

# NEW FORMULAE AND TABLES FOR STEAM (REPORT 1.)

By

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## INTRODUCTION.

There are many formulae and tables for the properties of steam and they differ more or less from one another. On the other hand the problems occurring in steam engineering become more international year after year, and the demand for an international standard steam table becomes more and more imperative. To fulfil this demand, authorities from Great Britain, U.S.A., Germany and Czechoslovakia assembled in July 1929 in London<sup>(1)</sup> and consulted upon the units and tables of steam for international use. As a result of this conference, the first skeleton tables proposed for international purposes were prepared, the metric system being used for the units. In the following year, i.e. in June 1930, the second International Steam-Table Conference was held in Berlin<sup>(2)</sup> and representatives from Great Britain, U. S. A., Germany, Czechoslovakia, Switzerland and Sweden improved the said tables and produced the second skeleton tables as the standard steam tables for international purposes. These standard tables, which are shown as Tables 1, 2 and 3, in this paper contain the mean values and tolerances of saturation pressure, specific volume and heat content of saturated liquid water, saturated steam and superheated steam for certain intervals of temperature.

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(1) Mechanical Engineering, Feb. 1930.

(2) V. D. I. Feb. 1931.

Table 1. Values in Saturated Condition

temp. °C	sat. pressure kg/cm <sup>2</sup>	specific volume m <sup>3</sup> /kg.		heat content int. kcal/kg.	
		water	steam	water	steam
0	0.006225 ±0.000005	0.001000 ±0.000000	206.4 ±0.2	0 0	595.5 ±1.0
50	0.1258 ±0.0001	0.001012 ±0.000000	12.05 ±0.01	49.95 ±0.02	618.5 ±1.0
100	1.0332 ±0.0000	0.001043 ±0.000000	1.673 ±0.002	100.04 ±0.04	639.2 ±0.5
150	4.855 ±0.003	0.001090 ±0.000000	0.392 ±0.001	150.92 ±0.05	656.0 ±1.5
200	15.86 ±0.01	0.001156 ±0.000001	0.1273 ±0.0004	203.55 ±0.10	667.0 ±2.5
250	40.6 ±0.1	0.001252 ±0.000003	0.0502 ±0.0004	259.2 ±0.5	669 ±4
275	60.7 ±0.1	0.001317 ±0.000004	0.0329 ±0.0005	289.0 ±1.0	666 ±5
300	87.7 ±0.1	0.001403 ±0.000005	0.0215 ±0.0005	322 ±2	657 ±5
325	123.0 ±0.1	0.00153 ±0.00001	0.0142 ±0.0004	360 ±3	643 ±6
350	168.7 ±0.15	0.00174 ±0.00001	0.00875 ±0.00020	404 ±5	615 ±8

Table 2. Specific Volume of Superheated Steam m<sup>3</sup>/kg.

t °C p kg/cm <sup>2</sup>	t °C										
	100	150	200	250	300	350	400	450	500	550	
1	1.730 ±0.003	1.974 ±0.003	2.214 ±0.003	2.452 ±0.003	2.689 ±0.004	2.925 ±0.004	3.161 ±0.005	3.396 ±0.005	3.632 ±0.005	3.868 ±0.005	
5			0.4332 ±0.0005	0.4833 ±0.0005	0.5326 ±0.0005	0.5811 ±0.0005	0.6290 ±0.0005	0.6768 ±0.0005	0.7243 ±0.0005	0.7719 ±0.0005	
10			0.2102 ±0.0003	0.2374 ±0.0003	0.2631 ±0.0003	0.2880 ±0.0003	0.3124 ±0.0003	0.3366 ±0.0003	0.3604 ±0.0004	0.3843 ±0.0005	
25				0.0890 ±0.0003	0.1010 ±0.0003	0.1120 ±0.0003	0.1224 ±0.0003	0.1325 ±0.0003	0.1424 ±0.0004	0.1521 ±0.0005	
50					0.0465 ±0.0004	0.0531 ±0.0004	0.0590 ±0.0003	0.0644 ±0.0003	0.0697 ±0.0004	0.0747 ±0.0005	
100						0.0231 ±0.0005	0.0270 ±0.0005	0.0304 ±0.0004	0.0333 ±0.0005	0.0361 ±0.0005	
150						0.0119 ±0.0005	0.0160 ±0.0005	0.0189 ±0.0004	0.0212 ±0.0004	0.0233 ±0.0005	
200							0.01028 ±0.00005	0.01305 ±0.00030	0.01515 ±0.00035	0.01685 ±0.00040	
250							0.00636 ±0.00005	0.00940 ±0.00015	0.01140 ±0.00025	0.01290 ±0.00040	

**Table 3.** Heat Content of Superheated Steam int. kcal/kg.

t °C ρ kg/cm <sup>2</sup>	100	150	200	250	300	350	400	450	500	550
	1	639.4 ±0.5	663.5 ±1.0	687.0 ±1.5	711.0 ±2.0	735.0 ±2.0	759.0 ±2.0	783.5 ±2.0	808.0 ±2.0	833.0 ±2.0
5			682.5 ±1.5	707.5 ±2.0	732.5 ±2.0	757.0 ±2.0	781.5 ±2.0	807.0 ±2.0	832.0 ±2.0	857.5 ±2.0
10			676.0 ±1.5	703.0 ±2.0	729.5 ±2.0	755.0 ±2.0	780.0 ±2.0	805.5 ±2.0	831.0 ±2.0	857.0 ±2.0
25				689.0 ±2.0	719.5 ±2.0	747.5 ±2.0	775.0 ±2.0	801.5 ±2.0	827.5 ±2.5	854.0 ±3.0
50					700.0 ±3.0	734.0 ±2.5	765.0 ±2.5	793.5 ±2.5	822.5 ±2.5	850.5 ±3.5
100						701.5 ±3.0	742.5 ±2.5	777.5 ±2.5	810.0 ±2.5	841.0 ±4.0
150						648.5 ±4.0	715.0 ±2.5	759.0 ±2.5	796.5 ±2.5	830.5 ±4.0
200							676.0 ±2.5	737.0 ±3.0	782.0 ±4.0	820 ±6
250							623.0 ±4.0	710.0 ±5.0	765.0 ±5.0	807 ±10

As the standard tables are only skeltons, they are not sufficient for practical use, and it is desirable to make more detailed steam tables for international purposes and these tables should agree with the Skelton Tables within the region of tolerance.

The object of this paper and the following is to construct formulae and make tables fulfilling the said condition and to investigate several properties of steam on the basis of these formulae.

### I. RELATION BETWEEN THE TEMPERATURE AND PRESSURE OF SATURATED STEAM.

#### A) COMPARISON OF THE NUMERICAL VALUES OF SATURATION PRESSURE GIVEN BY SEVERAL AUTHORITIES WITH THOSE OF THE STANDARD SKELETON TABLES.

There are many tables and equations expressing the relation between the temperature and pressure of saturated steam, one of which, the equation of Regnault, published in 1847, was adopted for long years as accurate.

In 1905 Dieterici<sup>(3)</sup> proposed the empirical equation,

$$p = p_c e^{7.4 \left(1 - \frac{T_c}{T}\right)} \dots\dots\dots (1)$$

where  $T = 273 + t$ , is the absolute temperature of saturated steam in the Centigrade scale

$p$ , the saturation pressure at  $t$  in kg/cm<sup>2</sup>

$T_c = 638^\circ$ , the absolute critical temperature

$p_c = 200.5$  kg/cm<sup>2</sup>, the critical pressure.

In 1908 Henning gave comparisons of the temperature-pressure relations of several authorities published up to that time. This comparison was given graphically and as the standard of comparison he adopted the equation of Thiesen, which is as follows,

$$(t + 273) \log_{10} \frac{p}{1.033} = 5.409(t - 100) - 0.508 \cdot 10^{-8} [(365 - t)^4 - 265^4] \dots (2)$$

L. Holborn and Henning gave a correction curve for Thiesen's formula from the results of experiments made at the Physikalisch Technische Reichsanstalt, and Marks and Davis<sup>(4)</sup> adopted this relation for their steam tables.

W. Schüle<sup>(5)</sup> gives the following 3 approximate equations for the temperature-pressure relation of saturated steam.

$$\left. \begin{array}{l} \text{for } t = 20^\circ\text{C to } 100^\circ\text{C} \quad \log_{10} p = 5.9778 - \frac{2224.4}{T} \\ \text{for } t = 100^\circ\text{C to } 200^\circ\text{C} \quad \log_{10} p = 5.6485 - \frac{2101.1}{T} \\ \text{for } t = 200^\circ\text{C to } 350^\circ\text{C} \quad \log_{10} p = 5.45142 - \frac{2010.8}{T} \end{array} \right\} \dots\dots\dots (3)$$

In 1919 L. Holborn, F. Henning and K. Scheel published a booklet of heat tables<sup>(6)</sup>, in which Table 45 gives the relation between the pressure

(3) V. D. I. 1905, p. 365.

(4) Marks-Davis, Steam Tables and Diagrams. Longmans, Green and Co. 1922.

(5) W. Schüle, Technische Thermodynamik. Julius Springer 1923.

(6) Wärmetabellen der Physikal.—Technischen Reichsanstalt 1919.

and temperature of saturated steam. R. Mollier<sup>(7)</sup> and O. Knoblauch<sup>(8)</sup> adopted this table for the relation between the temperature and pressure of saturated steam in their steam tables.

Since 1922 F. G. Keyes and L. B. Smith in the Massachusetts Institute of Technology have been making experimental researches on the properties of steam and publishing the results in "Mechanical Engineering" every year. They gave the following equation as the relation between the temperature and pressure of saturated steam in "Mechanical Engineering" Feb. 1930.

$$\log_{10} p = 2.3395111 - \frac{1}{T} (ax + Yx^2) \dots\dots\dots (4)$$

where  $p$  is the saturation pressure in physical atmosphere,

$$T = 273.13 + t, \text{ the absolute temperature,}$$

$$a = 3.4763,$$

$$x = T_c - T, T_c \text{ being the absolute critical temperature}$$

$$= 273.13 + 374.11 = 647.24,$$

and  $Y$  is given as a function of  $x$  in the following form,

$$\text{for } t = 0^\circ \text{ to } 150^\circ \text{C, } 10^3 Y = -4.494947 + 2.000419 \cdot 10^{-2} \cdot x - 3.593284 \cdot 10^{-5} \cdot x^2 \\ + 3.143572 \cdot 10^{-8} \cdot x^3$$

$$\text{for } t = 150^\circ \text{ to } 290^\circ \text{C, } 10^3 Y = -7.371777 + 6.340868 \cdot 10^{-2} \cdot x - 2.572841 \cdot 10^{-4} \cdot x^2 \\ + 4.115226 \cdot 10^{-7} \cdot x^3$$

$$\text{for } t = 290^\circ \text{ to } 374^\circ \text{C, } 10^3 Y = -21.944157 + 1.022158x - 2.808741 \cdot 10^{-2} \cdot x^2 \\ + 4.225870 \cdot 10^{-4} \cdot x^3 - 3.224082 \cdot 10^{-6} \cdot x^4 \\ + 9.724372 \cdot 10^{-9} \cdot x^5$$

Table 4 gives the comparison of the numerical values of pressures corresponding to temperatures of saturated steam calculated by the said equations and those given in the form of tables by several authorities, the

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(7) R. Mollier, *Neue Tabellen und Diagramme für Wasserdampf*, Julius Springer 1930.

(8) O. Knoblauch, E. Reisch and H. Hausen, *Tabellen und Diagramme für Wasserdampf*. Oldenburg 1923.

temperature-pressure relation given in the second Skeleton Tables being taken as the standard.

Table 4. Sturation Pressure.

No.	t °C	0	50	100	150	200	250	275	300	325	350
1	$p$ kg/cm <sup>2</sup>	0.010123	0.1472	1.0444	4.663	15.17	39.4	59.5	86.6	122.2	167.8
	$\Delta p$ "	+0.003893	+0.0213	+0.0112	-0.189	-0.68	-1.1	-1.1	-1.0	-0.7	-0.75
2	$p$ kg/cm <sup>2</sup>	0.006229	0.1258	1.0333	4.866	15.95	40.9	61.2	88.3	123.4	167.9
	$\Delta p$ "	—	—	+0.0001	+0.008	+0.08	+0.2	+0.4	+0.5	+0.3	-0.65
3	$p$ kg/cm <sup>2</sup>	—	0.1233	1.0334	4.801	15.86	40.4	60.5	87.5	122.7	167.4
	$\Delta p$ "	—	-0.0024	+0.0002	-0.051	—	-0.1	-0.1	-0.1	-0.2	-1.15
4	$p$ kg/cm <sup>2</sup>	0.006226	0.1258	1.0332	4.854	15.85	40.5	60.6	87.5	123.1	168.8
	$\Delta p$ "	—	—	—	—	—	—	—	-0.1	—	—
5	$p$ kg/cm <sup>2</sup>	0.006226	0.1258	1.0333	4.855	15.85	40.5	60.6	87.6	123.0	168.6
	$\Delta p$ "	—	—	+0.0001	—	—	—	—	—	—	—
6	$p$ kg/cm <sup>2</sup>	0.006273	0.1258	1.0327	4.862	15.84	39.8	—	—	—	—
	$\Delta p$ "	+0.000043	—	-0.0005	+0.004	-0.01	-0.7	—	—	—	—
7	$p$ kg/cm <sup>2</sup>	—	—	—	—	15.99	40.9	61.0	87.7	122.5	167.5
	$\Delta p$ "	—	—	—	—	+0.12	+0.2	+0.2	—	-0.4	-1.05
8	$p$ kg/cm <sup>2</sup>	—	—	1.0332	4.854	15.85	40.5	60.6	87.6	123.0	168.8
	$\Delta p$ "	—	—	—	—	—	—	—	—	—	—

In Table 4, No. 1. gives the values calculated by Equation (1),

No. 2., the values calculated by Equation (2),

No. 3., the values calculated by Equation (3),

No. 4., the values calculated by Equation (4), reduced to kg/cm<sup>2</sup>.

No. 5., the values taken from the heat table<sup>(6)</sup> of P.T.R.,

No. 6., the values taken from the steam table of H.L. Callendar<sup>(9)</sup> reduced to kg/cm<sup>2</sup>.

No. 7., the same from the steam table of H.L. Callendar<sup>(10)</sup>

and

No. 8., the same from the paper of F.G. Keyes and L.B.

(9) H.L. Callendar, Properties of Steam., Edward Arnold, 1924.

(10) H.L. Callendar, Proc. of the Institution of Mechanical Engineers, London, 1929, Vol. 1 p. 519.

Smith<sup>(11)</sup>

The symbol  $p$  in each line denotes the value of saturation pressure given by the respective authority and  $\Delta p$ , the amount of deviation of the value of pressure *outside the region of tolerance* of the Standard Tables, both expressed in kg/cm<sup>2</sup>.

We can see from Table 4, that the values in No. 5 & No. 8 are all inside the region of tolerance of the Standard Tables except the value at 100°C in No. 5. The originals of both No. 5 & No. 8, as above mentioned, give the values in the form of tables, with no equations.

The values calculated by the equations mentioned above have generally a comparatively large amount of deviation from the Standard Tables. Only the equation of F.G. Keyes & L.B. Smith gives exact values, but it has a very complex form and it is very troublesome to calculate with their equation.

B) A NEW EQUATION FOR THE RELATION BETWEEN  $p$  AND  $t$ .

If, now, we had an equation expressing in a simple way the saturation pressure of steam in terms of its temperature, and giving the results falling within the region of tolerance of the Standard Tables, at least throughout the whole range of temperatures contained in the said Tables, we could find with tolerable accuracy not only the pressure  $p$ , but also the differential coefficient  $dp/dT$  for any temperature, which latter occurs in many problems relating to steam. Such an equation would be very useful, and so the author tried to make one starting from the equation,

$$\log_e \frac{p_c}{p} = a \left( \frac{T_c}{T} - 1 \right)$$

and arrived at the following result.

$$\log_e \frac{p_c}{p} = \left[ a_0 + \frac{(T - T')^2}{a + \beta T} \right] \left( \frac{T_c}{T} - 1 \right) \dots\dots\dots (I)$$

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(11) F.G. Keyes & L.B. Smith, "Some Final Values for the Properties of Saturated and Superheated Water.", Mechanical Engineering, Feb. 1931.

where  $p$  is the saturation pressure corresponding to temperature in  $\text{kg}/\text{m}^2$ ,

$T = t + 273.20^{(12)}$ , the absolute temperature,

$p_c = 225.05 \cdot 10^4 \text{ kg}/\text{m}^2$ ,<sup>(13)</sup> the critical pressure,

$T_c = 374 + 273.20 = 647.20$ , the absolute critical temperature

and  $a_0$ ,  $a$ ,  $\beta$  and  $T'$  are constants having the following numerical values:

$$a_0 = 7.21280,$$

$$T' = 483.20,$$

$$a = 87060, \quad \beta = 36.9, \quad \text{for } t = 0^\circ \text{ to } 210^\circ\text{C}$$

$$a = 318660, \quad \beta = -395. \quad \text{for } t = 210^\circ \text{ to } 374^\circ\text{C}$$

From the form of this equation it is evident that when  $T = T_c$ ,  $p = p_c$ . It applies, therefore, to the critical point.

The values of saturation pressure calculated by this equation are given in Table 5.

**Table 5.** Values of Saturation Pressure by Eq. (I)

t °C	0	50	100	150	200	250	275	300	325	350
$p \text{ kg}/\text{cm}^2$	0.006225	0.1257	1.0332	4.855	15.86	40.6	60.7	87.7	123.0	168.7

We see that the values calculated by Equation (I) all lie in the region of tolerance of the Skeleton Tables.

Table 6. shows the values calculated by Equation (I) at  $10^\circ\text{C}$  intervals compared with those of Wärmstabellen d. P.T.R. and those of the table of F.G. Keyes & L.B. Smith<sup>(14)</sup>.

(12) The value of absolute temperature at  $0^\circ\text{C}$  i.e. 273.20 is taken from Henning's paper, Z.S. f. Phys. Vol. 5 1921 p. 313.

(13) The values of critical pressure and temperature are taken from Wärmstabellen der Physikalisch Technischen Reichsanstalt.

(14) See the explanation of Table 4 No. 8.



**Table 6.** Comparison of the Values of Saturation Pressure

t °C	p kg/cm <sup>2</sup>			t °C	p kg/cm <sup>2</sup>		
	Eq. (I)	P.T.R.	Keyes-Smith		Eq. (I)	P.T.R.	Keyes-Smith
0	0.006225	0.006226		200	15.859	15.854	15.850
10	0.012506	0.012520		210	19.460	19.453	19.444
20	0.023814	0.023840		220	23.662	23.655	23.641
30	0.043231	0.043267		230	28.539	28.527	28.507
40	0.075172	0.075218		240	34.157	34.132	34.109
50	0.12573	0.12577		250	40.589	40.547	40.522
60	0.20306	0.20309		260	47.907	47.850	47.823
70	0.31767	0.31771		270	56.189	56.110	56.092
80	0.48284	0.4828		280	65.522	65.42	65.417
90	0.71481	0.7148		290	75.989	75.88	75.889
100	1.0332	1.0333	1.0332	300	87.691	87.63	87.600
110	1.4610	1.4610	1.4608	310	100.72	100.68	100.66
120	2.0247	2.0247	2.0246	320	115.20	115.17	115.17
130	2.7550	2.7549	2.7547	330	131.26	131.25	131.28
140	3.6859	3.685	3.6853	340	149.03	149.04	149.11
150	4.8551	4.855	4.8542	350	168.70	168.64	168.83
160	6.3043	6.303	6.3028	360	190.49	190.26	190.61
170	8.0788	8.079	8.0764	370	215.07	214.47	214.78
180	10.227	10.225	8.1570				
190	12.801	12.799	12.796				

We can see from Table 6, that the values calculated by Equation (I) coincide very well with the experimental values, which are at present considered to be very accurate.

C) AN EXPRESSION FOR  $dp/dT$  AND ITS NUMERICAL VALUES.

Let us now derive the expression for  $dp/dT$  from Equation (I) and check the accuracy of it by using the equation of Clapeyron,

$$\frac{L}{T} = A(v_s - v_w) \frac{dp}{dT},$$

or 
$$\frac{dp}{dT} = \frac{1}{A(v_s - v_w)} \frac{L}{T} \dots\dots\dots (5)$$

- where  $v_s$  is the specific volume of dry saturated steam at any temperature  $T$  in  $\text{m}^3/\text{kg}$ .,  
 $v_w$  the specific volume of saturated liquid water at the same temperature in  $\text{m}^3/\text{kg}$ .,  
 $L$ , the latent heat of evaporation at the same temperature in int. kcal/kg. and it can be expressed as the difference between the heat content of dry saturated steam and that of liquid water at the same temperature,  
 $A$ , the thermal equivalent of work and has a value of  $\frac{1}{427.08}$  int. kcal/kg.<sup>(15)</sup>

In the Standard Skeleton Tables the values of specific volume and heat content of both dry saturated steam and saturated liquid water are given. From these data we can calculate the value of  $dp/dT$  for any temperature contained in the same tables by using the relation expressed by Equation (5).

On the other hand Equation (I) expresses  $p$  as an explicit function of  $t$  and it is easy to derive an equation for  $dp/dT$  from it, and then we can calculate the value of  $dp/dT$  for any temperature by the equation so derived.

Differentiating an equation with the form,

$$\log_e \frac{p_c}{p} = f(T) \frac{T_c - T}{T}$$

we get

$$-\frac{1}{p} \frac{dp}{dT} = \frac{d}{dT} \left[ f(T) \cdot \frac{T_c - T}{T} \right]$$

(15) According to the first International Steam-Table Conference,

$$1 \text{ int. kcal.} = \frac{\text{int. kw-hr}}{860} = \frac{1000 \cdot 3600}{860} \text{ int. joules.}$$

But the international joule (electrically based) is not exactly equal to absolute joule ( $10^7$ erg). According to Grüneisen (Ann. d. Physik 63 179-200 1920.)

$$1 \text{ int. joule} = (1.00051 \pm 0.00005) \cdot 10^7 \text{ erg}$$

$$\therefore 1 \text{ int. kcal.} = \frac{(1.00051 \pm 0.00005) \cdot 10^7 \cdot 3.6 \cdot 10^6}{860 \cdot 1000 \cdot 980.665 \cdot 100} = 427.076 \pm 0.021 \text{ mkg.}$$

Therefore we take

$$A = \frac{1}{427.08} \frac{\text{int. kcal}}{\text{mkg.}}$$

If now  $f(T) = a_0 + \frac{(T - T')^2}{a + \beta T}$ ,

then

$$\begin{aligned} -\frac{1}{p} \frac{dp}{dT} &= \frac{d}{dT} \left[ \left( \frac{T_c}{T} - 1 \right) \left\{ a_0 + \frac{(T - T')^2}{a + \beta T} \right\} \right] \\ &= \frac{d}{dT} \left[ \frac{a_0 T_c}{T} - a_0 + \frac{T_c}{T} \frac{(T - T')^2}{a + \beta T} - \frac{(T - T')^2}{a + \beta T} \right] \\ &= -\frac{a_0 T_c}{T^2} + \frac{2(T - T')}{a + \beta T} \frac{T_c - T}{T} + \frac{(T - T')^2}{(a + \beta T)^2} \left[ \beta - \frac{T_c}{T^2} (a + 2\beta T) \right]. \\ \therefore \frac{dp}{dT} &= p \left[ \frac{a_0 T_c}{T^2} - \frac{2(T - T')}{a + \beta T} \frac{T_c - T}{T} + \frac{(T - T')^2}{(a + \beta T)^2} \left\{ \frac{T_c}{T^2} (a + 2\beta T) - \beta \right\} \right] \dots (II) \end{aligned}$$

In Table 7 the values of  $dp/dT$  calculated from the values of heat content and specific volume given in the Standard Skeleton Tables and those calculated by Equation (II) are compared.

**Table 7.** Comparison of the Values of  $dp/dT$  in kg/m<sup>2</sup> °C

t °C	by Eq. (5) and the standard tables		by Eq. (II)
	Min.	Max.	
0	4.4983	4.5222	4.5214
50	62.189	62.516	62.398
100	368.21	369.83	368.93
150	1296.6	1311.3	1302.0
200	3287.0	3345.2	3317.8
250	6703.8	6966.4	6861.9
275	9007.7	9600.7	9328.8
300	11862	13000	12347
325	14956	17004	16046
350	18794	22575	20690

We see from the above comparison that the values of  $dp/dT$  calculated by Equation (II) all lie between the maximum and minimum values, which are calculated from the values of the heat content and specific volume, both inside the region of tolerance of the Standard Skeleton Tables by using Equation (5).

Table 8 shows the values of  $dp/dT$  calculated by Equation (II) at 10°C intervals, compared with the tabulated values of some authorities.

**Table 8.** Comparison of the Values of  $dp/dT$  in  $\text{kg}/\text{m}^2 \text{ } ^\circ\text{C}$ 

t °C	I	II	III	IV	t °C	I	II	III	VI
0	4.5214	4.534		4.501	200	3317.8		3317	3313
10	8.3775	8.399		8.380	210	3890.7		3890	3885
20	14.749	14.76		14.75	220	4529.3		4526	4520
30	24.814	24.82		24.83	230	5235.7		5227	5223
40	40.078	40.06		40.08	240	6012.1		5996	5996
50	62.398	62.36		62.38	250	6861.9		6846	6843
60	94.001	93.91		93.98	260	7787.9		7764	7771
70	137.44	137.3		137.4	270	8793.9		8769	8783
80	195.60	195.5		195.6	280	9885.3		9861	9883
90	271.63	271.5		271.6	290	11067		11067	11075
100	368.93	369.0		369.0	300	12347		12400	12366
110	491.07	491.0		491.0	310	13734		13730	1376 <sub>2</sub>
120	641.65	641.5		641.6	320	15242			1529 <sub>1</sub>
130	824.52	824.1		824.4	330	16889			1694 <sub>4</sub>
140	1043.4	1042.4		1043.1	340	18693			1875 <sub>3</sub>
150	1302.0	1300		1301.4	350	20690			2071 <sub>6</sub>
160	1604.0	1600		1603.2	360	22924			2287 <sub>5</sub>
170	1953.0	1945		1951.9	370	25503			2525 <sub>1</sub>
180	2352.6	2341	2352	2350.9					
190	2806.2		3805	2803.7					

In Table 8, the values in column I are the values calculated by Eq. (II); the values in column II, are the values from Table 47 of *Wärmetabellen der P.T.R.*, the unit being converted from mm. of Hg. per degree Centigrade to  $\text{kg}/\text{m}^2 \text{ } ^\circ\text{C}$ ; the values in column III, are taken from the table in Jakob's paper<sup>(16)</sup>, and the values in column IV, are the tabulated values of Keyes and Smith<sup>(17)</sup>, (the unit being converted from physical atmosphere per degree Centigrade to  $\text{kg}/\text{m}^2 \text{ } ^\circ\text{C}$ ) which are calculated by a very elaborate method by them.

We can see from this comparison that the values calculated by

(16) M. Jakob & W. Fritz, *Technische Mechanik u. Thermodynamik*. June 1930.

(17) See foot note (11)

Equation (II), which process is very simple, coincide very well with the tabulated values, which are calculated by a very elaborate process and considered very accurate so far as the experimental data obtained up to the present are concerned.

## II. CHARACTERISTIC EQUATIONS OF SUPERHEATED STEAM.

### A) COMPARISON OF THE NUMERICAL VALUES OF SPECIFIC VOLUME OF SUPERHEATED AND DRY SATURATED STEAM CALCULATED BY THE CHARACTERISTIC EQUATIONS OF SUPERHEATED STEAM OF SEVERAL AUTHORITIES, WITH THOSE OF THE STANDARD SKELETON TABLES.

Many formulae for the characteristic equation of superheated steam have been proposed, of which the following equations are chosen for purposes of comparison.

#### 1) H. L. Callendar

H.L. Callendar has ascertained by the results of his own experiments, that the equation for the adiabatic change of steam can be expressed by the equation  $p v^{\frac{n+1}{n}} = \text{constant}$  with  $n+1 = \frac{13}{3}$ . He started with this equation, and using the general thermodynamical relations and experimental data derived the following characteristic equation :

$$v - 0.001 = \frac{47.0 \cdot T}{p} - 0.0263 \left( \frac{373.1}{T} \right)^{\frac{10}{3}} \dots\dots\dots (6a)^{(18)}$$

where  $v$  is the specific volume in  $\text{m}^3/\text{kg}^{(19)}$ .

$p$ , the pressure in  $\text{kg}/\text{m}^2$ ,

$T = t + 273.1$ , the absolute temperature in the Centigrade scale.

(18) See foot note (9)

(19) In the following equations, the units of specific volume, pressure and temperature are  $\text{m}^3/\text{kg}$ ,  $\text{kg}/\text{m}^2$  and  $^{\circ}\text{C}$  respectively and they are denoted by the symbols  $v$ ,  $p$  and  $t$  or  $T$ .

This equation gives satisfactory values of specific volume for moderate pressures and temperatures and was used for a long time in England, but for high pressures and temperatures it gives values that deviate from the results of recent experiments. In 1929 he published another characteristic equation for superheated steam,<sup>(20)</sup> which is as follows,

$$v + 0.000175 = \frac{47.0T}{p} - \frac{f(T)}{1 - \left[ \frac{0.0508 \cdot p \cdot f(T)}{T} \right]^2} \quad \dots\dots\dots (6b)$$

where  $f(T) = 0.0263 \left( \frac{373.1}{T} \right)^{\frac{10}{3}}$

He used Equation (6b) for temperatures above 200°C and made new steam tables.

## 2) R. Mollier

R. Mollier adopts the following equation:—

$$\frac{pv}{T} = 47.1 - \frac{0.02p}{\left( \frac{T}{100} \right)^{\frac{13}{3}}} - \frac{1.9}{10^6} \frac{p^3}{\left( \frac{T}{100} \right)^{15}}, \quad \dots\dots\dots (7)$$

where  $T = 273 + t$

This equation is simple and he used it to make his steam tables.<sup>(21)</sup>

## 3) O. Knoblauch, R. Raisch and H. Hausen

O. Knoblauch with his collaborateurs constructed an equation for the specific heat under constant pressure of superheated steam, which gives values that coincide satisfactorily with the results of the experiments they made in the München Laboratory, up to the pressure of 30 kg/cm<sup>2</sup>. This equation is as follows,

$$c_p = f(T) + \frac{C}{T - \varphi(p)}$$

From this equation, combined with the thermodynamical relation,

(20) See foot note (10), but the units are changed. See also "Properties of Steam at High Pressure and Temperature" by G.S. Callendar, World Power Oct. 1930, There are some misprints in that paper.

(21) See foot note (7)

they deduced the following characteristic equation of superheated steam :—

$$v = \frac{RT}{p} - \frac{C\varphi'(p)}{A[\varphi(p)]^2} \left[ T \log_e \frac{T}{T - \varphi(p)} - \varphi(p) \right] + \psi(p) \dots\dots (8)$$

where they put  $\varphi(p) = 588.97 + 0.37354 p \cdot 10^{-4} - \frac{6310.3}{18.165 + p \cdot 10^{-4}}$ ,

$$\psi(p) = -0.01151 + 0.0000257 p \cdot 10^{-4} + \frac{0.438}{15.3 + p \cdot 10^{-4}},$$

$$R = 47.10, \quad T = 273.1 + t,$$

$$A = \frac{1}{426.9}, \quad C = 20.33.$$

This equation is very inconvenient to use. However, they constructed their steam tables<sup>(22)</sup> extending up to the pressure of 60 kg/cm<sup>2</sup>, using this equation.

4) **G. Eichelberg**

G. Eichelberg found an expression for the specific heat under constant pressure of superheated steam, which satisfies very well the München Laboratory experimental values of specific heat under constant pressure up to the pressure of 20 kg/cm<sup>2</sup>.

$$c_p = c_{p_0} + \frac{C_1 p}{T^4} + \frac{C_2 (p + 2 \cdot 10^4)^{3.2} - C_3}{T^{15}}$$

From this equation for specific heat under constant pressure of superheated steam he deduced thermodynamically with suitable constants the following characteristic equation :—

$$v = \frac{47.06 T}{p} - \frac{1.139}{\left(\frac{T}{100}\right)^5} - \frac{11615 (p \cdot 10^{-4} + 2)^{2.2}}{\left(\frac{T}{100}\right)^{14}} \dots\dots\dots (9)$$

G. Eichelberg did not construct any steam table with this equation, but he investigated several properties of steam by it.<sup>(23)</sup>

(22) See foot note (8)

(23) Forschungsarbeiten. Heft 220 1920. "Die Eigenschaften des Wasserdampfes im technisch wichtigen Gebiet."

## 5) J. Keenan

J. Keenan, using the empirical formula of H. Davis and R. Kleinschmidt for the Joule-Thomson Effect, derived thermodynamically, with suitable constants, the following equation as the characteristic equation of superheated steam.

$$v = \left( \frac{47.066}{p} + \frac{59}{10^7} \right) T - \frac{13015 \cdot 10^5}{T^{4.2}} - \frac{7574 \cdot 10^{15} \cdot p^{\frac{5}{4}}}{T^{10.5}} - \frac{6.37}{10^3} \dots \quad (10)^{(24)}$$

where

$$T = t + 273.13$$

J. Keenan used this equation, with slight modifications, combined with other data, for the construction of his new steam tables.<sup>(25)</sup>

The following five tables are prepared by the present author in order to compare the values of specific volume of superheated steam calculated by Equations (6b), (7), (8), (9) and (10), with those given in the Standard Skeleton Tables.

In these tables, the symbol  $v$  denotes the value of specific volume calculated by the respective equation and  $\Delta v$  the amount of deviation of the value of specific volume thus calculated *outside the region of tolerance* of the Standard Skeleton Tables, both expressed in  $\text{m}^3/\text{kg}$ , and  $\Delta$  the percentage of  $\Delta v$  referred to the standard value.

We see from Table 9, that the values calculated by Equation (6b) lie almost all inside the region of tolerance, but at and near saturation we see some deviations, the maximum of which (about 15%) is at the pressure of  $250 \text{ kg/cm}^2$  and the temperature of  $400^\circ\text{C}$ .

We see from Table 10, that the values calculated by Equation (7), all lie inside the region of tolerance in the dry saturated condition, but in the region of superheat they almost all lie outside the region of tolerance of the Standard Skeleton Tables. The amounts of deviation are comparatively

(24) J. Keenan, Progress Report on the Development of Steam Charts and Tables from the Harvard Throttling Experiments. Mechanical Engineering. Feb. 1926.

(25) J. Keenan, Steam Tables and Mollier Diagram. 1930.



Table 9. H.L. Callendar.

t °C	0	50	100	150	200	250	275	300	325	350
$p$ kg/cm <sup>2</sup>	0.006225	0.1258	1.0332	4.855	15.86	40.6	60.7	87.7	123.0	168.7
$v$	206.1	12.03	1.671	0.392	0.1276	0.0508	0.0335	0.0223	0.0146	0.00886
$\Delta v$	-0.1	-0.01	—	—	—	+0.0002	+0.0001	+0.0003	—	—
$\Delta$	-0.05	-0.08	—	—	—	+0.40	+0.30	+1.40	—	—

t °C		100	150	200	250	300	350	400	450	500	550
1	$v$	1.727	1.971	2.212	2.450	2.687	2.924	3.160	3.400	3.631	3.867
	$\Delta v$	—	—	—	—	—	—	—	—	—	—
	$\Delta$	—	—	—	—	—	—	—	—	—	—
5	$v$			0.4326	0.4830	0.5322	0.5808	0.6289	0.6766	0.7242	0.7717
	$\Delta v$			-0.0001	—	—	—	—	—	—	—
	$\Delta$			-0.02	—	—	—	—	—	—	—
10	$v$			0.2101	0.2371	0.2629	0.2879	0.3125	0.3368	0.3609	0.3848
	$\Delta v$			—	—	—	—	—	—	+0.0001	—
	$\Delta$			—	—	—	—	—	—	+0.03	—
25	$v$				0.0893	0.1012	0.1122	0.1227	0.1329	0.1428	0.1527
	$\Delta v$				—	—	—	—	+0.0001	—	+0.0001
	$\Delta$				—	—	—	—	+0.1	—	+0.1
50	$v$					0.0469	0.0534	0.0593	0.0649	0.0702	0.0753
	$\Delta v$					—	—	—	+0.0002	—	+0.0001
	$\Delta$					—	—	—	+0.3	—	+0.1
100	$v$						0.0235	0.0275	0.0308	0.0338	0.0366
	$\Delta v$						—	—	—	—	—
	$\Delta$						—	—	—	—	—
150	$v$						0.0121	0.0165	0.0193	0.0216	0.0237
	$\Delta v$						—	—	—	—	—
	$\Delta$						—	—	—	—	—
200	$v$							0.01032	0.01335	0.01544	0.01718
	$\Delta v$							—	—	—	—
	$\Delta$							—	—	—	—
250	$v$							0.00538	0.00951	0.01165	0.01324
	$\Delta v$							-0.00093	—	—	—
	$\Delta$							-14.6	—	—	—

Table 10. R. Mollier.

t °C	0	50	100	150	200	250	275	300	325	350
$\rho$ kg/cm <sup>3</sup>	0.006225	0.1258	1.0332	4.855	15.86	40.6	60.7	87.7	123.0	168.7
$v$	206.5	12.05	1.675	0.392	0.1275	0.0500	0.0325	0.0213	0.0139	0.00882
$\Delta v$	—	—	—	—	—	—	—	—	—	—
$\Delta$	—	—	—	—	—	—	—	—	—	—

t °C		100	150	200	250	300	350	400	450	500	550
1	$v$	1.732	1.976	2.217	2.455	2.693	2.930	3.166	3.403	3.639	3.875
	$\Delta v$	—	—	—	—	—	+0.001	—	+0.002	+0.002	+0.002
	$\Delta$	—	—	—	—	—	+0.03	—	+0.06	+0.05	+0.05
5	$v$			0.4341	0.4846	0.5338	0.5824	0.6305	0.6783	0.7260	0.7735
	$\Delta v$			+0.0004	+0.0008	+0.0007	+0.0008	+0.0010	+0.0010	+0.0012	+0.0011
	$\Delta$			+0.09	+0.17	+0.13	+0.14	+0.16	+0.15	+0.17	+0.14
10	$v$			0.2108	0.2381	0.2639	0.2880	0.3135	0.3378	0.3619	0.3859
	$\Delta v$			+0.0003	+0.0004	+0.0005	+0.0006	+0.0008	+0.0009	+0.0011	+0.0011
	$\Delta$			+0.14	+0.17	+0.19	+0.21	+0.26	+0.27	+0.31	+0.29
25	$v$				0.0894	0.1017	0.1128	0.1233	0.1335	0.1434	0.1533
	$\Delta v$				+0.0001	+0.0004	+0.0005	+0.0006	+0.0007	+0.0006	+0.0007
	$\Delta$				+0.1	+0.4	+0.4	+0.5	+0.5	+0.4	+0.5
50	$v$					0.0469	0.0538	0.0598	0.0653	0.0706	0.0757
	$\Delta v$					—	+0.0003	+0.0005	+0.0006	+0.0005	+0.0005
	$\Delta$					—	+0.6	+0.8	+0.9	+0.7	+0.7
100	$v$						0.0234	0.0277	0.0311	0.0342	0.0370
	$\Delta v$						—	+0.0002	+0.0003	+0.0004	+0.0004
	$\Delta$						—	+0.7	+1.0	+1.2	+1.1
150	$v$						0.0118	0.0166	0.0196	0.0219	0.0240
	$\Delta v$						—	+0.0001	+0.0003	+0.0003	+0.0002
	$\Delta$						—	+0.6	+1.6	+1.4	+0.9
200	$v$							0.01043	0.01358	0.01574	0.01749
	$\Delta v$							+0.00010	+0.00023	+0.00024	+0.00024
	$\Delta$							+1.0	+1.8	+1.6	+1.4
250	$v$								0.00617	0.00977	0.01194
	$\Delta v$								-0.00014	+0.00022	+0.00029
	$\Delta$								-2.2	+2.3	+2.5

Table 11. O. Knoblauch, R. Raisch, H. Hausen.

t °C	0	50	100	150	200	250	275	300	325	350
$p$ kg/cm <sup>2</sup>	0.006225	0.1258	1.0332	4.855	15.86	40.6	60.7	87.7	123.0	168.7
$v$	206.6	12.06	1.674	0.393	0.1272	0.0498	0.0324	0.0214	0.0143	0.00958
$\Delta v$	—	—	—	—	—	—	—	—	—	+0.00043
$\Delta$	—	—	—	—	—	—	—	—	—	+4.9

t °C		100	150	200	250	300	350	400	450	500	550
1	$v$	1.731	1.976	2.217	2.457	2.695	2.933	3.171	3.407	3.644	3.885
	$\Delta v$	—	—	—	+0.002	+0.002	+0.004	+0.005	+0.006	+0.007	+0.012
5	$\Delta$	—	—	—	+0.08	+0.07	+0.14	+0.16	+0.18	+0.19	+0.31
	$v$			0.4340	0.4851	0.5349	0.5839	0.6325	0.6810	0.7289	0.7768
10	$\Delta v$			+0.0003	+0.0013	+0.0018	+0.0023	+0.0030	+0.0037	+0.0041	+0.0044
	$\Delta$			+0.07	+0.27	+0.34	+0.40	+0.48	+0.55	+0.57	+0.57
25	$v$			0.2105	0.2381	0.2642	0.2895	0.3143	0.3388	0.3632	0.3873
	$\Delta v$			—	+0.0004	+0.0008	+0.0012	+0.0016	+0.0019	+0.0024	+0.0025
50	$\Delta$			—	+0.17	+0.34	+0.42	+0.51	+0.56	+0.67	+0.65
	$v$				0.0891	0.1010	0.1117	0.1221	0.1321	0.1420	0.1518
100	$\Delta v$				—	—	—	—	-0.0001	—	—
	$\Delta$				—	—	—	—	-0.1	—	—
150	$v$					0.0460	0.0519	0.0573	0.0624	0.0674	0.0724
	$\Delta v$					-0.0001	-0.0008	-0.0014	-0.0017	-0.0019	-0.0018
200	$\Delta$					-0.2	-1.5	-2.4	-2.6	-2.7	-2.4
	$v$						0.0218	0.0249	0.0276	0.0301	0.0327
250	$\Delta v$						-0.0008	-0.0016	-0.0024	-0.0027	-0.0029
	$\Delta$						-3.5	-5.9	-7.9	-8.1	-8.0
300	$v$						0.0122	0.0148	0.0167	0.0185	0.0201
	$\Delta v$						—	-0.0007	-0.0018	-0.0023	-0.0027
350	$\Delta$						—	-4.4	-9.5	-10.9	-11.6
	$v$							0.01010	0.01173	0.01311	0.01442
400	$\Delta v$							-0.00013	-0.00102	-0.00169	-0.00203
	$\Delta$							-1.3	-7.8	-11.2	-12.1
450	$v$								0.00746	0.00918	0.01038
	$\Delta v$								+0.00105	-0.00007	-0.00077
500	$\Delta$								+16.5	-0.7	-6.8
	$\Delta$								—	-6.8	-0.3

Table 12. G. Eichelberg.

t °C	0	50	100	150	200	250	275	300	325	350
$\rho$ kg/cm <sup>2</sup>	0.006225	0.1258	1.0332	4.855	15.86	40.6	60.7	87.7	123.0	168.7
$v$	206.4	12.04	1.676	0.394	0.1272	0.0488	0.0308	0.0191	0.0112	0.00554
$\Delta v$	—	—	+0.001	+0.001	—	—	-0.0016	-0.0019	-0.0026	-0.00301
$\Delta$	—	—	+0.06	+0.3	—	—	-5.2	-8.8	-18.3	-34.4

t °C		100	150	200	250	300	350	400	450	500	550
1	$v$	1.732	1.975	2.215	2.453	2.690	2.927	3.163	3.399	3.635	3.871
	$\Delta v$	—	—	—	—	—	—	—	—	—	—
5	$v$			0.4341	0.4842	0.5332	0.5817	0.6297	0.6775	0.7251	0.7726
	$\Delta v$			+0.0004	+0.0004	+0.0001	+0.0001	+0.0002	+0.0002	+0.0003	+0.0002
10	$v$			0.2109	0.2379	0.2635	0.2885	0.3130	0.3372	0.3613	0.3851
	$\Delta v$			+0.0004	+0.0002	+0.0001	+0.0002	+0.0003	+0.0003	+0.0005	+0.0003
25	$v$				0.0890	0.1014	0.1124	0.1229	0.1331	0.1430	0.1529
	$\Delta v$				—	+0.0001	+0.0001	+0.0002	+0.0003	+0.0002	+0.0003
50	$v$					0.0462	0.0534	0.0594	0.0650	0.0703	0.0754
	$\Delta v$					—	—	+0.0001	+0.0003	+0.0002	+0.0002
100	$v$						0.0223	0.0272	0.0307	0.0338	0.0366
	$\Delta v$						-0.0003	—	—	—	—
150	$v$						0.0093	0.0155	0.0190	0.0215	0.0237
	$\Delta v$						-0.0021	—	—	—	—
200	$v$							0.00860	0.01271	0.01522	0.01711
	$\Delta v$							-0.00163	-0.00004	—	—
250	$v$							0.00323	0.00851	0.01127	0.01311
	$\Delta v$							-0.00308	-0.00074	—	—
	$\Delta$							-48.4	-7.9	—	—

Table 13. J. Keenan.

t °C	0	50	100	150	200	250	275	300	325	350
p kg/cm <sup>2</sup>	0.006225	0.1258	1.0332	4.855	15.86	40.6	60.7	87.7	123.0	168.7
v	206.3	12.04	1.673	0.393	0.1272	0.0502	0.0330	0.0221	0.0150	0.01021
Δv	—	—	—	—	—	—	—	+0.0001	+0.0004	+0.00126
Δ	—	—	—	—	—	—	—	+0.5	+2.8	+14.4

t °C		100	150	200	250	300	350	400	450	500	550
p kg/cm <sup>2</sup>	v	1.731	1.975	2.215	2.453	2.691	2.927	3.163	3.400	3.635	3.871
	Δv	—	—	—	—	—	—	—	—	—	—
	Δ	—	—	—	—	—	—	—	—	—	—
5	v			0.4341	0.4841	0.5330	0.5814	0.6295	0.6772	0.7249	0.7725
	Δv			+0.0004	+0.0003	—	—	—	—	+0.0001	+0.0001
	Δ			+0.09	+0.06	—	—	—	—	+0.01	+0.01
10	v			0.2114	0.2379	0.2633	0.2881	0.3126	0.3369	0.3611	0.3851
	Δv			+0.0009	+0.0002	—	—	—	—	+0.0003	+0.0003
	Δ			+0.43	+0.08	—	—	—	—	+0.08	+0.08
25	v				0.0902	0.1015	0.1122	0.1226	0.1327	0.1428	0.1527
	Δv				+0.0009	+0.0002	—	—	—	—	+0.0001
	Δ				+1.0	+0.20	—	—	—	—	+0.1
50	v					0.0475	0.0535	0.0592	0.0647	0.0700	0.0752
	Δv					+0.0006	—	—	—	—	—
	Δ					+1.3	—	—	—	—	—
100	v						0.0242	0.0275	0.0306	0.0336	0.0365
	Δv						+0.0006	—	—	—	—
	Δ						+2.6	—	—	—	—
150	v						0.0144	0.0170	0.0193	0.0215	0.0236
	Δv						+0.0020	+0.0005	—	—	—
	Δ						+16.0	+3.1	—	—	—
200	v							0.01167	0.01362	0.01541	0.01711
	Δv							+0.00134	+0.00027	—	—
	Δ							+13.0	+2.1	—	—
250	v							0.00850	0.01021	0.01177	0.01323
	Δv							+0.00209	+0.00066	+0.00012	—
	Δ							+32.9	+7.0	+1.1	—

small, mostly under 1%, the maximum (2.5%) being at 250 kg/cm<sup>2</sup> and 500°C.

In Table 11, almost as many of the values calculated by Equation (8) lie outside the region of tolerance as of those calculated by Equation (7), but the amounts of deviation are larger, the maximum being 16.5% at 250 kg/cm<sup>2</sup> and 400°C.

In Table 12, the values calculated by Equation (9) are given. A larger number of the values lie inside the region of tolerance than in the case of those calculated by Equations (7) and (8). Moreover the amounts of the deviations are comparatively small, but near saturation at high pressure, the deviation increases very rapidly and amounts to as much as 48.4% at 250 kg/cm<sup>2</sup> and 400°C.

In Table 13, we see that the values calculated by Equation (10) almost all lie inside the region of tolerance in regard to steam with certain amount of superheating, but at and near saturation the values are far out, the maximum deviation amounting to 32.9% at 250 kg/cm<sup>2</sup> and 400°C.

Very recently H. Hausen<sup>(26)</sup> proposed a new characteristic equation of superheated steam which is based upon the experimental results of specific heat under constant pressure of superheated steam up to the pressure of 120 kg/cm<sup>2</sup> obtained in the München Laboratory. He constructed an empirical equation for  $c_p$  and deduced thermodynamically the characteristic equation of superheated steam with proper constants. He states that his equation gives satisfactory results up to very high pressures, only near saturation at high pressures the values calculated by his equation show a certain amount of deviation. Thus in constructing his new steam tables (not yet published), which should satisfy all the conditions of the Standard Skeleton Tables, he had to use, in addition to his equation, the graphical method and the method of stepwise integration.

The present author has calculated by the said equation the values of the specific volume for the temperatures and pressures given in the

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(26) H. Hausen, "Eine neue Zustandsgleichung des Wasserdampfes." *Forschung* Bd. 2 Heft 9, 1931.

Standard Skeleton Tables. But unfortunately, despite of Hausen's statement to the contrary, almost all the values thus calculated were found to fall outside the region of tolerance of the Standard Skeleton Tables. There may have been some mistakes in the calculation or misprints in that paper, so the author has omitted the results of calculation by H. Hausen's new formula.

B) A NEW CHARACTERISTIC EQUATION PROPOSED  
BY THE AUTHOR.

The author has found by the method of trial, that the following empirical equation gives satisfactory values of specific volume of superheated steam in the whole range of the Standard Skeleton Tables.

$$v = \frac{RT}{p} - \frac{0.60}{\left(\frac{T}{100}\right)^{2.6}} - \frac{42p}{\left(\frac{T}{100}\right)^{14}} - \frac{1.26 \cdot 10^{-7} p^3 - 8.16 \cdot 10^{-34} p^7}{\left(\frac{T}{100}\right)^{18}} - \frac{22}{p + 1000} \dots\dots\dots (III)$$

where  $v$  is the specific volume of superheated and dry saturated steam at  $p$  and  $T$  in  $m^3/kg$ ,

$p$ , the pressure in  $kg/m^2$ ,

$T = t + 273.20$ ,<sup>(27)</sup> the absolute temperature in the Centigrade scale.

$R$ , the gas constant, taking at  $47.05 m/^\circ C$ .<sup>(28)</sup>

Table 14 shows the values of  $v$  calculated by Equation (III), which lie all inside the region of tolerance of the Standard Skeleton Tables.

(27) See foot note (12)

(28) According to Henning (Z.S. f. Phys. Bd. 6 p. 69-72 1921)

$$R = (8.313 \pm 0.003) \cdot 10^7 \text{ erg/degree Mol.} \\ = \frac{(8.313 \pm 0.003) \cdot 10^7}{980.665} = 847.690 \pm 0.306 \text{ mkg/degree Mol.}$$

Therefore the gas constant for  $H_2O$  is,

$$R_{H_2O} = \frac{847.69 \pm 0.31}{18.016} = 47.052 \pm 0.017;$$

so we take

$$R = 47.05 m/^\circ C.$$

Table 14. S. Sugawara.

t °C	0	50	100	150	200	250	275	300	325	350
p <sup>kg/cm<sup>2</sup></sup>	0.006225	0.1258	1.0332	4.855	15.86	40.6	60.7	87.7	123.0	168.7
v	206.4	12.05	1.674	0.392	0.1270	0.0500	0.0327	0.0215	0.0140	0.00883

t °C	100	150	200	250	300	350	400	450	500	550
p <sup>kg/cm<sup>2</sup></sup>										
1	1.730	1.974	2.214	2.452	2.689	2.925	3.161	3.397	3.633	3.869
5			0.4336	0.4836	0.5325	0.5808	0.6288	0.6766	0.7242	0.7717
10			0.2103	0.2375	0.2630	0.2878	0.3123	0.3366	0.3606	0.3846
25				0.0891	0.1011	0.1120	0.1224	0.1325	0.1425	0.1523
50					0.0466	0.0532	0.0590	0.0645	0.0698	0.0749
100						0.0232	0.0272	0.0304	0.0334	0.0362
150						0.0119	0.0162	0.0190	0.0212	0.0233
200							0.01027	0.01312	0.01511	0.01681
250							0.00635	0.00950	0.01141	0.01292

III. EQUATIONS FOR HEAT CONTENT OF SUPERHEATED STEAM.

A) COMPARISON OF THE NUMERICAL VALUES OF THE HEAT CONTENT OF SUPERHEATED AND DRY SATURATED STEAM CALCULATED BY THE EQUATIONS GIVEN BY SEVERAL AUTHORITIES, WITH THOSE OF THE STANDARD SKELETON TABLES.

If we have an equation expressing the specific volume of steam in terms of its pressure and temperature, the equation for the heat content can be deduced thermodynamically. We have the following general thermodynamical relation :—

$$\left(\frac{\partial H}{\partial p}\right)_T = -AT^2 \left(\frac{\partial\left(\frac{v}{T}\right)}{\partial T}\right),$$

whence 
$$H = \varphi(T) - AT^2 \int_{T=\text{const.}} \left(\frac{\partial\left(\frac{v}{T}\right)}{\partial T}\right)_p dp \dots\dots\dots(11)$$



where  $H$  denotes the heat content,  $p$ ,  $v$ ,  $T$  denote as before the pressure, specific volume and absolute temperature respectively, and  $\varphi(T)$  is a function of the temperature alone.

Combining Equation (11) with the characteristic equations given by the several authorities mentioned above, and expressing  $\varphi(T)$  in suitable forms with suitable numerical constants, they got the following equations respectively.

1) **H. L. Callendar**

$$H = 464.17 + 0.477 T - \frac{101.5}{10^4} \frac{f(T) \cdot p}{1 - \left( \frac{0.0508 \cdot f(T) \cdot p}{T} \right)^2} - \frac{0.0041 p}{10^4} \left. \dots (12)^{(29)} \right\}$$

$$f(T) = 0.0263 \left( \frac{373.1}{T} \right)^{\frac{10}{3}}$$

2) **R. Mollier**

$$H = 595 + 0.47(T - 273) - \frac{202.96 \cdot 10^{-4} p}{\left( \frac{T}{100} \right)^{\frac{10}{3}}} - \frac{2.2248 \cdot 10^{-6} p^3}{\left( \frac{T}{100} \right)^{14}} \dots (13)$$

3) **O. Knoblauch, R. Raisch & H. Hausen.**

$$H = 436.13 + 0.3391 T + 0.0000985 T^2 - 29.473 \log_{10}(T - 256.4) + 20.33 \log_e [T - \varphi(p)] + A \int \psi(p) dp \dots (14)$$

4) **G. Eichelberg**

$$H = 513.2 + 0.345 T + 0.0000985 T^2 - \frac{5500}{T} - \frac{1.0667 \cdot 10^4 p}{T^2} - \frac{2.83 \cdot 10^{22} (p + 2 \cdot 10^4)^{3.2} - 1.64 \cdot 10^{36}}{14 T^{14}} \dots (15)$$

5) **J. Keenan**

$$H = 350.23 + 0.2708 T + 0.000135 T^2 + 66.775 \log_{10} T - 1.585 \cdot 10^7 \frac{p}{T^{4.2}} - 0.90653 \cdot 10^{17} \frac{p^{\frac{9}{4}}}{T^{10.5}} - 0.14918 \cdot 10^{-4} p \dots (16)$$

(29) Eq. (12) was deduced on the assumption that the adiabatic change of steam is given by the equation

$$pv^{\frac{n+1}{n}} = \text{constant.}$$

The following five tables have been prepared by the author in order to compare the values of heat content of superheated steam calculated by Equations (12), (13), (14), (15) and (16), with those given in the Standard Skeleton Tables.

Table 15. H.L. Callendar.

t °C	0	50	100	150	200	250	275	300	325	350
$p$ kg/cm <sup>2</sup>	0.006225	0.1258	1.0332	4.855	15.86	40.6	60.7	87.7	123.0	168.7
$H$	594.4	617.8	639.4	657.4	669.8	673.9	671.3	663.6	648.1	618.1
$\Delta H$	-0.1	—	—	—	+0.3	+0.9	+0.3	+1.6	—	—

t °C		100	150	200	250	300	350	400	450	500	550
$p$ kg/cm <sup>2</sup>											
1	$H$	639.5	664.2	688.6	712.8	736.9	760.9	784.9	808.8	832.7	856.6
	$\Delta H$	—	—	+0.1	—	—	—	—	—	—	—
5	$H$			683.8	709.3	734.3	759.0	783.4	807.6	831.8	855.8
	$\Delta H$			—	—	—	—	—	—	—	—
10	$H$			677.5	704.9	731.1	756.5	781.5	806.1	830.5	854.8
	$\Delta H$			—	—	—	—	—	—	—	-0.2
25	$H$				691.0	721.2	749.1	775.8	801.6	827.0	851.9
	$\Delta H$				—	—	—	—	—	—	—
50	$H$					702.7	736.1	766.0	794.0	820.9	847.0
	$\Delta H$					—	—	—	—	—	—
100	$H$						704.1	744.4	778.0	808.4	837.0
	$\Delta H$						—	—	—	—	—
150	$H$						651.2	716.9	759.8	795.1	826.6
	$\Delta H$						—	—	—	—	—
200	$H$							676.4	737.8	780.2	815.6
	$\Delta H$							—	—	—	—
250	$H$							604.0	708.8	763.1	803.6
	$\Delta H$							-15.0	—	—	—

Table 16. R. Mollier.

t °C	0	50	100	150	200	250	275	300	325	350
$p$ kg/cm <sup>2</sup>	0.006225	0.1258	1.0332	4.855	15.86	40.6	60.7	87.7	123.0	168.7
$H$	595.0	618.0	639.4	657.0	667.7	666.4	659.2	646.6	630.1	602.1
$\Delta H$	—	—	—	—	—	—	-1.8	-5.3	-6.9	-0.9

t °C		100	150	200	250	300	350	400	450	500	550
1	H	639.5	663.8	687.9	711.7	735.4	759.0	782.7	806.2	829.8	853.3
	ΔH	—	—	—	—	—	—	—	—	-1.2	-3.7
5	H			683.2	708.4	733.0	757.2	781.2	805.1	828.9	852.6
	ΔH			—	—	—	—	—	—	-1.1	-2.9
10	H			676.8	704.1	729.9	754.9	779.5	803.7	827.8	851.7
	ΔH			—	—	—	—	—	—	-1.2	-3.3
25	H				689.0	720.1	747.8	774.1	799.5	824.4	849.0
	ΔH				—	—	—	—	—	+0.6	-2.0
50	H					699.1	734.6	764.6	792.3	818.8	844.5
	ΔH					—	—	—	—	-1.2	-2.5
100	H						697.1	742.0	776.6	807.0	835.2
	ΔH						-1.4	—	—	-0.5	-1.8
150	H						634.5	710.9	757.8	793.9	825.4
	ΔH						-10.0	-1.6	—	-0.1	-1.1
200	H							666.9	734.2	779.0	814.8
	ΔH							-6.6	—	—	—
250	H							605.8	704.3	761.7	803.2
	ΔH							-13.2	-0.7	—	—

Table 17. O. Knoblauch, R. Raisch, H. Hausen.

t °C	0	50	100	150	200	250	275	300	325	350
p kg/cm <sup>2</sup>	0.006225	0.1258	1.0332	4.855	15.86	40.6	60.7	87.7	123.0	168.7
H	596.8	619.2	639.8	656.1	665.3	666.3	665.1	664.1	663.5	658.9
ΔH	+0.3	—	+0.1	—	—	—	—	+2.1	+14.5	+35.9

t °C		100	150	200	250	300	350	400	450	500	550
1	H	639.9	663.7	687.1	710.6	734.3	758.3	782.6	807.3	832.4	858.0
	ΔH	—	—	—	—	—	—	—	—	—	—
5	H			681.8	706.7	731.3	755.9	780.7	805.8	831.2	856.9
	ΔH			—	—	—	—	—	—	—	—
10	H			674.8	701.9	727.8	753.2	778.5	804.0	829.7	855.7
	ΔH			—	—	—	—	—	—	—	—
25	H				687.0	718.0	746.0	772.9	799.4	825.9	852.5
	ΔH				—	—	—	-0.1	-0.1	—	—
50	H					701.8	735.4	764.9	792.8	820.2	847.5
	ΔH					—	—	—	—	—	—
100	H						715.5	751.2	781.8	810.7	838.9
	ΔH						+11.0	+6.2	+1.8	—	—
150	H						684.7	735.6	769.5	799.8	828.8
	ΔH						+32.2	+18.1	+8.0	+0.8	—
200	H							719.2	758.5	790.7	820.6
	ΔH							+40.7	+18.5	+4.7	—
250	H							694.5	747.5	782.2	813.2
	ΔH							+67.5	+32.5	+12.2	—

Table 18. G. Eichelberg.

t °C	0	50	100	150	200	250	275	300	325	350
$p$ kg/cm <sup>2</sup>	0.006225	0.1258	1.0332	4.855	15.86	40.6	60.7	87.7	123.0	168.7
$\frac{H}{\Delta H}$	594.5 —	617.3 -0.2	638.4 -0.3	655.9 —	666.2 —	661.6 -3.4	649.8 -11.2	628.9 -23.1	596.7 -40.3	549.8 -57.2

t °C $p$ kg/cm <sup>2</sup>		100	150	200	250	300	350	400	450	500	550	
		1	$\frac{H}{\Delta H}$ 638.4 -0.5	662.3 -0.2	685.8 —	709.3 —	733.1 —	757.1 —	781.5 —	806.2 —	831.4 —	857.0 —
5	$\frac{H}{\Delta H}$ —	—	681.5 —	706.3 —	730.8 —	755.3 —	780.1 —	805.1 —	830.5 —	856.2 —		
10	$\frac{H}{\Delta H}$ —	—	675.4 —	702.3 —	727.9 —	753.1 —	778.3 —	803.7 —	829.3 —	855.3 —		
25	$\frac{H}{\Delta H}$ —	—	—	687.2 —	718.3 —	746.1 —	773.0 —	799.4 -0.1	825.8 —	852.4 —		
50	$\frac{H}{\Delta H}$ —	—	—	—	695.7 -1.3	732.5 —	763.3 —	792.0 —	819.9 -0.1	847.6 —		
100	$\frac{H}{\Delta H}$ —	—	—	—	—	687.7 -10.8	738.1 -1.9	775.1 —	807.3 -0.2	837.5 —		
150	$\frac{H}{\Delta H}$ —	—	—	—	—	—	599.2 -45.3	698.1 -14.4	752.7 -3.8	792.5 -1.5	826.6 —	
200	$\frac{H}{\Delta H}$ —	—	—	—	—	—	—	634.1 -39.4	721.6 -12.4	774.3 -3.7	814.3 —	
250	$\frac{H}{\Delta H}$ —	—	—	—	—	—	—	—	536.6 -82.4	678.1 -26.9	751.2 -8.8	799.9 —

Table 19. J. Keenan.

t °C	0	50	100	150	200	250	275	300	325	350
$p$ kg/cm <sup>2</sup>	0.006225	0.1258	1.0332	4.855	15.86	40.6	60.7	87.7	123.0	168.7
$\frac{H}{\Delta H}$	596.8 +0.3	618.7 —	638.9 —	655.6 —	666.4 —	668.9 —	666.5 —	661.3 —	653.1 +4.1	641.5 +18.5

$p$ kg/cm <sup>2</sup>	t °C	100	150	200	250	300	350	400	450	500	550
		1	H ΔH	639.0 —	662.7 —	686.1 —	709.6 —	733.3 —	757.5 —	782.1 —	807.2 —
5	H ΔH			681.5 —	706.4 —	731.1 —	755.7 —	780.7 —	806.0 —	831.8 —	858.0 —
10	H ΔH			675.1 —	702.3 —	728.1 —	753.5 —	778.8 —	804.4 —	830.4 —	856.8 —
25	H ΔH				687.8 —	718.5 —	746.4 —	773.2 —	799.8 —	826.4 —	853.2 —
50	H ΔH					699.2 —	733.2 —	763.2 —	791.7 —	819.5 -0.5	847.1 —
100	H ΔH						702.4 —	741.2 —	774.6 -0.4	805.2 -2.3	834.7 -2.3
150	H ΔH						659.5 +7.0	713.7 —	755.0 -1.5	789.7 -4.3	821.6 -4.9
200	H ΔH							682.4 +3.9	733.6 -0.4	773.3 -4.7	808.0 -6.0
250	H ΔH							646.0 +19.0	709.9 —	755.8 -4.2	793.9 -3.1

In these tables, the symbol  $H$  denotes the value of the heat content calculated by the respective equation and  $\Delta H$  the amount of deviation of the value of the heat content thus calculated *outside the region of tolerance* of the Standard Skeleton Tables, both expressed in kcal/kg.

We see from Table 15, that the values calculated by Equation (12) fall in the region of tolerance of the Standard Skeleton Tables except for a few values at and near saturation.

In Table 16, the values calculated by Equation (13) also coincide pretty well with those of the Standard Skeleton Tables. Only in the dry saturated conditions under high pressures and also at high temperatures, there are some deviations, but they are very small.

In Table 17, the values calculated by Equation (14) are satisfactory at moderate pressures, but in the high pressure region, the deviation is pretty large.

For the values in Table 18, which are calculated by Equation (15), we can say almost the same thing.

In Table 19, we can see that the values calculated by Equation (16) coincide very well with those of the Standard Skeleton Tables, next to the values given in Table 15, but at high pressures they are almost all outside the region of tolerance.

### B) A NEW EQUATION FOR THE HEAT CONTENT OF SUPERHEATED AND DRY SATURATED STEAM.

Now if we use Equation (III) for the deduction of the expression of the heat content of superheated steam, we shall get the equation for the heat content in the following manner.

From Eq. (III) we have

$$\frac{v}{T} = \frac{R}{p} - \frac{0.60 \cdot 100^{2.6}}{T^{2.6}} - \frac{42 \cdot 100^{14} p}{T^{15}} - \frac{(1.26 \cdot 10^{-7} p^3 - 8.16 \cdot 10^{-34} p^7) \cdot 100^{18}}{T^{19}} - \frac{22}{T(p+1000)}.$$

$$\therefore \left( \frac{\partial v}{\partial T} \right)_p = 0 + \frac{3.6 \cdot 0.60 \cdot 100^{2.6}}{T^{4.6}} + \frac{15 \cdot 42 \cdot 100^{14} p}{T^{16}} + \frac{19(1.26 \cdot 10^{-7} p^3 - 8.16 \cdot 10^{-34} p^7) \cdot 100^{18}}{T^{20}} + \frac{22}{T^2(p+1000)},$$

$$AT^2 \left( \frac{\partial v}{\partial T} \right)_p = \frac{5.0576 \cdot 10^{-3}}{\left( \frac{T}{100} \right)^{2.6}} + \frac{1.47513 p}{\left( \frac{T}{100} \right)^{14}} + \frac{5.605507 \cdot 10^{-9} p^3 - 3.63023 \cdot 10^{-35} p^7}{\left( \frac{T}{100} \right)^{18}} + \frac{0.05151}{p+1000},$$

$$AT^2 \int_{T=\text{const.}} \left( \frac{\partial v}{\partial T} \right)_p dp = \frac{5.0576 \cdot 10^{-3} p}{\left( \frac{T}{100} \right)^{2.6}} + \frac{0.73757 p^2}{\left( \frac{T}{100} \right)^{14}} + \frac{1.401377 \cdot 10^{-9} p^4 - 4.5378 \cdot 10^{-36} p^8}{\left( \frac{T}{100} \right)^{18}} + 0.05151 \log_e (p+1000).$$

$$\therefore H = \varphi(T) - \frac{5.0576 \cdot 10^{-8} p}{\left(\frac{T}{100}\right)^{2.6}} - \frac{0.7376 p^3}{\left(\frac{T}{100}\right)^{14}} - \frac{1.401 \cdot 10^{-9} p^4 - 4.538 \cdot 10^{-36} p^8}{\left(\frac{T}{100}\right)^{18}} - 0.119 \log_{10}(p + 1000) \dots (IV)$$

If we give the following expression for the function  $\varphi(T)$ , Equation (IV) gives values of  $H$  which fall in the whole range inside the region of the tolerance of the Standard Skeleton Tables :

$$\varphi(T) = 596.6 + 0.456(T - 273.20) + 7.4 \cdot 10^{-8}(T - 273.20)^3 \dots (IV_a)$$

Table 20 gives the values of the heat content  $H$  in int. kcal/kg calculated by Equation (IV) combined with the expression for  $\varphi(T)$  given by Equation (IV<sub>a</sub>).

Table 20. S. Sugawara.

t °C	0	50	100	150	200	250	275	300	325	350
p kg/cm <sup>2</sup>	0.006225	0.1258	1.0332	4.855	15.86	40.6	60.7	87.7	123.0	168.7
H	596.2	618.6	639.3	655.9	666.5	668.3	664.3	655.1	639.3	614.6

t °C \ p kg/cm <sup>2</sup>	100	150	200	250	300	350	400	450	500	550
1	639.4	663.5	687.0	710.6	734.4	758.5	782.9	807.8	833.1	859.0
5			682.7	707.6	732.1	756.6	781.4	806.5	832.1	858.1
10			676.2	703.7	729.2	754.4	779.6	805.0	830.8	857.0
25				689.4	720.0	747.5	774.1	800.5	827.0	853.8
50					701.3	735.2	764.7	792.9	820.7	848.4
100						702.8	743.9	777.2	807.9	837.7
150							646.3	716.8	794.6	825.9
200								678.1	738.8	816
250									623.1	713.4
										764.5
										804

In conclusion, the present author desires to say that he considers that his new characteristic equation of superheated steam and also the equation for the heat content of superheated steam, both of which fulfil all

the conditions of the Standard Skeleton Tables, are not very much more complicated than the others and that their form is as convenient for practical purposes as the others. These equations are, therefore, very useful at least for the construction of international steam tables and diagrams, in the region of superheated steam covered by the Skeleton Tables of the second Steam Table Conference.

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