On the Two-dimensional Flow around Slotted Wing Sections*

By Busuke Hudimoto

The flow of the perfect fluid around a circle and a circular arc is investigated. Then they are transformed conformally into a slotted wing section and the lift and the moment of the lift are calculated.

The problems connected with the two-dimensional flow of the perfect fluid around slotted wing sections have been investigated by some authors. The first of them was Tchapliguine,[1] who studied the two-dimensional flow around a wing section composed of a number of segments of one circular arc. The second was Lachmann,[2] who treated the problem of the two-dimensional flow around the Joukowski section with an auxiliary aerofoil. As a result of his investigation the auxiliary aerofoil was replaced by a number of vortices placed on the skeleton line of the auxiliary aerofoil. The last one was Watanabe.[3] He placed a sink and a source of the same strength on the surface of the Joukowski section, the points being the centres of the inlet and exit port of the slot.

In the present paper, the author deals with the two-dimensional flow around a circle and a circular arc and from them by the wellknown conformal transformation a kind of slotted wing section was obtained.

1. Conformal Transformation

The aerofoil of the wing section considered here is obtained entirely by conformal transformation.

Consider in the z-plane Fig. 1, a circle and a circular arc BB'. The axes of x and y are so chosen that the x-axis, i.e. the real axis, passes through the two intersections H and H' of the circle and the circle of the arc BB'. The y-axis, i.e. the imaginary axis, passes through the mid point of HH' and perpendicular to the x-axis. Then, z is expressed by

$$z=x+iy$$

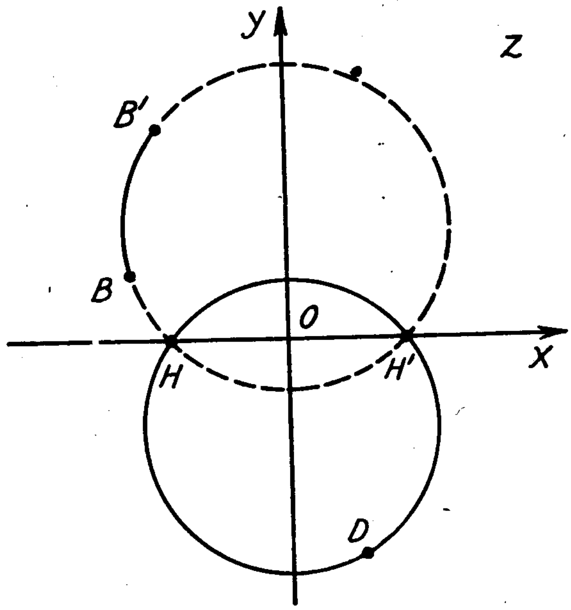
Now the z-plane is transformed into the f-plane (Fig. 2.) by the following relation

$$f=\log\frac{z+a}{z-a} \qquad (1)$$

where $a=$ the length of $OH=OH'$ (Fig. 1.)

Then, the segments of the circular arc BB' are transformed into a straight line parallel to the real axis of the f-plane and the distance between them is equal to the angle between two straight lines connecting any point on the arc BB' and H and H' respectivly in the z-plane. The circle is transformed into two straight lines also parallel to the real axis extending to infinity. The distance between them is equal to π and the outside region of the circle is transformed into a strip section

Fig. 1



between the above-mentioned two straight lines, and the points H and H' of the z-plane correspond to infinity.

Then if the f-plane is cut along the straight line CG or $C'G'$ passing through the mid point of BB' and perpendicular to the real axis (see Fig. 2.), the f-plane is transformed into a rectangular region of the s-plane by the following relation :[4]

$$\frac{df}{ds}=\frac{\wp'(\nu)}{\wp(\nu)-\wp(\mu)}\,\frac{\wp(s)-\wp(\mu)}{\wp(s)-\wp(\nu)} \qquad (2)$$

Fig. 2

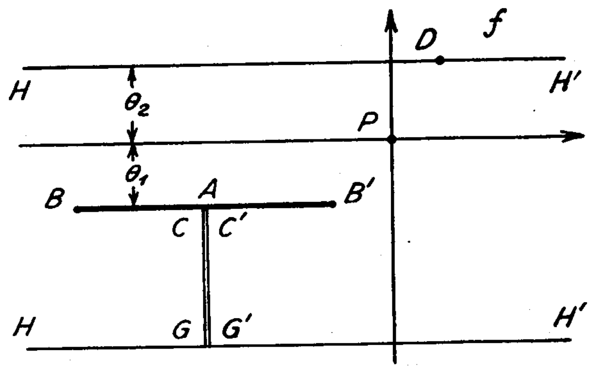


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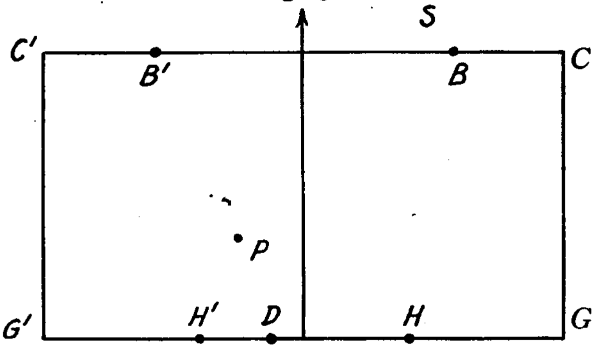
Fig. 3



where $\wp$ is the elliptic function of Weierstrass, and $\wp'$ is its derivative,

and $s=\nu$ for point H,

$s=-\nu$ H'.

and $s=\mu$ B.

Integrating the above relation, we get

$$f=[\zeta(\mu+\nu)-\zeta(\mu-\nu)](s-\mu)$$
$$-\log\frac{\sigma(s+\nu)\sigma(\mu-\nu)}{\sigma(s-\nu)\sigma(\mu+\nu)}+\text{constant}, \quad (3)$$

where ζ is the ζ-function of Weierstrass.

Now, let the distance between the real axis and BB' in the f-plane be equal to θ_1, and the co-ordinates of B be p and θ_1, i.e. $s=\mu$ corresdonds to $f=p+i\theta_1$.

Then the constant in eq. (3) is determined and

$$f=[\zeta(\mu+\nu)-\zeta(\mu-\nu)](s-\mu)$$
$$-\log\frac{\sigma(s+\nu)\sigma(\mu-\nu)}{\sigma(s-\nu)\sigma(\mu+\nu)}+p+i\theta_1 \quad (4)$$

We denote the breadth of this rectangle by $2\omega_1$ and the height by ω_2/i. Now we cut off the f-plane along CG or $C'G'$, and to prevent the discontinuity of the flow at this section, the function f defined by eq. (4) must be a periodic function whose period is equal to $2\omega_1$. So it is necessary that μ and ν satisfy the following condition,

$$\zeta(\mu+\nu)-\zeta(\mu-\nu)=\frac{2\eta_1\nu}{\omega_1}, \quad (5)$$

where $\eta_1=\zeta(\omega_1)$.

Now let the distance between the real axis of the f-plane and the straight line HDH' be equal to θ_2, and the values of f at the points C and G be equal to f_c and f_g, then

$$f_c-f_g=\frac{2\eta_1\nu}{\omega_1}\omega_2-2\nu\eta_2, \quad (6)$$

where $\eta_2=\zeta(\omega_2)$. By the relation

$$\eta_1\omega_2-\eta_2\omega_1=\frac{\pi i}{2},$$

we have,

$$f_c-f_g=\frac{\nu\pi}{\omega_1}i, \quad (7)$$

and this must be exual to $i(\pi+\theta_1-\theta_2)$, so

$$\nu=\omega_1\left(1+\frac{\theta_1-\theta_2}{\pi}\right). \quad (8)$$

θ_1 and θ_2 are determined when the circle and the circular arc are given, so ν can be determined by eq. (8).

And let the length of BB' in the f-plane be equal to $2l$, and the values of f at the points B and A be f_b and f_a, then, by eq. (4)

$$f_a-f_b=\frac{2\eta_1\nu}{\omega_1}(\omega_2-\mu)-\log\frac{\sigma(\omega_2+\nu)\sigma(\mu-\nu)}{\sigma(\omega_2-\nu)\sigma(\mu+\nu)} \quad (9)$$

and this must be equal to l. Then, if $\mu=m+\omega_2$ and if we put this into the above eq. (9) and introduce the ϑ-function, the relation between m, ν and l is determined.

$$l=\log\frac{\vartheta_0\left(\frac{m+\nu}{2\omega_1}\right)}{\vartheta_0\left(\frac{m-\nu}{2\omega_1}\right)} \quad (10)$$

And if the origin of the f-plane P corresponds to the point $s=s_0$ in the s-plane, then,

$$0=\frac{2\eta_1\nu}{\omega_1}(s_0-\mu)-\log\frac{\sigma(s_0+\nu)\sigma(\mu-\nu)}{\sigma(s_0-\nu)\sigma(\mu+\nu)}+p+i\theta_1,$$

and from this relation, we have

$$p=\log\frac{\vartheta_1\left(\frac{s_0+\nu}{2\omega_1}\right)}{\vartheta_1\left(\frac{s_0-\nu}{2\omega_1}\right)}-l+i\left(\frac{\nu\pi}{\omega_1}-\theta_1\right) \quad (11)$$

From eqs. (8), (10), and (11), ν, μ and s_0 can be determined, though it is very tedious work in practice.

Now returning to the z-plane, we transform the z-plane to the z_1-plane by

$$z_1=(z+m)e^{-i\theta} \quad (12)$$

the amount of the translation being m and generally a complex number, and θ is the angle of rotation. Then transform the z_1-plane into the z_2-plane by

$$z_2=z_1+\frac{r_0^2}{z_1} \quad (13)$$

This is the usual transformation applied when a circle is transformed into the Joukowski section and we get a Joukowski section with an auxiliary aerofoil in the z_2-plane.

2. The Potential Flow of the Perfect Fluid.

We consider now the steady irrotational flow of the perfect fluid. The flow in the rectangular regions is represented by using the elliptic functions.

The parallel flow at infinity in the z-plane is represented by the flow due to a doublet placed on the origin of the f-plane, and it is then equivalent to the flow due to a doublet placed on the point $s=s_0$. The circulation flow around the circle and the circular arc in the z-plane is represented by the flow due to an irrotational vortex placed on the origin in the f-plane and so at the point $s=s_0$ in the s-plane.

Now let W be a complex velocity potential of the flow, then $\frac{dW}{ds}$ is the conjugate complex of the actual velocity. The boundary condition to be fullfilled is that the normal component of the velocity on the sides CC' and GG' of the rectangle must vanish. And $\frac{dW}{ds}$ must be a periodic function with the period of $2\omega_1$.

The velocity due to the doublet in a rectangular region is expressed by the $\wp$-function and that of the vortex is expressed by the ζ-function. In the present problem,

$$\frac{dW}{ds}=\frac{i\Gamma}{2\pi}\left[\zeta(s-s_0)-\zeta(s-\bar{s}_0)\right]$$
$$-\left[w_s\wp(s-s_0)+\bar{w}_s\wp(s-\bar{s}_0)\right]+k, \quad (14)$$

where Γ is the strength of the vortex and w_s is that of the doublet and this is generally a complex number and we denote it by $w_s=u_s-iv_s$, and $\bar{w}_s$ is its conjugate complex i.e. $\bar{w}_s=u_s+iv_s$. Also we denote the conjugate complex of s_0 by the letter $\bar{s}_0$, and k is a real constant. It is easily verified that the above eq. (14) satisfies the boundary conditions.

In eq. (14) Γ and k are unknowns, and they are determined by two conditions. For these conditions we take the usual condition as in the case of a Joukowski section, namely, at the point corresponding to the trailing edge the velocity is equal to zero. In our case we have two points, one point corresponding to the trailing edge of the auxiliary aerofoil and this is the point B, and the other point is on the circle and we denote it by D.

By equating the velocity at the points B and D to zero, we get the value of Γ and k.

Then, it is necessary to express the strength of the doublet w_s by the velocity of the fluid at infinity in the z-plane. The parallel flow with the velocity w_z in the z-plane is equivalent to

$$-\frac{2aw_z}{f^2}$$

in the neighbourhood of the origin, and this is equivalent to $\quad -\frac{2aw_z}{(s-s_0)^2}\left(\frac{ds}{df}\right)_{s=s_0}$

in the s-plane, where $\left(\frac{ds}{df}\right)_{s=s_0}$ is the value of $\frac{ds}{df}$ at $s=s_0$. Comparing this and

$$-w_s\wp(s-s_0)_{s=s_0}\approx-\frac{w_s}{(s-s_0)^2},$$

we get

$$w_s=2aw_z\left(\frac{ds}{df}\right)_{s=s_0} \quad (15)$$

The flow is determined by eqs. (14) and (15)

3. Calculation of the Lift.

The lift acting on the aerofoil is calculated by the well-known Blasius formula. Let the lift be equal to $P_{x_2}+iP_{y_2}$, then,

$$P_{x_2}-iP_{y_2}=\frac{\rho i}{2}\oint\left(\frac{dW}{dz_2}\right)^2 dz_2 \quad (16.)$$

where ρ is the density of the fluid.
We evaluate the above integral in the s-plane instead of the z_2-plane.

$$\oint\left(\frac{dW}{dz_2}\right)^2 dz_2=\oint\left(\frac{dW}{ds}\right)^2\frac{ds}{dz_2}ds$$

Integration is performed around the point s_0 in the opposite direction.

Now in the neighbourhood of s_0

$$\frac{df}{ds}=A\left\{1+c_1(s-s_0)+c_2(s-s_0)^2+\dots\right\}, \quad (17)$$

where

$$A=\frac{\wp'(\nu)}{\wp(\nu)-\wp(\mu)}\,\frac{\wp(s_0)-\wp(\mu)}{\wp(s_0)-\wp(\nu)},$$
$$c_1=\wp'(s_0)\left\{\frac{1}{\wp(s_0)-\wp(\mu)}-\frac{1}{\wp(s_0)-\wp(\nu)}\right\},$$
$$c_2=\frac{\wp''(s_0)}{2}\left\{\frac{1}{\wp(s_0)-\wp(\mu)}-\frac{1}{\wp(s_0)-\wp(\nu)}\right\}$$
$$-c_1\frac{\wp'(s_0)}{\wp(s_0)-\wp(\nu)}. \quad (18)$$

Integrating this and by the condition that for $s=s_0$, $f=0$,

$$f=A\left\{(s-s_0)+\frac{c_1}{2}(s-s_0)^2+\dots\dots\right\} \quad (19)$$

By eq. (19) the expansion of $\frac{ds}{dz_2}$ in the neighbohurhood of s_0 can be expressed

$$\frac{ds}{dz_2}=-\frac{A}{2ae^{-i\theta}}\left\{(s-s_0)^2+b(s-s_0)^4+\dots\dots\right\}, \quad (20)$$

$$\text{where}\quad b=\frac{c_1^2}{4}-\frac{c_2}{3}+\frac{a^2+3r_0^2e^{2i\theta}}{12a^2}A^2$$

By eqs. (14) and (20) $\left(\frac{dW}{ds}\right)^2\frac{ds}{dz_2}$ can be expanded in the neighbourhood of s_0 in the power series of $(s-s_0)$, and only the coefficient of the term $(s-s_0)^{-1}$ contributes to the result of the integral. The coefficient is equal to

$$\frac{i\Gamma Aw_s}{2\pi ae^{-i\theta}},$$

so the integral is equal to $\frac{\Gamma A w_s}{a e^{-i\theta}}$. Now the velocity w_{z_2} in the z_2-plane is equal to

$$w_{z_2} = w_z e^{i\theta}, \quad \text{and} \quad \frac{w_s A}{a} = 2w_z,$$

so the value of the integral is equal to $2\Gamma w_{z_2}$.

The relation between the circulation around the aerofoil Γ_f and the circulation Γ about the point s_0, i.e. around the infinitely distant point in the z-plane, is

$$\Gamma_f = -\Gamma.$$

So the integral is equal to $-2\Gamma_f w_{z_2}$ and the lift becomes

$$P_{x_2} - iP_{y_2} = -\rho \Gamma_f w_{z_2} i \tag{21}$$

The result is quite the same as in the ordinary Joukowski section; the lift acts perpendicularly to w_{z_2} and is equal to $|\rho \Gamma_f w_{z_2}|$, and there is no resistance.

4. Calculation of the Moment of the Lift.

In the same way as the lift, the moment of the lift about the origin in the z_2-plane can be calculated by the Blasius formula,

$$\mathfrak{M} = -\frac{\rho}{2} \Re \oint \left(\frac{dW}{dz_2}\right)^2 z_2 dz_2 \tag{22}$$

where $\Re$ means to take the real part of the integral.

$z_2 \frac{ds}{dz_2}$ can be expanded in the neighbourhood of s_0 in the following way,

$$z_2 \frac{ds}{dz_2} = -(s-s_0) - \frac{mA}{2a}(s-s_0)^2 + \frac{c_1}{2}(s-s_0)^2$$
$$-\left\{\frac{1}{2}c_1^2 - \frac{2}{3}c_2 + \frac{a^2 + 3r_0^2 e^{2i\theta}}{6a^2} A^2\right\}(s-s_0)^3$$
$$-\cdots\cdots \tag{23}$$

By eqs. (14) and (23) $\left(\frac{dW}{ds}\right)^2 z_2 \frac{ds}{dz_2}$ can be expanded in the power series of $(s-s_0)$, and the coefficient of the term $(s-s_2)^{-1}$ is as follows,

$$-w_s^2 \left\{\frac{1}{2}c_1^2 - \frac{2}{3}c_2 + \frac{a^2 + 3r_0^2 e^{2i\theta}}{6a^2} A^2\right\}$$
$$+\frac{i\Gamma}{\pi} w_s \left(\frac{mA}{2a} - \frac{c_1}{2}\right) + \frac{\Gamma^2}{4\pi^2} - 2w_s \bar{w}_s \wp(s_0 - \bar{s}_0)$$
$$-\frac{i\Gamma}{\pi} w_s \zeta(s_0 - \bar{s}_0) + 2k w_s \tag{24}$$

Let this coefficient be denoted by M, then the integral is equal to $-2\pi i M$, and the moment $\mathfrak{M}$ is

$$\mathfrak{M} = -\rho\pi \Im(M), \tag{25}$$

$\Im$ meaning to take the imaginary part of M.

5. Example.

An example was calculated. Fig. 4 shows this section. The Joukowski section is the so-called Göttingen Nr. 580, tested in the Aerodynamischen Versuchsanstalt zu Göttingen. The lift coefficients of the slotted and un-slotted sections are as follows;

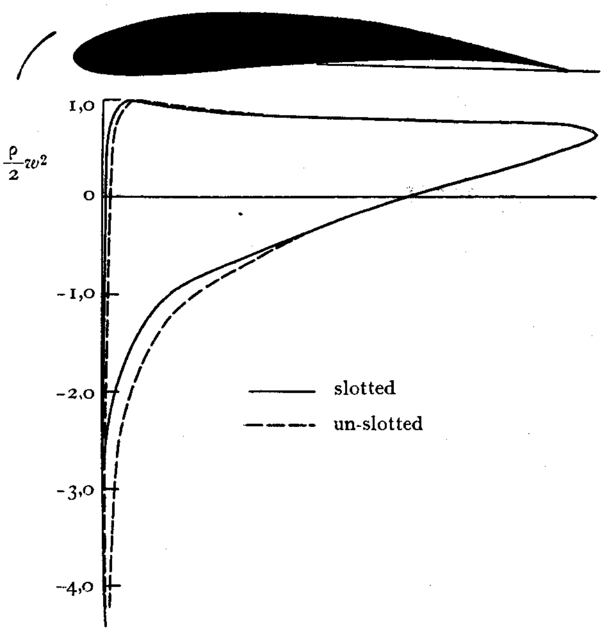
$$c_z = \pi \cdot 2{,}346 \sin(\alpha_0 + 4{,}853°) \qquad \text{slotted,}$$
$$c_z = \pi \cdot 2{,}210 \sin(\alpha_0 + 5{,}712°) \qquad \text{un-slotted,}$$

defined as

$$c_z = \frac{A}{\frac{\rho}{2} w^2 F},$$

where A = lift,
α_0 = angle of incidence,
ρ = air density,
w = air velocity,
F = area of the wing,

Fig. 4



In Fig. 4 the pressure distribution over the surface of the aerofoil at an incidence angle of about 16° is shown. At this angle the un-slotted aerofoil almost reaches the maximum lift coefficient and the negatiue pressure and the pressure gradient at the nose of the aerofoil are very great. By adding the auxiliary aerofoil the negative pressure and the pressure gradient at the nose are lessened considerably, and the flow condition at this angle is equivalent to the condition of the un-slotted section at a much lower incidence angle.

6. Another Conformal Transformation.

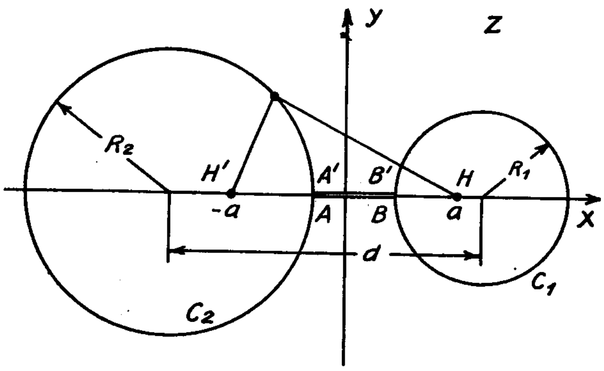
In the preceding paragraphs, the author treated the case of a Joukowski section with an auxiliary aerofoil, but the calculation is very tedious, so he proposes another much easier transformation.

In the first place we consider two circles in the z-plane whose centres are on the x-axis, and so situated that the ratio of the length of the two straight lines joining any point on the circle and H and H', whose coordinates are $(a, 0)$ and $(-a, 0)$ respectively, is constant. (see Fig. 5)

Now by the following relation the outside region of those two circles is transformed into a rectangular region of the s-plane,

$$s=\log\frac{z+a}{z-a} \qquad (26)$$

Fig. 5



The circle C_1 in Fig. 5 is transformed into a vertical straight line passing through the point $s=\alpha$ and the circle C_2 is transformed into a vertical straight line passing through the point $s=-\beta$, the height of the rectangle being 2π and symmetrical to the real exis of the s-plane. When the radii of these circles and the distances between these centres are given, a, α and β can be calculated by the following formulae which were deduced by Lagally.[5)]

$$\mathfrak{Cof}\,J=\frac{d^2-R_1^2-R_2^2}{2R_1R_2},$$

$$a=\frac{R_1R_2}{d}\,\mathfrak{Sin}\,J, \qquad \mathfrak{Sin}\,\alpha=\frac{R_2}{d}\,\mathfrak{Sin}\,J,$$

$$\mathfrak{Sin}\,\beta=\frac{R_1}{d}\,\mathfrak{Sin}\,J \qquad (27)$$

Now returning to the z-plane we transform it to the z_4-plane by the following relations.

First step $\quad z_1=ze^{i\theta}-me^{i\theta}, \qquad (28)$

second stop $\quad z_2=z_1+\dfrac{r_1^2}{z_1}, \qquad (29)$

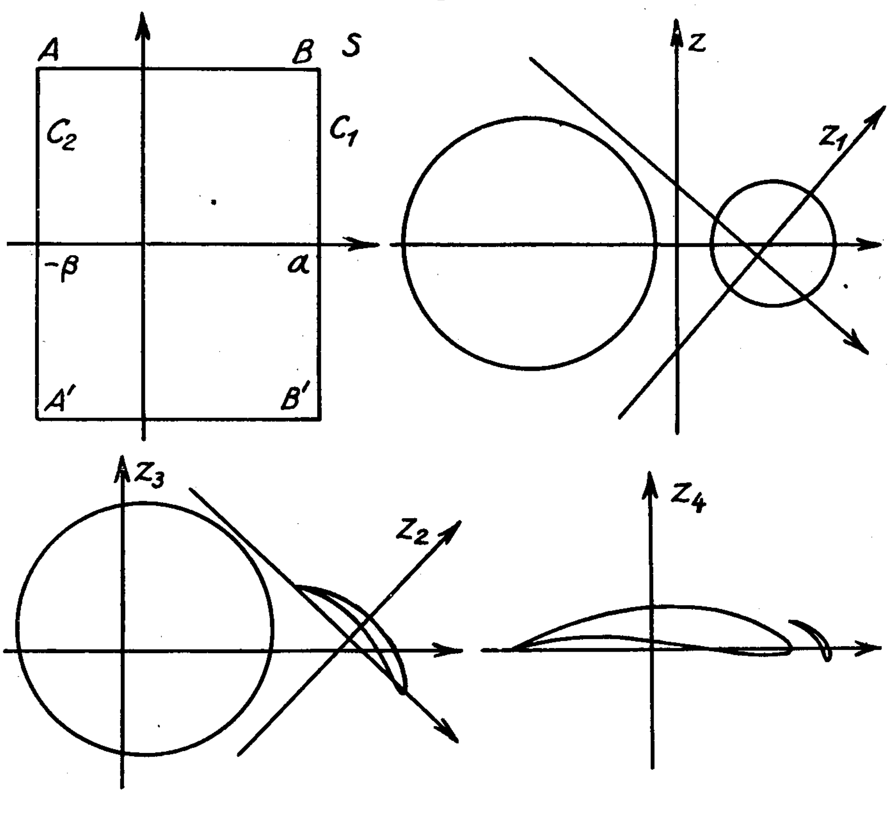
third step $\quad z_3=z_2e^{-i\theta'}-ne^{-i\theta'}, \qquad (30)$

fourth step $\quad z_4=z_3+\dfrac{r_2^2}{z_3}. \qquad (31)$

Then,

$$z=(m+ne^{-i\theta})+z_4e^{i(\theta'-\theta)}$$
$$-\frac{1}{z_4}\{r_2^2e^{i(\theta'-\theta)}+r_1^2e^{-i(\theta'+\theta)}\}$$
$$+\frac{1}{z_4^2}nr_1^2e^{-i(\theta+2\theta')}+\dots\dots\dots \qquad (32)$$

Fig. 6



$$\frac{dz}{dz_4}=e^{i(\theta'-\theta)}+\frac{1}{z_4^2}\{r_2^2e^{i(\theta'-\theta)}+r_1^2e^{-i(\theta+\theta')}\}$$

$$-\frac{2}{z_4^3}nr_1^2e^{-i(2\theta'+\theta)}+\ldots\ldots \quad (33)$$

7. Flow of the Fluid and the Lift and Moment.

The flow in the s-plane can be expressed as follows,

$$\frac{dW}{ds}=-\frac{i\Gamma_f}{2\pi}\Big[\zeta(s)-\zeta(s+2\beta)\Big]$$

$$-2a\Big[w_z\wp(s)-\overline{w}_z\wp(s+2\beta)\Big]+ik, \quad (34)$$

where $w_z=u_z-iv_z$, the conjugate velocity in the z-plane at infinity, and the magnitude of w_z is equal to the velocity in the z_4-plane, and

Γ_f=circulation around the aerofoil,

k=real constant.

Γ_f and k can be determined by the two conditions, i.e. $\frac{dW}{ds}=0$, at the two points which correspond to the trailing edges of the aerofoils. By eqs. (34) and (32),

$$\frac{dW}{dz}=\frac{i\Gamma_f}{2\pi}\frac{1}{z}-\frac{i\Gamma_f}{2\pi}\frac{2a\zeta(2\beta)}{z^2}$$

$$-\frac{i\Gamma_f}{2\pi}\Big(\frac{1}{3}-4\wp(2\beta)\Big)\frac{a^2}{z^3}-\ldots$$

$$+u_z+\frac{a^2u_z}{3z^2}-\frac{4a^2\wp(2\beta)u_z}{z^2}-\frac{8a^3\wp'(2\beta)u_z}{z^3}-\ldots$$

$$-v_zi-\frac{a^2v_zi}{3z^2}-\frac{4a^2\wp(2\beta)v_zi}{z^2}-\frac{8a^3\wp'(2\beta)v_zi}{z^3}-\ldots$$

$$-\frac{2aki}{z^2}-\ldots\ldots \quad (35)$$

From eqs. (33) and (35)

$$\frac{dW}{dz_4}=A_0+\frac{A_1}{z_4}+\frac{A_2}{z_4^2}+\ldots\ldots\ldots \quad (36)$$

where

$$A_0=w_ze^{i(\theta'-\theta)},$$

$$A_1=\frac{i\Gamma_f}{2\pi},$$

$$A_2=\Big\{-\frac{i\Gamma_f}{2\pi}2a\zeta(2\beta)+\frac{a^2}{3}w_z-4a^2\wp(2\beta)\overline{w}_z-2aki$$

$$-\frac{i\Gamma_f}{2\pi}(m+ne^{-i\theta})\Big\}e^{i(\theta-\theta')}$$

$$+w_z(r_2^2e^{i(\theta'-\theta)}+r_1^2e^{-i(\theta+\theta')})$$

From eq. (36), the coefficient of z_4^{-1} in the expansion of

$$\Big(\frac{dW}{dz_4}\Big)^2 \text{ is } \frac{i\Gamma_f}{2\pi}2w_ze^{i(\theta'-\theta)}=\frac{i\Gamma_f}{\pi}w_{z_4},$$

and the lift can be calculated by the Blasius formula and we get

$$\frac{\rho i}{2}\oint\Big(\frac{dW}{dz_4}\Big)^2dz_4=-\rho i\Gamma_fw_{z_4} \quad (37)$$

Let the coefficient of z_4^{-2} be M, then the moment of the lift about the origin of the z_4-plane is

$$\mathfrak{M}=\rho\pi\,\mathfrak{I}(M) \quad (38)$$

$$\mathfrak{I}(M)=-2u_z\frac{\Gamma_f}{\pi}a\zeta(2\beta)-\frac{4}{3}a^2u_zv_z-4au_zk$$

$$-\frac{\Gamma_f}{\pi}\Big\{u_zp+u_zr\cos\theta+u_zs\sin\theta+v_zq-v_zr\sin\theta$$

$$+v_zs\cos\theta\Big\}+2r_2^2(u_z^2-v_z^2)\sin 2(\theta'-\theta)$$

$$-4u_zv_zr_2^2\cos 2(\theta-\theta')-2r_1^2(u_z^2-v_z^2)\sin 2\theta$$

$$-4u_zv_zr_1^2\cos 2\theta,$$

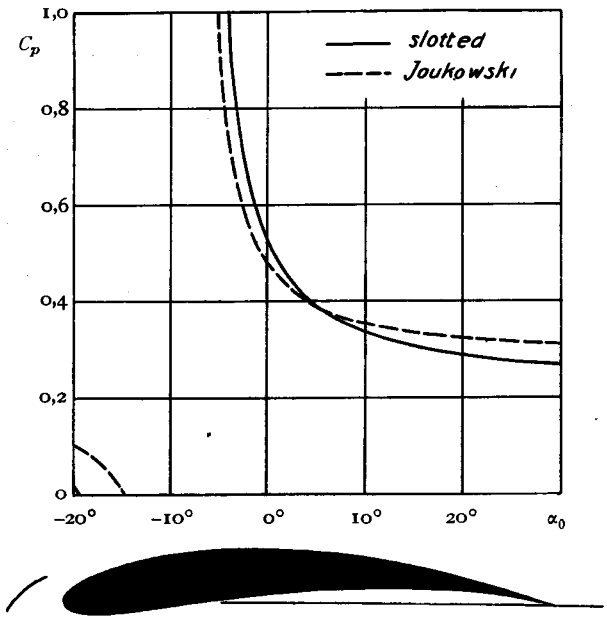
where $m=p+iq,$

$n=r+is.$

8. Example.

In Fig. 7 an example of this transformation is shown, and Figs. 7 and 8 show its lift coefficient c_z, moment coefficient c_m and the

Fig. 7

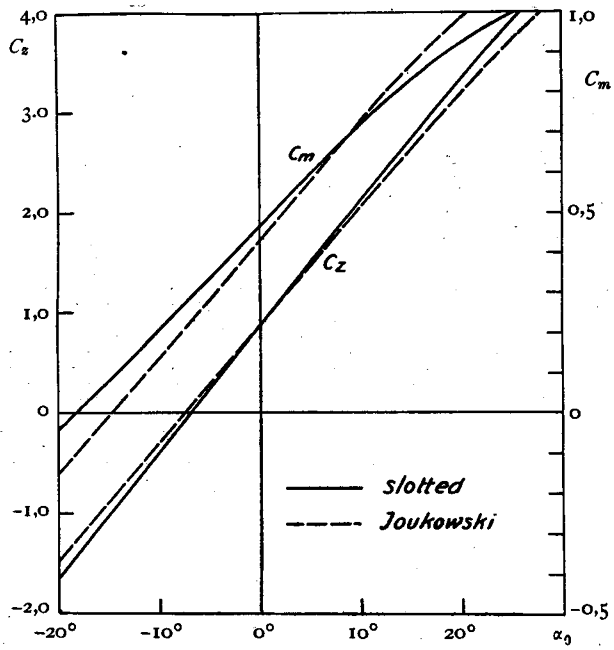


position of the centre of pressure c_p as a function of the incidence angle. In Figs. 7 and 8 these values of the Juokowski section, approximately equal to the calculated slotted wing section, are plotted in dotted lines for comparison. In these figures, c_z and c_m are defined as follows,

$$c_z=\frac{A}{\frac{\rho}{2}w^2F},$$

$$c_m=\frac{\mathfrak{M}_0}{\frac{\rho}{2}w^2Ft}.$$

Fig. 8



$\mathfrak{M}_0$ is the moment about the point on the real axis of the z_4-plane at a distance of $2r_2$ from the origin, and $t=4r_2$, and

$$c_p=\frac{\mathfrak{M}_0}{At\cos\alpha_0}=\frac{c_m}{c_z\cos\alpha_0}\ .$$

These calculated results agree with the experimental results where the incidence angle is fairly large.

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