Investigation on the Earthquake Damage of the Goshō Suspension Bridge

By

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Synopsis

This investigation shows the earthquake damage of the Goshō Suspension Bridge, Fukui Prefecture. The general appearance of the failure is that the upper chords of the stiffening trusses buckled horizontally and the bridge deck flattened out during the earthquake, with the north span flattening out more than the south span.

These damages were theoretically investigated and the results obtained explained the damages fairly well.

1. Introduction

The Goshō Suspension Bridge spans the Kuzuryū River and is located near the town of Matsuoka at the eastern edge of the meizo-seismic area of the Fukui earthquake, June 28, 1948.

Fig. 1 shows the general view of the Goshō Bridge.

Classification of Road: Prefecture Road.

Name of Route: Matsuoka–Maruoka line.

Location:

Left Bank; Matsuoka town, Yoshida County, Fukui Prefecture.

Right Bank; Goryogashima Village, Yoshida County, Fukui Prefecture.

Total Length of Bridge: 248 m.

Effective Width: 3.64 m.

Effective Area of Bridge: 899.17 m^2.

No. of Span: 2

Length of one span: 124 m.

No. of Cables: 2

Composition of one cable: 6 special flexible steel wire ropes.

Dia. 44.5 mm, (1.78 in.). Wire ropes are arranged horizontally.
Back Stay : Straight.

Each stiffening truss has a wind bracing which is made of steel wire ropes and is arranged in an inclined plane as shown in **Fig. 1.** The stiffening truss has no bracing in the upper horizontal plane. The construction of the floor system is shown in **Fig. 2** and the bridge floor is made of planking.

The bridge towers are made of reinforced concrete. The centre tower acts as an anchor for the cables of the spans on both sides of it.

Both the abutments and anchorage blocks aıe made of plain concrete.

The design loads are as follows:

Uniform live load

244.13 kg/m², (50 lbs/ft²).

Dead load

268.54 kg/m², (55 lbs/ft²).

2. Condition of Damages

It was found by direct measurement immediately after the Fukui earthquake that the upper chords of the stiffening truss of both spans buckled horizontally in the central portion of each span as shown in **Fig. 3** and that the vertical members connected to it were bent. The saddles on the bridge towers displaced

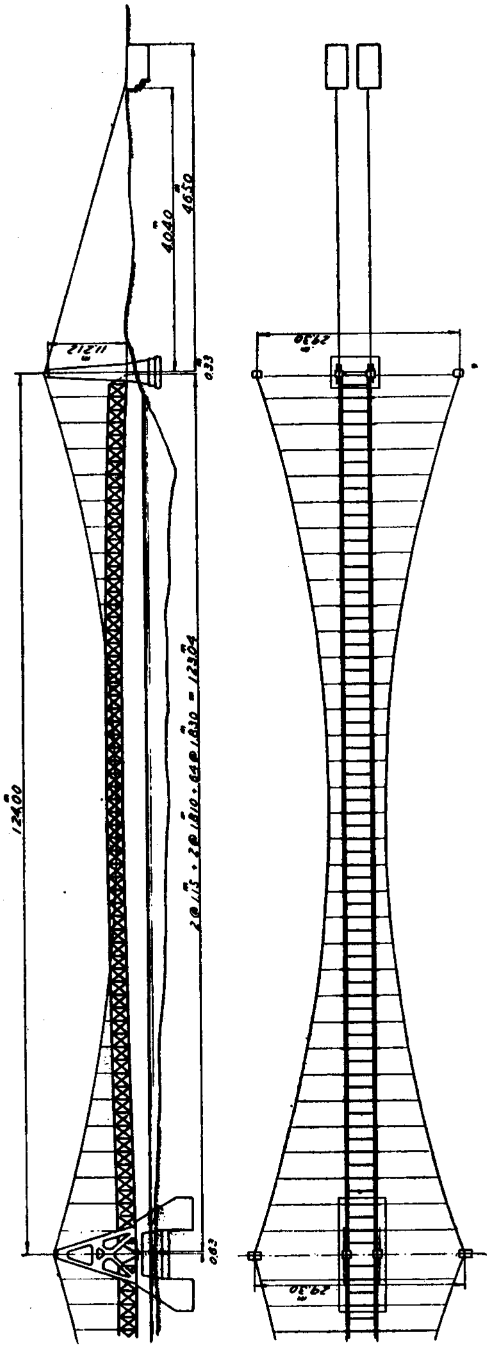


Fig. 1 General View of the Goshū Suspension Bridge

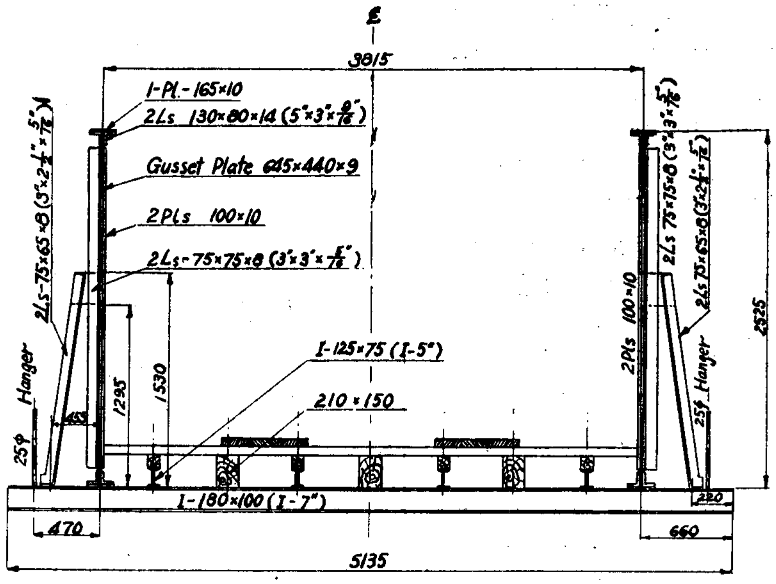


Fig. 2.1 Details of Stiffening Truss; Cross Section of Panel Point with Hanger

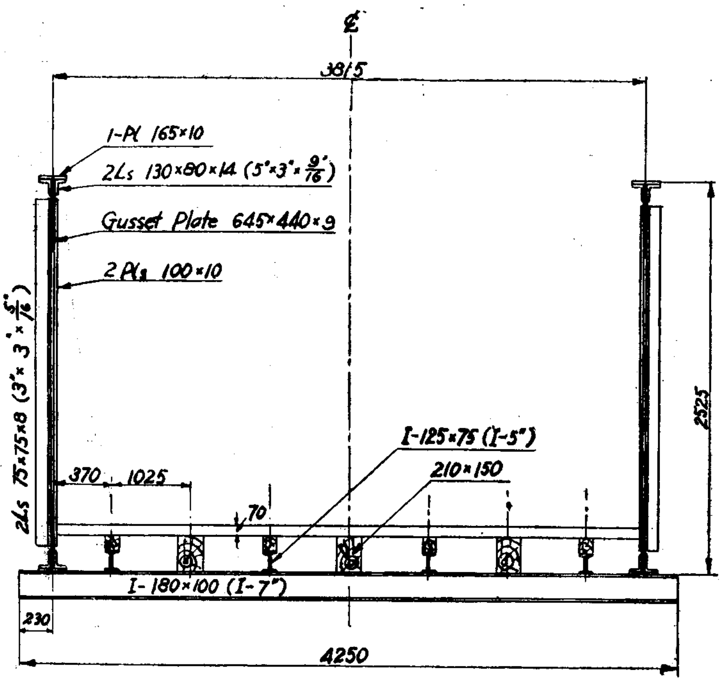
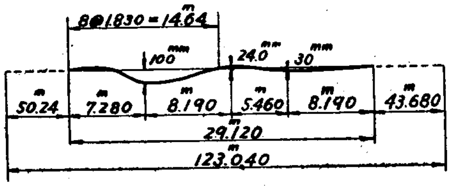


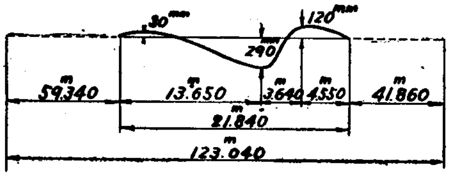
Fig. 2.2 Details of Stiffening Truss; Cross Section of Panel Point without Hanger

towards the river centre. The displacements on the upstream side and downstream side at the right bank tower (the north side tower) were 17.5 cm and 15 cm respectively. But in the central and left bank towers, no displacement was observed. Besides, the top of the I beams of the supports in the left bank stiffening truss inclined towards the left bank side, as if the whole of the stiffening truss was pushed towards the left bank side i. e. south side. On the other hand, it was found that the tension steel round bars connecting the cross beams buckled at the end of the span.

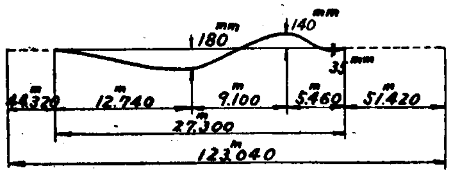
Fig. 3 Details of the Buckling of Upper Chord of Stiffening Truss



Left Bank Span, Upstream Side



Right Bank Span, Upstream Side



Right Bank Span, Downstream Side

3. Buckling of the Upper Chord of the Stiffening Truss due to the Displacement of the Anchorage Block

(1) Bending moments and member stresses of the stiffening truss due to the displacement of the anchorage block.

In **Fig. 4** the anchorage block D displaced ΔL_1, toward the straight backstay DA$'$. In this case the horizontal component of the cable tension, H_r, is obtained as follows:

$$H_r = -\frac{E I_t \Delta L_1 \sec\alpha_1}{\frac{8}{15} f^2 \cdot l + \frac{E\cdot I_t}{E_c\cdot A_c}\left\{l(1+8n^2)+l_1\cdot\sec^3\alpha_1\right\}}, \qquad \ldots\ldots (1)$$

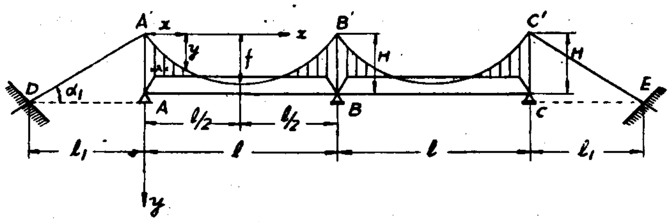


Fig. 4 Double Span Suspension Bridge with Two-hinged Stiffening Truss

where E=Young's modulus of the material of the stiffening truss, i. e. steel,
E_e=Young's modulus of the steel wire rope,
I_t=Moment of inertia of the stiffening truss about the horizontal axis,
A_e=Cross sectional area of the steel wire rope,
$n=f/l$.

Let M_r be the bending moment of the stiffening truss, produced by the displacement of the anchorage block, then

$$M_r = -y\cdot H_r = \frac{E\cdot I_t\cdot \Delta L_1\cdot \sec\alpha_1}{\frac{8}{15}f^2\cdot l + \frac{E\cdot I_t}{E_e\cdot A_e}\left[l(1+8n^2)+l_1\cdot\sec^3\alpha_1\right]}\cdot y \quad \dots (2)$$

As the stiffening truss has parallel chords as shown in **Fig. 1**, putting the vertical distance between the centroids of the chord members, h_0, the chord member stresses produced by the displacement of the anchorage block are obtained as follows:

$$P = \mp\frac{M_r}{h_0} \quad \dots\dots\dots\dots (3)$$

In **Table 1** necessary dimensions of the bridge and constants of the materials are tabulated.

Using the numerical values in **Table 1**, M_r and P can be calculated by eq. (2) and (3) as follows:

$$\left.\begin{aligned} M_r &= 5.424\times10^4\cdot\Delta L_1\cdot y \quad (\text{kg. m.}),\\ P &= \mp 22{,}250\cdot\Delta L_1\cdot y \quad (\text{kg.}), \end{aligned}\right\} \quad \dots\dots\dots (4)$$

where ΔL_1, y in m.

(2) Modulus of equivalent elastic foundation of the upper chord for horizontal displacement.

As shown in **Fig. 2** the stiffening truss lies on the cross beam and so the cross beam forms a part of the lower lateral truss. The hangers are connected to the panel points of the stiffening truss at every other panel, as shown in **Fig. 1.** These are connected to the cross beam by extending the cross beam to the outer side of the floor system, as shown in **Fig. 2.** To obtain the horizontal stability of the upper chord of the stiffening truss, the lower ends of the vertical member are fixed to the cross beam and at the same time to improve the lateral stability of the upper chord, the brackets are inserted between the vertical member and cross beam at the panel points with hangers.

In this way the upper chords are supported at each panel point by the vertical members. As the upper chord is in the condition of a compressed bar, the lateral buckling is resisted by the elastic reactions of the vertical and diagonal members. These elastic resistances are calculated in the following at both the panel points, with and without hangers.

(a) Panel points with hangers:

In calculating the modulus β of the elastic foundation, equivalent to the elasitic resistance of the verticals, it is necessary to establish the relation between the force P, applied at the top of a vertical and the deflection that would be produced if the upper chord were removed. It can easily be seen from the symmetry of deformation that the horizontal load P is resisted by a structure formed by the vertical

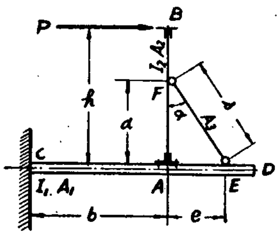


Fig. 5

Table 1.
Notations and Dimensions

l	m	124
l_1	m	40.4
$n=f/l$		1/12
f	m	10.333
h_0	m	2.438
H	m	11.212
$2\cdot\lambda$	m	3.66
d	m	162
e	m	0.455
b	m	1.908
h	m	2.62
s	m	1.683
E	kg/cm^2	2.1×10^6
E_c	kg/cm^2	1.05×10^6
I_t	cm^4	2.098×10^6
A_c	cm^2	54
I_1	cm^4	1755
I_2	cm^4	124.8
A_1	cm^2	38
A_2	cm^2	22.94
A_3	cm^2	20.9
I	cm^4	921

member, cross beam and bracket, and fixed at the centre point of the cross beam.

Then considering the structure of the panel point as the following two cases, the modulus of elastic foundation will be calculated in the following.

(i) Lower ends of the vertical member and cross beam are connected rigidly to each other (**Fig. 5**). This is an indeterminate structure of the 1st. order. Taking the member stress S of the bracket as the indeterminate force X_1, X_1 can be obtained as follows:

$$X_1=-\frac{\frac{I_1}{I_2}\left[\frac{d^2(h-d)}{2}+\frac{d^3}{3}\right]\cdot\sin\alpha}{\frac{d^3\sin^2\alpha}{3}\cdot\frac{I_1}{I_2}+\frac{e^3\cdot\cos^2\alpha}{3}+d\frac{I_1}{A_2}\cdot\cos^2\alpha+e\cdot\frac{I_1}{A_1}\cdot\sin^2\alpha+s\cdot\frac{I_1}{A_3}}\cdot P \qquad \text{......... (5)}$$

Using the values in **Table 1**, X_1 for this structure can be calculated as

$$X=-6.8596\ P \qquad \text{......... (6)}$$

The horizontal deflection δ of the top of a vertical produced by the horizontal force P applied at the top of vertical can be ob'ained by the principle of virtual work and X_1 from eq. (6). In this case

$$\delta=7.101\times10^{-3}\cdot P \text{ cm}, \qquad P \text{ in kg.}$$

If P_0 is the necessary force to produce $\delta=1$ cm, then

$$P_0=140.8 \text{ kg.}$$

Therefore the modulus of the equivalent elastic foundation can be obtained as follows:

$$\beta_1=\frac{P_0}{\lambda}=\frac{140.8}{183}=0.770 \text{ kg/cm}^2, \ldots\ldots (7)$$

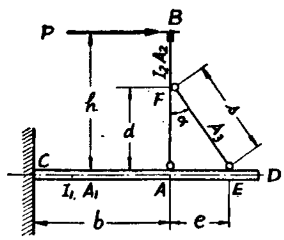


Fig. 6

where λ=panel length.

(ii) Lower ends of the vertical member and cross beam are connected by a hinged joint (**Fig. 6**). In this case, the structure is statically determinate. So the deflection δ, above mentioned, can be obtained immediately.

$$1\cdot\delta=\left[\frac{h^2}{I_1}\left(b+\frac{e}{3}\right)+\frac{h}{3I_2}\left(h-d\right)^2+\frac{1}{A_1}\left\{\left(\frac{h-d}{d}\right)^2\cdot b+\left(\frac{h}{d}\right)^2\cdot e\right\}\right.$$
$$\left.+\frac{1}{A_2}\left(\frac{h}{e}\right)^2\cdot d+\left(\frac{h\cdot s}{d\cdot e}\right)^2\cdot\frac{s}{A_3}\right]\cdot\frac{P}{E} \quad \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

Using **Table 1,**

$$\left.\begin{aligned} &1\cdot\delta=7.420\times10^{-3}\cdot P \text{ cm}, \qquad P \text{ in kg,}\\ &P_0=134.8 \text{ kg,}\\ &\beta_2=0.737 \text{ kg/cm}^2 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (9)$$

By comparing β_2 from eq. (9) with β_1 from eq. (7), we can see that as for β there is no remarkable difference whether the connection of the lower end of the vertical and cross beam is a fixed joint or a hinged joint.

(b) Panel points without hangers, (**Fig. 7**):

In this case, assuming that the connection between the lower end of the vertical and cross beam is rigid, the horizontal deflection δ can be obtained as follows.

$$\delta=\left(\frac{bh^2}{I_1}+\frac{h^3}{3I_2}\right)\cdot\frac{P}{E} \quad \ldots\ldots\ldots (10)$$

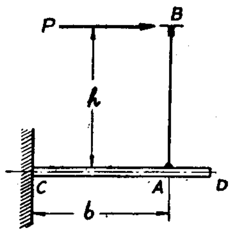


Fig. 7

Using the values of **Table 1,**

$$\left.\begin{aligned} \delta &= 2.643\times10^{-3}\cdot P \text{ cm}, \quad P \text{ in kg}, \\ P_0 &= 37.8 \text{ kg}, \\ \beta_3 &= 0.207 \text{ kg/cm}^2 \end{aligned}\right\} \qquad \cdots\cdots\cdots\cdots (11)$$

As can be understood from eq. (11), the lateral rigidity in this case is inferior to the structure having the bracket, even if the lower end of the vertical is connected to the cross beam rigidly.

(3) Anchorage displacement ΔL_1, necessary to produce the buckling of the upper chord member.

As 3.66 m, the interval length of hangers, is very small in comparison with the bridge span $l=124$ m and the half wave length of the buckled form is large compared with the panel length as shown in **Fig. 3**, it can be said that the axial compressive force in any section x of the upper chord is given by eq. (4)

$$P=22{,}250\ y\cdot\Delta L_1 \qquad \cdots\cdots\cdots\cdots (4)$$

and the upper chord is resisted by a continuous lateral elastic resistance β for horizontal displacement.

Thus the upper chord may be considered as a bar with hinged ends compressed by forces distributed along its length and elastically supported by an equivalent elastic foundation.

If dP is the increase of P on an element dx of the upper chord and q the continuous distributed axial compression, then

$$\left.\begin{aligned} q &= \frac{dP}{dx} = 22{,}250\cdot\Delta L_1\cdot\frac{dy}{dx} \\ &= 7416\cdot\Delta L_1\cdot\left(1-2\frac{x}{l}\right) \\ &= q_0\left(1-2\frac{x}{l}\right), \end{aligned}\right\} \qquad \cdots\cdots\cdots\cdots (12)$$

where $\quad q_0=7416\cdot\Delta L_1.$

Therefore the continuous axial compression q is greatest at both ends, i. e. q_0, and zero at the middle, and is proportional to the distance from the middle of the span as shown in **Fig. 8.** Then the equivalent compressive load P acting in the upper chord is max. at the middle of the span and its value $P_{x=l/2}$ is given by the shaded areas in **Fig. 8** or by putting $y=f$ in eq. (4). From these,

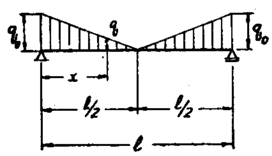


Fig. 8
Equivalent Compressive Load Distribution for Upper Chord

$$P_{x=l/2}=\frac{1}{4}\cdot q_0\cdot l$$

In this manner the problem of the buckling of the upper chord due to the displacement of the anchorage block is reduced to one of the buckling of a bar with hinged ends, supported laterally by a continuous elastic medium and axially loaded by a continuous load, the intensity of which is proportional to the distance from the middle.

The condition of loading in this problem of buckling is the same as that discussed by S. Timoshenko.(1) But the application of the above solution to this problem is laboursome, so we solved it by an approximate method. We will solve this problem by a rigorous solution in the near future.

As indicated in **Fig. 3**, the portion of the buckling of the upper chord (we denote the length of it by $\bar{l}$) appears in the middle part of the span and its length is approximately from $l/6 \fallingdotseq 20$ m to $l/4 \fallingdotseq 30$ m.

On the other hand as the distributed axial compression q is small in this central portion of the upper chord as shown in **Fig. 8**, we can assume without making a considerable error that the compressive force of the upper chord acting at the middle section of the span $P_{x=l/2}=\frac{1}{4}\cdot q_0\cdot l$ acts throughout the length $\bar{l}$. Then the problem is reduced to the problem of the buckling of a compressed bar subject to a constant compression of $P_{x=l/2}$ at both ends and a lateral elastic resistance proportional to the displacement as shown in **Fig. 9**. This problem can be solved by the following method.(2)

$$\left(P_{x=\frac{l}{2}}\right)_{cr}=\frac{\pi^2 EI}{\bar{l}^2}\left(m^2+\frac{\beta\bar{l}^4}{m^2\pi^4 EI}\right), \quad (13)$$

where I is the moment of inertia of the upper chord with respect to the vertical axis through its centroid, and m is obtained by the next equation,

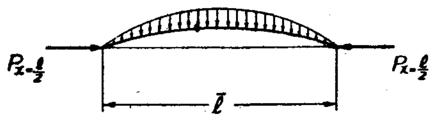


Fig. 9

Buckling of Compressed Bar subject to a Constant Compression $P_{x=l/2}$ at both Ends and a Lateral Elastic Resistance proportional to the Displacement

$$\frac{\beta\bar{l}^4}{\pi^4 EI}=m^2(m+1)^2 \quad \text{.........................} \quad (14)$$

On the other hand as $P_{x=l/2}$ is calculated by eq. (4)

$$\left.\begin{aligned} P_{x=l/2}&=[22{,}250\cdot y\cdot \Delta L_1]_{y=f} \\ &=229{,}900\cdot\Delta L_1=\omega\cdot\Delta L_1, \end{aligned}\right\} \quad \text{..................} \quad (15)$$

where $\omega=229{,}900$.

Equating eq. (13) with eq. (15), we obtain the next equation determining the displacement ΔL_1 which produces the buckling of the upper chord of the stiffening truss.

$$\Delta L_1 = \frac{\pi^2 EI}{\omega \cdot \bar{l}^2}\left(m^2 + \frac{\beta \cdot \bar{l}^4}{m^2 \pi^4 EI}\right) \qquad \cdots\cdots\cdots\cdots\cdots\cdots \text{(16)}$$

Referring to **Fig. 3**, we calculated the value of ΔL_1 by eq. (14), (16), using $\bar{l}=20$ m, 25 m, 30 m and $\beta=0.1, 0.2, \ldots\ldots, 1$ kg/cm^2.

These results are plotted in the coordinate plane $\beta \sim \Delta L_1$ as shown in **Fig. 10.** As can be understood clearly from **Fig. 10**, the effect of $\bar{l}$ upon ΔL_1 is very small. The relations between the number of half waves m and β are shown in **Fig. 11** for each value of $\bar{l}$. As the number of half waves m must be an integer, we are able to determine the relation between $\bar{l}$ corresponding to each integer m and the modulus of equivalent elastic foundation β by interpolating $\bar{l}$ in the three

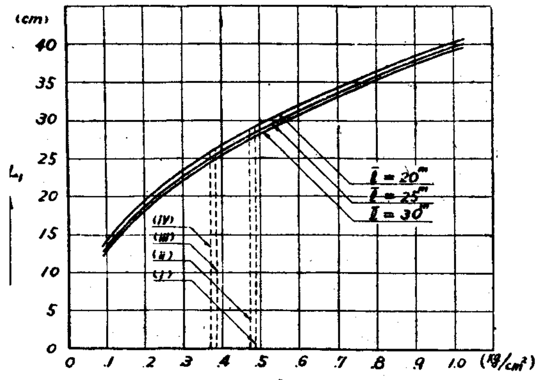


Fig. 10 Anchorage Block Displacement ΔL_1, necessary to produce the Buckling of Upper Chord for Various Value of the Modulus of Elastic Fundation β, taking $\bar{l}=20$, 25, 30 m respectively

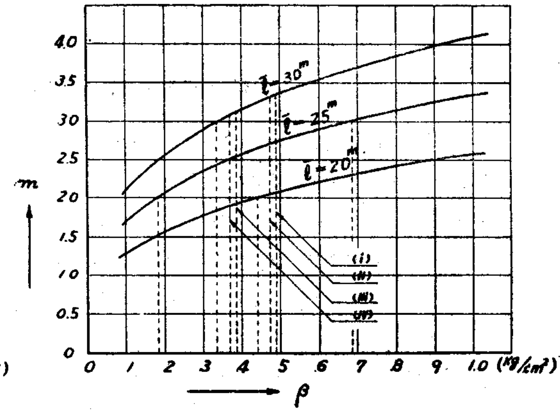


Fig. 11 Number of Half-wave m for Various Value of β when the Buckling of Upper Chord occurs, taking $\bar{l}=20$, 25, 30 m respectively

curves shwon in **Fig. 11.** Next, the relations between β and half wave length $\frac{\lambda}{2}$ are shown in **Fig. 12**, where $\frac{\lambda}{2}$ is obtained by $\bar{l}$ and m from eq. (14). As can be understood from the figure, the length of the half wave decreases rapidly as β increases, so the buckled form becomes as if it ripples.

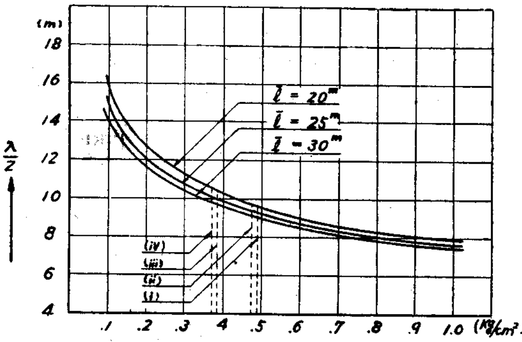


Fig. 12 Length of Half-wave $\lambda/2$ for Various Value of β when the Buckling of Upper Chord occurs, taking $\bar{l}=20$, 25, 30 m respectively

To discuss the buckling of the upper chord of this bridge, taking into accout the rigidity of the vertical members, above introduced, we take as the modulus of elastic foundation β the values given in **Table 2.** Taking these values, the displacements ΔL_1, necessary to produce the buckling of the upper chord are obtained from **Fig. 10** and these values are also tabulated in **Table 2.** From the above considerations we can conclude that the displacement of the anchorage block, ΔL_1, is about 24~29 cm (9.4~11.4 in.), if the bucklings occurred as a result of the displacement of thé anchorage block only.

Table 2.

Moduli of Equivalent Elastic Foundation β, and Displacements of the Anchorage Block, necessary to produce the Buckling of the Upper Chord of the Stiffening Truss

(kg/cm²)	ΔL_1 (cm)
(i) $(\beta_1+\beta_3)/2=0.489$	27.6~29.0
(ii) $(\beta_2+\beta_3)/2=0.472$	27.1~28.6
(iii) $\beta_1/2=0.385$	24.5~26.1
(iv) $\beta_2/2=0.369$	24.1~25.7

4. Consideration about the Damages

Assuming $\bar{l}$ the length of the buckled chord equal to 20~30 m, and taking the values of β, calculated above for this bridge, we can obtain the number of half waves m and the length of one half wave $\frac{\lambda}{2}$. Comparing these values with the actual condition of buckling shown in **Fig. 3,** we find that the conditions of the damage generally agree with the calculated values. But when we assume that the anchorage displacement is the only cause of buckling, we consider that the displacement which is about 24~29 cm as shown in **article 3** is too large.

As explained above the observed values of the saddle displacements are 15 cm and 17.5 cm for the right bank towers. Although it may be unreasonable to connect directly the saddle displacement with the displacement of the anchorage, we may safely presume that at least an anchorage displacement of the same degree as the saddles occurred as a result of the earthquake. The damages of the anchorage blocks were also investigated by a party headed by Prof. Fukuda, Tokyo University and he expressed his view that the anchorage block of the right bank is constructed in the paddy field and so the anchorage might be displaced during the earthquake.

Thus the main cause for the damage of the Goshō Suspension Bridge, that is the buckling of the upper chord of stiffening truss, is a displacement of the anchorage block and at the same time as the secondary causes for the damages, we can give the next factors:

(i) In the stiffening truss the initial stresses had occurred even when there were no live loads acting. The reason for this, we presume, is that at the time of erection the cables were not loaded with all the dead loads of the stiffening truss and moreover the cables had lengthened during the long period after the construction.

(ii) Vertical and horizontal vibration of the suspension bridge during the earthquake.

(iii) Unsatisfactory anchorage of the cables on the central tower, etc.

5. Conclusion

(1) Referring to the values obtained by measurements, **Fig. 3**, we calculated the displacement of the anchorage block, ΔL_1, assuming that the buckling of the upper chord occurred only by the displacement of the anchorage. These values are plotted in **Fig. 10** in the relation to the moduli of equivalent elastic foundation β. Using **Fig. 10** and the values of β, calculated above for this bridge, we obtain the displacement of the anchorage block $\Delta L_1 = 24 \sim 29\,\text{cm}$.

(2) We clarified the axial distributed compressive force in the upper chord due to the displacement ΔL_1 of the anchorage block. These results are as shown in **Fig. 8.** In the calculation of ΔL_1, the displacement of the anchorage block necessary to produce the buckling of the upper chord, we solved a compression bar resisted by a continuous lateral elastic foundation and subject to a concentrated axial load $P_{x=l/2}$, which occurs in the middle section of the span, because the buckling occurs in the central portion of the span and moreover as the continuous distributed axial compression q is small as shown in **Fig. 8.**

(3) Judging from the observed values of the saddle displacements, the displacement of the anchorage block due to the earthquake is somewhat small in comparison with the results mentioned in (1), but we believe that the direct cause of the damages is due to the displacement of the anchorage block. Next the consideration about the secondary causes are also explained.

(4) The moduli of equivalent elastic foundation β for the upper chord of this stiffening truss are approximately $0.37 \sim 0.49\,\text{kg/cm}^2$, which we obtained by the calculations. Applying the bracket to the verticals, we found that the value of β with bracket is improved about four times the value of β without bracket.

Acknowledgement

It may be added that as we could not obtain the detailed data for this bridge prepared at the time of design and construction, we used the data obtained by direct measurement immediately after the earthquake and "The Collection of the

Highway Bridges in Japan" (Enlarged Edition) published by the Civil Engineering Research Institute of the Home Ministry in March, 1928. The research cost necesary to cover this investigation was defrayed from the Hokuriku Earthquake Damage Investigation Special Committee sponsored by the National Research Council of the Japanese Government. The author avails himself of this opportunity to express his grateful thanks to those mentioned above.

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