The Lift on an Aerofoil with a Circular Arc Section placed near the Ground.*

By Busuke Hudimoto

The outside region of a segment of a circular arc and a straight line is transformed conformally into a rectangular region and then the flow of the perfect fluid and the lift and its moment acting on the circular arc are expressed by using the elliptic functions.

The theory of the effect of ground interference upon the aerofoil has been discssed by many authors. Betz,[1] Wieselsberger,[2] Tani[3] and Sasaki[4] investigated the problems relating to the aerofoil with finite span, and the problems relating to the infinite span or the case of the two-dimensional flow were investigated by Sasaki,[5] Rosenhead,[6] Bonder,[7] Tani[8] and recently by Tomotika, Nagamiya and Takenouti.[9] The cases discussed by all of these except Bonder were the effect of the interference of the ground upon a flat plate placed near the ground; Bonder treated the problem of an aerofoil with a nearly Joukowski section.

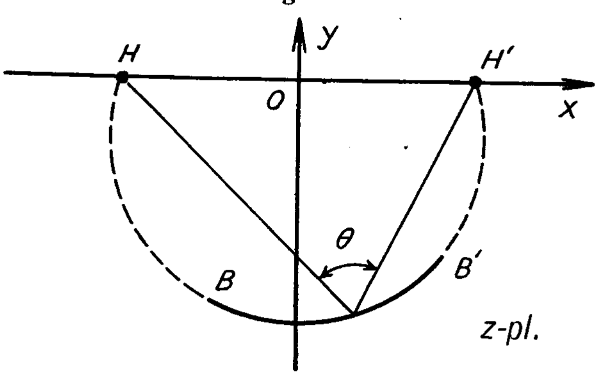
The effect of the ground upon aerofoils in general is probably not much different from that upon a flat plate, so the results of the calculations made by Tomotika, Nagamiya and Takenouti can be used very conveniently for practical calculations. But in this paper from the standpoint of theoretical interest, the author deals with the case of an aerofoil with a circular arc section placed near the surface of the ground.

1. Conformal Transformations.

The conformal transformations used here are quite the same as in the author's previous paper on the slotted wing section.

In Fig. 1 we consider the circular arc BB' and a straight line, i.e. the surface of the ground. We denote this plane the z-plane and take this straight line as the x-axis or the real axis. We consider here only the case when the circle of

Fig. 1



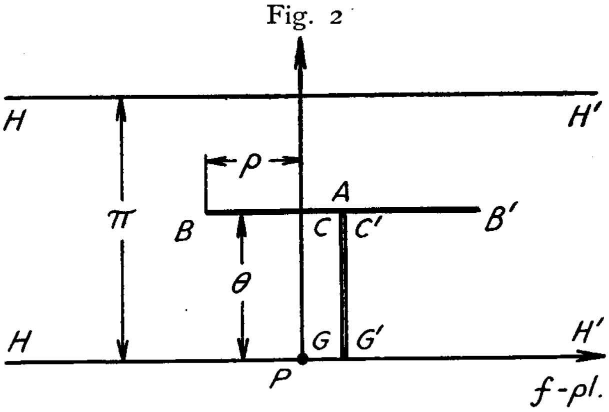
the arc BB′ intersects the straight line or at least touches the straight line. The intersections are denoted H and H' in Fig. 1. The origin O is taken at the mid-point of HH' and the direction of the imaginary axis y is so chosen that the arc BB' lies on the side of the negative values of y.

Let $OH=OH'=a$ and by

$$f=\log\frac{z+a}{z-a} \qquad (1)$$

the z-plane is transformed into the f-plane. By this transformation the half plane on the negative side of y is transformed into a strip section parallel to the real axis of f and extending to both infinities in the f-plane, one side of this strip passing through the origin of the f-plane (see Fig. 2). The height of the strip is π and the

Fig. 2



points H and H' correspond to infinity and the point of infinity of the z-plane corresponds to the origin of the f-plane. The arc BB' is transformed into a straight line parallel to the real axis.

Then we cut off the f-plane by CG or $C'G'$ passing through the mid-point of BB' and perpendicular to the real axis, and transform the f-plane into a rectangle in the s-plane by the relation,[10]

$$\frac{df}{ds}=\frac{\wp'(\nu)}{\wp(\nu)-\wp(\mu)}\,\frac{\wp(s)-\wp(\mu)}{\wp(s)-\wp(\nu)} \qquad (2)$$

where $\wp$ is the $\wp$-function of Weierstrass and $\wp'$ is its derivative and

$$\begin{aligned} &s=\nu \quad \text{for point } H,\\ &s=-\nu \quad\ ,, \quad H'\\ \text{and}\quad &s=\mu \quad\ ,, \quad B. \end{aligned}$$

* received Oct. 20, 1933.

Integrating the above relation (2), we get

$$f=\left\{\zeta(\mu+\nu)-\zeta(\mu-\nu)\right\}(s-\mu)$$
$$-\log\frac{\sigma(s+\nu)\sigma(\mu-\nu)}{\sigma(s-\nu)\sigma(\mu+\nu)}+\text{const.} \quad (3)$$

where ζ is the ζ-function af Weierstrass.
Let the coordinates of the point B in the f-plane be (p, θ), then the integration constant is determined and

$$f=\left\{\zeta(\mu+\nu)-\zeta(\mu-\nu)\right\}(s-\mu)$$
$$-\log\frac{\sigma(s+\nu)\sigma(\mu-\nu)}{\sigma(s-\nu)\sigma(\mu+\nu)}+p+i\theta \quad (3')$$

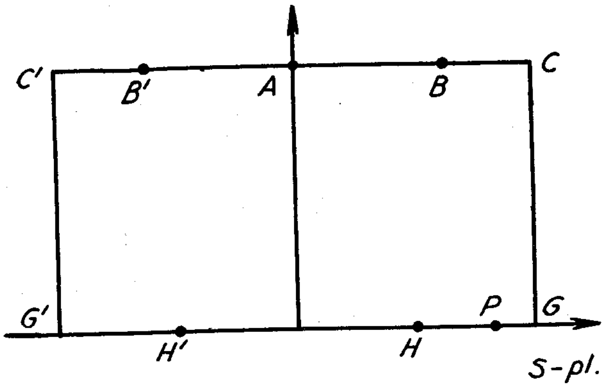
We denote the breadth of this rectangle by $2\omega_1$ and the height by $\frac{\omega_2}{i}$, then ω_1 and ω_2 are the half-periods of the above mentioned $\wp$ and ζ functions.

Now we cut off the region in the f-plane along the straight line CG or $C'G'$, so f must be a periodic function with the period $2\omega_1$, unless discontinuity occurs along this straight line, and consequently μ and ν must satisfy the following condition:

$$\zeta(\mu+\nu)-\zeta(\mu-\nu)=\frac{2\eta_1\nu}{\omega_1}, \quad (4)$$

where $\quad \eta_1=\zeta(\omega_1)$

Fig. 3



The circular arc BB' is transformed into a straight line BB' in the f-plane and its distance from the real axis is equal to θ which is the angle between two straight lines joining H and H' to any point on the circular arc BB' in the z-plane.

Let the values of f at the points C and G be denoted by f_c and f_g, then

$$i\theta=f_c-f_g=\frac{2\eta_1\nu}{\omega_1}\omega_2-2\eta_2\nu,$$

where $\quad \eta_2=\zeta(\omega_2)$,

then by the relation $\quad \eta_1\omega_2-\eta_2\omega_1=\frac{\pi i}{2}$,

$$\nu=\frac{\theta}{\pi}\omega_1 \quad (5)$$

And let the values of f at the points A and B be denoted by f_a and f_b, and the length of BB' be equal to $2l$, then

$$l=f_a-f_b$$
$$=\frac{2\eta_1\nu}{\omega_1}(\omega_2-\mu)-\log\frac{\sigma(\omega_2+\nu)\sigma(\mu-\nu)}{\sigma(\omega_2-\nu)\sigma(\mu+\nu)}.$$

Now if $\mu=m+\omega_2$ and the ϑ-function is introduced,

$$l=\log\frac{\vartheta_0\left(\frac{m+\nu}{2\omega_1}\right)}{\vartheta_0\left(\frac{m-\nu}{2\omega_1}\right)} \quad (6)$$

The origin P of the f-plane corresponds to a point of the s-plane on the real axis and we denote this point $s=s_0$; then by eq. (3)

$$\frac{2\eta_1\nu}{\omega_1}(s_0-\mu)-\log\frac{\sigma(s_0+\nu)\sigma(\mu-\nu)}{\sigma(s_0-\nu)\sigma(\mu+\nu)}+p+i\theta=0,$$

or by using the ϑ-function and by eq. (6)

$$p=\log\frac{\vartheta_1\left(\frac{s_0+\nu}{2\omega_1}\right)}{\vartheta_1\left(\frac{s_0-\nu}{2\omega_1}\right)}-l \quad (7)$$

When the circular arc BB' in the z-plane is given, p, l and θ can be calculated; then, by eqs. (5), (6) and (7) ν, μ and s_0 are determined.

2. Flow of the Perfect Fluid in the Rectangular Region.

We consider the two-dimensional steady flow of the perfect fluid in the rectangular region in the s-plane.

The parallel flow along the real axis in the z-plane is equivalent to the flow due to a doublet placed at s_0. Let the complex velocity potential of the flow be W_1, then the conjugate complex of the velocity, i.e. $\frac{dW_1}{ds}$, can be expressed by the $\wp$-function,

$$\frac{dW_1}{ds}=-w_s\wp(s-s_0) \quad (8)$$

where w_s is an unknown constant, and it is determined in the following way. The parallel flow along the real axis with the velocity w_z in the z-plane is equivalent to the flow due to a doublet placed at the origin in the f-plane and in the neighbourhood of the origin the conjugate complex of the velocity can be expressed in the following form:

$$-\frac{2aw_z}{f^2}+\cdots\cdots$$

When this flow is transformed into the s-plane, it is expressed in the neighbourhood of the point

s_0 in the following form,

$$-\frac{2aw_z}{(s-s_0)^2}\left(\frac{ds}{df}\right)_{s=s_0}+\cdots\cdots\cdots$$

where $\left(\frac{ds}{df}\right)_{s=s_0}$ is the value of $\frac{ds}{df}$ at $s=s_0$.

Comparing this and eq. (8), w_s is determined and

$$w_s=2aw_z\left(\frac{ds}{df}\right)_{s=s_0} \tag{9}$$

The other type of the flow is the flow circulating around the circular arc. If this complex velocity potential is denoted by W_2, the conjugate complex of the velocity, i.e. $\frac{dW_2}{ds}$, can be expressed in the following form:

$$\frac{dW_2}{ds}=k, \tag{10}$$

where k is a real constant.

Combining eqs. (8) and (10) the conjugate complex of the velocity $\frac{dW}{ds}$ is expressed by

$$\frac{dW}{ds}=\frac{dW_1}{ds}+\frac{dW_2}{ds}$$

$$=-w_s\wp(s-s_0)+k \tag{11}$$

The value of k is determined by the condition that at the point corresponding to the trailing edge of the circular arc, in our case at the point B', $\frac{dW}{ds}$ must vanish. Let $s=\lambda$ at the point B', then

$$k=w_s\wp(\lambda-s_0),$$

or this is equivalent to

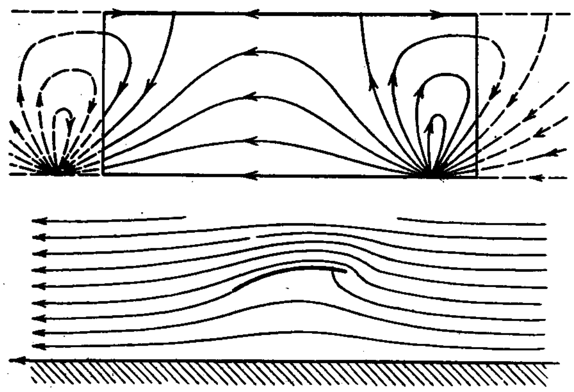
$$k=w_s\wp(-\mu-s_0).$$

and

$$\frac{dW}{ds}=-w_s\left\{\wp(s-s_0)-\wp(\lambda-s_0)\right\} \tag{12}$$

Integrating $\frac{dW}{ds}$ around the circular arc, the circulation Γ around this circular arc is determined:

Fig. 4

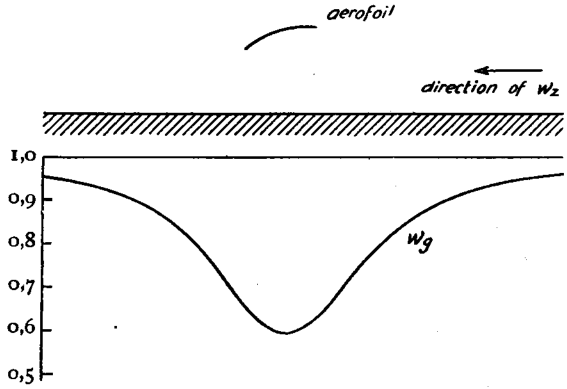


$$\Gamma=\int_{c'}^{c}\frac{dW}{ds}\,ds=2w_s\eta_1+2w_s\omega_1\wp(\lambda-s_0) \tag{13}$$

In Fig. 4 the stream lines of the flow in the s and z-planes are shown in free-hand writing. The direction of the velocity at the trailing edge coincides with the direction of the tangent to the circular arc at the trailing edge. This is easily verified and so is not described here.

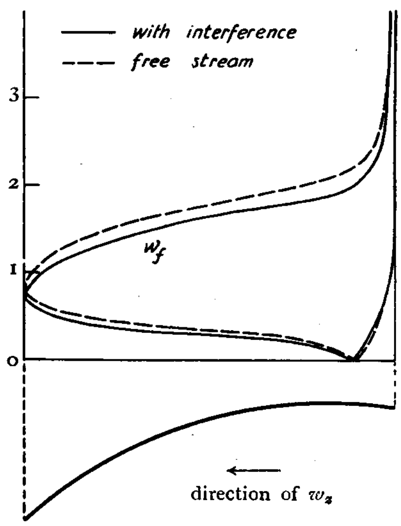
An example was calculated and in Fig. 5, the circular arc and the plane wall, i.e. the surface

Fig. 5



of the ground, are shown, the ratio of the length of the chord to the height of the leading edge from the surface of the ground being 0.885 and the ratio of the camber to the chord being 0.115 and the angle of incidence is 17° 2,28′. The velocity over the ground surface, denoted in this figure w_g, is shown, with the velocity at infinity, i.e. w_z, taken as unity. In Fig. 6 the velocity

Fig. 6



over the circular arc, denoted in this figure w_f, is shown, with the velocity w_z taken as unity and the velocity when the same circular arc is placed in the free stream at the same incidence angle is shown by the dotted line for comparison.

3. The Lift acting on the Aerofoil.

The lift acting on the aerofoil calculated by Blasius' formula. Let the components of the force in the directions of the x-axis and the y-axis be denoted by P_x and P_y respectively, then

$$P_x - iP_y = \frac{\rho i}{2} \oint \left(\frac{dW}{dz}\right)^2 dz \qquad (14)$$

the integration being performed in the counter-clockwise direction in the z-plane. Instead of in the z-plane, the integration may be performed in the s-plane or more conveniently in the Z-plane, where the relation between s and Z being as follows:

$$s = \omega_1 + \omega_2 - \frac{\omega_1}{i\pi} \log Z, \qquad (15)$$

hence

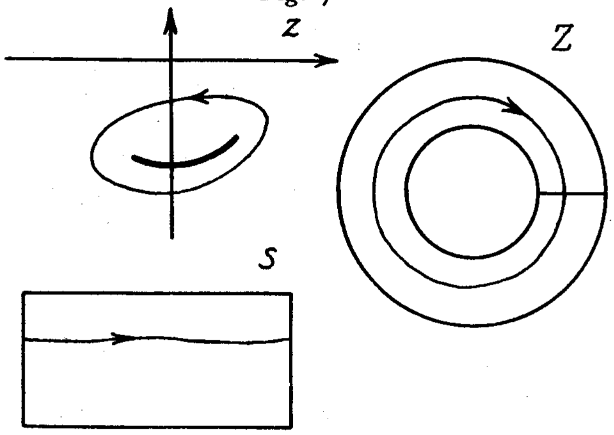
$$\int \left(\frac{dW}{dz}\right)^2 dz = \int \left(\frac{dW}{ds}\right)^2 \frac{ds}{df} \frac{df}{dz} ds$$
$$= -\frac{\omega_1}{i\pi} \int \left(\frac{dW}{ds}\right)^2 \frac{ds}{df} \frac{df}{dz} \frac{dZ}{Z}. \qquad (16)$$

The paths and the directions of the integration are shown in Fig. 7.

Now

$$\left(\frac{dW}{ds}\right)^2 = w_s^2 \{ \wp^2(s-s_0) - 2k_1 \wp(s-s_0) + k_1^2 \},$$

Fig. 7



where $\qquad k_1 = \wp(\lambda - s_0),$

and $$\frac{ds}{df} = \frac{\wp(\nu)-\wp(\mu)}{\wp'(\nu)} \frac{\wp(s)-\wp(\nu)}{\wp(s)-\wp(\mu)}$$

$$\frac{df}{dz} = -\frac{(e^f - 1)^2}{2ae^f}$$

$$\therefore \quad \frac{ds}{dz} = -(e^f + e^{-f} - 2) \frac{\wp(\nu)-\wp(\mu)}{2a\wp'(\nu)} \frac{\wp(s)-\wp(\nu)}{\wp(s)-\wp(\mu)}$$

and

$$e^f = \frac{\sigma(\mu+\nu)\sigma(s-\nu)}{\sigma(\mu-\nu)\sigma(s+\nu)} e^{\frac{2\eta_1\nu}{\omega_1}(s-\mu)+p+i\theta}$$

Inserting these values into the integral of eq. (16), we separate the integrand into three parts, the first that multiplied by $e^{\frac{2\eta_1\nu}{\omega_1}s}$, the second, that multiplied by $e^{-\frac{2\eta_1\nu}{\omega_1}s}$, and the last, that not multiplied by any of them. The first part i.e. that multiplied by $e^{\frac{2\eta_1\nu}{\omega_1}s}$, is an elliptic function of the second kind and is expressed by the sum of the function

$$A(t) = -\frac{\sigma(t-2\nu)}{\sigma(t)\sigma(2\nu)} e^{\frac{2\eta_1\nu}{\omega_1}t},$$

and expanded as follows;

$$a_1 A(s-s_0) + a_2 A'(s-s_0) + a_3 A''(s-s_0) + a_4 A'''(s-s_0)$$
$$+ a_5 A(s+\mu) + a_6 A(s-\mu),$$

where $a_1, a_2, \ldots\ldots$ are constants and $A', A'', \ldots\ldots$ are the derivatives of the function A. After some calculation the coefficients $a_1, a_2, \ldots\ldots$ can be determined and the necessary coefficients a_1, a_5 and a_6 are as follows;

$$a_1 = -\frac{\wp(\nu)-\wp(\mu)}{2a\wp'(\nu)} \frac{\wp(s_0)-\wp(\nu)}{\wp(s_0)-\wp(\mu)} \times$$
$$\left\{ c_0 \left(\frac{c_0^2}{6} + \frac{c_1^2}{2} - \frac{2c_2}{3} - 2k_1 \right) - (c_3 - 2c_1c_2 + c_1^3 - 2k_1c_1) \right\},$$

$$a_5 = \frac{\{\wp(\nu)-\wp(\mu)\}^2}{2a\wp'(\nu)\wp'(\mu)} \{\wp(\mu - s_0) - k_1\}^2 e^p e^{i\frac{\nu\pi}{\omega_1}},$$

$$a_6 = \frac{\wp(\nu)-\wp(\mu)}{2a\wp'(\nu)} \frac{\sigma^2(\mu)\sigma^3(\mu+\nu)}{\sigma^2(\nu)\sigma(2\mu)\sigma(\mu-\nu)} e^{-\frac{4\eta_1\nu}{\omega_1}\mu+p+i\theta} \times$$
$$\{\wp(\mu+s_0) - k_1\}^2$$
$$= 0$$

where

$$c_0 = \frac{\wp'(\nu)}{\wp(\nu)-\wp(\mu)} \frac{\wp(s_0)-\wp(\mu)}{\wp(s_0)-\wp(\nu)},$$

$$c_1 = \wp'(s_0) \left\{ \frac{1}{\wp(s_0)-\wp(\mu)} - \frac{1}{\wp(s_0)-\wp(\nu)} \right\},$$

$$c_2 = \frac{\wp''(s_0)}{2} \left\{ \frac{1}{\wp(s_0)-\wp(\mu)} - \frac{1}{\wp(s_0)-\wp(\nu)} \right\} - \frac{c_1\wp'(s_0)}{\wp(s_0)-\wp(\nu)},$$

$$c_3 = \frac{\wp'''(s_0)}{6} \left\{ \frac{1}{\wp(s_0)-\wp(\mu)} - \frac{1}{\wp(s_0)-\wp(\nu)} \right\} - \frac{c_2\wp'(s_0)}{\wp(s_0)-\wp(\nu)} - \frac{c_1\wp''(s_0)}{2\{\wp(s_0)-\wp(\nu)\}}.$$

The second part i.e. multiplied by $e^{-\frac{2\eta_1\nu}{\omega_1}s}$ is also an elliptic function of the second kind and, in the same way, it can be expressed by the sum of the function

$$B(t) = \frac{\sigma(t+2\nu)}{\sigma(t)\sigma(2\nu)} e^{-\frac{2\eta_1\nu}{\omega_1}t},$$

and expanded as follows;

$$\beta_1 B(s-s_0) + \beta_2 B'(s-s_0) + \beta_3 B''(s-s_0) + \beta_4 B'''(s-s_0)$$
$$+ \beta_5 B(s+\mu) + \beta_6 B(s-\mu),$$

where

$$\beta_1=-\frac{\wp(\nu)-\wp(\mu)}{2a\wp'(\nu)}\frac{\wp(s_0)-\wp(\nu)}{\wp(s_0)-\wp(\mu)}\times$$
$$\left\{-c_0\left(\frac{c_0^2}{6}+\frac{c_1}{2}-\frac{2c_2}{3}-2k_1\right)-(c_3-2c_1c_2+c_1^3-2k_1c_1)\right\},$$

$$\beta_5=\frac{\{\wp(\nu)-\wp(\mu)\}^2}{2a\wp'(\nu)\wp'(\mu)}\{\wp(\mu-s_0)-k_1\}^2e^{-p}e^{-i\frac{\nu\pi}{\omega_1}},$$

$$\beta_6=\frac{\wp(\nu)-\wp(\mu)}{2a\wp'(\nu)}\frac{\sigma^2(\mu)\sigma(\mu+\nu)\sigma(\mu-\nu)}{\sigma^2(\nu)\sigma(2\mu)}e^{\frac{4\eta_1\nu}{\omega_1}\mu-p-i\theta}\times$$
$$\{\wp(\mu+s_0)-k_1\}^2$$
$$=0$$

The third part is an elliptic function of the first kind and it can be expressed by the sum of the $\wp$, $\wp'$, $\wp''$ and ζ-functions and we express it

$$\gamma_1\wp''(s-s_0)+\gamma_2\wp'(s-s_0)+\gamma_3\wp(s-s_0)+\gamma_4\zeta(s-s_0)+\gamma_5\zeta(s-\mu)+\gamma_6\zeta(s+\mu)+\gamma_7,$$

The coefficients $\gamma_1, \gamma_2, \ldots\ldots$ are determined and are as follows;

$$\gamma_1=\frac{\wp(\nu)-\wp(\mu)}{6a\wp'(\nu)}\frac{\wp(s_0)-\wp(\nu)}{\wp(s_0)-\wp(\mu)},$$

$$\gamma_2=c_1\frac{\wp(\nu)-\wp(\mu)}{2a\wp'(\nu)}\frac{\wp(s_0)-\wp(\nu)}{\wp(s_0)-\wp(\mu)},$$

$$\gamma_3=-\frac{\wp(\nu)-\wp(\mu)}{a\wp'(\nu)}\frac{\wp(s_0)-\wp(\nu)}{\wp(s_0)-\wp(\mu)}(2k_1+c_2-c_1^2),$$

$$\gamma_4=\frac{\wp(\nu)-\wp(\mu)}{a\wp'(\nu)}\frac{\wp(s_0)-\wp(\nu)}{\wp(s_0)-\wp(\mu)}\times$$
$$\{2k_1c_1-(c_3-2c_1c_2+c_1^3)\},$$

$$\gamma_5=-\{\wp(\mu-s_0)-k_1\}^2\frac{\{\wp(\mu)-\wp(\nu)\}^2}{a\wp'(\nu)\wp'(\mu)},$$

$$\gamma_6=\{\wp(\mu+s_0)-k_1\}^2\frac{\{\wp(\mu)-\wp(\nu)\}^2}{a\wp'(\nu)\wp'(\mu)},$$

$$\gamma_7=\{\wp(s_0)-k_1\}^2\frac{\wp(\nu)-\wp(\mu)}{a\wp'(\nu)}-\gamma_1\wp''(s_0)+\gamma_2\wp'(s_0)-\gamma_3\wp(s_0)+\gamma_4\zeta(s_0)+\gamma_5\zeta(\mu).$$

The integration of the first and second parts, i.e. the parts of the elliptic functions of the second kind, are performed in the Z-plane[11] and the results are as follows;

$$+\frac{\pi}{a}\operatorname{ctg}\frac{\nu\pi}{\omega_1}\left\{\frac{c_0^2}{6}+\frac{c_1^2}{2}-\frac{2c_2}{3}-2k_1\right\}, \qquad (17_1)$$

$$-\frac{\pi}{\sin\frac{\nu\pi}{\omega_1}}\{\wp(\mu-s_0)-k_1\}^2\frac{\{\wp(\mu)-\wp(\nu)\}^2}{2a\wp'(\nu)\wp'(\mu)}\times(e^p-e^{-p}), \qquad (17_2)$$

$$-2\pi i\frac{\wp(\nu)-\wp(\mu)}{2a\wp'(\nu)}\frac{\wp(s_0)-\wp(\nu)}{\wp(s_0)-\wp(\mu)}\times(c_3-2c_1c_2+c_1^3-2k_1c_1) \qquad (17_3)$$

The evaluation of the integral containing $\wp$, $\wp'$, $\wp''$ and ζ-functions is easily performed in the s-plane and the results are as follows;

$$-\frac{\wp(\nu)-\wp(\mu)}{a\wp'(\nu)}\frac{\wp(s_0)-\wp(\nu)}{\wp(s_0)-\wp(\mu)}$$
$$\{2\eta_1(\omega_1-s_0)(c_3-2c_1c_2+c_1^3-2k_1c_1)-2\eta_1(c_2-c_1^2+2k_1)\}, \qquad (18_1)$$

$$-\frac{\{\wp(\mu)-\wp(\nu)\}^2}{a\wp'(\nu)\wp'(\mu)}\{\wp(\mu-s_0)-k_1\}^2\times$$
$$\left\{2\eta_1(\omega_1-m)+\omega_1\left[2\zeta(m)+\frac{\wp'(m)}{\wp(m)-e_3}\right]\right\}, \qquad (18_2)$$

$$+2\omega_1(\gamma_7-\gamma_5\zeta(\mu)) \qquad (18_3)$$

$$+2\pi i\frac{\wp(\nu)-\wp(\mu)}{2a\wp'(\nu)}\frac{\wp(s_0)-\wp(\nu)}{\wp(s_0)-\wp(\mu)}\times(c_3-2c_1c_2+c_1^3-2k_1c_1) \qquad (18_4)$$

where $e_3=\wp(\omega_2)$.

Then the air force is

$$P_x-iP_y=\frac{\rho w_s^2}{2}i\{(17_1)+(17_2)+(17_3)+(18_1)+(18_2)+(18_3)+(18_4)\}. \qquad (19)$$

Now $(17_3)+(18_4)=0$, i.e. the imaginary parts of the terms in the bracket of the above equation cancel out each other, hence

$$P_x=0,$$
$$P_y=-\frac{\rho w_s^2}{2}\{(17_1)+(17_2)+(18_1)+(18_2)+(18_3)\}, \qquad (20)$$

so there acts no resistance, as was expected beforehand.

In the case of the example in paragraph 2, the ratio of the lift coefficient with ground interference, c_z, to that in the free stream, c_{z_0}, is

$$\frac{c_z}{c_{z_0}}=0.825.$$

4. The Moment of the Lift.

The moment of the lift about the origin of the z-plane can be calculated by Blasius' formula. Let the moment be $\mathfrak{M}$ then

$$\mathfrak{M}=-\frac{\rho}{2}\Re\oint\left(\frac{dW}{dz}\right)^2zdz \qquad (21)$$

where $\Re$ means to take the real part of the integral.

Now $\left(\frac{dW}{dz}\right)^2zdz=\left(\frac{dW}{ds}\right)^2\frac{ds}{df}\frac{df}{dz}zds,$

and by eq. (1) $z=a\frac{e^f+1}{e^f-1},$

hence

$$z\frac{ds}{df}\frac{df}{dz}=-\frac{1}{2}\frac{\wp(\nu)-\wp(\mu)}{\wp'(\nu)}\frac{\wp(s)-\wp(\nu)}{\wp(s)-\wp(\mu)}(e^f-e^{-f}).$$

So the integral of eq. (21) is the same as that of the lift and moreover the part containing the elliptic function of the first kind disappears. Consequently

$$\mathfrak{M}=-\frac{\rho a w_s^2}{2}M \tag{22}$$

where

$$M=-\pi \operatorname{ctg}\frac{\nu\pi}{\omega_1}\frac{\wp(\nu)-\wp(\mu)}{a\wp'(\nu)}\frac{\wp(s_0)-\wp(\nu)}{\wp(s_0)-\wp(\mu)}\times$$
$$(c_3-2c_1c_2+c_1^3-2k_1c_1)$$
$$-\frac{\pi}{\sin\frac{\nu\pi}{\omega_1}}\{\wp(\mu-s_0)-k_1\}^2\times$$
$$\frac{\{\wp(\nu)-\wp(\mu)\}^2}{2a\wp'(\nu)\wp'(\mu)}(e^p+e^{-p}).$$

When $s_0=\omega_1$ i.e. when the incidence angle of the aerofoil is zero,

$$c_1=0,$$
$$c_3=0$$

and $$\wp(\mu-s_0)-k_1=0,$$

and accordingly $\mathfrak{M}=0$. That means that the lift acts along the y-axis, as was expected from the symmetry of the flow about this axis.

5. Special Cases.

(i) When the circle of the circular arc BB' touches the surface of the ground we transform the z-plane into the f-plane by the relation

$$f=\frac{1}{z}.$$

(ii) In the case of a flat plate we take the origin of the z-plane at the intersection of two straight lines, one that of the ground surface, the other that of the flat plate, and we transform the z-plane into the f-plane by the relation

$$f=\log z.$$

6. Summary.

In this paper the author treats the problem of the effect of ground interference upon an aerofoil with a circular arc section, when the circle of this aerofoil intersects or touches the surface of the ground. By the conformal transformation the region outside the circular arc and the straight line is transformed into a rectangle. Then the velocity of the flow of the perfect fluid is determined and hence the lift and the moment of the lift acting on the aerofoil are calculated by Blasius' formulas.

In conclusion, the author expresses his cordial thanks to Prof. Nisihara for his kind advice and encouragement throughout this investigation.

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