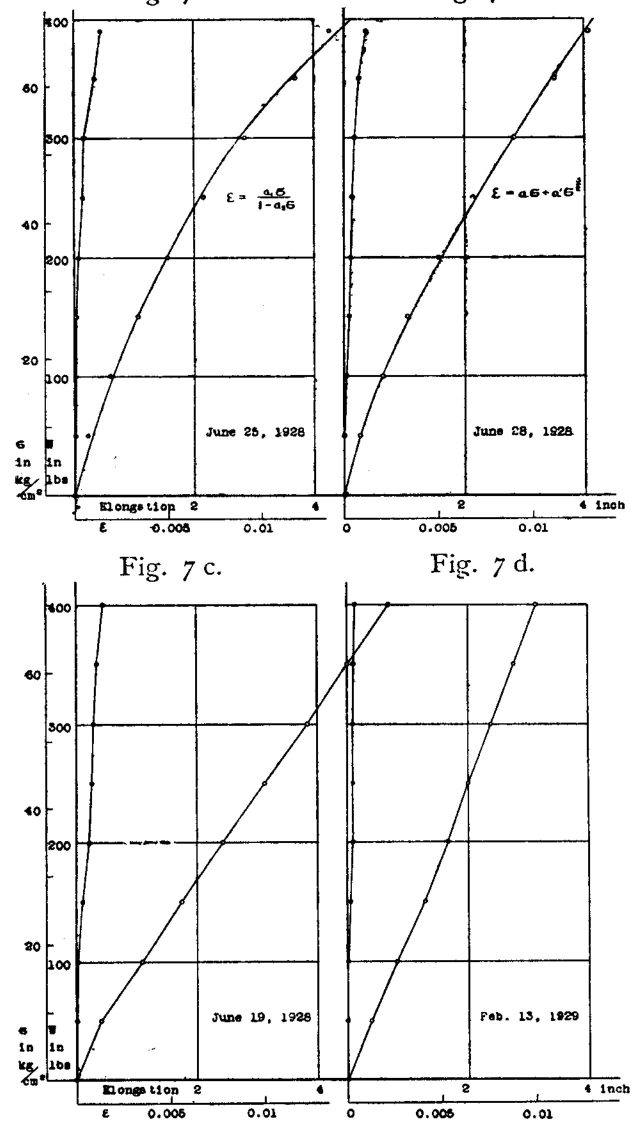
Fig. 6. Tension Test of Leather Belt.



Tension Test of Balata Belt.

Fig. 7 a. Fig. 7 b.

Fig. 7 c. Fig. 7 d.



Time Effect on the Elongation of Leather Belt.

Fig. 8 a.

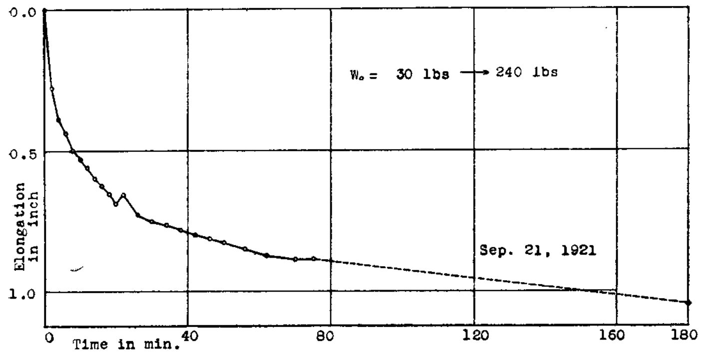


Fig. 8 b.

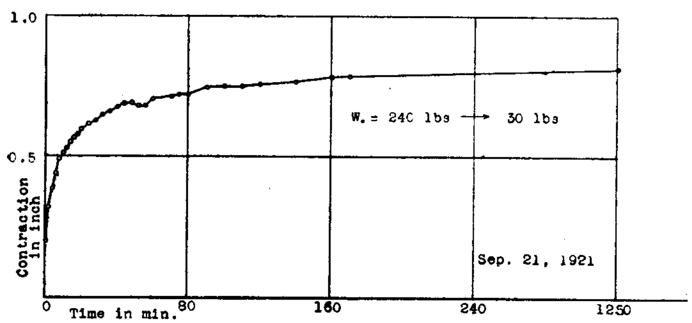
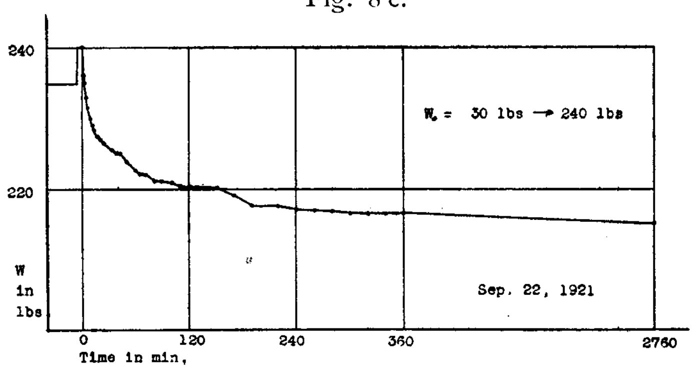


Fig. 8 c.



Time Effect on the Elongation of Cotton Belt.

Fig. 9 a.

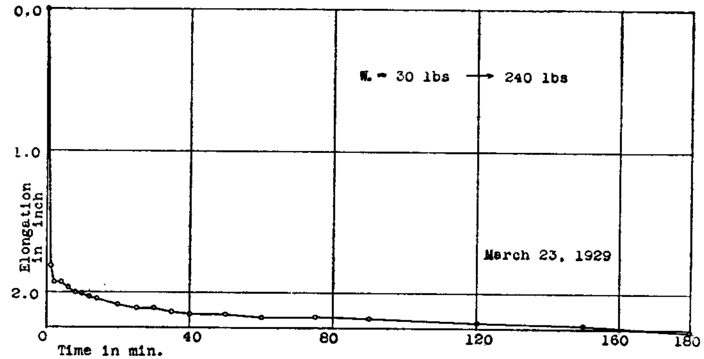


Fig. 9 b.

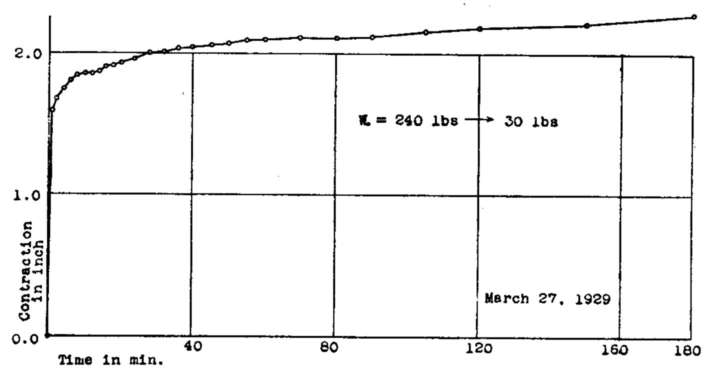


Fig. 9 c.

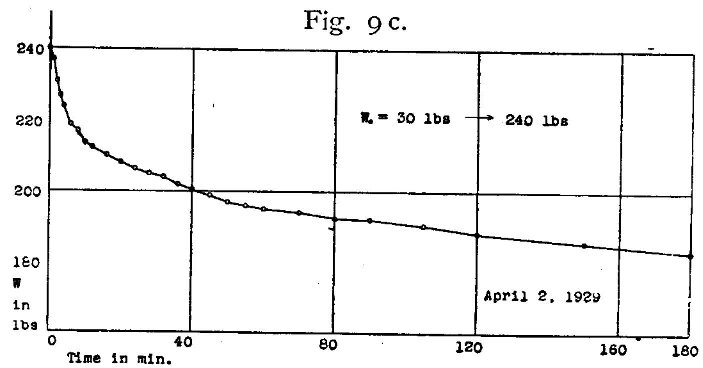
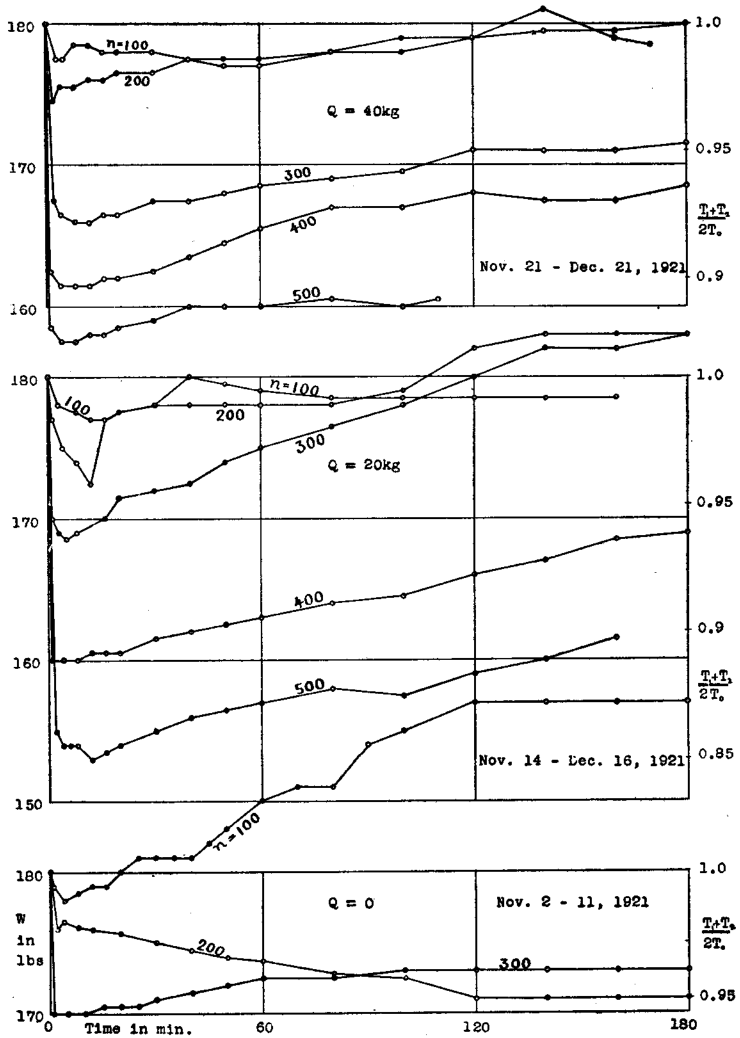


Fig. 10. Variation of Tension for d=30″.



Variation of Belt Tension.

Fig. 11 a.

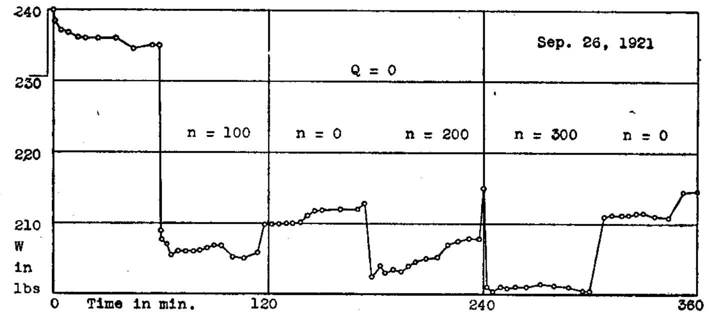


Fig. 11 b.

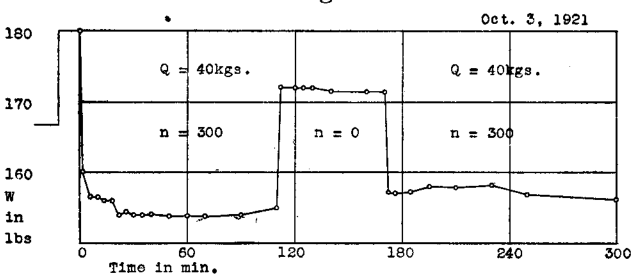


Fig. 11 c.

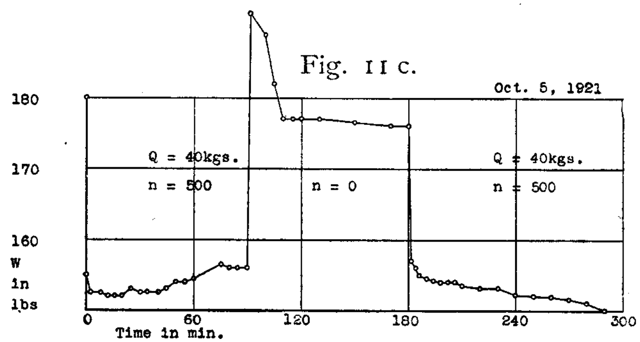


Fig. 12 a. Elastic Hysterisis of Leather Belt.

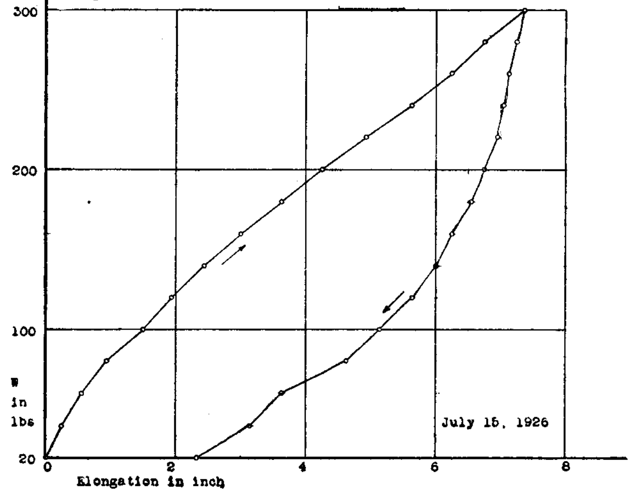


Fig. 12 b. Elastic Hysterisis of Balata Belt.

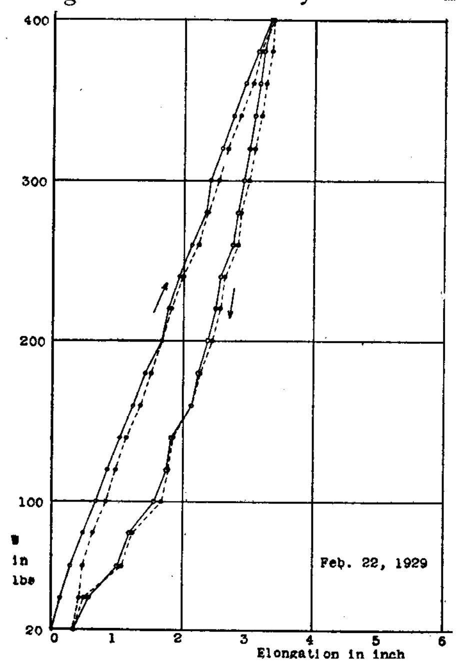


Fig. 13. Slip Test of Leather Belt for
d=30″ and n=100.

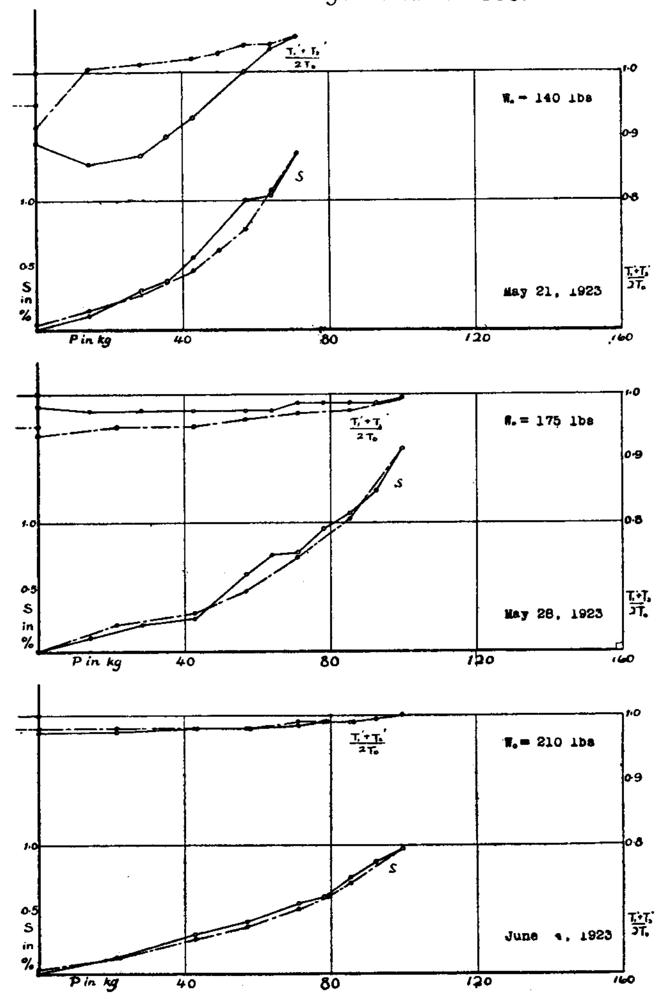


Fig. 14. Slip Test of Leather Belt for d=30″ and n=200.

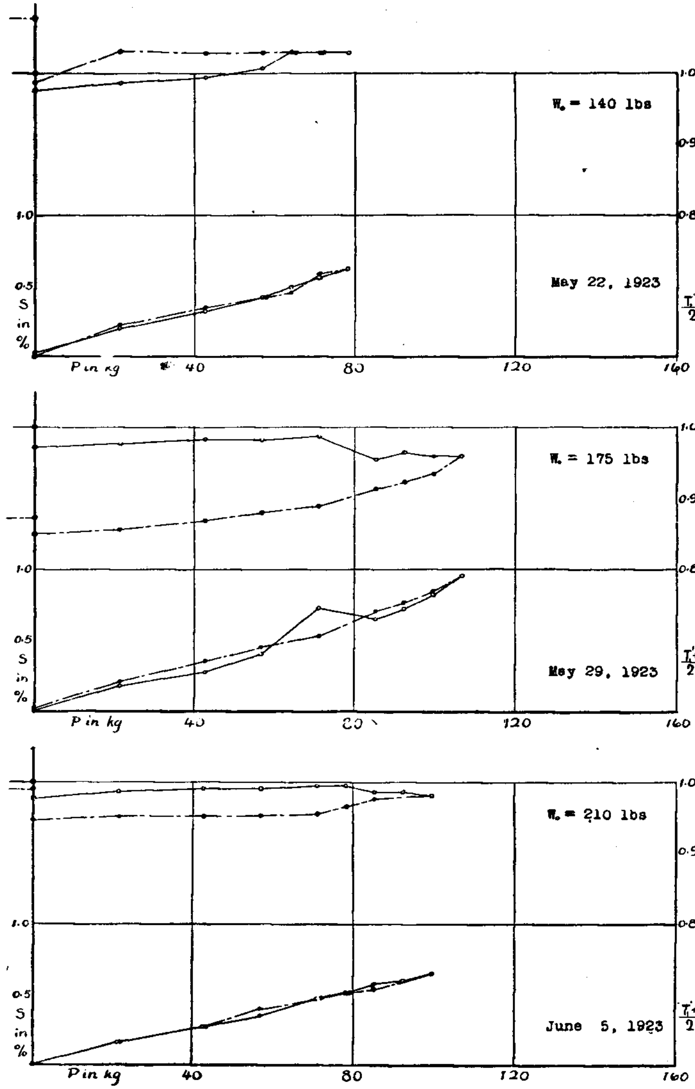


Fig. 15. Slip Test of Leather Belt for d=30″ and n=300.

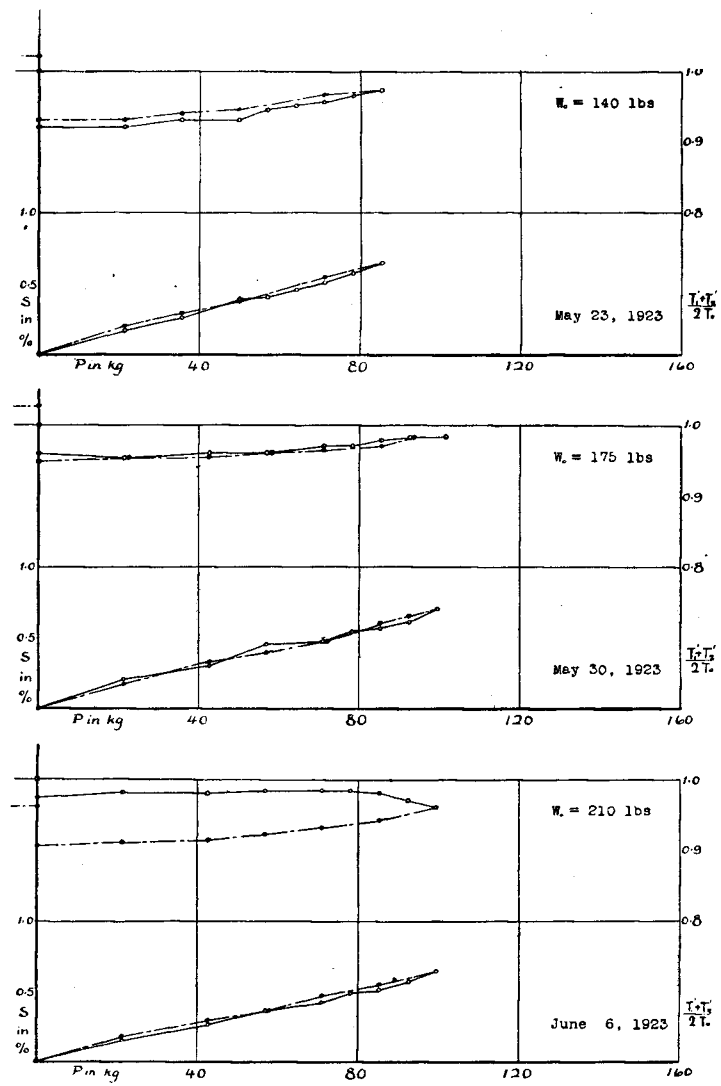


Fig. 16. Slip Test of Leather Belt for d=30″ and n=400.

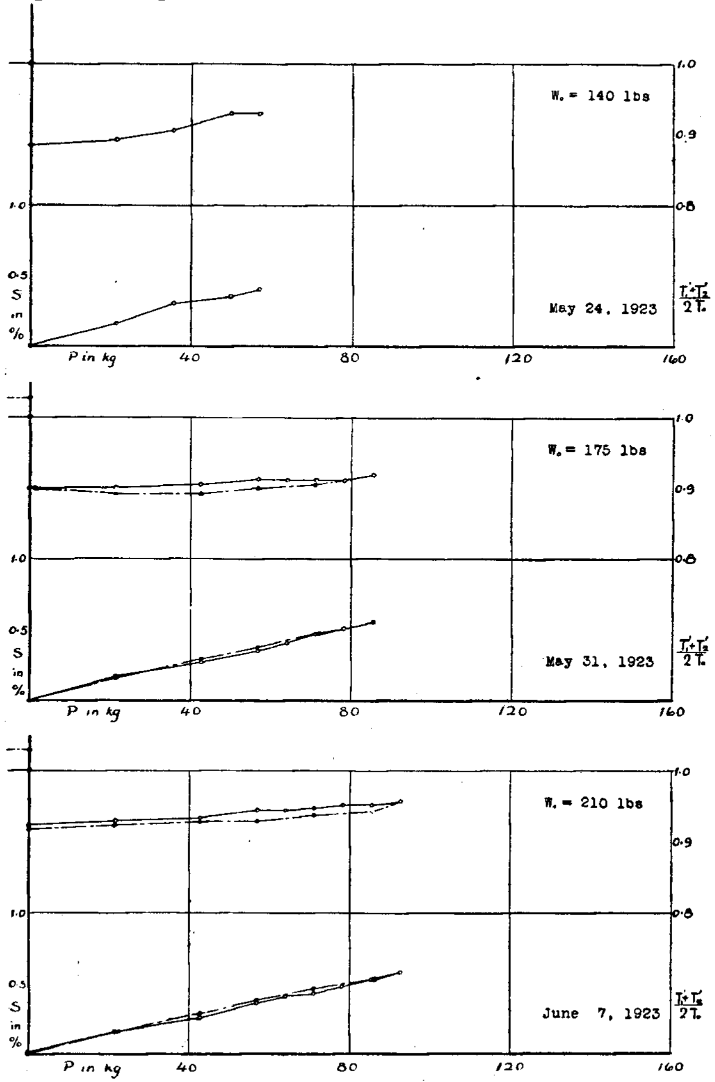


Fig. 17. Silp Test of Leather Belt for d=30″ and n=500.

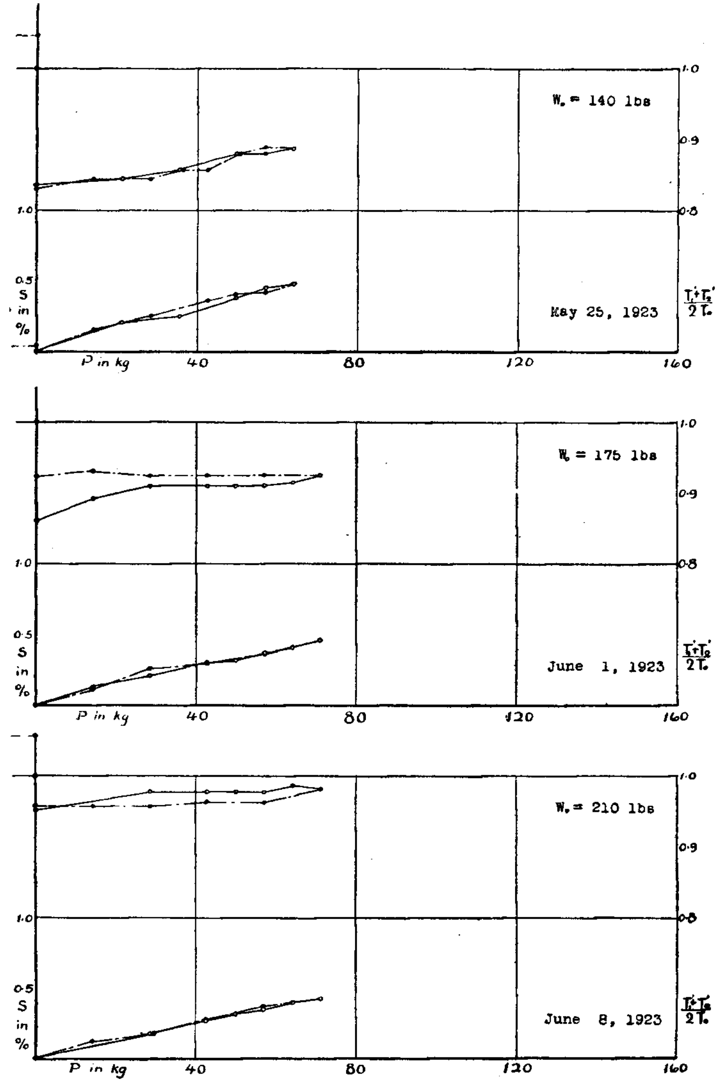


Fig. 18. Slip Test of Leather Belt for d=24″ and n=100.

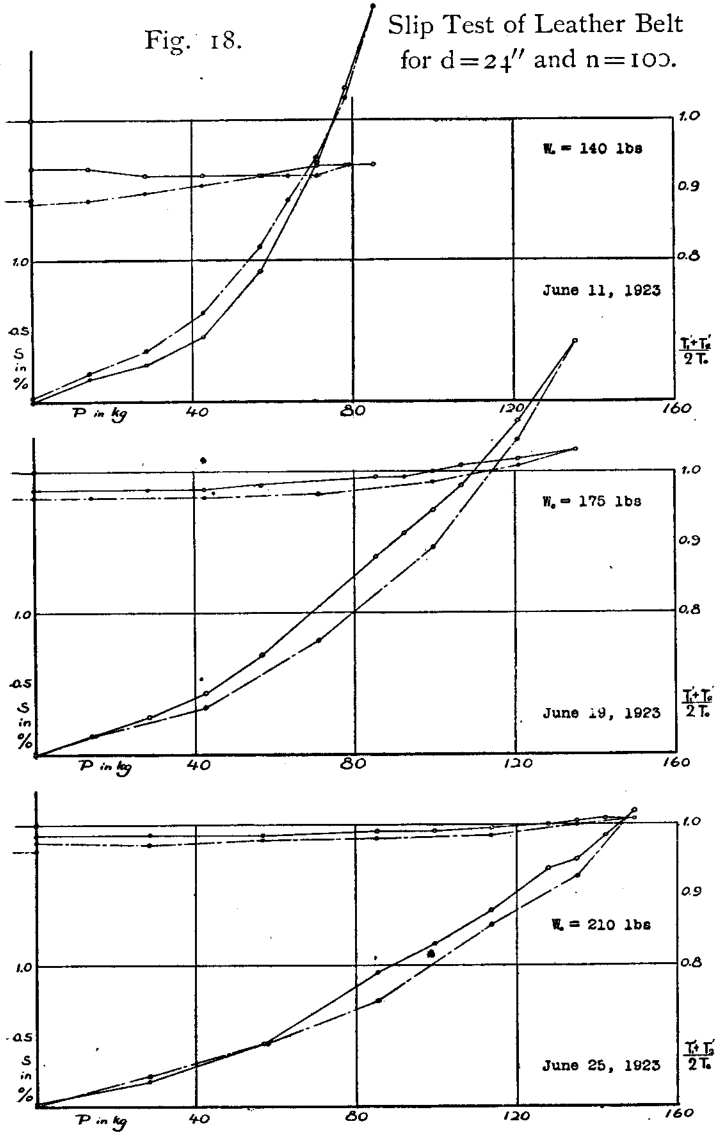


Fig. 19. Slip Test of Leather Belt for d=24″ and n=200.

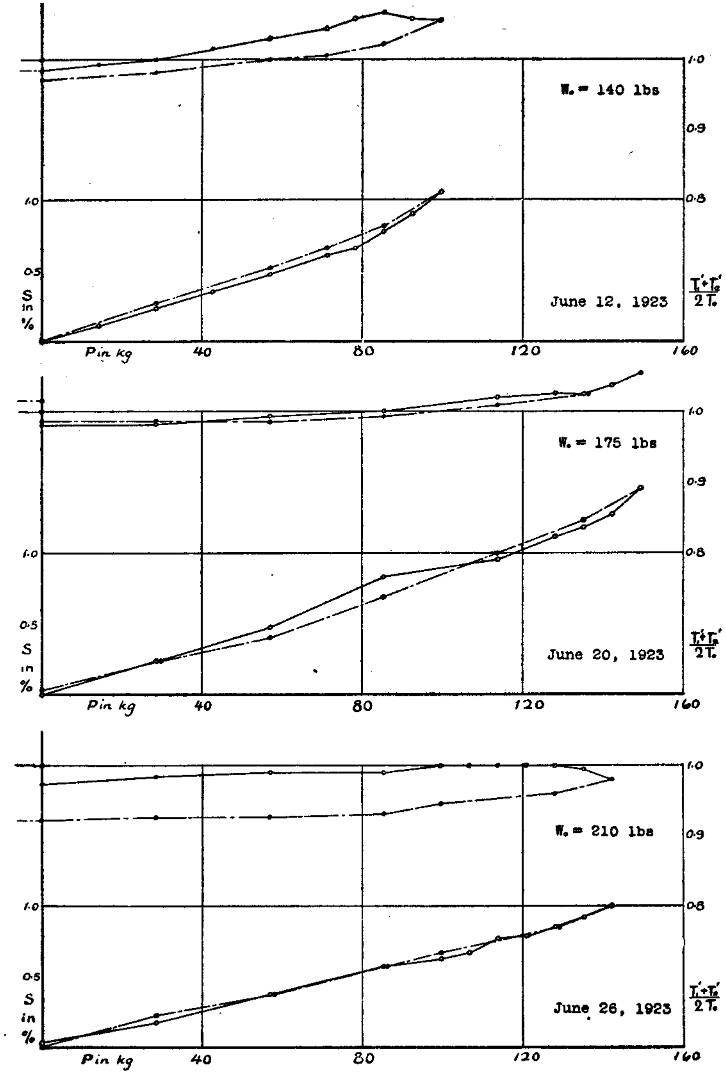


Fig. 20. Slip Test of Leather Belt for d=24″ and n=300.

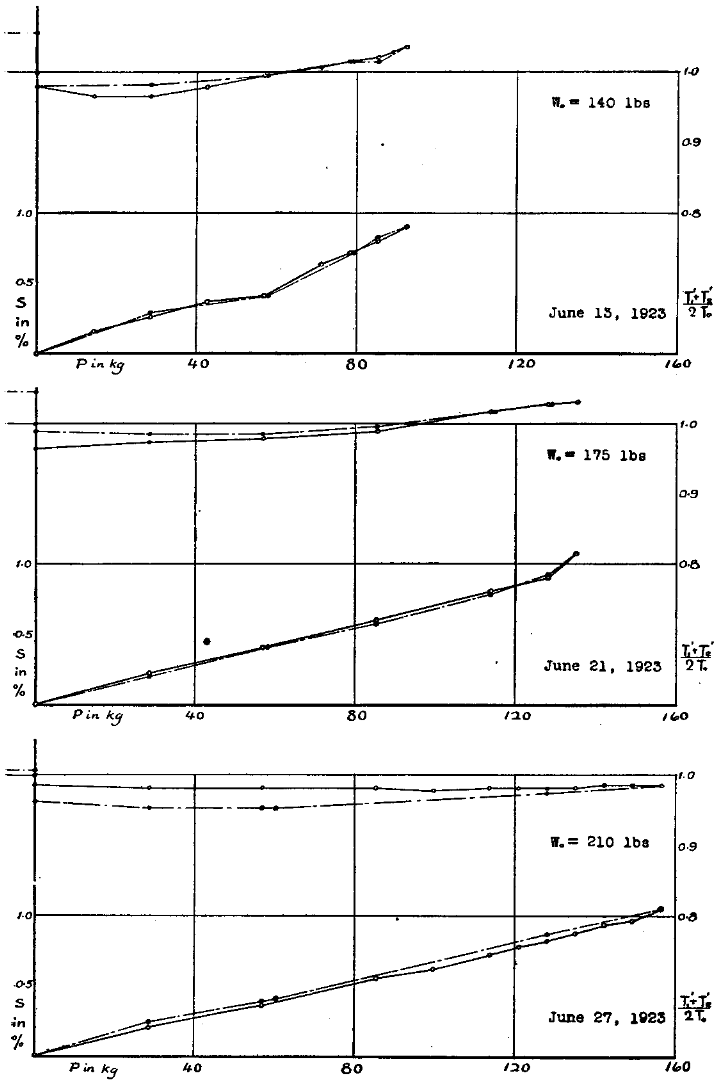


Fig. 21. Slip Test of Leather Belt for d=24″ and n=400.

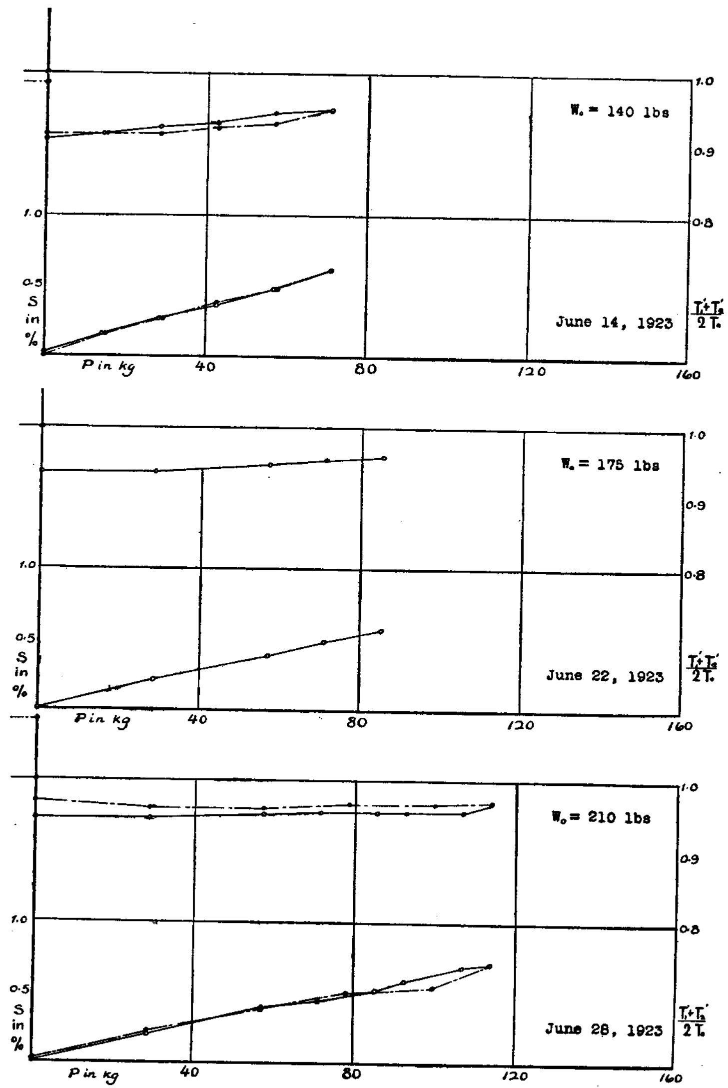


Fig. 22. Slip Test of Leather Belt for d = 24″ and n = 500.

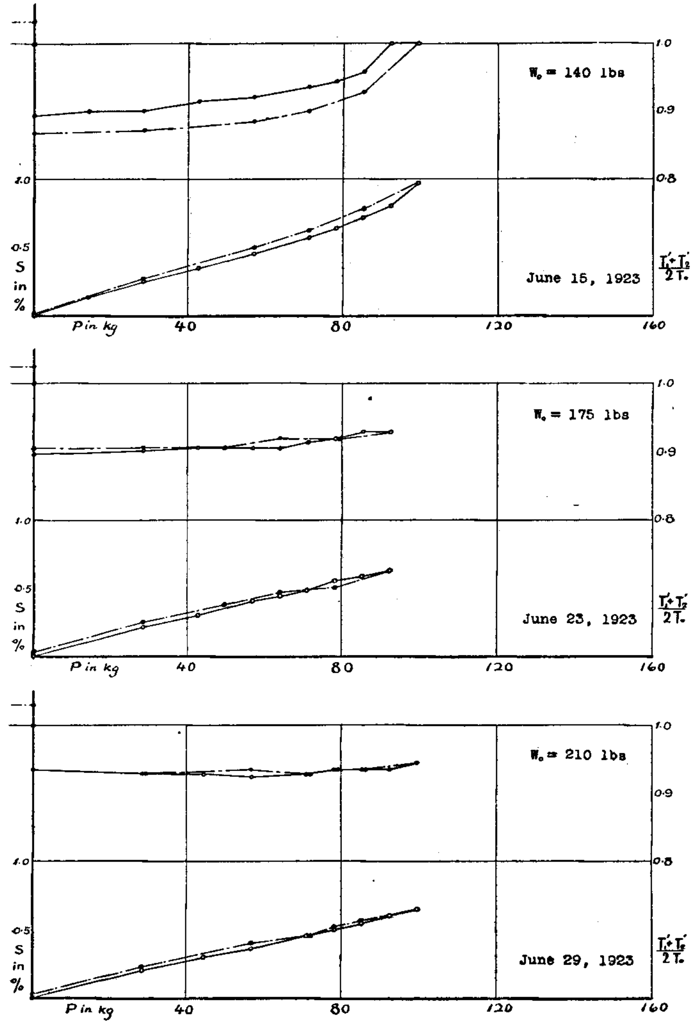


Fig. 23. Slip Test of Leather Belt for d = 19″ and n = 100.

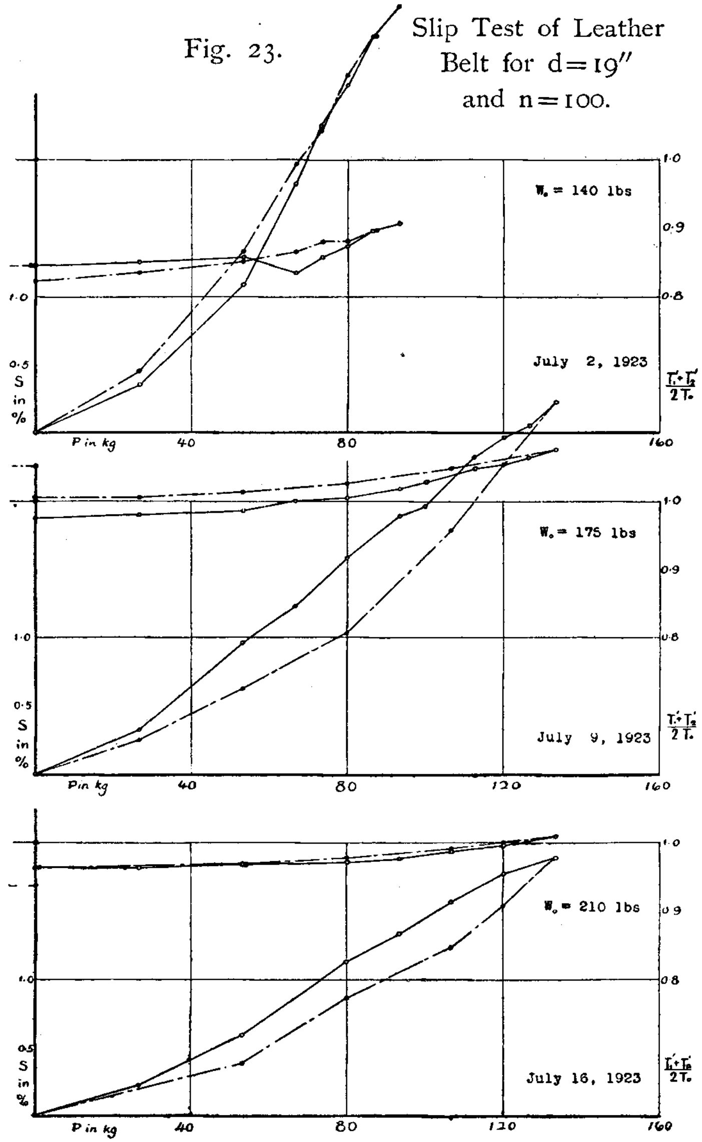


Fig. 24. Slip Test of Leather Belt for d = 19″ and n = 200.

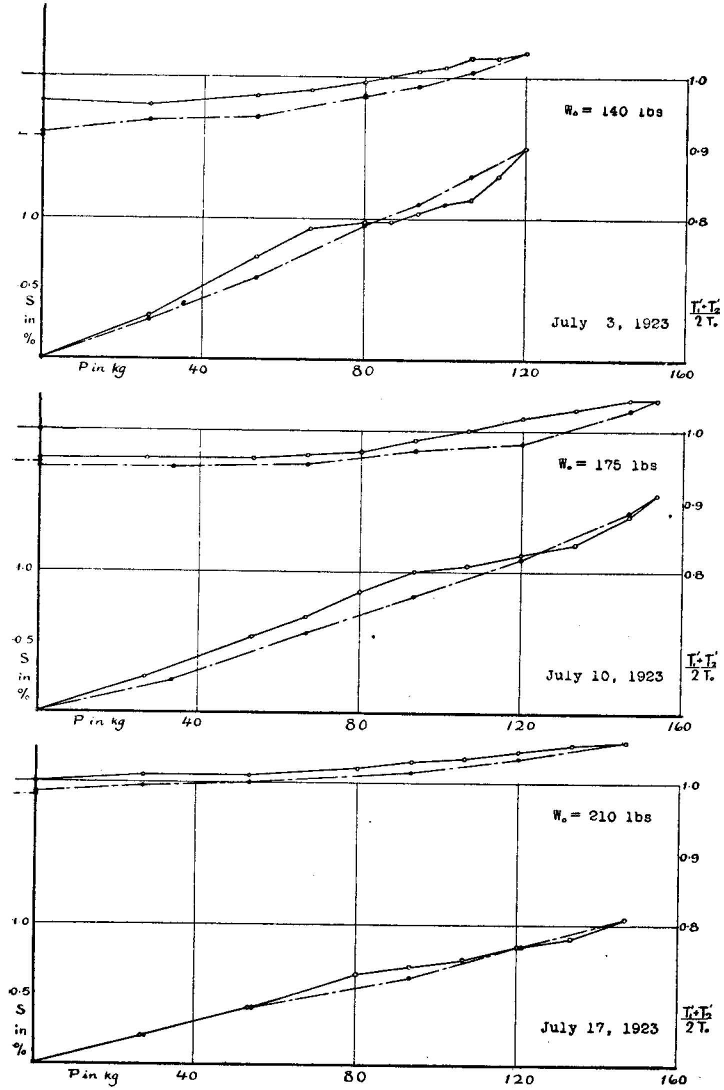


Fig. 25. Slip Test of Leather Belt for d = 19″ and n = 300.

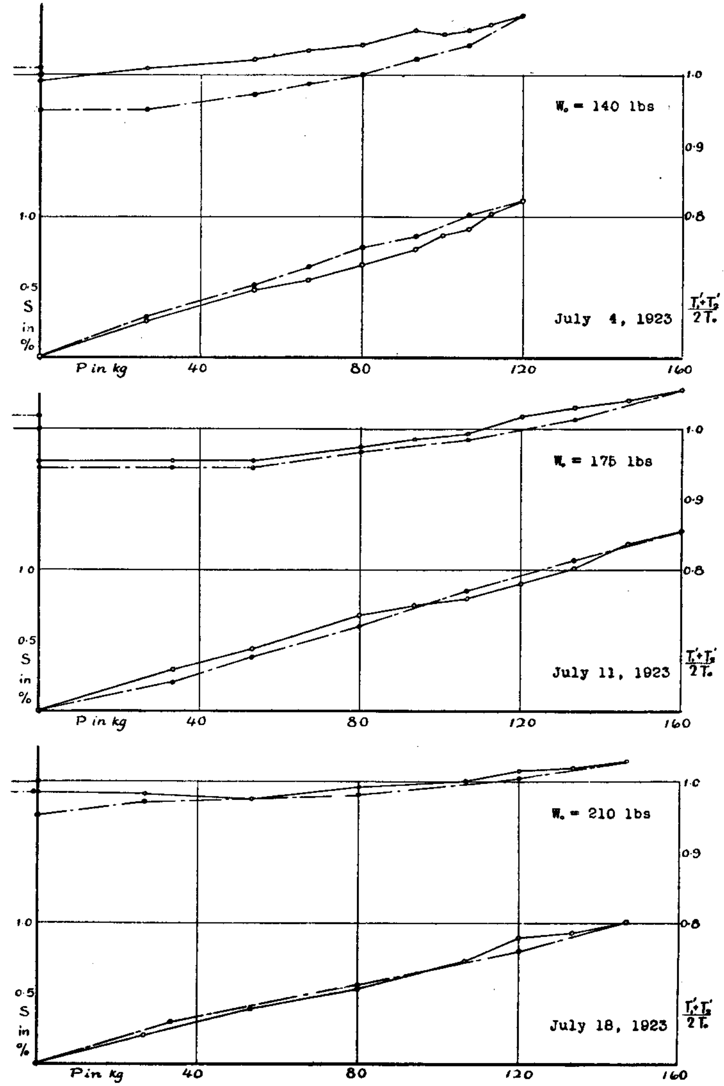


Fig. 26. Slip Test of Leather Belt for d=19″ and n=400.

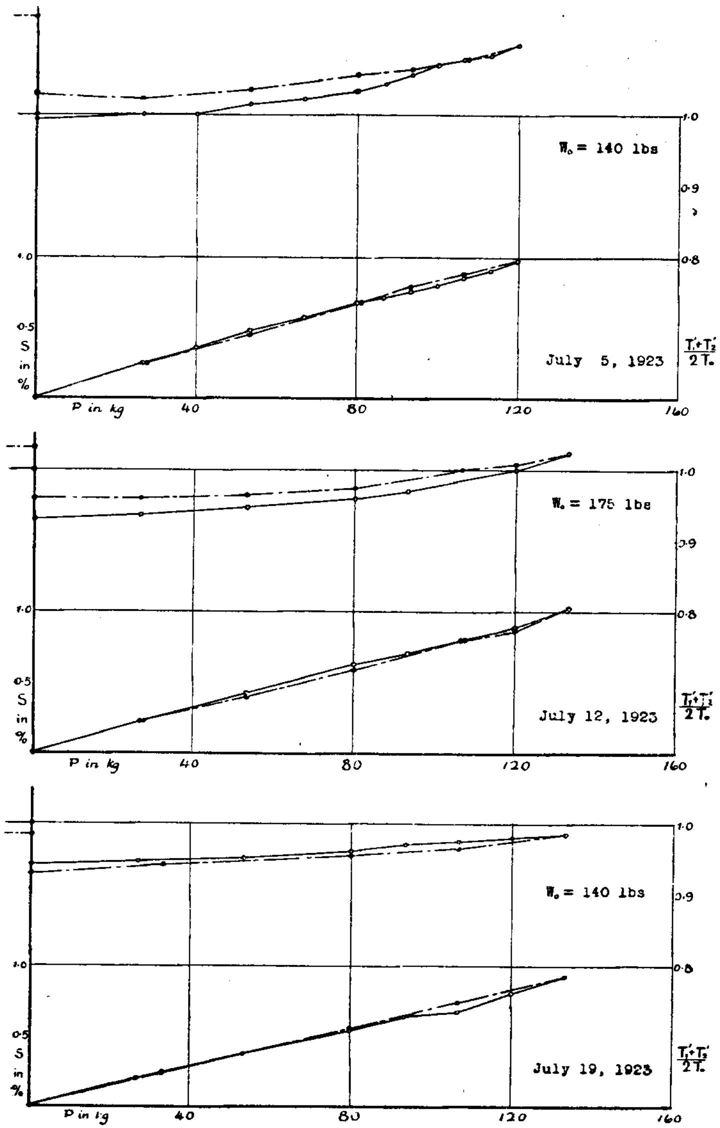


Fig. 27. Slip Test of Leather Belt for d=10″ and n=500.

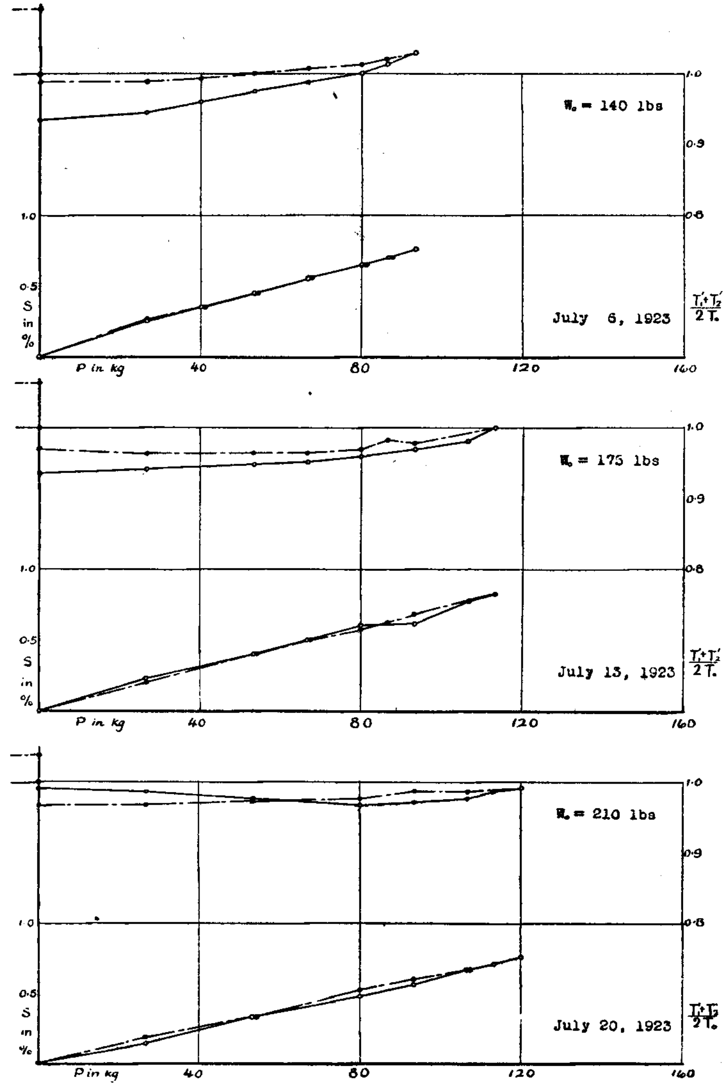


Fig. 28. Slip Test of Leather Belt for d=15″ and n=100.

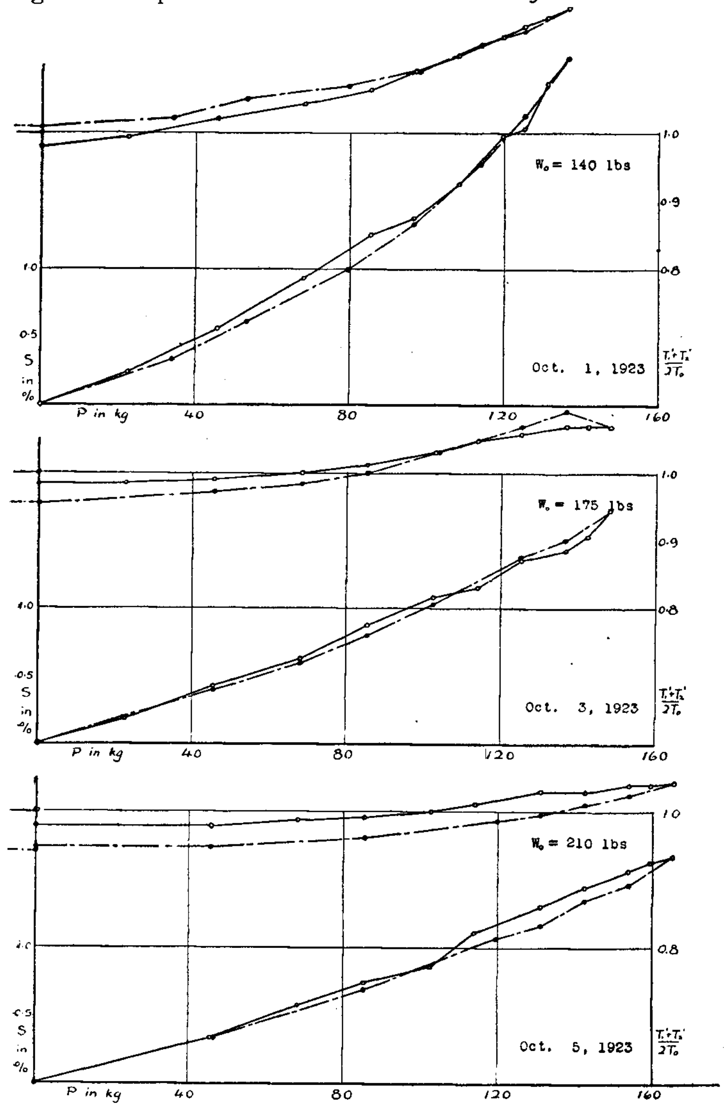


Fig. 29. Slip Test of Leather Belt for d=15″ and n=200.

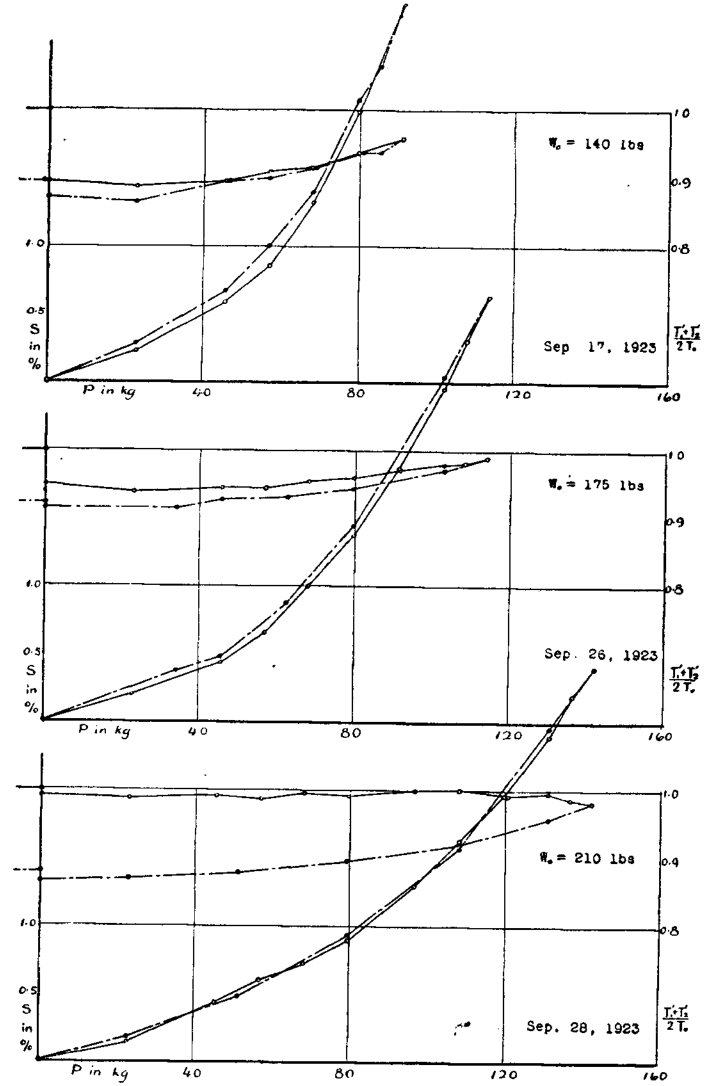


Fig. 30. Slip Test of Leather Belt for d=15″ and n=300.

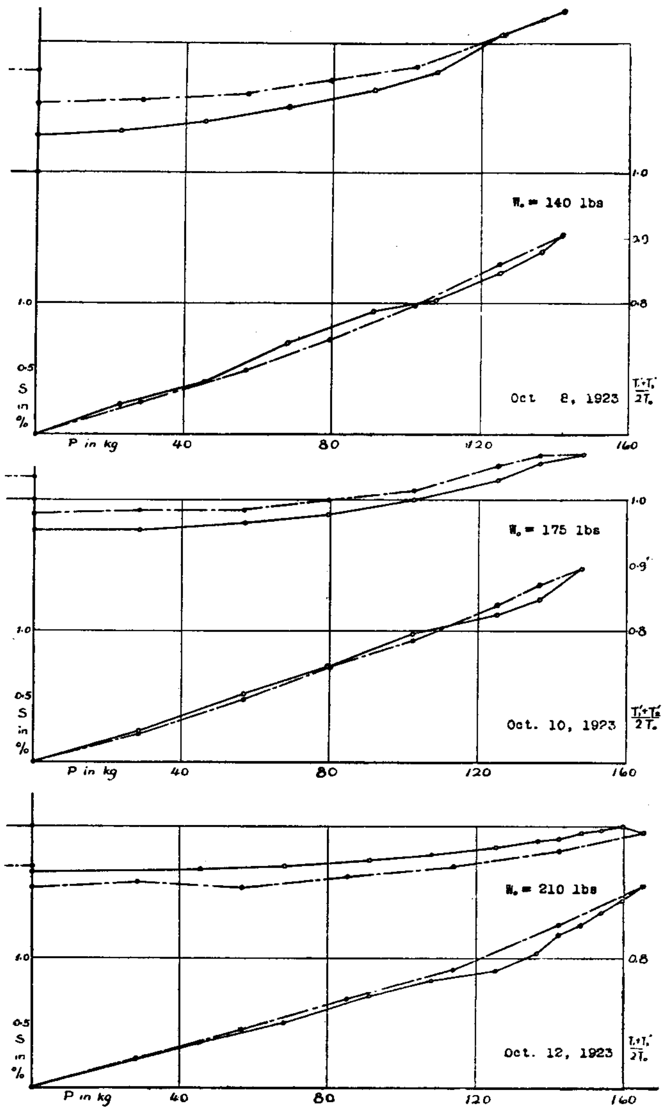


Fig. 31. Slip Test of Leather Belt for d=15″ and n=400.

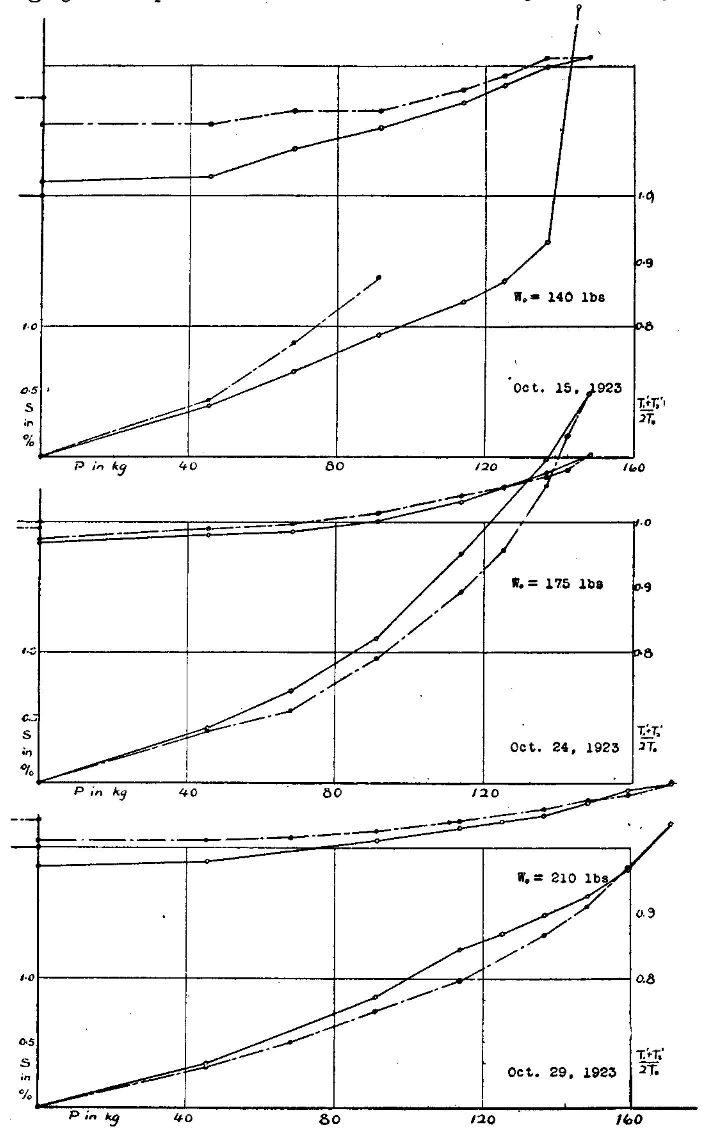


Fig. 32. Slip Test of Leather Belt for d = 15″ and n = 500.

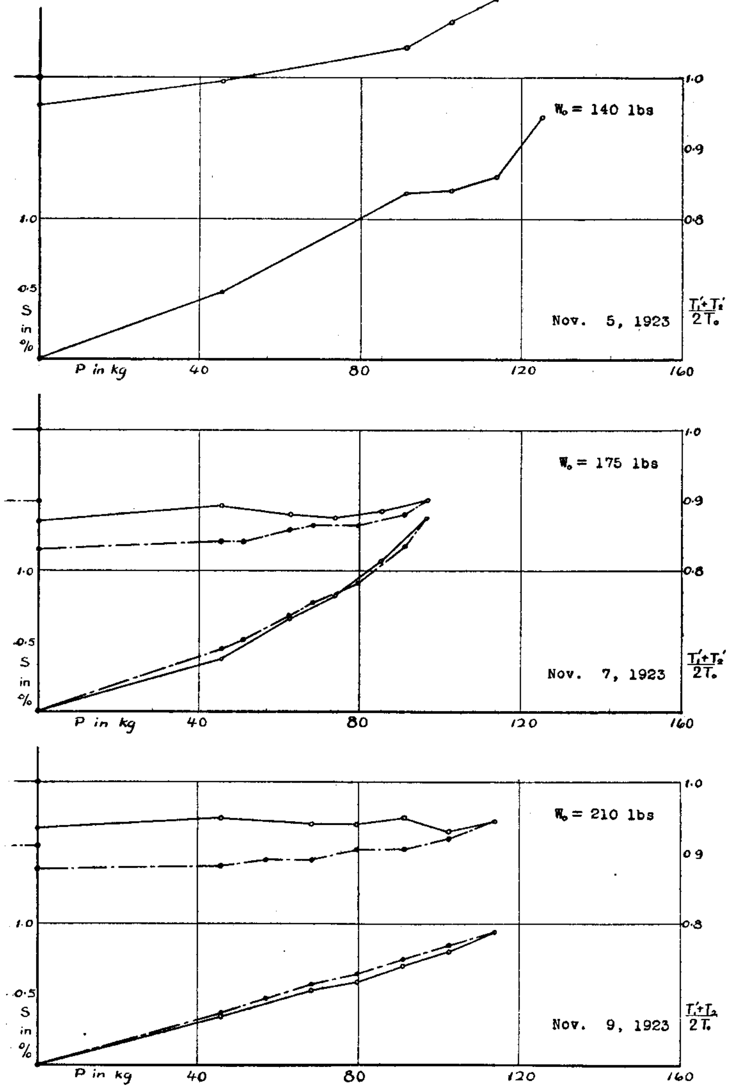


Fig. 33. Slip Test of Balata Belt for d = 30″ and n = 100.

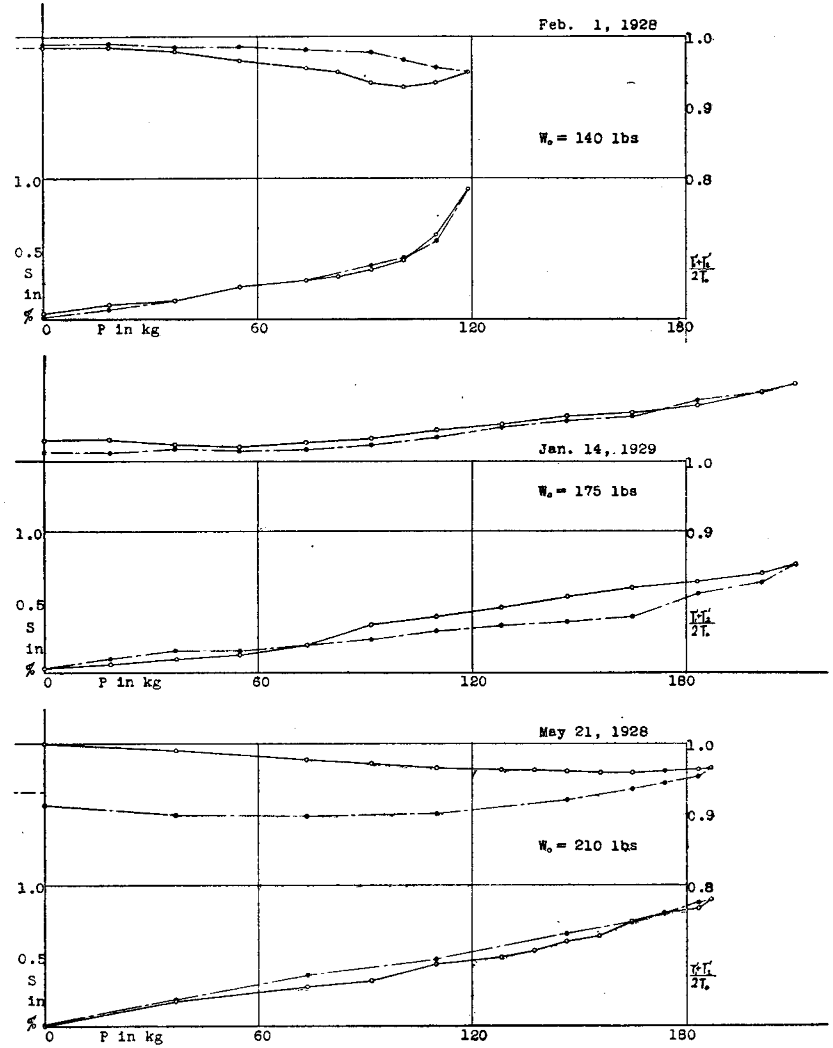


Fig. 34. Slip Test of Balata Belt for d=30″ and n=200.

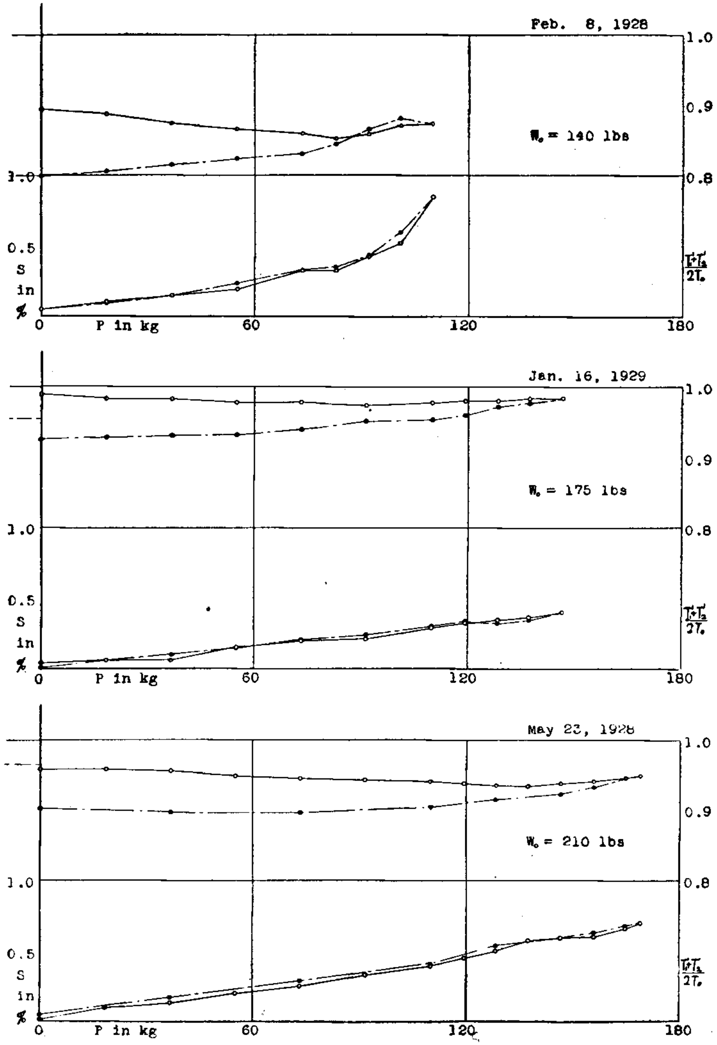


Fig. 35. Slip Test of Balata Belt for d=30″ and n=300.

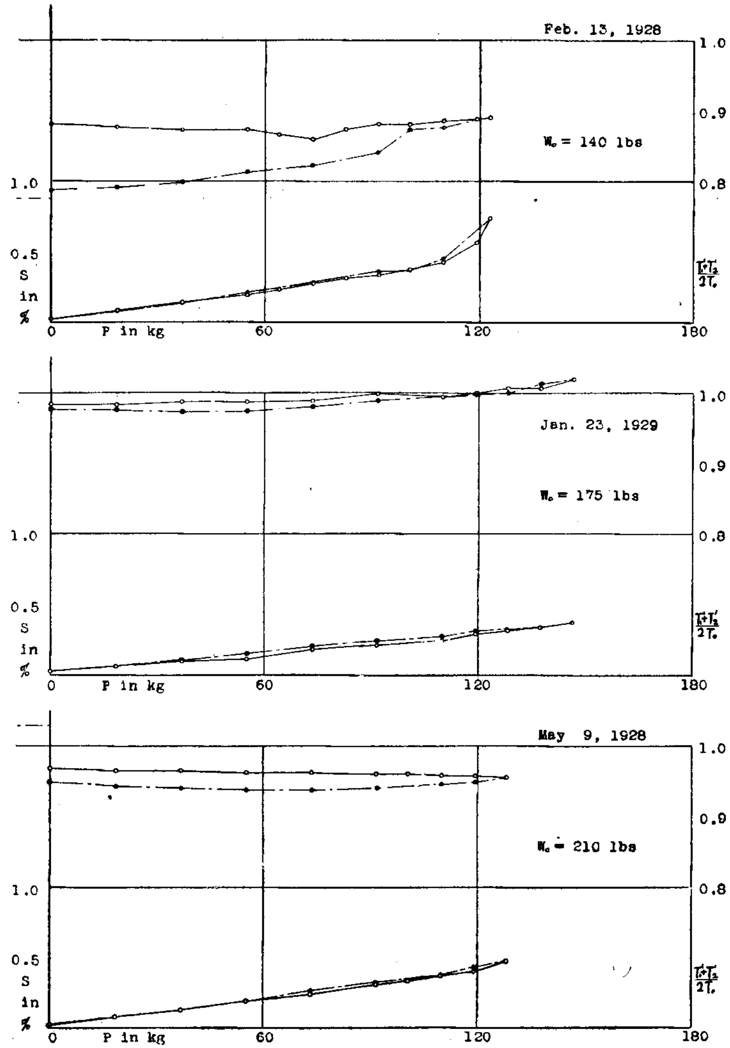


Fig. 36. Slip Test of Balata Belt for d=30″ and n=400.

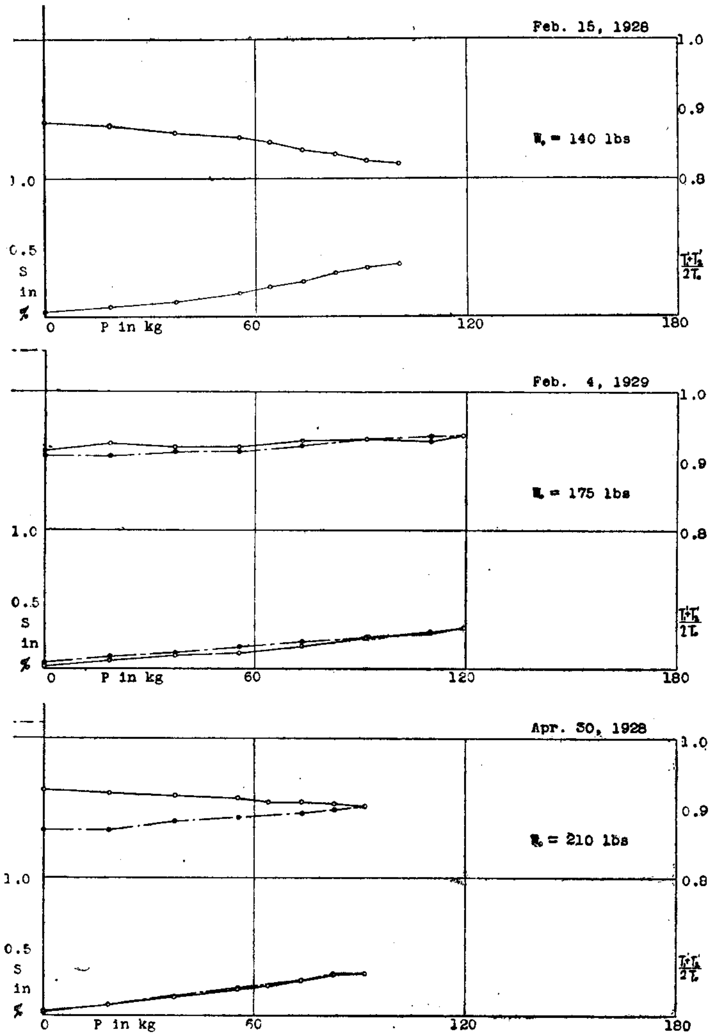


Fig. 37. Slip Test of Balata Belt for d=30″ and n=500.

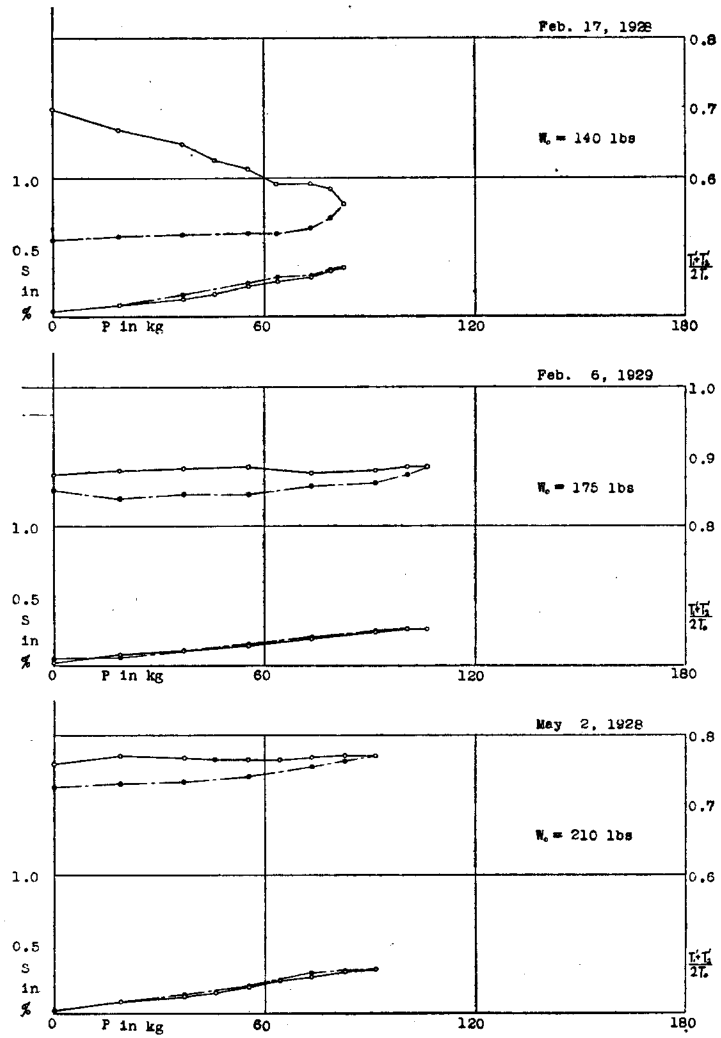


Fig. 38. Slip Test of Balata Belt for d=30″.

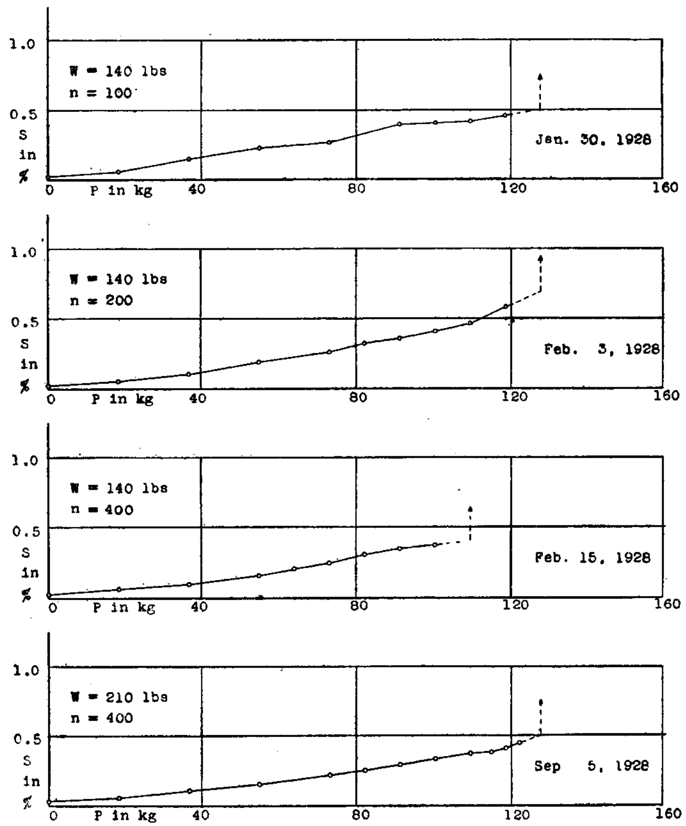


Fig. 39. Slip and Speed for d=30″.

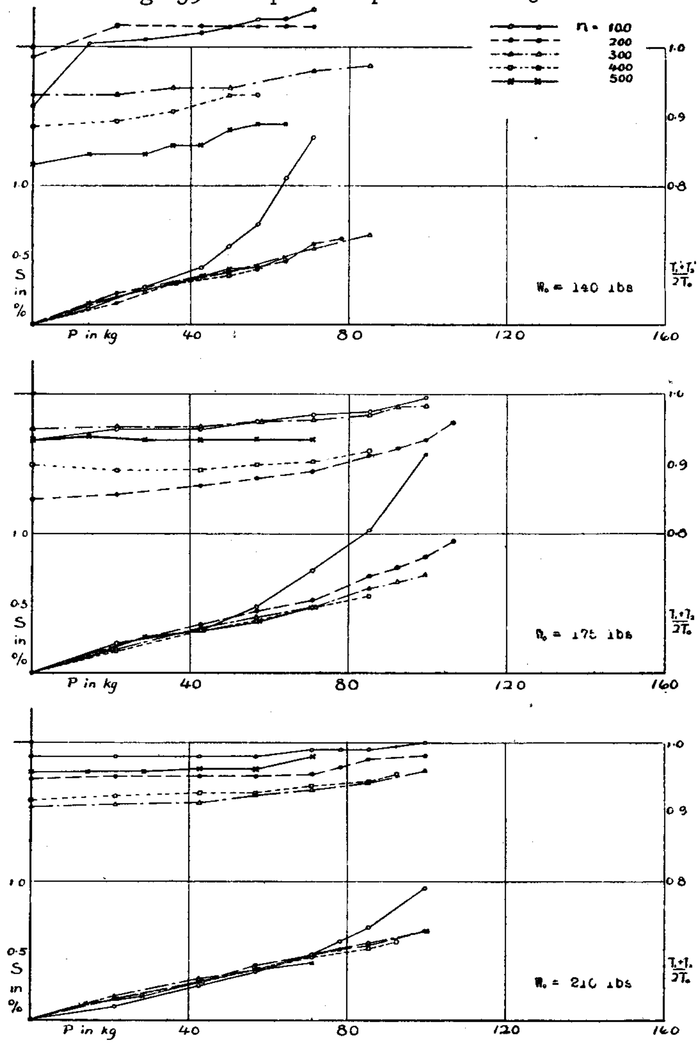


Fig. 40. Slip and Speed for d=24″.

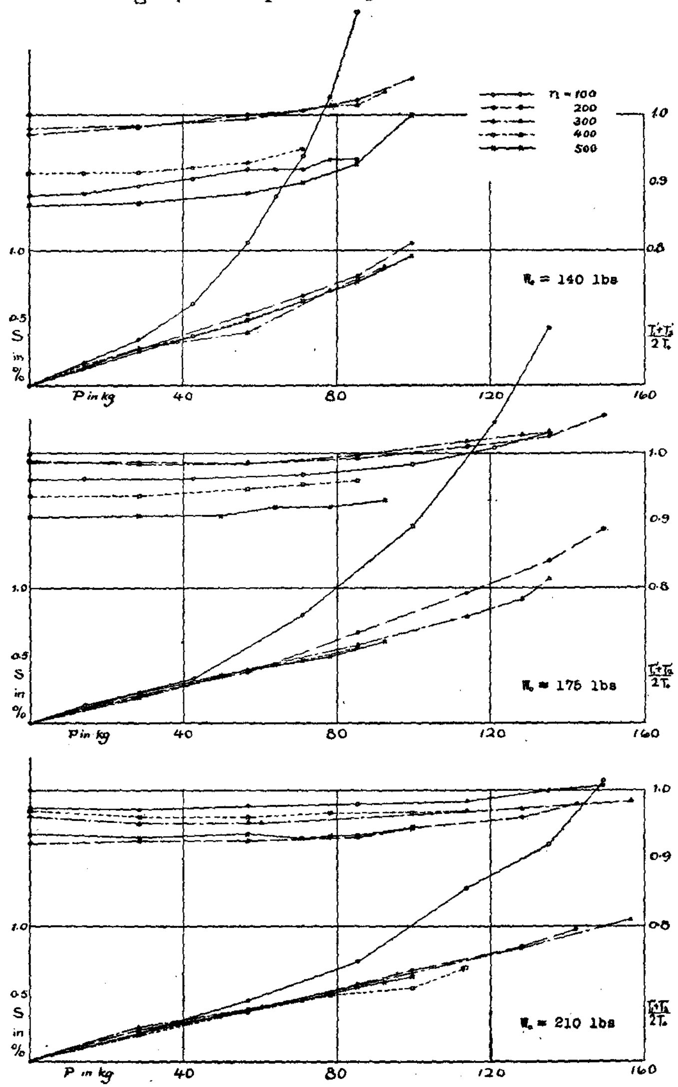


Fig. 41. Slip and Speed for d=19″.

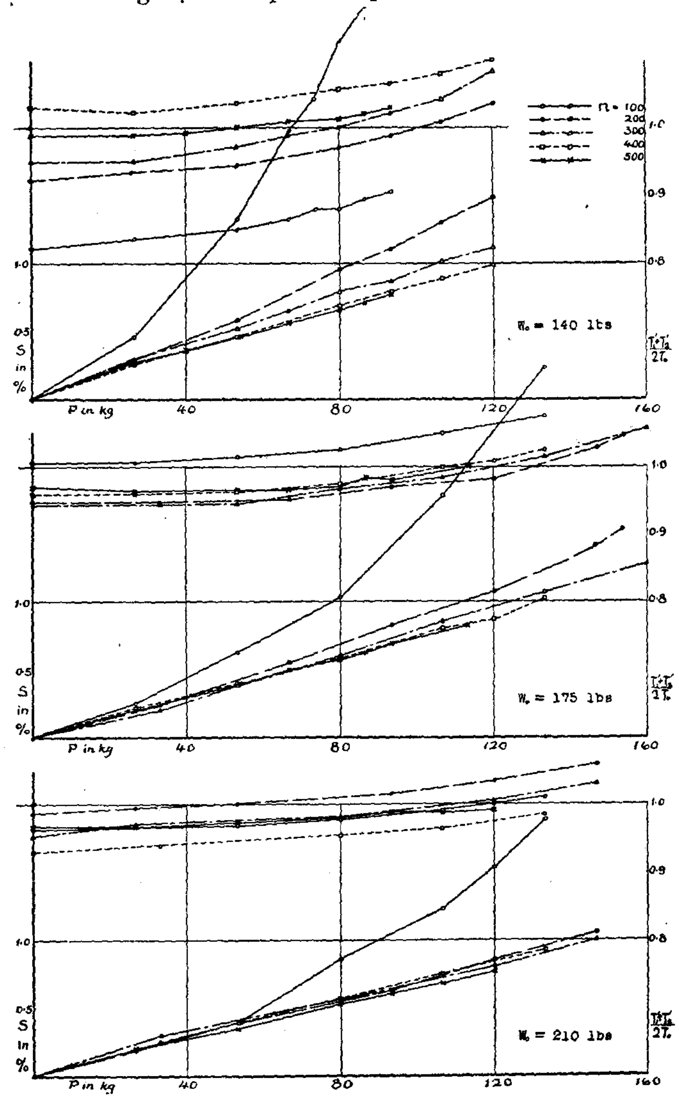


Fig. 42. Slip and Speed for d=15″.

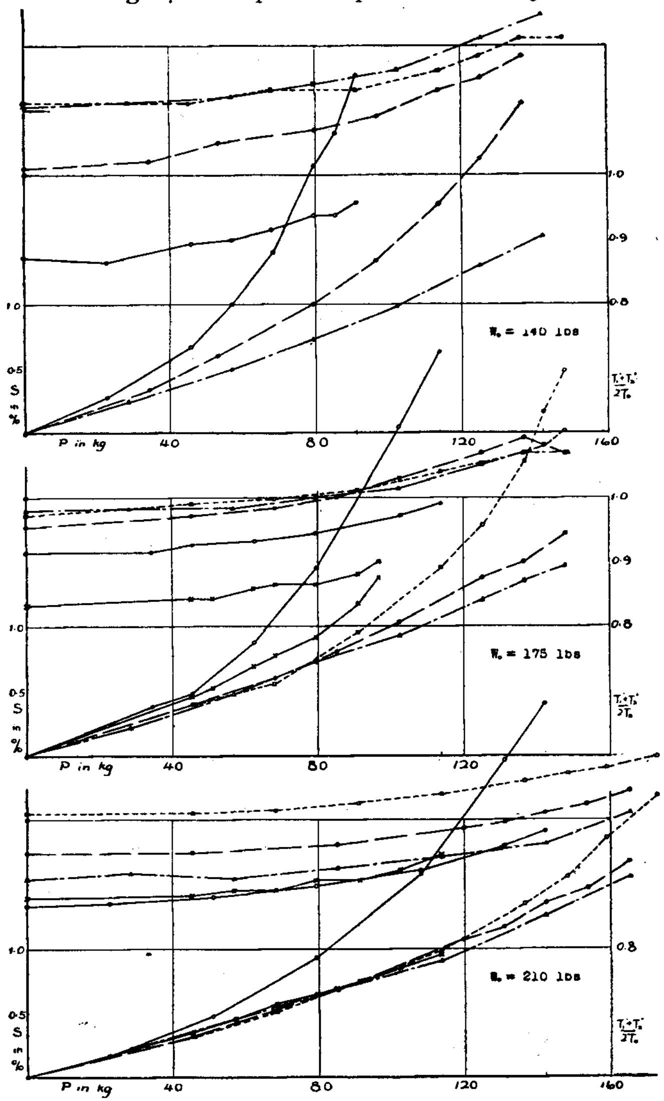


Fig. 43. Slip and Speed for d=30″.

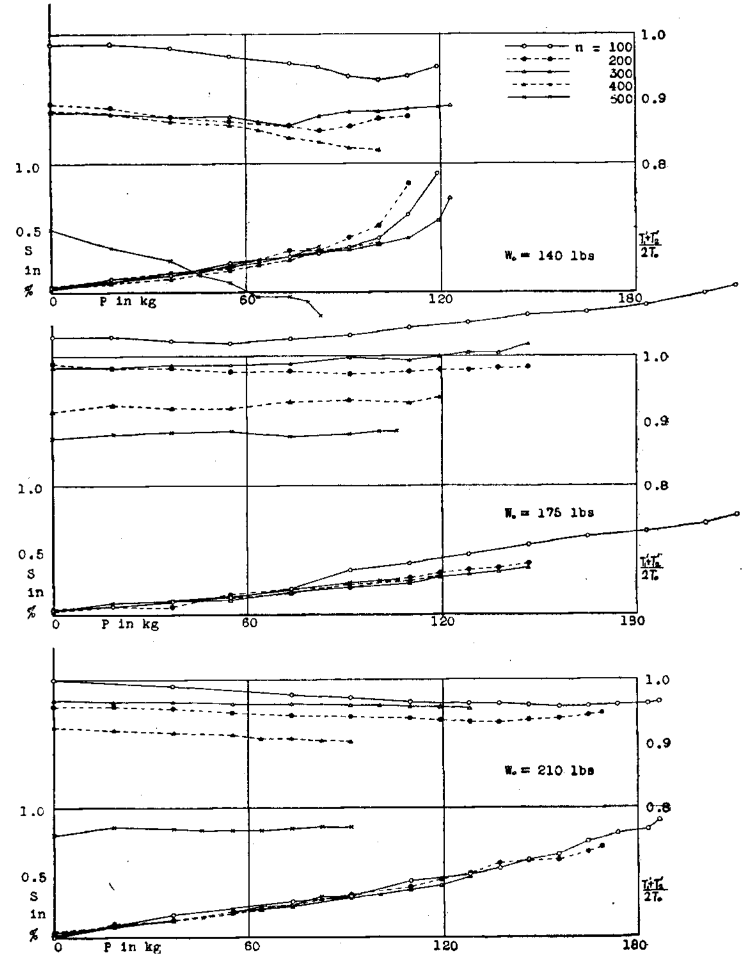


Fig. 44. Slip and Belt Speed at P=40 kg.

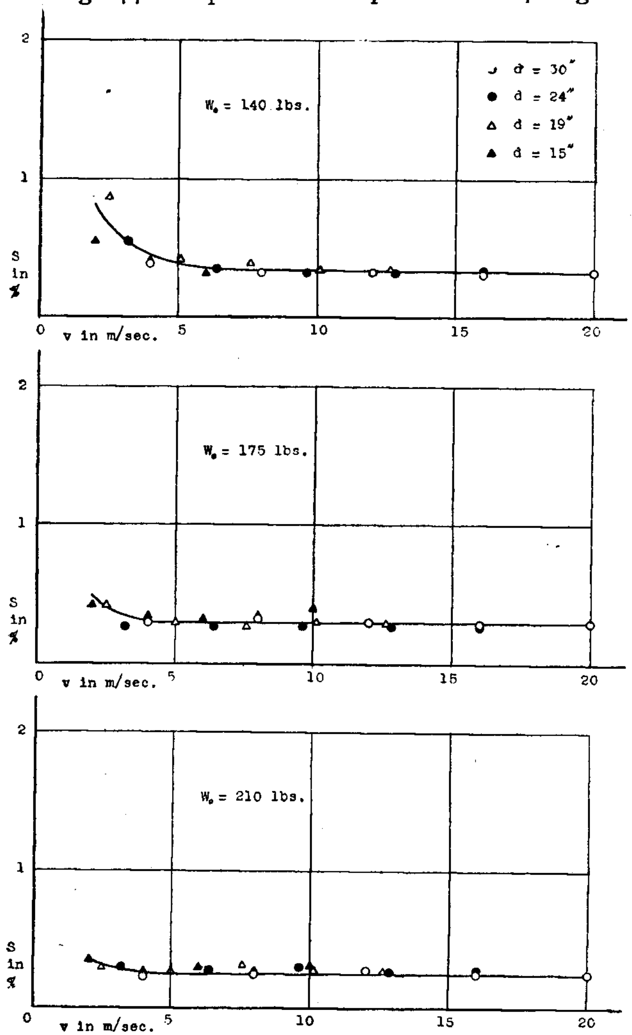


Fig. 45. Slip and Belt Speed at P=60 kg.

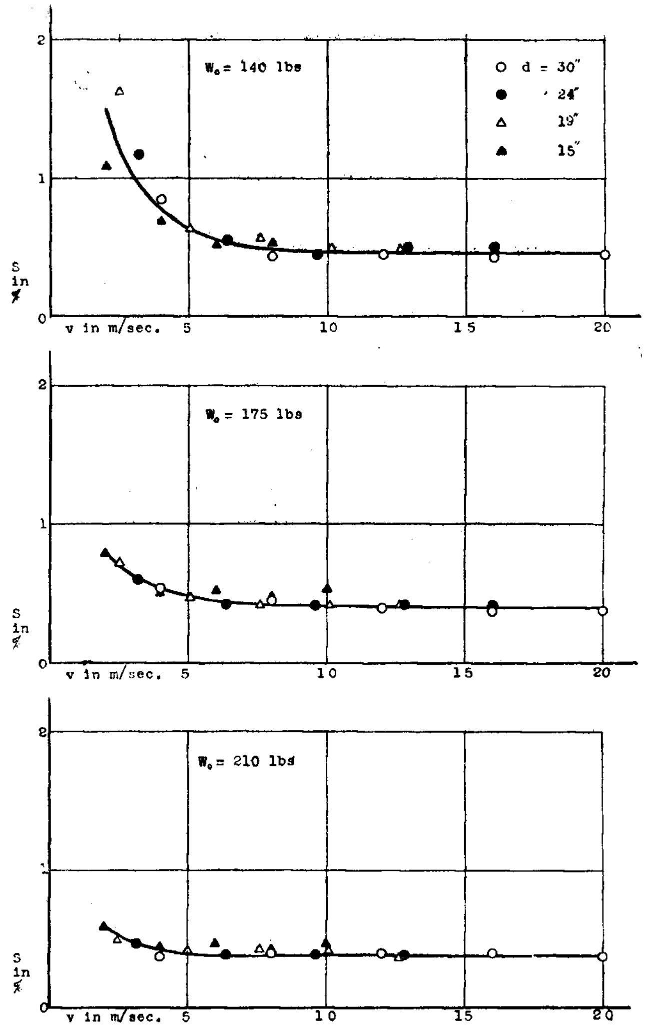


Fig. 46. Slip and Belt Speed at P=80 kg.

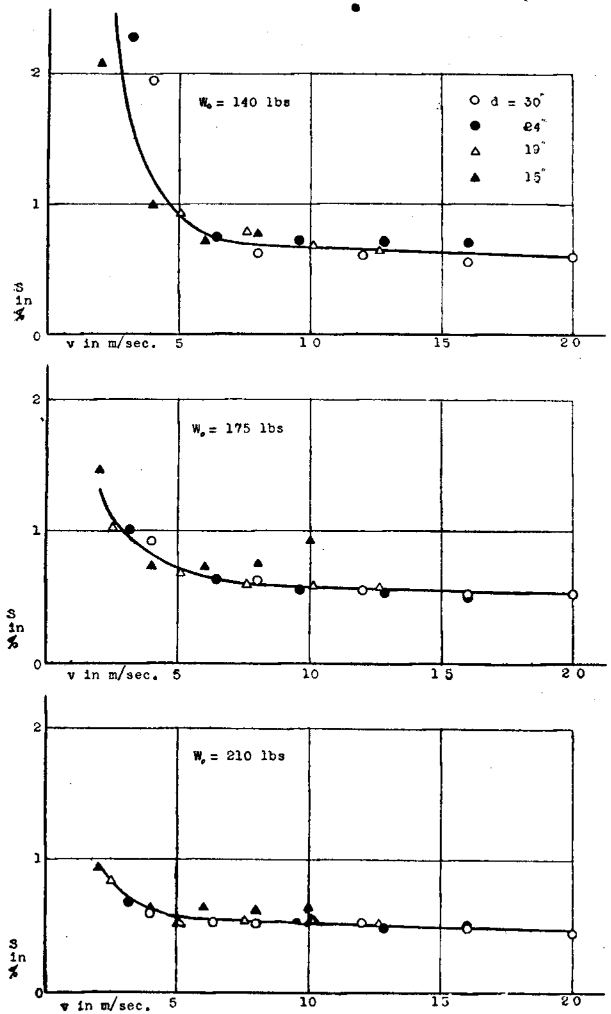


Fig. 47. Slip and Pulley Diameter for n=100.

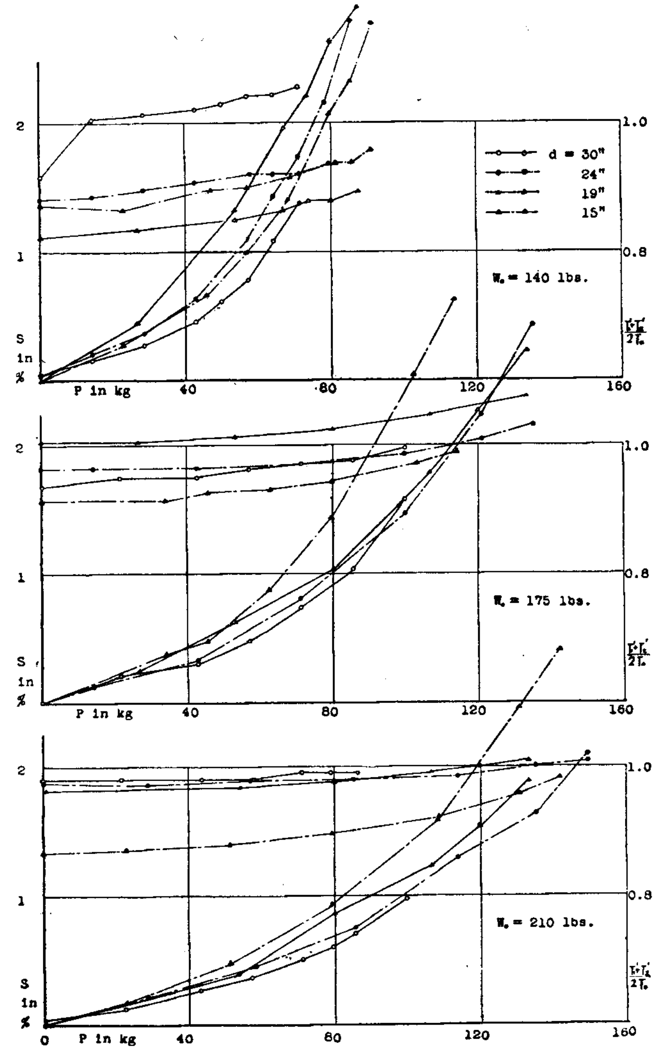


Fig. 48. Slip and Pulley Diameter for n=300.

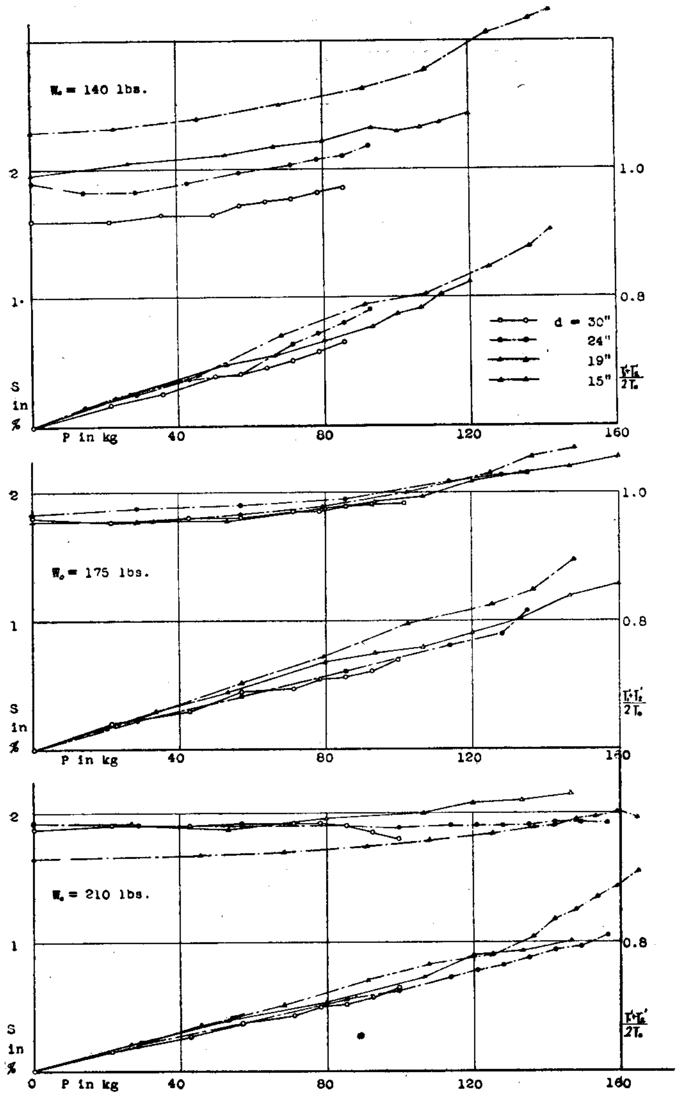


Fig. 49. Slip and Initial Tension for d=24".

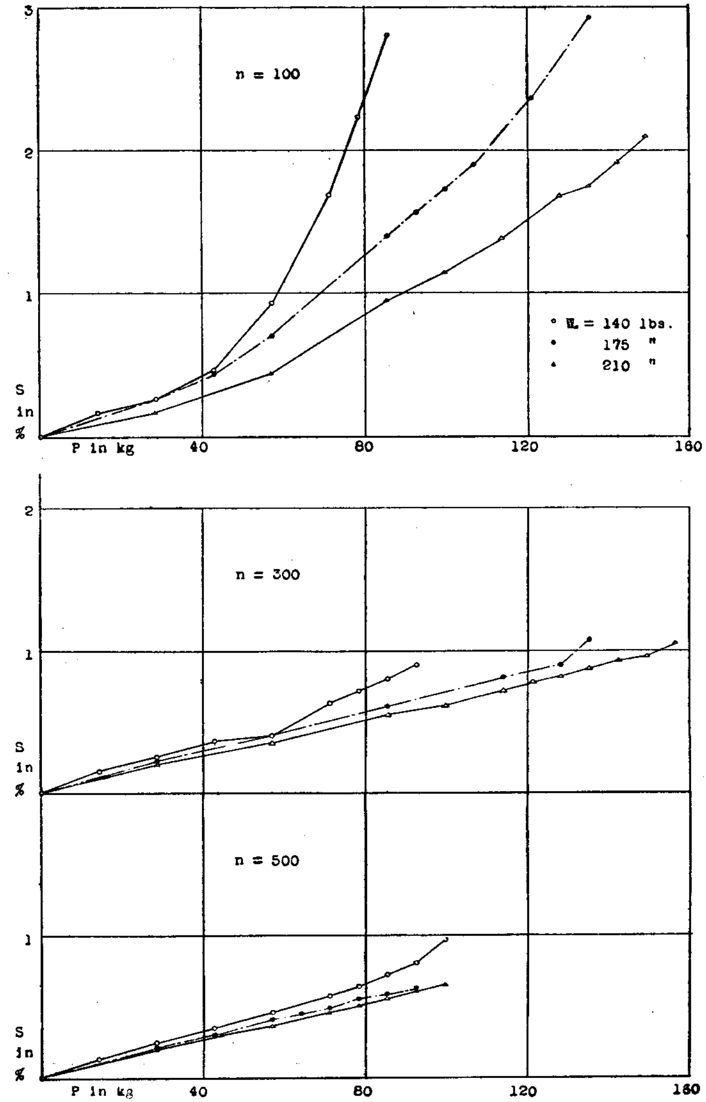


Fig. 50. Slip and Initial Tension for d=30″.

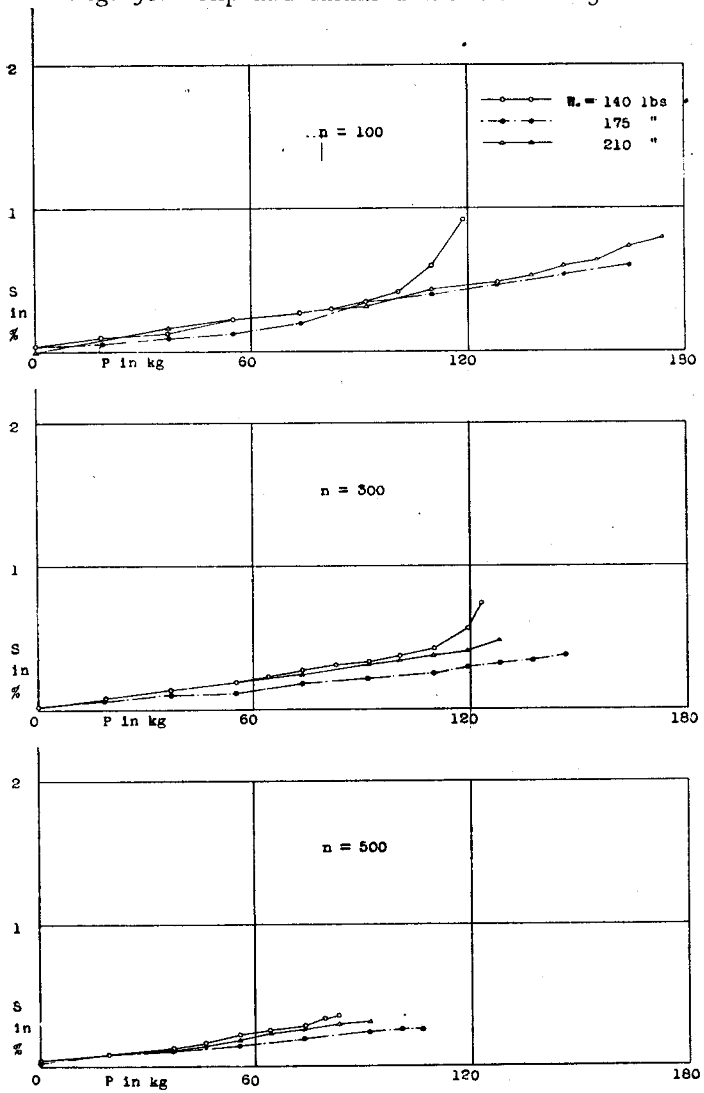


Fig. 51. Slip and Coefficient of Friction for d=30″.

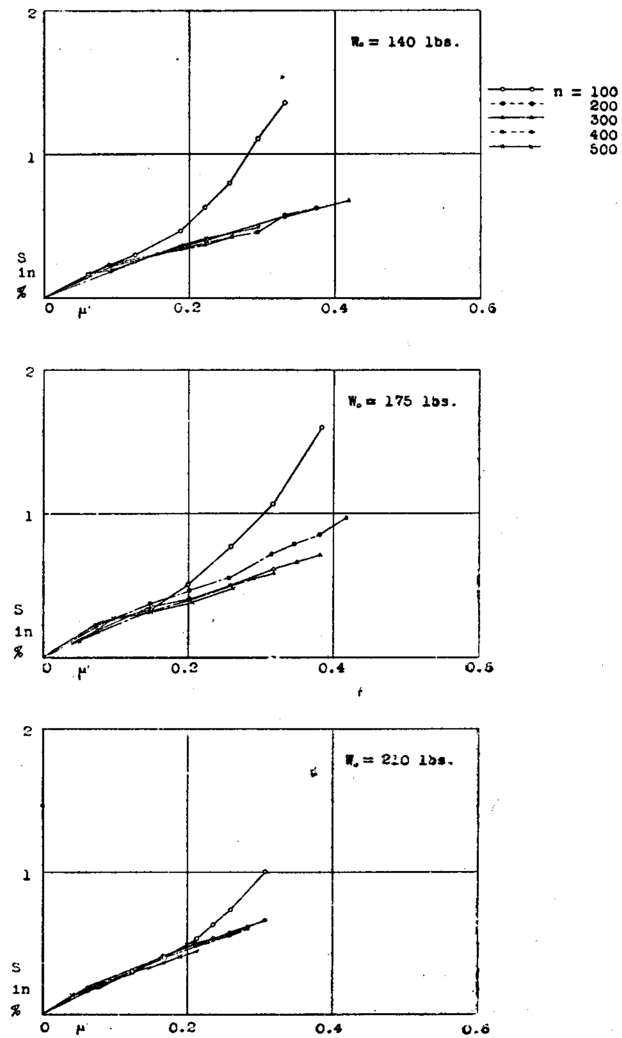


Fig. 52. Slip and Coefficient of Friction for d=24″.

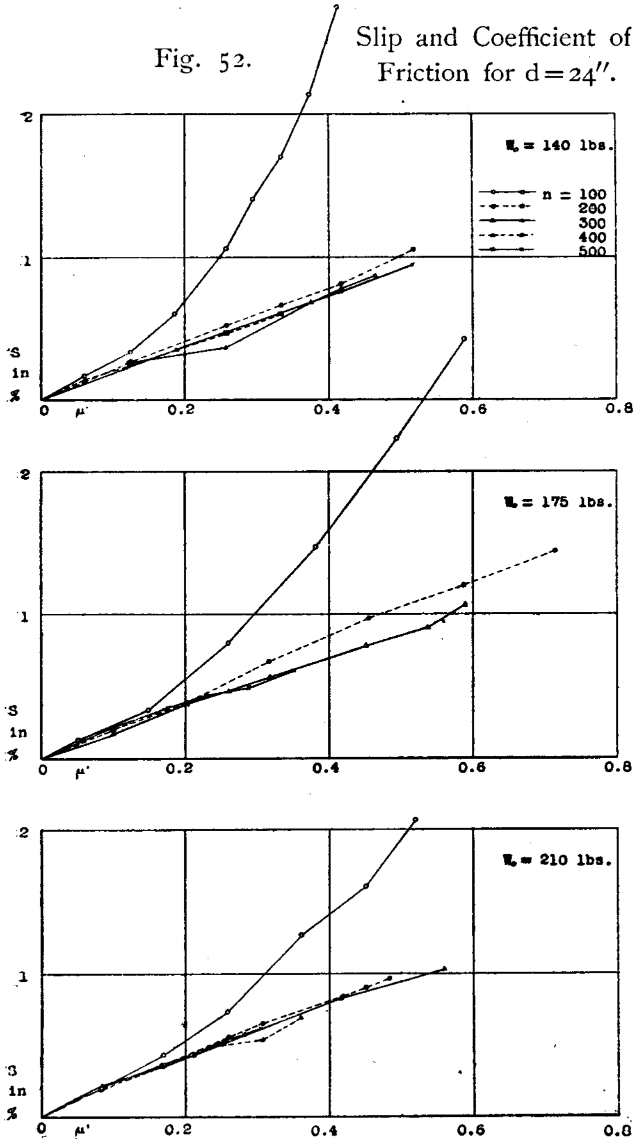


Fig. 53. Slip and Coefficient of Friction for d=19″.

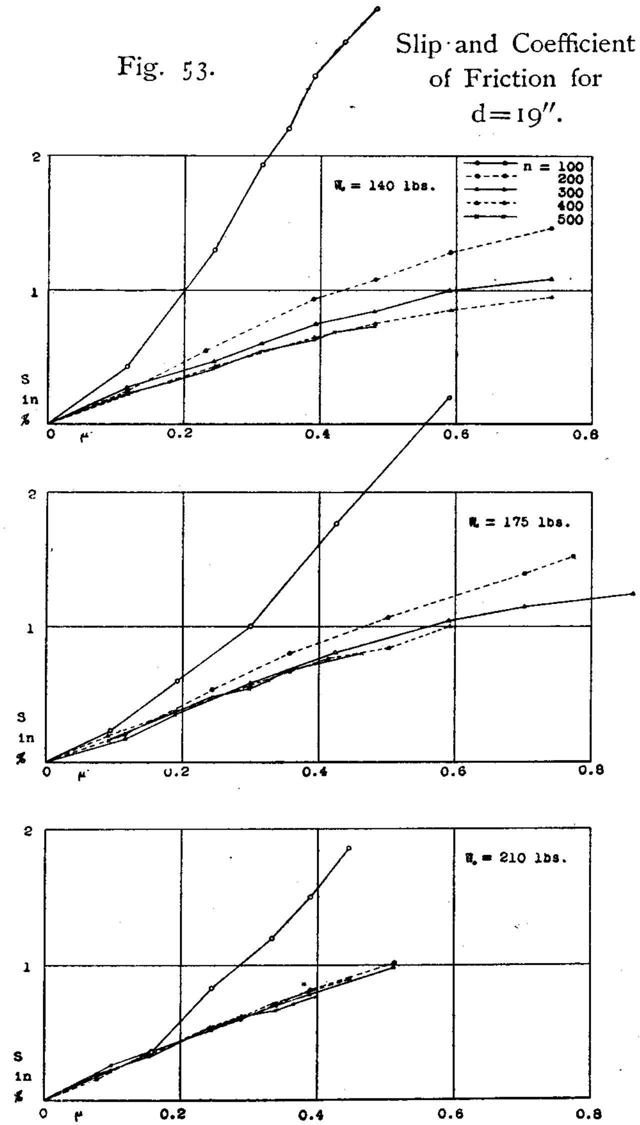


Fig. 54. Slip and Coefficient of Friction for d = 15″.

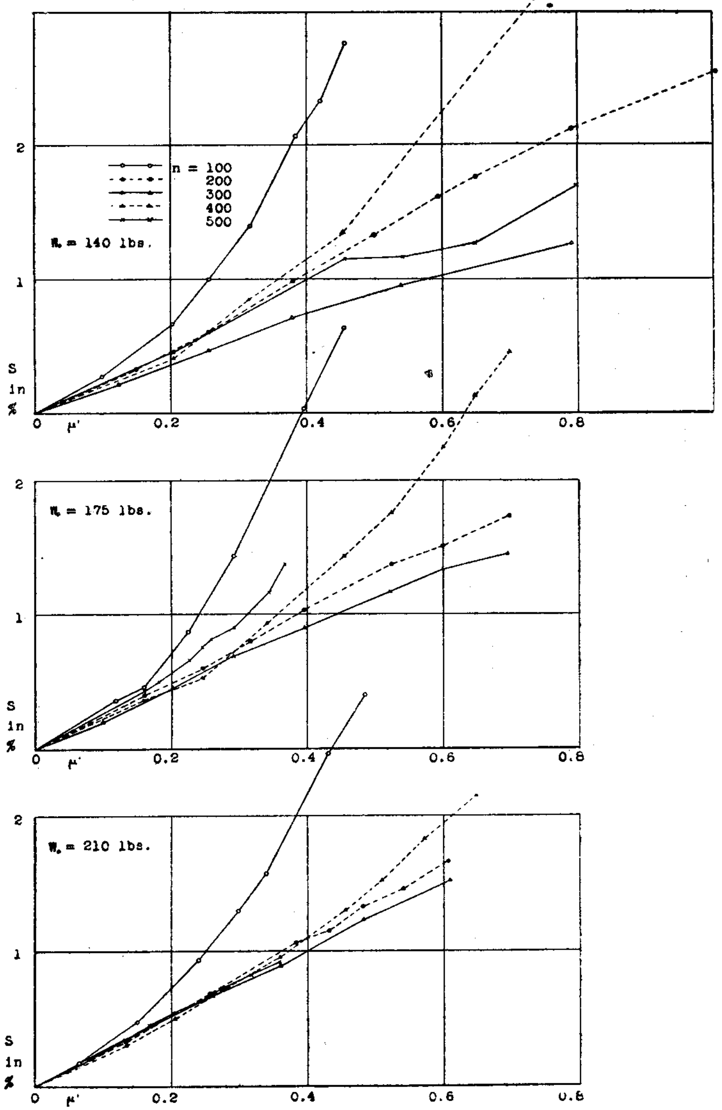


Fig. 55. Belt Speed and Coefficient of Friction for S = 0.5%.

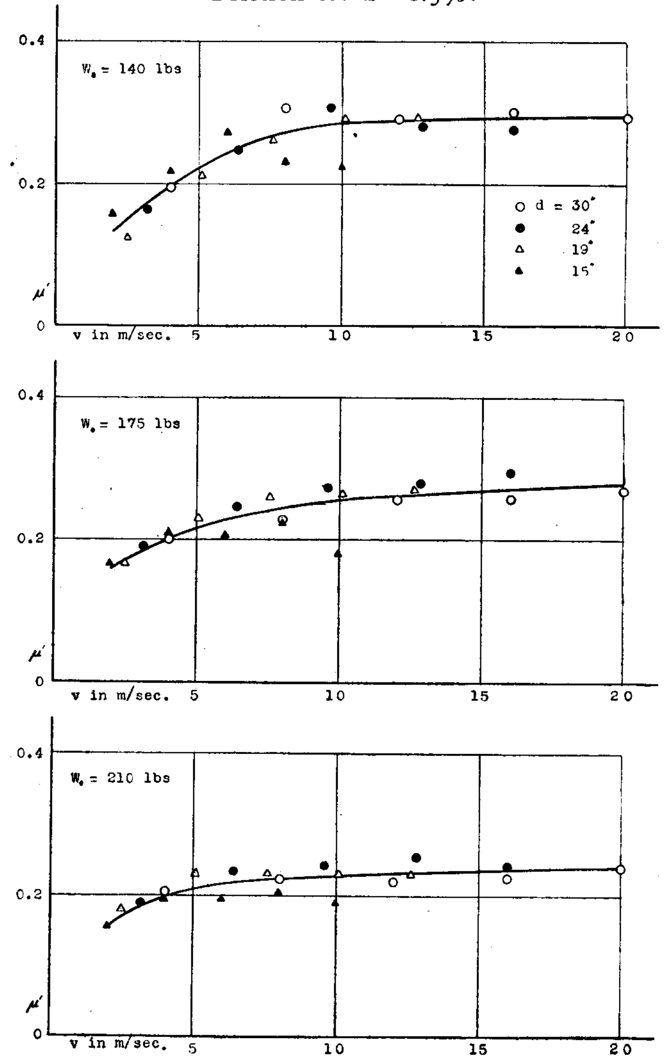


Fig. 56. Variation of Tention for Balata Belt.
W=140 lbs., d=30″.

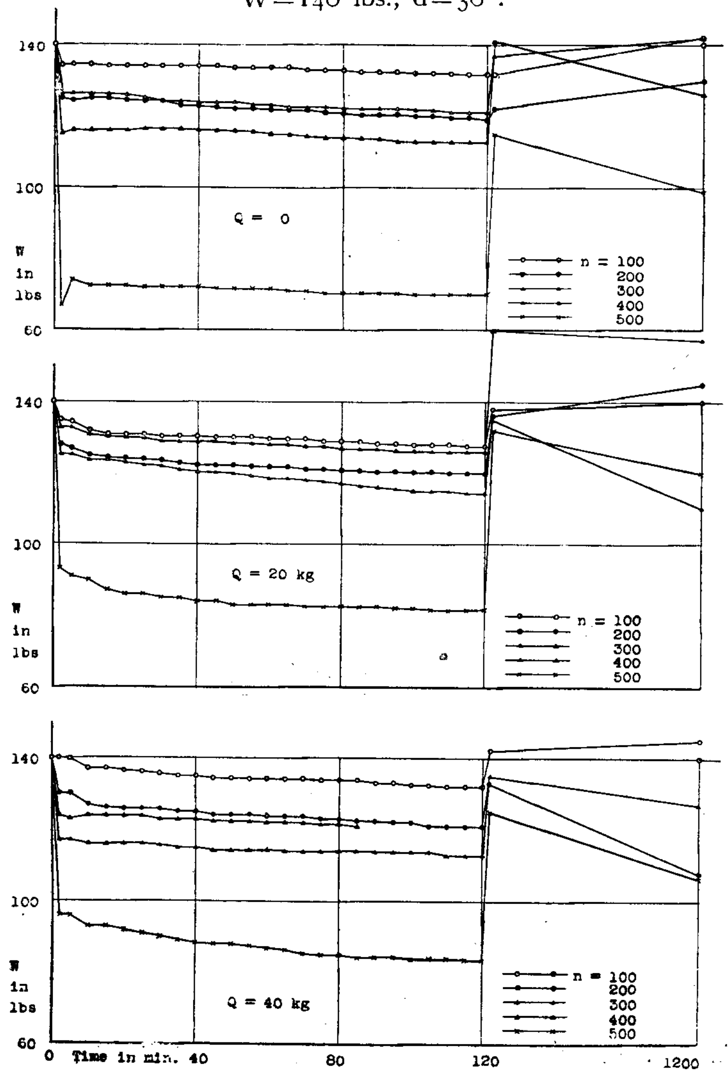


Fig. 57. Variation of Tension for Balata Belt
W=210 lbs. and d=30″.

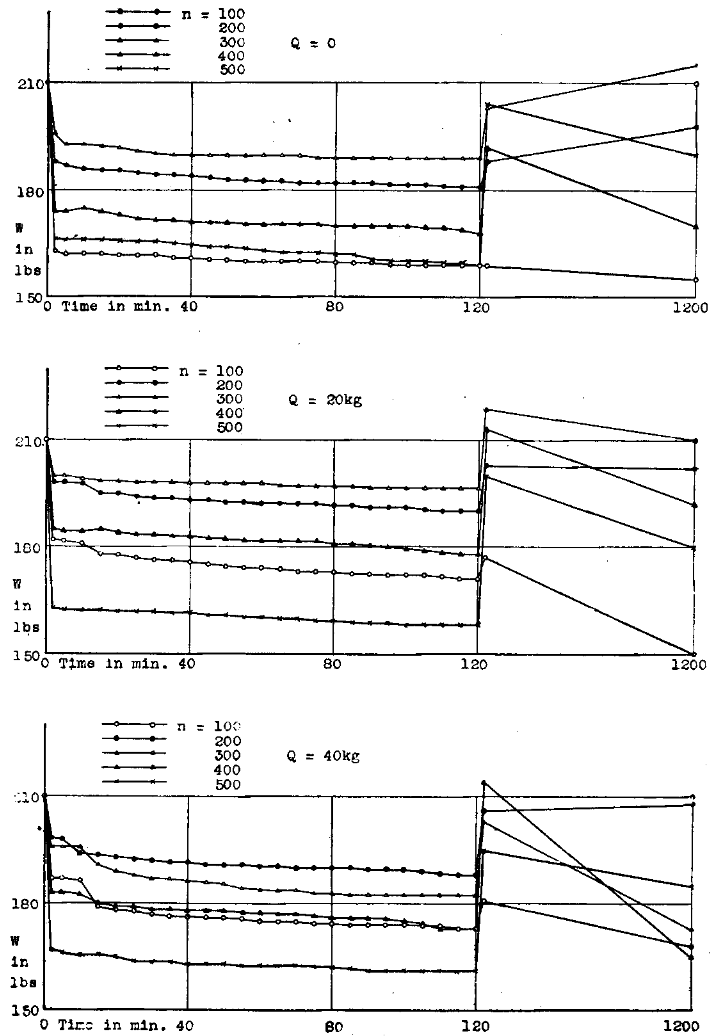


Fig. 58. Slinding Load for Leather Belt.

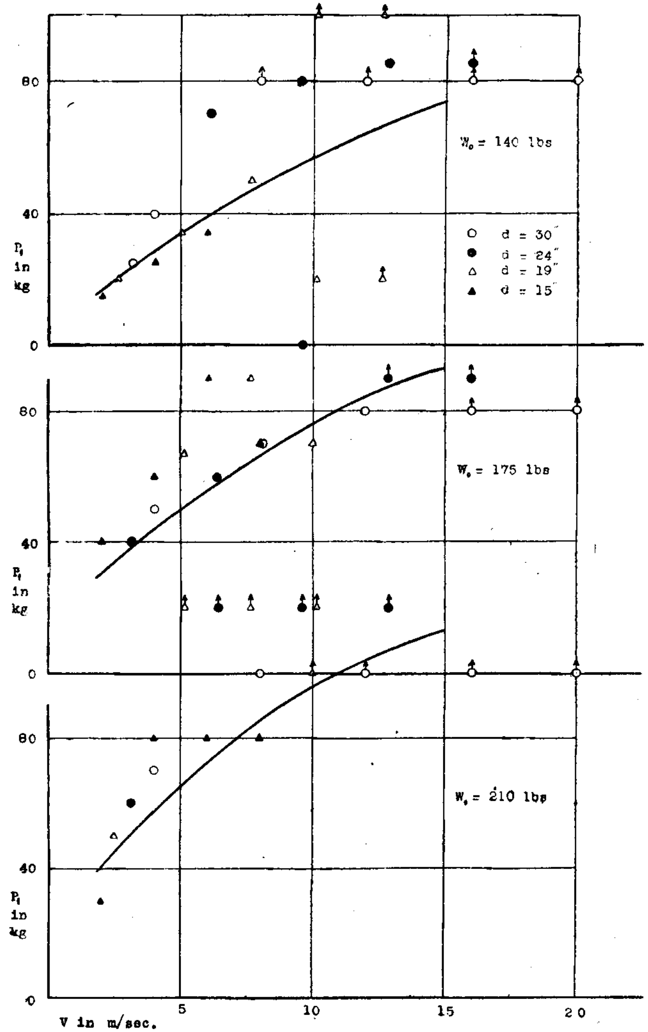


Fig. 59. Sliding Load and Initial Tenstion for Leather Belt.