Simultaneous Heat and Mass Transfer
The Relation between Heat transfer and Mass transfer Coefficients

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General

Simultaneous heat and mass transfer is an important problem in studying the unit operations in chemical engineering such as drying, humidification, dehumidification and so on.

In order to solve the problem of heat and mass transfer, the analogy among fluid friction, heat transfer and mass transfer is very useful.

With the development of the theory of fluid flow, this has been improved too. Mizushina has slso offered a new one [5].

Using this the authors found the relation between coefficients of heat and mass transfer in common gaseous systems (σ, $\phi = 0.5 \sim 2.5$) as follows

$$\frac{h}{\kappa'} = \left(\frac{\phi}{\sigma}\right)^{\frac{1}{2}} \cdot c_p$$

and the equation was found to fit experimental data by means of a wet and dry-bulb hygrometer using water and several kind of organic solvents.

Theoretical consideration

The analogy between heat transfer and fluid friction in turbulent flow was originally offered by Reynolds, and developed by Taylor, Prandtl, von Kármán, Chilton and Colburn, assuming that fluid friction is produced by exchange of momentum, heat transfer by exchange of heat, and both are similar phenomena. Similarly this analogy may be applied to the relation between fluid friction and mass transfer. Consequently, fluid friction, heat transfer and mass transfer can be considered to be quite analogous as Sherwood [1] suggested.

Reynolds' analogy on heat transfer for fluid flow in a round pipe is shown as follows:

$$h = \frac{1}{2} f c_p \rho u_0 \qquad\qquad (1)$$

Taylor and Prandtl [2] tried to expand this theory introducing:

$$h = \frac{\frac{1}{2} f c_p \rho u_0}{1 + (\sigma - 1) 6.2 \sqrt{f}} \qquad (2)$$

where $\sigma = c_p \mu / \lambda$

Kár/mán [3] gave the following equation:

$$h = \frac{\frac{1}{2} f c_p \rho u_0}{1 + 5 \sqrt{\frac{f}{2}} \left\{ (\sigma - 1) + \ln \left(\frac{1 + 5\sigma}{6} \right) \right\}} \qquad (3)$$

Chilton and Colburn [4] reduced the relation as follows:

$$h = \frac{\frac{1}{2} f c_p \rho u_0}{\sigma^{2/3}} \qquad (4)$$

Similarly concerning mass transfer

$$\kappa' = \frac{\frac{1}{2} f \rho u_0}{\phi^{2/3}}$$

where $\phi = \mu / \rho D$

$$\therefore \quad \frac{h}{\kappa'} = \left(\frac{\phi}{\sigma} \right)^{2/3} \cdot c_p \qquad (5)$$

The power index 2/3 in Eq. (5) is the mean value in the range of σ, $\phi =$ 0.5~100. While within the limit of σ, $\phi = 0.5$~2.5 as in the case of gas humidifier and dehumidifier, Eq. (5) deviates slightly from experimental values.

Now, Mizushina [5] reexamined the above–mentioned analogy of Tayler and Prandtl and proposed the following equation:

$$h = \frac{\frac{1}{2} f c_p \rho u_0}{1 + (\sigma - 1) \frac{6.3 \sqrt{f}}{\sigma^{0.2}}} \qquad (6)$$

Similarly

$$\kappa' = \frac{\frac{1}{2} f \rho u_0}{1 + (\phi - 1) \frac{6.3 \sqrt{f}}{\phi^{0.2}}}$$

$$\therefore \quad \frac{h}{\kappa'} = \frac{1 + (\phi - 1) 6.3 \sqrt{f} / \phi^{0.2}}{1 + (\sigma - 1) 6.3 \sqrt{f} / \sigma^{0.2}} c_p \qquad (7)$$
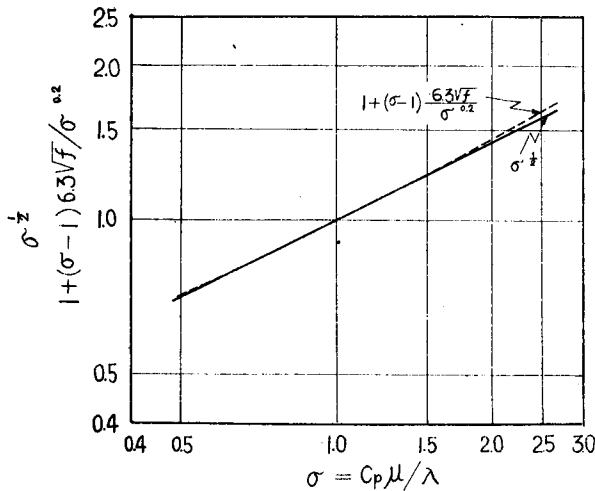
Fig. 1. Comparison of denominator of equ. (6) with $\sigma^{1/2}$

Fig. 1 shows relations between $\left[1+(\sigma-1)\,6.3\sqrt{f}\,/\sigma^{0.2}\right]$, the denominator of Eq. (6), and Prandtl number σ within the limits of $\sigma=0.5\sim2.5$, taking $6.3\sqrt{f}=0.5$. This line can be substituted by a straight line which represents $\sigma^{1/2}$. Then the denominator of Eq. (6) becomes $\sigma^{1/2}$. The same thing may be said about ϕ in case of mass transfer. Then in the range of σ, $\phi=0.5\sim2.5$ Eq. (7) becomes

$$\frac{h}{\kappa'} = \left(\frac{\phi}{\sigma}\right)^{\frac{1}{2}} \cdot c_p \tag{8}$$

This relation is for the stream in a round pipe. But Eq. (8) may be applicable to the case of fluid flow at right angle to a simple rod as in the case of wet–bulb thermometer.

Wet and dry bulb hygrometer

If a drop of liquid is left in the unsaturated gas, it rapidly assumes the wet–bulb temperature t_w, then the heat required for evaporation will be supplied from gas since the temperature of gas t is higher than that of the drop t_w i. e.

$$Q = hA(t-t_w)$$

where A: Surface area of the drop.

Under adiabatic conditions, taking the saturated humidity at the temperature of t_w as H_w and the humidity of gas at the temperature of t as H,

$$hA(t-t_w) = r_w\kappa'A(H_w-H) \tag{10}$$

where r_w: Letent heat of liquid at t_w,

$$\therefore \left(\frac{h}{\kappa'}\right)^*_{obs} = \frac{r_w(H_w-H)}{t-t_w} \tag{11}$$

Now it is possible to calculate the value of $(h/\kappa')_{obs}$ when t and t_w are measured. But Eq. (10) can not be applied actually with perfect accuracy because some factors influence the reading of wet–bulb temperature.

* (Note) Many investigators have calculated (h/κ') by Eq. (11). The authors will use, therefore, the notation $(h/\kappa')_{obs}$ for the value of (h/κ') given by Eq. (11).

One of these factors is the radiation. In the usual case the temperature of sourroundings is near the dry–bulb temperature t higher than that of the wet–bulb. So representing the coefficient of heat transfer by radiation as h_r, the heat radiated to the wet–bulb is to be $h_r A(t-t_w)$.

Secondly, we must consider the heat input by conduction through the wet–bulb thermometer stem. Take h_c as the coefficient of heat transfer by conduction, then the heat conducted is to be $h_c A(t-t_w)$.

Usually a capillary phenomenon is used in order to wet the surface of wet–bulb. Arnold asserts himself in the discussion of Sherwood's paper [6] that capillarity is not operative unless a partial drying out of the wick occurs. In these cases we must consider that the surface area of evaporation of the wet–bulb is a little smaller than that of heat transfer. Let us take the ratio of both area as a, then $a \leqq 1$ according to the conditions of wick and the method of liquid supply.

In the experiments of investigators referred here and that of authors, the pre–cooling of supplied liquid has been devised so as to limit the sensible heat input with it to be only 1~2% of total quantity of heat transfer. Therefore, this influence can be neglected.

Considering what is mentioned above, the heat balance on the wet–bulb surface becomes

$$(h+h_r+h_c)(t-t_w) = r_w \kappa' a(H_w-H) \tag{12}$$

From Eqs. (11) and (12)

$$\left(\frac{h}{\kappa'}\right)_{obs} = \frac{r_w(H_w-H)}{t-t_w} = \frac{h+h_r+h_c}{\kappa' a} = \frac{h}{\kappa'}\frac{h+h_r+h_c}{ha}$$

Since the velocity of air flow is considered constant during one experiment, h is constant, and h_r and h_c are also almost constant as to one apparatus. Furthermore, a may be considered as a constant irrespective of the kind of liquid in one experiment.

Therefore

$$\frac{h+h_r+h_c}{ha} = \beta \tag{13}$$

is almost constant in one experiment.

Then

$$\left(\frac{h}{\kappa'}\right)_{obs} = \frac{r_w(H_w-H)}{t-t_w} = \beta\frac{h}{\kappa'} = \beta\left(\frac{\phi}{\sigma}\right)^{\frac{1}{2}}c_p \tag{14}$$

As $\beta > 1$, the value of $(h/\kappa')_{obs}$ obtained by wet–and dry–bulb thermometer is a little larger than the theoretical value of h/κ'.

In our experiment the value of β is 1.26. As to the experiments of Sherwood

and Comings[6], Mark[7] and Arnold[8] it is 1.31, 1,16, 1.31 respectively. In all cases β is calculated from data for toluene.

Apparatus and operations

The air is sent into a glass tube of 1.7 cm inner diameter from a compressor through several calcium dichloride tubes and an active carbon tube to remove the water and organic vapors. (Fig. 2)

The amount of air flow is controlled at the orifice. While the experiment is being done, the velocity of air flow is kept constant (about 4 m/sec.). Larger velocity was desirable in order to minimize the influence by radiation and other factors but it could not be realized.

At the end of the glass tube two copper–constantan thermo–couples protected with small glass tubes of about 1~2 mm inner diameter were fixed as shown in Fig. 3.

A: Compressor
B: Valve
C: CaCl₂ Tube
C': Active Carbon Tube
D: Orifice
E: Manometer
F: Electric Heater
G: Transformer
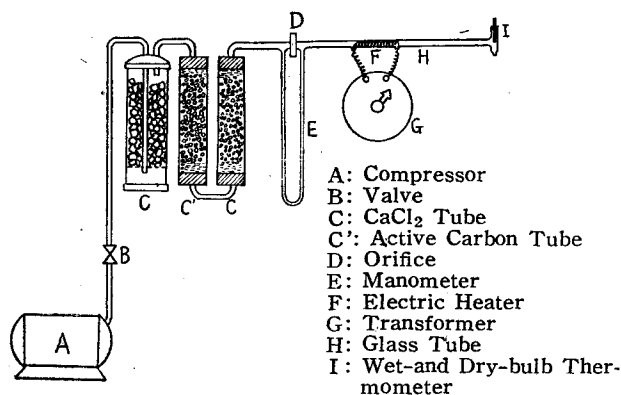H: Glass Tube
I: Wet-and Dry-bulb Ther-
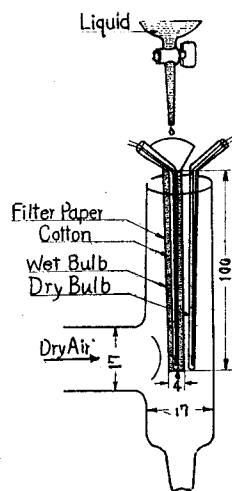mometer

Fig. 2. Schematic drawing of apparatus

Fig. 3. Dry and wet bulb thermometer

The wet–bulb thermo–couple is covered with cotton and filter paper, which are kept wet with liquid drops from a burette. The outer diameter of the wet–bulb is about 4 mm.

The liquid flows down along the cotton, and a part of it evaporates through the surface of the filter paper. The liquid is cooled gradually losing the latent heat of vaporization, and at the end of the wet–bulb it will take the wet–bulb temperature. This considerably long wick is also useful to prevent the heat input caused by conduction through the thermometer stem. The rate of liquid dripping is controlled so that drops may be held at the lower end of the wick and not fall down. If it drops, by accident, it will soon be driven out from the

bottom hole and its vapor will be blown off by the downward air flow. There-fore, the main flow of air will not contain the vapor of liquid.

Wet and dry bulb temperatures can be measured with a potentiometer.

With a electric heater the temperature of air flow is varied and $(h/\kappa')_{obs}$ is obtained for each temperature using Eq. (11).

Thus experiments were repeated by using water and several kind of liquids, obtaining the mean value of $(h/\kappa')_{obs}$ for each liquid.

Result of Experiment

Table 1 shows the mean values of $(h/\kappa')_{obs}$ for each liquid obtained by Sher-wood and Comings [6], Mark [7], Arnold [8] and the authors.

Prandtl number σ was taken as 0.74 for air.

Schmidt number can be calculated using $\mu/\rho=0.132$ cm²/sec for pure air and the diffusion coefficient. All of these values are at 0°C and 1 atm.

The diffusion coefficient for water, Methyl alcohol, ethyl alcohol, n–propyl alcohol, benzene, ethyl acetate, toluene, ethylpropionate and xylene of 0.220, 0.133, 0.102, 0.085, 0.077, 0.072, 0.071, 0.065 and 0.059 cm²/sec respectively were taken from the International Critical Tables. As there were no proper data for propyl acetate, chlorobenzene, carbon tetrachloride and ethylen tetrachloride, they were calculated by Arnold's equation [9] and the following values were obtained respec-tively, 0.065, 0.065, 0.065 and 0.064.

Table 1 also shows the calculated values of $\beta(h/\kappa')=\beta(\phi/\sigma)^{\frac{2}{3}}\cdot c_p$ which are in fair agreement with the values of $(h/\kappa')_{obs}$.
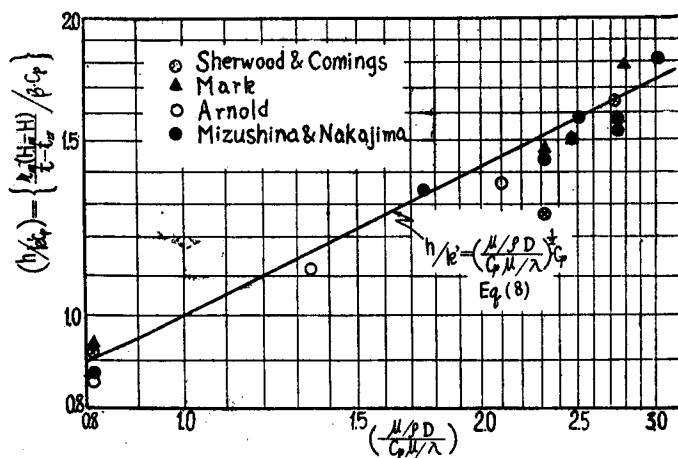

Fig. 4. Correlation of data for dry and wet bulb thermometers
by theoretical equation (8)

Now, $\left(\dfrac{h}{\kappa' c_p}\right) = \left\{\dfrac{r_w(H_w - H)}{(t - t_w)\beta c_p}\right\}$ are plotted vs. (ϕ/σ) using the logarithmic coordinates in Fig. 4. Eq. (8) is represented by a straight line.

From the above the theoretical relation expressed by Eq. (8) is found to fit experimental data quite satisfactorily.

Summary

As the result of the theoretical consideration concerning with the analogy among fluid friction, heat transfer and mass tranfer in common gaseous systems (σ, $\phi = 0.5 \sim 2.5$), the following relation between the coefficients of heat and mass transfer was obtained:

$$\frac{h}{\kappa'} = \left(\frac{\phi}{\sigma}\right)^{\frac{1}{2}} \cdot c_p$$

and this was ascertained experimentally, using the wet and dry bulb thermometer.

Acknowledgment

(Nomenclature)

c_p : specific heat of fluid
D : diffusion coefficient
f : friction factor $0.079\ Re^{-\frac{1}{4}}$
h : coefficient of heat transfer
h_c : coefficient of heat transfer by conduction through the thermometer stem
h_r : coefficient of heat transfer by radiation
H : absolute humidity of gas
H_w : saturated absolute humidity at t_w
κ' : coefficient of mass transfer
r_w : latent heat of vaporization of liquid at t_w
t : temperature of gas
t_w : wet-bulb temperature
u_0 : mean velocity of fluid
α : ratio of evaporation surface area to heat transfer surface area of wet-bulb
β : constant defined by Eq. (13)
λ : heat conductivity of fluid
μ : absolute viscocity of fluid
ρ : density of fluid
σ : Prandtl number $c_p \mu/\lambda$
ϕ : Schmidt number $\mu/\rho D$

Table 1

Liquid used	$\left(\dfrac{h}{\kappa'}\right)_{obs}$	$\dfrac{h}{\kappa' c_p}=\dfrac{(h/\kappa')_{obs}}{\beta \cdot c_p}$	D cm²/sec	$\psi=\dfrac{\mu}{\rho D}$	$\dfrac{\psi}{\sigma}$	$\beta\left(\dfrac{\psi}{\sigma}\right)^{\frac{1}{2}}c_p$	
Water	0.29	0.92	0.220	0.60	0.81	0.28	Sherwood &
Benzene	0.40	1.27	0.077	1.72	2.32	0.48	Comings[5]
Toluene	0.50	1.59	0.071	1.86	2.51	0.50	$\beta=1.31$
n-Propyl acetate	0.52	1.65	0.065	2.03	2.75	0.52	
Carbon tetrachloride	0.50	1.59	0.065	2.03	2.75	0.52	
Water	0.26	0.94	0.220	0.60	0.81	0.25	Mark[7]
Benzene	0.41	1.48	0.077	1.72	2.32	0.42	$\beta=1.16$
Ethyl acetate	0.42	1.51	0.072	1.83	2.48	0.44	
Toluene	0.44	1.59	0.071	1.86	2.51	0.44	
Chlorobenzene	0.44	1.59	0.065	2.03	2.75	0.46	
Carbon tetrachloride	0.44	1.59	0.065	2.03	2.75	0.46	
Ethylen tetrachloride	0.50	1.80	0.064	2.06	2.79	0.46	
Water	0.27	0.86	0.220	0.60	0.81	0.28	Arnold[8]
Methyl alcohol	0.35	1.11	0.133	0.99	1.34	0.36	$\beta=1.31$
n-Propyl alcohol	0.43	1.37	0.085	1.55	2.10	0.46	
Toluene	0.50	1.59	0.071	1.86	2.51	0.50	
Ethyl propionate	0.50	1.59	0.065	2.03	2.75	0.52	
Water	0.26	0.88	0.220	0.60	0.81	0.27	Mizushina
Ethyl alcohol	0.41	1.35	0.102	1.29	1.75	0.40	& Nakajima
Benzene	0.44	1.44	0.077	1.72	2.32	0.46	$\beta=1.26$
Ethyl acetate	0.46	1.51	0.072	1.83	2.48	0.47	
Toluene	0.48	1.59	0.071	1.86	2.51	0.48	
Carbon tetrachloride	0.47	1.54	0.065	2.03	2.75	0.50	
Xylene	0.55	1.83	0.059	2.24	3.02	0.52	

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