The Investigation on the "X-ray Stress Measurement".

By

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To measure the stress by X-ray was proposed in 1930 by G. Sachs and J. Weerts in the case of duralumin[1].

Since, in 1935, R. Glocker ascertained the fact that the surface stress is perfectly measurable by means of X-ray[2], it interested researchers of stress problems and achieved far greater development especially in our country and Germany.

"X-ray stress measurement" is excellent in the following points:

1) not only the stress of a test piece but also of machine parts can be measured directly without any additional or harmful working for objects.

2) local stress of objects is more clearly demonstrated as compared with any other method hitherto tried.

We personnel performed laboratory experiments using the principle of "X-ray stress measurement" and examined the results. Moreover, we have designed and produced an "X-ray stress measuring instrument", which is very convenient for actual use in applying the method beyond the laboratory, and ascertained the possibility of the shop-measurement.

I. X-ray and crystals.

X-ray is a kind of electromagnetic wave caused by the impact of cathode rays upon matter. It is divided into two kinds with regard to wave length as in the case of ordinary light. Ordinary white light is proved to be a mixture of various rays of visible light of different wave lengths each corresponding to pure colours, because a light beam spreads into a spectrum from violet to red by refraction in a prism. In analogous fashion the spectrum of a beam of X-ray identifies the quality. Corresponding to white light we name the general, "white" or continuous X-ray and to pure colour "characteristic" X-ray. The latter is produced when the potential on the X-ray tube is sufficiently high and its wave length depends upon the kind of anticathode matter, upon which cathode rays impinge.

The metals, which form many machine parts, are composed of many crystals. They are natural gratings in which parallel planes of regularly marshaled atoms, shortly to say atomic planes, spread from each other at distance of the same order of magnitude as X-ray wave lengths. The distance we call "spacing".

We now proceed to consider the reflection of X-ray. Let the crystal structure be represented, in Fig. 1, by the series of planes Y_1, Y_2, Y_3......, d being "spacing". A, B, C, D...... compose a train of advancing wave of wave length λ, making the angle θ with atomic planes. Consider those waves which, after reflection, join in moving along OT, and compare the distances which they must travel from some line such as OK before they reach the point T. Consider the route AOT and BPT, their path difference is equal to P_1PO, so $P_1PO = P_1PP_2 = 2d \cdot \sin\theta$. If the path difference is equal to the wave length or any whole multiple of it, the wave trains are in the same phase and their amplitudes are added together. So, if the condition

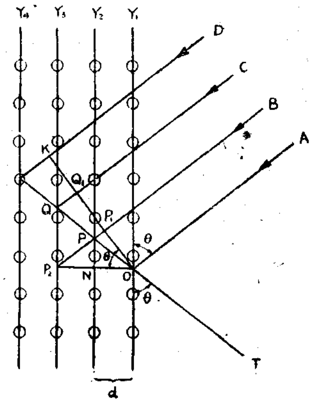

Fig. 1.

$$2d \cdot \sin\theta = n\lambda \qquad n = 1,\ 2,\ 3\ \ldots\ldots \qquad (1)$$

is satisfied, the reflected wave in the direction of OT, making an angle θ with atomic planes, is very strong. That is to say, only when the condition (1) is satisfied, the reflected X-ray is obtainable. This condition is called "Bragg's Law".

As the crystals of usual metals used for machine parts are very small and take random orientations, the atomic planes, as Y_1, Y_2...... which satisfy the condition (1), are considered not only as in the case of Fig. 1 but also as in any case, obtained by rotating the figure around the incident X-ray beam. So, as reflected X-ray becomes conical as in Fig. 2, when the film is put perpendicular to the incident X-ray beam, circular image of reflected X-ray is obtained.

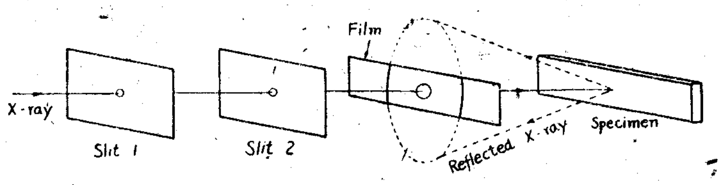

Fig. 2.

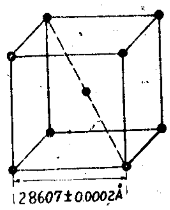

Fig. 3.

Usual metals, such as iron, aluminium or its alloys, have cubic lattices as in Fig. 3, which shows the unit cell of α-iron. In such cubic lattice, between the "lattice constant" a, i.e. the length of an edge of the cube, and the "spacing" d of any set of parallel atomic planes, there is the following relation:

$$d=\frac{a}{\sqrt{h^2+k^2+l^2}} \tag{2}$$

where $(h.\ k.\ l)$ is Miller's indices of crystal.

II. The theory of the "X-ray stress measurement".

In brief, the principle of the "X-ray stress measurement" stands on the basis of the fact that the "spacing" of metal varies according to the stress applied to it and it consists in the accurate measurement of the change of "spacing".

In Fig. 4, take any point O on the surface of test piece and let σ_1, σ_2, σ_3 be the principal stresses at the point O and ε_1, ε_2, ε_3 the strains in the direction of the principal stresses σ_1, σ_2, σ_3.

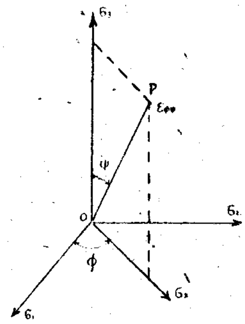

Fig. 4.

For the case of the "X-ray stress measurement", $\sigma_3=0$, because, as only surface stresses are treated, the problems are two dimentional.

The relation between σ and ε becomes as follows:

$$\left.\begin{aligned} \varepsilon_1 E &= \sigma_1-\nu\sigma_2 \\ \varepsilon_2 E &= \sigma_2-\nu\sigma_1 \\ \varepsilon_3 E &= -\nu(\sigma_1+\sigma_2) \end{aligned}\right\} \tag{3}$$

where $E=$ Elastic constant
$\nu=$ Poisson's ratio

Next, at the point O, let σ_x, σ_y, ε_x, ε_y be the stresses and strains in any two directions perpendicular to each other, which lie in the surface of the object. The relation between them are as follows:

$$\left.\begin{aligned} \varepsilon_x E &= \sigma_x-\nu\sigma_y \\ \varepsilon_y E &= \sigma_y-\nu\sigma_x \\ \varepsilon_3 E &= -\nu(\sigma_x+\sigma_y)=-\nu(\sigma_1+\sigma_2) \end{aligned}\right\} \tag{4}$$

The strain, in the direction whose direction cosines are γ_1, γ_2, γ_3, are represented by

$$\varepsilon=\gamma_1^2\varepsilon_1+\gamma_2^2\varepsilon_2+\gamma_3^2\varepsilon_3 \tag{5}$$

Denoting the strain in the direction of OP in Fig. 4 as $\varepsilon_{\phi,\psi}$, of which the direction cosines are $\sin\psi\cdot\cos\phi$, $\sin\psi\cdot\sin\phi$ and $\cos\psi$, $\varepsilon_{\phi,\psi}$ is represented as (6):

$$\varepsilon_{\phi,\psi}=\varepsilon_1\cos^2\phi\cdot\sin^2\psi+\varepsilon_2\sin^2\phi\cdot\sin^2\psi+\varepsilon_3\cos^2\psi$$
$$=\sin^2\psi(\varepsilon_1\cos^2\phi+\varepsilon_2\sin^2\phi)+\varepsilon_3-\varepsilon_3\sin^2\psi \tag{6}$$

At the same time, the strain ε_x, which lies in the plane $\sigma_1O\sigma_2$, and makes an angle ϕ with the direction of σ_1, is as follows:

$$\varepsilon_x=\varepsilon_1\cos^2\phi+\varepsilon_2\sin^2\phi \tag{7}$$

Substituting (7) for the bracket of the first term of (6), the formula (6) becomes as follows:

$$\varepsilon_{\phi,\psi}-\varepsilon_3=(\varepsilon_x-\varepsilon_3)\sin^2\psi \tag{8}$$

On the other hand, (4) leads to

$$E(\varepsilon_x-\varepsilon_3)=(1+\nu)\sigma_x \tag{9}$$

therefore (8) changes as follows:

$$E(\varepsilon_{\phi,\psi}-\varepsilon_3)=(1+\nu)\sigma_x\sin^2\psi \tag{10}$$

Using this formula, if $\varepsilon_{\phi,\psi}$, ε_3 and ψ are found by experiments, the stress σ_x, taking any direction on the surface of object, can be determined.

In the actual case, however, as will be explained later, what is measured is not ε_3. So, (10) must be amended as follows. In Fig. 5, let the strain in the direction of (ϕ, ψ_1) and (ϕ, ψ_2) be $\varepsilon_{\phi,\psi_1}$ and $\varepsilon_{\phi,\psi_2}$ respectively, (10) may be written as follows:

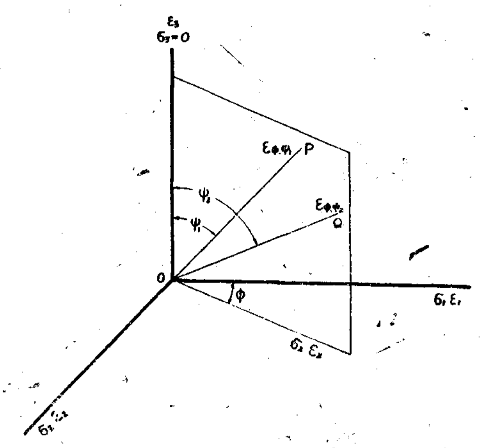

Fig. 5.

$$(\varepsilon_{\phi,\psi_1}-\varepsilon_3)E=(1+\nu)\sigma_x\sin^2\psi_1$$
$$(\varepsilon_{\phi,\psi_2}-\varepsilon_3)E=(1+\nu)\sigma_x\sin^2\psi_2$$

hence it follows

$$E(\varepsilon_{\phi,\psi_2}-\varepsilon_{\phi,\psi_1})=(1+\nu)\sigma_x(\sin^2\psi_2-\sin^2\psi_1) \tag{11}$$

Now, denoting the "spacing" of the atomic planes discussed now as d_0 when no stress exists in material and the one of the same kind of atomic planes, the normal of which makes an angle ϕ with σ_1 and ψ with the normal of the surface of the object, as $d_{\phi,\psi}$, the straines $\varepsilon_{\phi,\psi_1}$ and $\varepsilon_{\phi,\psi_2}$ in (11) may be regarded respectively as follows:

$$\varepsilon_{\phi,\psi_1}=\frac{d_{\phi,\psi_1}-d_0}{d_0},\quad \varepsilon_{\phi,\psi_2}=\frac{d_{\phi,\psi_2}-d_0}{d_0} \tag{12}$$

therefore

$$\varepsilon_{\phi,\psi_2}-\varepsilon_{\phi,\psi_1}=\frac{d_{\phi,\psi_2}-d_{\phi,\psi_1}}{d_0} \tag{13}$$

Putting $d_0=d_{\phi,\psi_1}+\delta$ and neglecting the higher order of δ, (13) is given by

$$\varepsilon_{\phi,\psi_2}-\varepsilon_{\phi,\psi_1}=\frac{d_{\phi,\psi_2}-d_{\phi,\psi_1}}{d_{\phi,\psi_1}}\left(1-\frac{\delta}{d_{\phi,\psi_1}}\right)$$

where $\delta/d_{\phi,\psi_1}$ is the order of 1/1000, so neglecting it compared with 1, (13) may be written in the following form:

$$\varepsilon_{\phi,\psi_2}-\varepsilon_{\phi,\psi_1}=\frac{d_{\phi,\psi_2}-d_{\phi,\psi_1}}{d_{\phi,\psi_1}} \tag{14}$$

and, from (11), σ_x takes finally the following form:

$$\sigma_x=\frac{E}{1-\nu}\,\frac{1}{\sin^2\psi_2-\sin^2\psi_1}\,\frac{d_{\phi,\psi_2}-d_{\phi,\psi_1}}{d_{\phi,\psi_1}} \tag{15}$$

This formula gives the magnitude of σ_x, when we obtain d_{ϕ,ψ_1} and d_{ϕ,ψ_2} by experiment. In a corresponding way, σ_y can be obtained from

$$\sigma_y = \frac{E}{1-\nu} \frac{1}{\sin^2 \psi_2 - \sin^2 \psi_1} \frac{d_{\phi+\frac{\pi}{2},\psi_2} - d_{\phi+\frac{\pi}{2},\psi_1}}{d_{\phi+\frac{\pi}{2},\psi_1}} \tag{16}$$

The full analysis of the stress distribution at the point O is, in a word, to know the principal stresses σ_1 and σ_2 in their magnitudes and directions, of which we have, as is known from (4),

$$\sigma_x + \sigma_y = \sigma_1 + \sigma_2 \tag{17}$$

and from the relation of the elasticity,

$$\sigma_x - \sigma_y = (\sigma_1 - \sigma_2) \cos 2\phi \tag{18}$$

Solving (17) and (18) for σ_1 and σ_2, it follows:

$$\left.\begin{aligned} \sigma_1 &= \frac{\sigma_x(1+\cos 2\phi) - \sigma_y(1-\cos 2\phi)}{2\cos 2\phi} \\ \sigma_2 &= \frac{-\sigma_x(1-\cos 2\phi) + \sigma_y(1+\cos 2\phi)}{2\cos 2\phi} \end{aligned}\right\} \tag{19}$$

Where, since σ_x and σ_y are able to be obtained from (15) and (16), the principal stresses σ_1 and σ_2 are determined in magnitude, if the angle ϕ is found. On the other hand, as the finding of ϕ leads to the determination of σ_1 as well as σ_2 at the same time, the question is how to find out the angle ϕ.

The solution is as follows: consider σ_x', making an angle α wlth σ_x and consequently $(\phi+\alpha)$ with σ_1, then, in the same way as in (18),

$$\sigma_x' - \sigma_y' = (\sigma_1 - \sigma_2) \cos 2(\phi+\alpha) \tag{20}$$

and putting

$$V = \frac{\sigma_x - \sigma_y}{\sigma_x' - \sigma_y'} \frac{\cos 2\phi}{\cos 2(\phi+\alpha)} \tag{21}$$

we can calculate V because σ_x, σ_y, σ_x' and σ_y' are all measurable. And from the relation

$$\sigma_x + \sigma_y = \sigma_x' + \sigma_y' = \sigma_1 + \sigma_2 \tag{22}$$

it is necessary to know only three of them for determination of V. Solving (21) for ϕ, we obtain

$$\cos 2\phi = \frac{\pm V \sin 2\alpha}{\sqrt{(V\cos 2\alpha - 1)^2 - V^2 \sin^2 2\alpha}} \tag{23}$$

from which ϕ is determined. If $\alpha = 45°$ in this formula, it becomes

$$\cos 2\phi = \frac{\pm V}{\sqrt{1+V^2}} \quad \text{or} \quad V = \pm \cot 2\phi \tag{24}$$

The double sign $\pm$ in (23) or (24) depends upon the relative sense of ϕ to α, i.e. as is shown in Fig. 6 it takes plus when they are of the same sense and takes minus sign when the case is opposite. Generally, by solving (23), four values of ϕ are given, two of which form a pair respectively and have opposite sign and each pair makes an angle of 90 degrees to each other. So it is unknown whether σ_1 or σ_2 makes an angle ϕ with σ_x. For this reason σ and ϕ are solved independently. However, in this place, the direction making an angle ϕ with σ_x must be of σ_1. For the purpose of solving this question, the magnitude of any one principal stress must be obtained once more by the above method, then by (17) or (22) the other is obtained and the greater is σ_1. Thus the full analysis of the stress distribution on any point of the surface of the object is carried out.

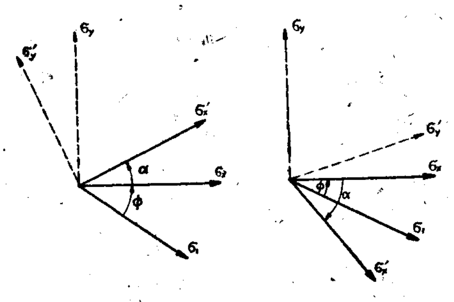

Fig. 6.

III. The practical operation of "X-ray stress measurement".

For the practical operation of measuring stresses, at least two photographs are required, of

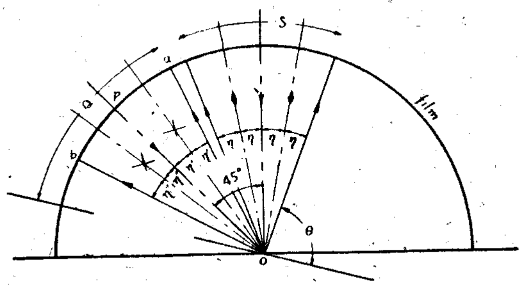

Fig. 7.

which the one is obtained by the X-ray beam incident perpendicularly to the surface of the object and the other by the beam which makes an angle of 45 degrees with the normal of the surface of the object, for the purpose of obtaining high accuracy.

As will easily be seen from Fig. 7, the angle between the surface normal and the normal of the atomic planes, reflecting the X-ray, is η for the former and $45°-\eta'$ or $45°+\eta''$ for the latter. That is to say,

$$\psi_1=\eta, \quad \psi_2=45°-\eta' \quad \text{or} \quad \psi_2=45°+\eta''$$

Though, of course, η, η' and η'' are not equal to one another and vary according to the acting stresses, their difference are so small that they can be regarded as equal and their mean value, 9.5° in the case of iron and steel, is used in the formula (15). When the combination of $\psi_1=\eta$ and $\psi_2=45°+\eta''$ is used in (15) it is denoted as $\perp+$) measurement for convenience sake and when $\psi_1=\eta$, $\psi_2=45°-\eta'$ as $\perp-$) measurement. As above, putting $\eta=\eta'=\eta''=9.5°$ approximately, (15) becomes as follows in $\perp+$) measurement:

$$\sigma_x=\frac{E}{1-\nu}\frac{1}{\sin^2 54.5°-\sin^2 9.5°}\frac{d_{\phi,54.5°}-d_{\phi,9.5°}}{d_{\phi,9.5°}}$$

$$=25814\frac{d_{\phi,54.5°}-d_{\phi,9.5°}}{d_{\phi,9.5°}}\ \text{kg/mm}^2 \qquad (25)$$

where $E=21000\ \text{kg/mm}^2$ and $\nu=0.28$

and in $\perp-$) measurement

$$\sigma_x=\frac{E}{1-\nu}\frac{1}{\sin^2 35.5°-\sin^2 9.5°}\frac{d_{\phi,35.5°}-d_{\phi,9.5°}}{d_{\phi,9.5°}}$$

$$=52959\frac{d_{\phi,35.5°}-d_{\phi,9.5°}}{d_{\phi,9.5°}}\ \text{kg/mm}^2 \qquad (26)$$

The above methods are of common use because of their accuracy, especially $\perp+$) measurement as will be mentioned later[2]. However, as they need at least two films for obtaining d_{ϕ,ψ_1} and d_{ϕ,ψ_2}, it is very troublesome. When we need not so much accuracy as that of the $\perp+$) measurement the following method may be used. That is to say, choosing as follows:

$$\psi_1=45°-\eta' \quad \text{and} \quad \psi_2=45°+\eta''$$

(15) becomes

$$\sigma_x=\frac{E}{1-\nu}\frac{1}{\sin^2 54.5°-\sin^2 35.5°}\frac{d_{\phi,54.5°}-d_{\phi,35.5°}}{d_{\phi,35.5°}}$$

$$=50390\frac{d_{\phi,54.5°}-d_{\phi,35.5°}}{d_{\phi,35.5°}}\ \text{kg/mm}^2 \qquad (27)$$

As can easily be seen, if only one film is obtained by obliquely incident X-ray, σ_x is measurable. This is denoted as $+-$) method[3].

When the oblique photographs can not be obtained on the account of the defect of apparatus, the following method may be used. It is proposed by F. Weber and H. Möller[4] in the early course of the development of the "X-ray stress measurement". Easily, from (3) or (4) we can write

$$\varepsilon_3=-\frac{\nu}{E}(\sigma_1+\sigma_2) \qquad (28)$$

and

$$\varepsilon_3=\varepsilon_{\phi\,0}=\frac{d_{\phi,0}-d_0}{d_0} \qquad (29)$$

where d_0 is "spacing" of the atomic planes, of which the normal coincides with that of the surface of the object, under the condition that no stress exists and $d_{\phi,0}$ is the one under stress. (28) takes the following form:

$$\sigma_1+\sigma_2=-\frac{E}{\nu}\,\frac{d_{\phi,0}-d_0}{d_0}$$

$$=-75000\frac{d_{\phi,0}-d_0}{d_0}\ \text{kg/mm}^2 \qquad (30)$$

In this method, X-ray beam is perpendicular to the surface of the object. If the values of $d_{\phi,0}$ and d_0 are obtained by X-ray, the sum of the principal stresses is determined.

As will be seen from the above, it is indispensable to obtain d_0 previously and the value of stress obtained is the sum of principal stress. Moreover, by this method, the direction of stress, the one element of the conception of the stress, is not contained at all. And, even if the X-ray beam is perpendicular to the surface of the object, the value of $d_{\phi,0}$ can not be obtained, but $d_{\phi,\eta}$ is measured, that is to say, between the normal of the surface of the object and the one of the atomic planes reflecting X-ray it contains an angle η, so ψ is not zero. This method, denoted as $\perp$) method, contains such defects as above.

IV. The determination of "spacing" d and its measuring accuracy.

By the explanation above mentioned, the distribution of the surface stress at any point is analysed. The next problem is by what means and how accurately we can measure the "spacing" d. Now, we will discuss this point. Obviously from (1), when the wave length λ of the incident X-ray is known, d is determined by measuring the angle θ between the reflecting atomic planes and reflected X-ray. From (2), the value of lattice constant a is obtained. Taking d as a function of θ and differentiating (1), it follows,

$$\Delta\theta=\frac{-\tan\theta}{d}\Delta d \qquad (31)$$

where $\Delta\theta$ is the slight variation of θ and Δd is that of d. So, it is clear that, as the value of d is approximately constant for metals of same kind, the absolute value of θ must be adjusted to be as large as possible for measuring $\Delta\theta$ as large as possible compared with Δd. In other words, it is desirable to choose the X-ray so as to make θ as large as possible in relation to the value of d according to the kind of metals used.

Now, in the case of α-iron, whose lattice is body centered cubic and the lattice constant is 2.8607 ± 0.0002 Å as is shown in Fig. 3, the combinations of the Miller's indices $(h.k.l.)$ of reflecting atomic planes and wave length λ of used X-ray which makes θ comparatively large is shown in Table 1. As will be known from it, it seems to attain our object to make θ the greatest to utilize the Mo-$K\alpha_1$ radiation as the X-ray which is reflected by (8.0.0) atomic planes of iron. However, as the spectrum by (8.0.0) atomic planes is the same meaning as the eighth spectrum of reflection of (1.0.0) atomic planes, it takes many hours to take the photograph as the intensity of the reflected X-ray is weakened very much. In this point, it is rather desirable to utilize the reflection of (3.1.0) atomic planes by Co-$K\alpha_1$ radiation. Moreover, it is profitable in the following point, that is to say, the wave length of the K-radiation of characteristic X-ray of Co is

$$K\alpha_2=1789.19\ \text{x.u.} \qquad K\alpha_1=1785.29\ \text{x.u.}$$

$$K\beta_2=1617.44\ \text{x.u.} \qquad K\beta_1=1605.72\ \text{x.u.}$$

and the K-critical absorption wave length of α-iron is 1739.4 x.u., which lies between the wave length of $K\alpha$ and $K\beta$ doublet of iron by Co-radiation. So, as $K\beta$ radiation is absorbed in iron, reflected X-ray is almost only by $K\alpha$ doublet, and as the ratio of intensity of $K\alpha_1$ and $K\alpha_2$ is nearly 2 : 1, reflected X-ray seems to be monochromatic. Here, the value of θ being as large as nearly 80 degrees, the reflected X-ray fulfills the condition of so-called "back reflection". (Fig. 2)

Now, the calculation of d may be dealt with. Therein the film is circular. In Fig. 8, let the distance between the interferential line be S and the one between the surface of the object

Table 1.

	λ (x.u.)	*h. k. l.*	θ
Mo $K\alpha_1$	707.831	(8, 0, 0)	83°
Co $K\alpha_1$	1785.29	(3, 1, 0)	80°50.5′
Cr $K\alpha_1$	2285.03	(2, 1, 1)	78°30′
Fe $K\alpha_1$	1932.076	(2, 2, 0)	73°
Cu $K\alpha_1$	1537.395	(2, 2, 2)	69°

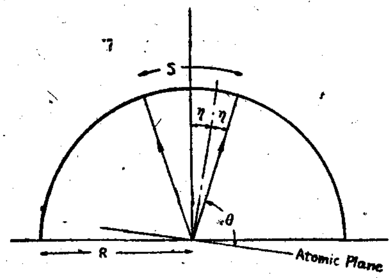

Fig. 8.

and film be R, as will easily be seen from the figure, it follows :

$$\eta=\frac{S}{4R}\ \text{rad.} \tag{32}$$

Citing the case of utilizing the reflection from (3. 1. 0) atomic planes by Co-$K\alpha_1$ radiation, by (1) and (2), it follows

$$\lambda=\frac{2a}{\sqrt{3^2+1^2+0}}\sin\theta=\frac{2a}{\sqrt{10}}\cos\eta$$

$$\therefore\quad a=\frac{\sqrt{10}}{2}\,\frac{\lambda}{\cos\eta} \tag{33}$$

Substituting $\lambda=1785.29$ x.u. for (33), it takes the following form

$$a=\frac{1.785.29\sqrt{10}}{2}\cdot\frac{1}{\cos\eta}$$

$$=2822.8\,\frac{1}{\cos\eta} \tag{34}$$

In this manner, measuring S from the films and calculating η from (32), we can determine a from the equation (34). Moreover, d is obtained from (35),

$$d=892.65\,\frac{1}{\cos\eta} \tag{35}$$

Now, in order to calculate the value of η from (32), the measurement of the value of R as well as S is necessary. For this purpose, the diffraction line of gold or silver on the same film with iron ring, which is obtained by the reflection from (4. 2. 0) atomic planes by Co-$K\alpha_1$ radiation and is little outside of the iron ring, is used. For instance, in the case of gold diffraction line, as the lattice constant a of gold is 4.0699 Å, it follows corresponding to (33)

$$1.78529=\frac{2\times4.0699}{\sqrt{4^2+2^2+0}}\cos\eta$$

$$\cos\eta=0.9808668$$

$$\eta=11°13'22'' \tag{36}$$

Measuring S by the gold interferential ring, the value of R is calculated from (32).

As will be shown in Fig. 7, in the case of oblique incidence, however, the X-ray reflected by (3. 1. 0) atomic planes makes an angle $45°-\eta'$ and $45°+\eta''$ with the surface normal of the object, and falls upon a and b on the film. Therefore, it will easily be seen $\widehat{Pa}\neq\widehat{Pb}$, where PO is incident X-ray beam. Because the position of P on the film is unknown, d or a on the direction of $45°-\eta'$ as well as $45°+\eta''$ can not be obtained. To remove this defect for the oblique incidence a standard line in the cover of film is photographed on the same film with the iron and gold interferential rings. That is to say, this line on the film gives us the settled position of the used camera. Therein the diameter of both gold rings

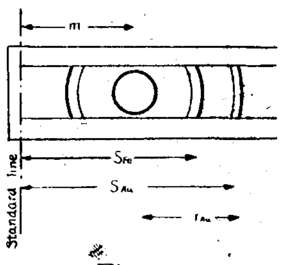

Fig. 9.

by perpendicularly incident X-ray and by obliquely incident X-ray gives equal value. Because $\widehat{Pa}=\widehat{Pb}$ for gold ring, the position of P can be determined. (See Fig. 9)

In the above, what is measured actually in order to attain the value of d or of a is S and R. So, the measuring accuracy of d or a consist in how accurately S and R are measured at all. From (32),

$$2\eta=\pi-2\theta=\frac{S}{2R}$$

$$\therefore \quad 2\theta=\pi-\frac{S}{2R}$$

Partially differentiating it we get

$$\frac{\partial\theta}{\partial S}=-\frac{1}{4R} \qquad \frac{\partial\theta}{\partial R}=\frac{S}{4R^2}$$

Denoting the measuring error of θ as $\Delta\theta$ when there exist the measuring error ΔS and ΔR for S and R respectively, it is represented as follows:

$$\Delta\theta=\frac{1}{4R}\left(\frac{S}{R}\Delta R-\Delta S\right)$$

$$=\frac{1}{2R}\left\{(\pi-2\theta)\Delta R-\frac{1}{2}\Delta S\right\} \qquad (37)$$

Next, the error that accompanies the calculation of d when the measurement of θ is attended by the error $\Delta\theta$ may be introduced[5]. Putting $n=1$ in (1), we get

$$d=\frac{\lambda}{2\sin\theta},$$

$$\therefore \quad \Delta d=-\frac{\lambda}{2}\frac{\cos\theta}{\sin^2\theta}\Delta\theta$$

$$=-d\cot\theta\,\Delta\theta \qquad (38)$$

where Δd is measuring error of d when there exists $\Delta\theta$ for the measurement of θ. Substituting (37),

$$\Delta d=\frac{d}{2R}\cot\theta\left\{\frac{1}{2}\Delta S-(\pi-2\theta)\Delta R\right\}$$

$$\therefore \quad \frac{\Delta d}{d}=\frac{1}{2R}\cot\theta\left\{\frac{1}{2}\Delta S-(\pi-2\theta)\Delta R\right\} \qquad (39)$$

therein $\frac{\Delta d}{d}=\frac{\Delta a}{a}$.

Now, to measure S, it is most convenient to contact glass scale, graduated by 0.1 mm, to films. So, ΔS may be about 0.1 mm. Putting $\Delta S=0.1$ mm, the value of $\Delta a/a$ for $\theta=50°, 55°\ldots\ldots 90°$ and $\Delta R=0.05$ mm are shown in Table 2. As is obvious from the table, even if $\Delta R=0.05$ mm,

Table 2.

θ deg	$\cot\theta$	$\pi-2\theta$ rad	$(\pi-2\theta)\Delta R$ mm	$\{\Delta S/2-(\pi-2\theta)\Delta R\}\times\frac{1}{2R}\times 10^4$ mm	$\Delta a/a\times10^5$, $\Delta S=0.1$ mm: $\Delta R=0$	$\Delta R=0.05$ mm
50	0.8390	1.3927	0.0698	6.7882	23.75	56.96
55	0.7002	1.2217	0.0611	6.2939	19.84	44.07
60	0.5773	1.0472	0.0524	5.7994	16.36	33.48
65	0.4663	0.8727	0.0436	5.3050	13.21	27.74
70	0.3640	0.6981	0.0349	4.8106	10.31	17.51
75	0.2679	0.5236	0.0262	4.3161	7.59	11.56
80	0.1763	0.3491	0.0175	3.8217	5.00	6.74
85	0.0874	0.1745	0.0087	3.3273	2.48	2.91
90	0.0000	0.0000	0.0000	0.0000	0.00	0.00

for the case of iron by Co-$K\alpha_1$ radiation θ being 80 degrees, $\Delta a/a$ or $\Delta d/d$ is comparatively small.

$$\frac{\Delta a}{a}=6.74\times 10^{-5} \tag{40}$$

Putting $a=2.86$ Å

$$\Delta a=2.86\times 6.74\times 10^{-5}=1.9\times 10^{-4}\text{ Å} \tag{41}$$

If $\Delta R=0$

$$\Delta a/a=5.00\times 10^{-5}$$
$$\therefore\ \Delta a=2.86\times 5.00\times 10^{-5}$$
$$=1.43\times 10^{-4}\text{ Å} \tag{42}$$

V. The theoretical accuracy of measured stress.

With the accuracy of measurement of d or a is dealt in the last chapter. So far as the stress is calculated from the value of d, the accuracy of calculated stress depends upon that of d. The theoretical error range of measured stress may be calculated in every case of the above methods.

In general, from the equation (15)

$$\sigma_x=C_i\frac{d_{\phi,\psi_2}-d_{\phi,\psi_1}}{d_{\phi,\psi_1}}\ \text{kg/mm}^2\quad i=1,2,3. \tag{43}$$

where for $\perp+)$ measurement $C_1=25814$

$\perp-)$ „ $C_2=52959$

$+-)$ „ $C_3=50390$

Differentiating (43), we get

$$\Delta\sigma_x=C_i\frac{1}{d_{\phi,\psi_1}}\left(\Delta d_{\phi,\psi_2}-\frac{d_{\phi,\psi_2}}{d_{\phi,\psi_1}}\Delta d_{\phi,\psi_1}\right) \tag{44}$$

where $\Delta\sigma_x$ is measuring error of stress, when the measurement of d is attended by the error Δd. Therein, assuming

$$d_{\phi,\psi_1}=d_{\phi,\psi_2}=d$$

and so

$$\Delta d_{\phi,\psi_1}=\Delta d_{\phi,\psi_2}\doteq\Delta d$$

in high order of approximation, (44) becomes as

$$\Delta\sigma_x=C_i\frac{1}{d}(2\Delta d\sim 0)$$

From (40) $\dfrac{\Delta a}{a}=\dfrac{\Delta d}{d}=6.74\times 10^{-5}$

$$\Delta\sigma_x=\pm C_i\times 1.348\times 10^{-4}\ \text{kg/mm}^2 \tag{46}$$

So, the range of error is as follows:

for $\perp+)$ measurement	3.48	kg/mm²
$\perp-)$ „	7.14	„
$+-)$ „	6.80	„

For $\perp)$ measurement, we will deal with the case when σ_2 is zero. From (30)

$$\sigma_x=-75000\frac{d_{\phi,0}-d_0}{d_0}\ \text{kg/mm}^2$$

In the analogous way as above,

$$\Delta\sigma_x=-75000\left(\frac{2\Delta d}{d}\sim 0\right)=\pm 10.12\ \text{kg/mm}^2$$

Of course, the range of error varies according to the condition of the experiment, that is to say, whether the film used is of circular form or plate and the distance between the film and the surface of the object.

VI. The variation of d caused by stress.

That the stress acting on the surface of the object can be determined if the value of a or d is measured by X-ray, is mentioned in Chapter II and its measuring accuracy is in Chapter IV. It becomes to be the next problem whether, as is supposed, the variation of d of the materials under stress is in the range of measuring error of d or not.

At first, an example of bending is carried out for this problem[6]. As in Fig. 10, X-ray is incident perpendicularly on the test piece under bending stress paralleled through two slits S_1 and S_2. For giving bending, of the supports A, B, D, we fasten D and move A and B in

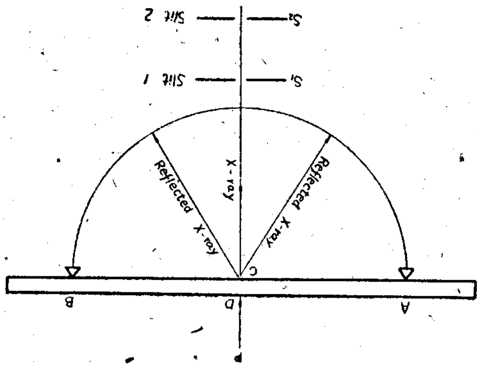

Fig. 10.

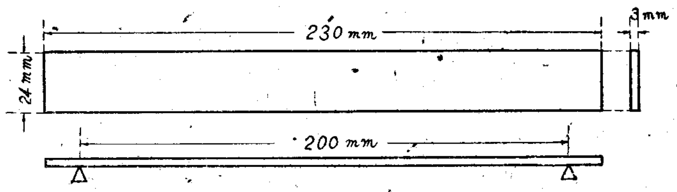

Fig. 11.

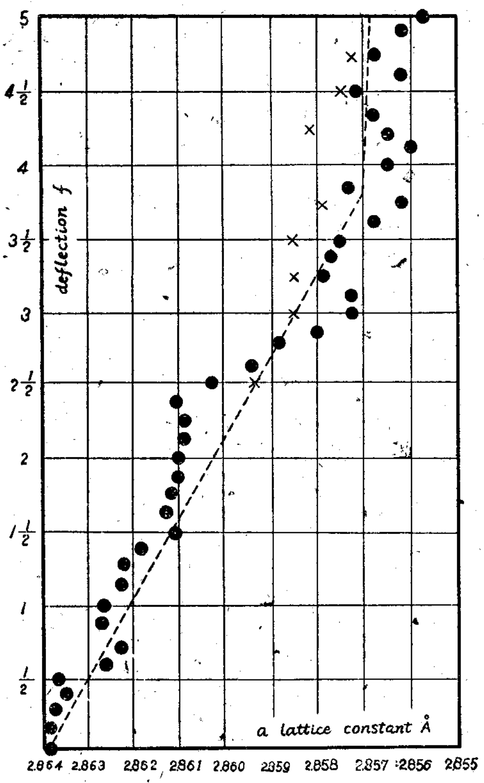

Fig. 12.

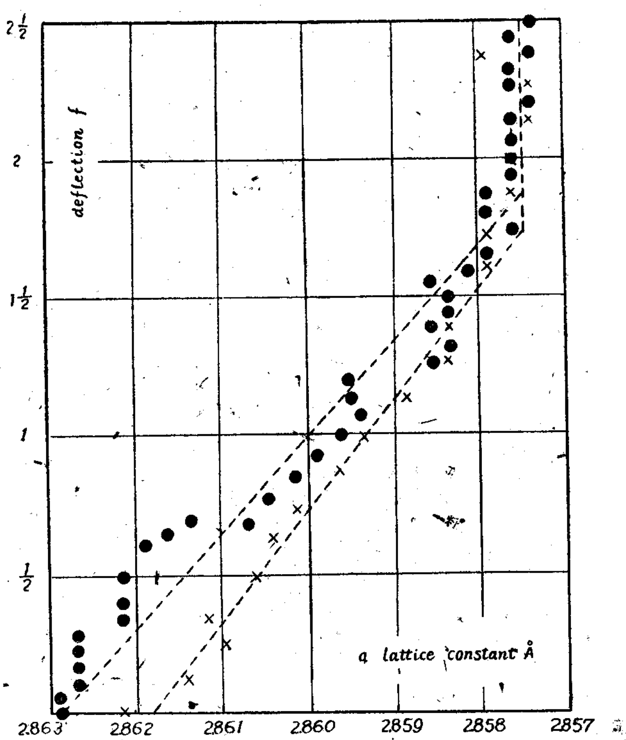

Fig. 13.

parallel stepwisely. The film is circular with the center at C, from which d or a is calculated at each step of bending. As an instance, the authors experimented on the case of pure iron, of which the test piece is as in Fig. 11 and the result of experiment is shown in Fig. 12. Moreover, they experimented on the case of cast iron, of which the result is shown in Fig. 13. As be seen in both figures the value of d or a decreases linearly according to the increase of the amount of deflection in pure iron as well as in cast iron, which contains comparatively much carbon, and the variation of d, now in question, is satisfactorily in the error range of X-ray measurement of d. Therein, in the case of pure iron they used the reflection from (3. 1. 0) atomic planes by Co-$K\alpha_1$ radiation, while for cast iron the reflection from (2. 2. 0) atomic planes by Fe-$K\alpha_1$ radiation is used. As the angle θ is about $80°50.5'$ for the former and about $73°$ for the latter, the error range for the former is 1.9×10^{-4} Å and for the latter is 3.3×10^{-4} Å, calculating them by the analogous method as in Chapter IV.

VII. The result of experiment.

The theory and technique of the " X-ray stress measurement " is as above mentioned. The authors carried out some experiments on it for the purpose to examine whether the value of measured stress by X-ray is equal to the stress acting on the object or not. The tried experiments were on the case of tension[7], compression[8] and bending[9]. Next, they will explain the results and try some investigation on them.

1) Tension.

Giving stepwise load on the test piece of pure iron as is shown in Fig. 14, they investigated the relation between the stress σ_x measured by X-ray and the one σ_M calculated from load. The setting of the measuring apparatus is shown in Fig. 15, where a Shearer tube was used. The

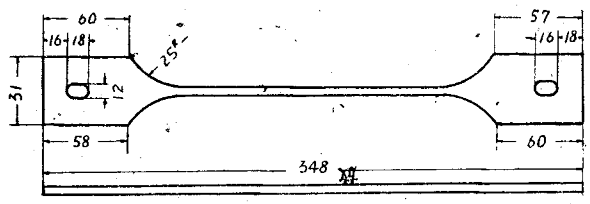

Fig. 14.

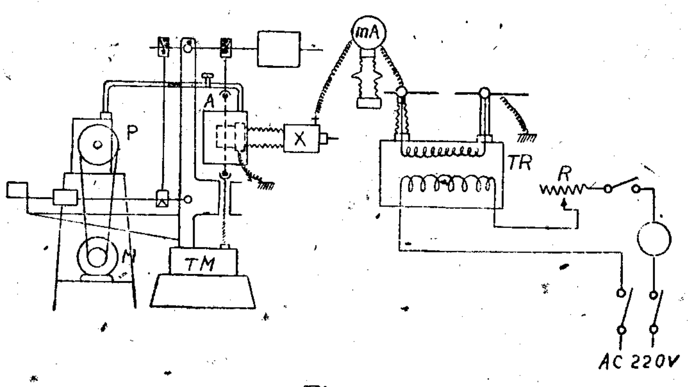

Fig. 15.

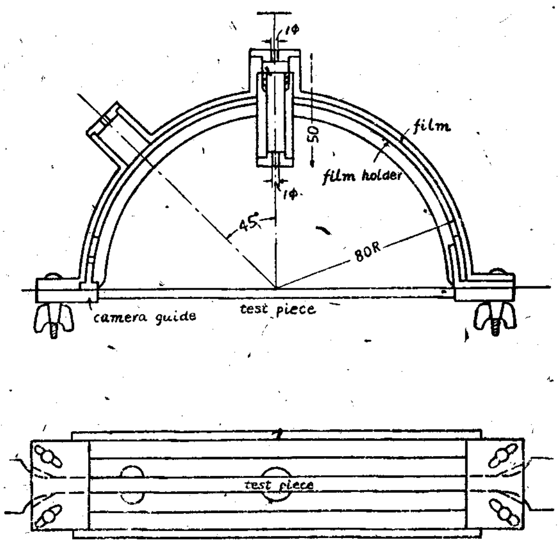

Fig. 16.

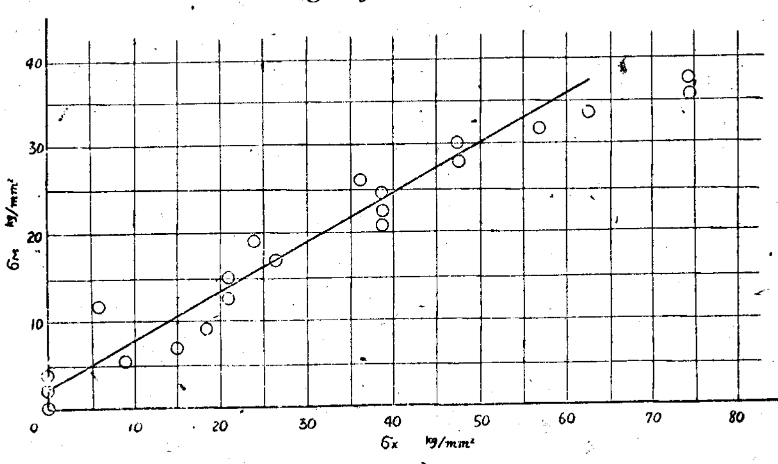

Fig. 17.

used camera is as in Fig. 16, which was attached to the test piece and X-ray beam was incident through each slit either perpendicularly or at an angle of 45° with the surface normal of the test piece. Thus, they obtained both perpendicular and oblique photographs, from which stress was measured. The result of $\perp$ —) measurement is shown in Fig. 17 and that of $\perp$) measurement in Fig. 18. The error range of measured stress under the condition here used is 2.9 kg/mm² for the former and 4.7 kg/mm² for the latter. Moreover, a specimen (Fig. 19) of mild steel (about 0.1% C) was stretched within the elastic limit and the load was removed and then the stress on the surface of the specimen was measured by X-ray. The experimental result is shown in Fig. 20, which shows no stress remains within the error range of measurement (4.7 kg/mm²).

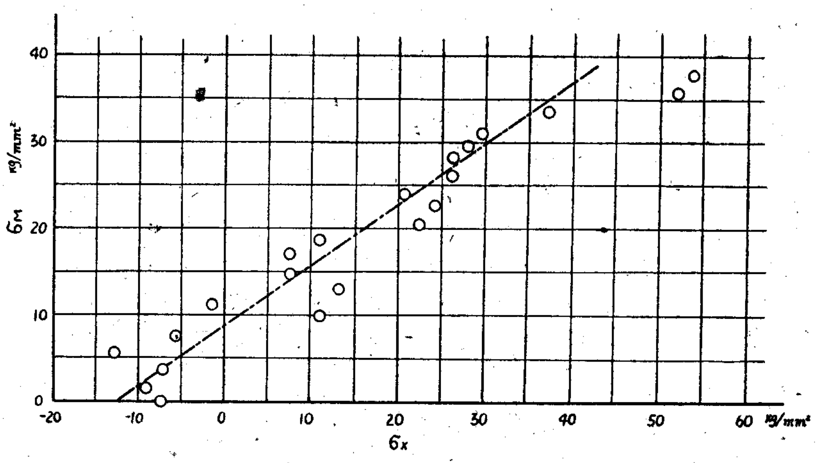

Fig. 18.

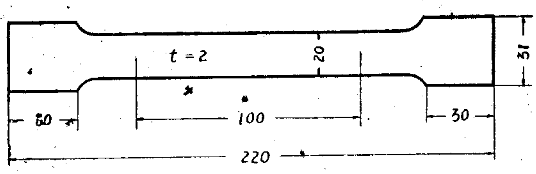

Fig. 19.

2) Compression.

They experimented with the test piece of 0.07%C mild steel as is shown in Fig. 21. The used X-ray tube is Sealex type and the result of this experiment is shown in Fig. 22.

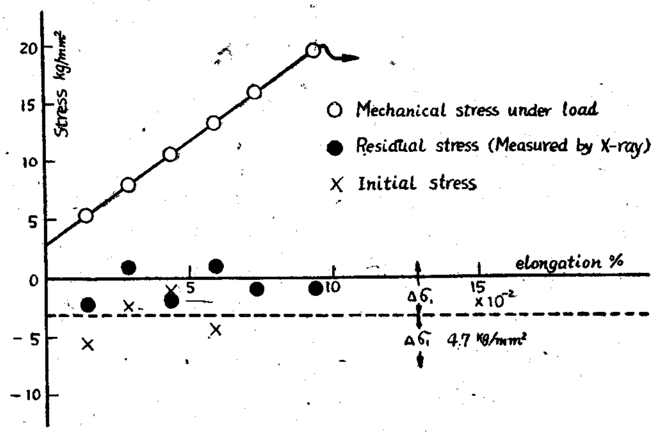

Fig. 20.

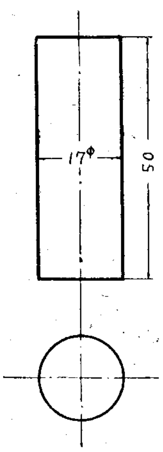

Fig. 21.

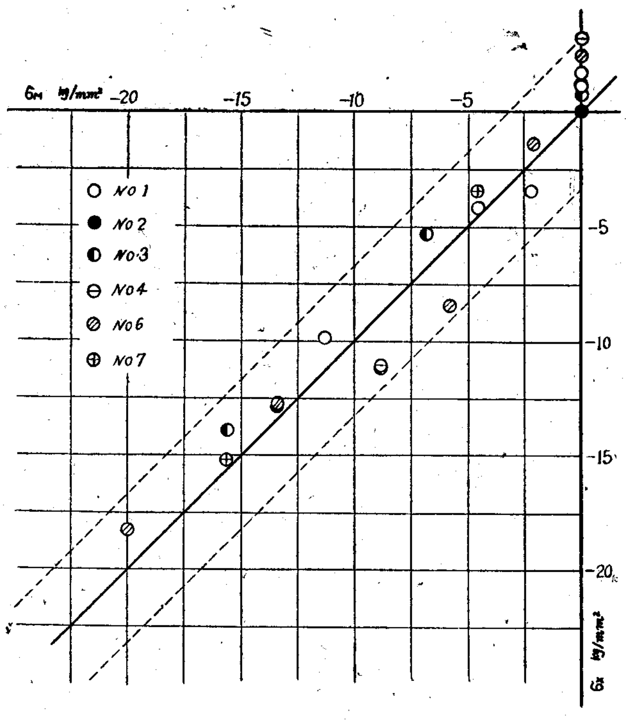

Fig. 22.

3) Bending.

In this case, they used specially designed camera as Fig. 23, where R and D are both rotatable and the principle for giving bending is the same as in Fig. 10. Fig. 24 shows the used specimen. When two photographs as are shown in Fig. 25 are obtained, the stress can be measured. On the other hand, the stress on the point P is represented as follows:

$$\sigma_M = \frac{6Eh}{l^2} f \qquad (47)$$

where h: the thickness of the test piece
l: the distance between two supports
f: the deflection at the middle point of the test picce
E: Elastic constant

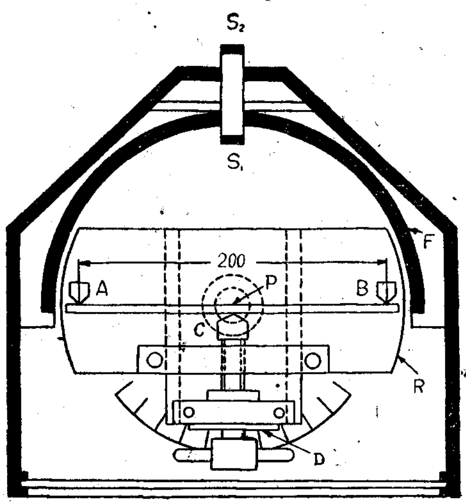

Fig. 23.

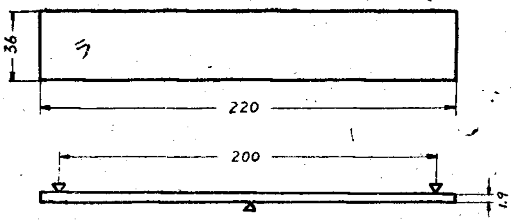

Fig. 24.

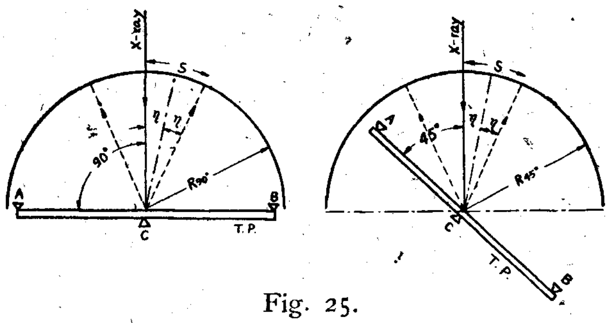

Fig. 25.

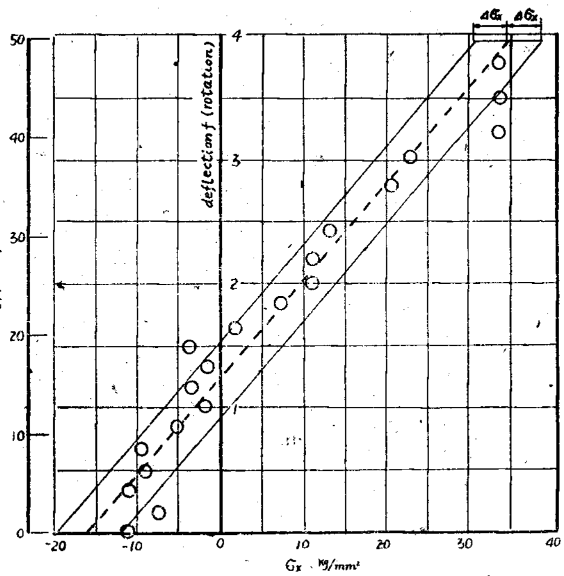

Fig. 26.

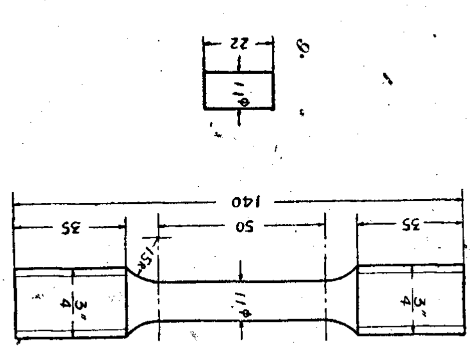

Fig. 27.

Therefore the stress is calculated by (47). The relation between σ_M and σ_X is shown in Fig. 26. The dotted lines in the figure represent the error range.

Surveying the above results, we can see that the value of stress measured by X-ray is approximately equal to the so called stress value in the range of measuring error. Therefore, it is clear

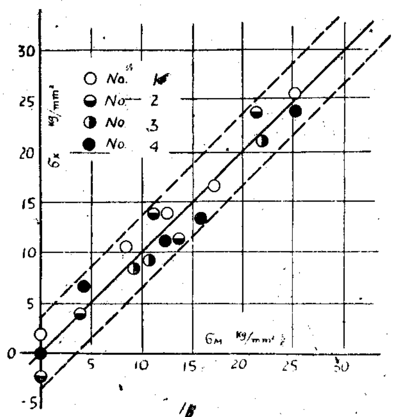

Fig. 28.

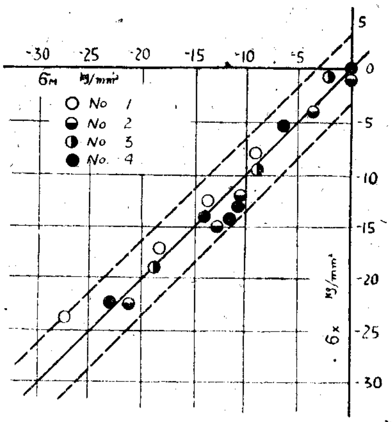

Fig. 29.

that, so far as elastic stress is concerned, it is measurable by " X-ray stress measurement " in the case of iron and steel.

Moreover, as an example of light metal, the authors tried an experiment on super-duralumin under tensile and compressive stress[10]. The test pieces used are shown in Fig. 27. The result by ⊥) measurement is shown in Figs. 28 and 29. The former is the case of tension and the latter of compression, in which the dotted lines show the range of error. In this case also, the stress measured by X-ray is approximately equal to mechanical stress in the error range. So, the possibility of the " X-ray stress measurement " was asceitained in the case of light metal also so far as elastic stress is concerned.

VIII. The application of the " X-ray stress measurement ".

As was written in the preface, the exellence of the " X-ray stress measurement " consists in the possibility of measuring stress without any harmful and additional working to the objects and of measuring highly local stress. Utilizing these points, the authors tried a few experiments for application.

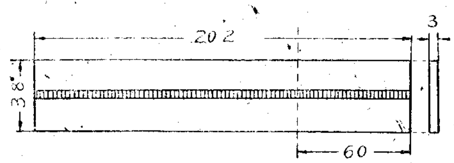

Fig. 30.

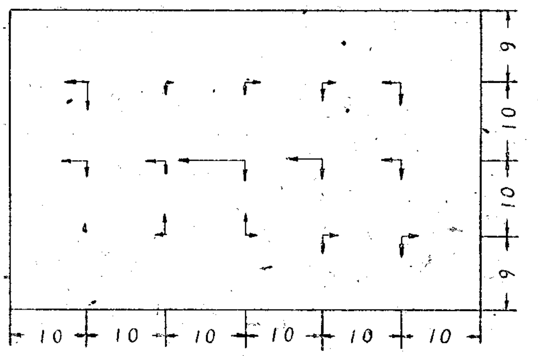

Fig. 31.

As an example, the measurement of residual stresses in welded part may be shown[11]. Fig. 30 shows the specimen, obtained by *V*-welding. The stresses parallel as well as perpendicular to the welding direction are measured in a few positions on the specimen. The result is shown in Fig. 31 where the length of arrows corresponds to the magnitude of stresses and the direction of them to the sign of stresses, i.e. the arrows in the first quadrant are plus, so they are tension.

IX. The " X-ray stress measuring instrument ".[12]

Hitherto, the " X-ray stress measurement " was executed only in laboratories, so was of a narrow range of application.

Making a step forward the authors designed and produced a " X-ray stress measuring instrument " so as to measure the stresses of the object wherever it may be. Besides this point, it was designed to be convenient for the full analysis of the surface stresses of the object, of

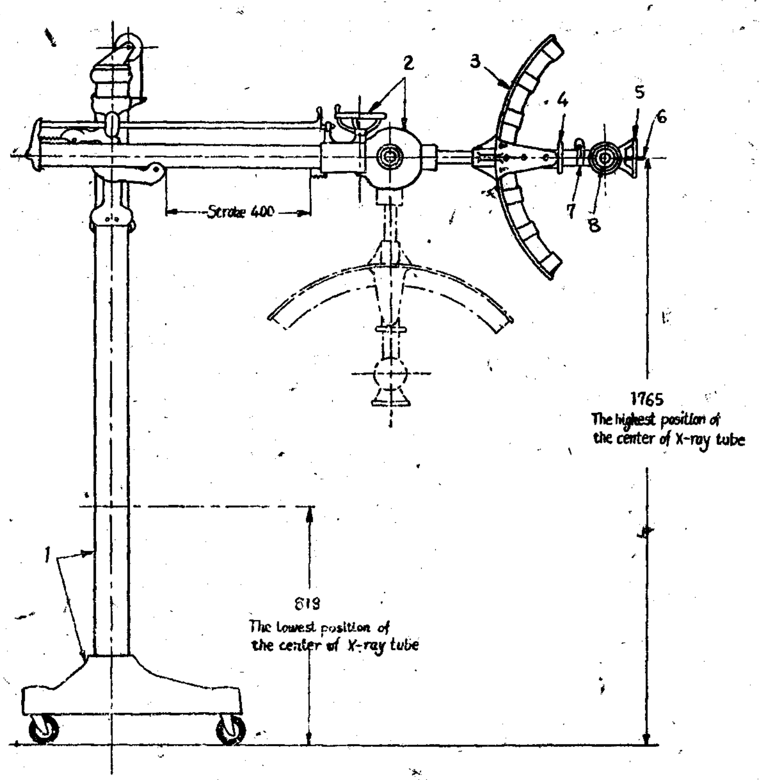

Fig. 32.

which the theory is explained in Chapter II. The outside view of this instrument is shown in Fig. 32. In the figure,

1 : tube stand
2 : main part of measuring instrument and inclination adjuster
3 : main part of arch type stress measuring instrument
4 : distance adjuster
5 : film holder
6 : slit
7 : X-ray tube supporter
8 : X-ray tube

It is carried to any place as we like by the rollers and the inclination of film is adjustable according to that of the surface of the object in wide range. They ascertained that it is very convenient for the actual use when the " X-ray stress measurement " is applied beyond laboratory and that sufficient accuracy is expected in shop-measurement.

X. Summary.

So far as the theory of the " X-ray stress measurement " is concerned, it seems established as was explained. From the stand point of metals as an aggregation of crystals, however, there remain many problems. Looking back upon the result obtained from this study, it may be remarked as follows :

1) within the range of the elastic limit of mild steel, the stresses measured by X-ray under tension, compression and bending coincide with the mechanical stress, in the measuring error range.

2) the " X-ray stress measurement " can be applied even for cast iron which is a special example of ferric substance.

3) in the case of super-duralumin, as an example of lignt metal, the possibility for its application is ascertained.

4) the residual stress, for example, in welded parts can be measured.

References.

1) Zeits f. Physik, 64 (1930) S. 344.
2) Zeits f. Tech. Physik, 16 (1935) S. 237.
3) Trans. Soc. Mech. Engr. Japan Vol. 8, No. 31 (1942) p. 53.
4) Arch. Eisenhüttenwesen, 5 (1931/1932) S. 215.
Mitt. Kaiser-Wilhem Institut Eisenforsch. Düsseldorf, 15 (1933) S. 59.
Zeits f. Tech. Physik, 14 (1933) S. 217.
5) Trans. Soc. Mech. Engr. Japan, Vol. 3, No. 12 (1937) p. 203.
6) " Vol. 4, No. 15 (1938) p. 110.
7) " Vol. 6, No. 24 (1940) p. 11.
" Vol. 7, No. 28 (1941) p. 15.
8) " Vol. 9, No. 34 (1943) p. 12.
9) " Vol. 6, No. 24 (1940) p. 11.
10) " Vol. 10, No. 38 (1944) p. 17.
11) " Vol. 3, No. 12 (1937) p. 203.
" Vol. 5, No. 18 (1939) p. 159.
12) " Vol. 8, No. 30 (1942) p. 32.