A Dynamical Consideration on Earthquake Damages of Bridge Piers (First Report)

By

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Synopsis

As was also noted from the results of the Fukui Earthquake of June 1948, it is now quite evident that the earthquake damage of the bridge substructure decisively affects the damage of the whole bridge, directly or indirectly, with almost no exception. In view of this fact, it is attempted in this paper to make a dynamical study from both theoretical and experimental points of view on the mechanism of earthquake damages of the bridge substructure and thereby contribute to the earthquake-proof design and construction of bridge piers.

1. Introduction

As it has become clear that the earthquake damage of the bridge substructure eventually produces a decisive effect on the damage of the whole bridge, the features of the vibration of the substructure must, first of all, be studied before proceeding with the dynamical study of earthquake damages. When substructures, namely very rigid structures like piers and wells, are rooted deeply in the ground, it can easily be understood that the elasticity of the ground itself has a great influence on the features of the vibration of the substructures.

From this point of view, vibration tests (first test in Sept. 1948, second test in Nov. 1948) were carried out on the piers of the Nakatsuno Bridge of the Keifuku Electric Railway Company spanning the Kuzuryu River which suffered a considerable damage during the Fukui Earthquake of June 1948, and it was found that the piers showed a phenomenon which may be called rocking vibration due to the elasticity of the base ground. Thus, taking this rocking vibration caused by the elasticity of the ground into consideration, a numerical calculation was performed on the results obtained by treating the free vibration of the bridge piers theoretically. As a result, a very reasonable explanation of the measured periods of the Nakatsuno Bridge piers could be given by the theoretical equations given in this paper which could not be explained by any of the former formulas.

Next, upon analyzing the forced vibrations of the piers due to seismic motion by using generalized coordinates and applying this to the Nakatsuno Bridge, a numerical calculation was performed, and the mechanism of the earthquake damages of the bridge due to the Fukui Earthquake was clarified to some extent.

2. Free Vibration of Bridge Piers

(1) Fundamental equation of free vibration.

As mentioned in the introduction, bridge piers, being greatly affected by the base ground, show a rocking vibration phenomenon, and, the reaction on the bottom surface also produces a considerable big effect. In order to study such vibration of a pier, the pier will be considered as a pillar on an elastic ground as shown in Fig. 1. Then potential energy V, kinetic energy T and dissipation function F can be expressed as follows.

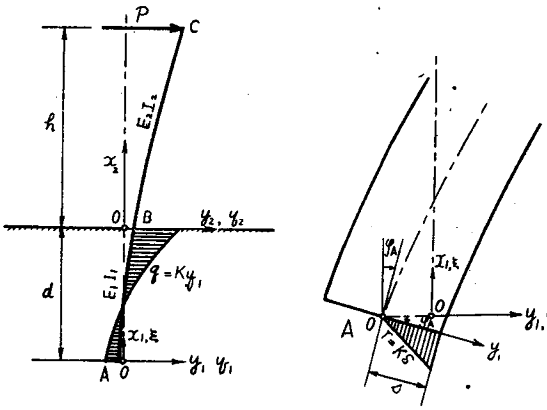


Fig. 1

$$\left.\begin{aligned}
V &= \frac{b_1 K m}{8}\int_0^a\left(\frac{\partial^2 y_1}{\partial \xi^2}\right)^2 d\xi+\frac{b_1 K m}{2}\int_0^a y_1^2 d\xi+\frac{b_1 K}{2}\int_0^s \delta^2 dy_1+\frac{E_2 I_2}{2}\int_0^h\left(\frac{\partial^2 y_2}{\partial x_2^2}\right)^2 dx_2\\
T &= \frac{w_1 a_1 m}{2g}\int_0^a\left(\frac{\partial y_1}{\partial t}\right)^2 d\xi+\frac{w_2 a_2}{2g}\int_0^h\left(\frac{\partial y_2}{\partial t}\right)^2 dx_2+\frac{W}{2g}\left(\frac{\partial y_2}{\partial t}\right)^2_{x_2=h}\\
F &= \frac{k_1 w_1 a_1 m}{2g}\int_0^a\left(\frac{\partial y_1}{\partial t}\right)^2 d\xi+\frac{b_1 R m}{2g}\int_0^a\left(\frac{\partial y_1}{\partial t}\right)^2 d\xi+\frac{k_2 w_2 a_2}{2g}\int_0^h\left(\frac{\partial y_2}{\partial t}\right)^2 dx_2\\
&\quad+\frac{b_2 f}{2g}\left(\frac{\partial y_2}{\partial t}\right)^2_{x_2=h}
\end{aligned}\right\}\quad (1)$$

where EI: bending rigidity of pier, a: cross sectional area, b: width, K: reaction coef. of ground, W: total weight of superstructure, g: gravity acceleration, k: internal frictional coef. of pier material, R: resistant coef. of base ground, f: frictional coef. between pier and superstructure, suffix 1 represents the part in the ground, suffix 2 represents the part above the ground,

$$\xi = \frac{x_1}{m}, \qquad m = \sqrt[4]{\frac{4E_1I_1}{b_1K}}, \qquad \frac{d}{m} = a.$$

The shapes of y_1 and y_2, vibration curves of eq. (1), are not yet determined, but as the phenomenon of rocking vibration is to be considered, it can be assumed that y_1 and y_2 are similar in shape with the statical deflection curves η_1 and η_2 in which the bottom surface reaction is neglected which appears when a horizontal force P acts on the top of the pier as shown in Fig. 1. This being the same as restricting the degree of freedom to one, corresponds to Rayleigh's method which, as is well known, gives a considerable high accuracy.

Thus the time-displacement curve can be expressed as follows representing the time function q_t as generalized coordinates.

$$y_j = \eta_j q_t \tag{2}$$

$j=1$ represents the part in the ground

$j=2$ represents the part above the ground

If A and B are integration constants determined from the boundary conditions, then

$$\left.\begin{aligned} \eta_1 &= \frac{1}{2}\left\{(A_1 e^{\xi} + A_2 e^{-\xi})\cos\xi + (A_3 e^{\xi} + A_4 e^{-\xi})\sin\xi\right\} P = \eta_1' P \\ \eta_2 &= \left\{B_1x_2^3 + B_2x_2^2 + B_3x_2 + B_4\right\} P = \eta_2' P \end{aligned}\right\} \tag{3}$$

The eight integration constants A and B can all be obtained from the conditions of equilibrium of forces and the conditions of continuity of deformation, at points A, B and C (Fig. 1).

Next, proceeding with the calculation by substituting eq. (2) into eq. (1) and applying Lagrange's equation of motion:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_t}\right) - \frac{\partial T}{\partial q_t} + \frac{\partial F}{\partial \dot{q}_t} = -\frac{\partial V}{\partial q_t}$$

the following fundamental equation is obtained.

$$\ddot{q}_t + 2\varepsilon\dot{q}_t + n^2 q_t = 0 \tag{4}$$

(2) Free vibration of bridge pier.

When each pier is considered to vibrate as an individual body, this corresponds to the case when $f=0$ in eq. (1) and so it is sufficient only to solve eq. (4). As is known, the free vibration of bridge piers is solved by assuming eq. (2) and by putting in the initial condition $t=0$, $y=y_0=\eta_0$, $\dot{y}=\dot{y}_0=\dot{\eta}_0$, but with actual piers, as $n>\varepsilon$, the following equation is obtained by using notation $\sqrt{n^2-\varepsilon^2}=\sigma$.

$$y_j = e^{-\varepsilon t}\left\{\eta_{0j}\cos\sigma t + \frac{1}{\sigma}(\varepsilon\eta_{0j} + \dot{\eta}_{0j})\sin\sigma t\right\} \tag{5}$$

Circular frequency n and damping coefficient ε in eq. (4) and (5) are calculated by the following equations.

$$\left.\begin{aligned} n^2 &= \frac{\dfrac{b_1Km}{8}\displaystyle\int_0^a\left(\frac{d^2\eta_1}{d\xi^2}\right)^2 d\xi+\frac{b_1Km}{2}\int_0^a\eta_1^2 d\xi+\frac{b_1Ks^3}{6}\left(\frac{d\eta_1}{d\xi}\right)^2_{\xi=0}+\frac{E_2I_2}{2}\int_0^h\left(\frac{d^2\eta_2}{dx_2^2}\right)^2 dx_2}{\dfrac{1}{2g}\left[w_1a_1m\displaystyle\int_0^a\eta_1^2 d\xi+w_2a_2\int_0^h\eta_2^2 dx_2+\frac{W}{2g}\left(\eta_2\right)^2_{x_2=h}\right]} \\ \varepsilon &= \frac{k_1w_1a_1m\displaystyle\int_0^a\eta_1^2 d\xi+k_2w_2a_2\int_0^h\eta_2^2 dx_2+b_1Rm\int_0^a\eta_1^2 d\xi}{\dfrac{1}{2}\left[w_1a_1m\displaystyle\int_0^a\eta_1^2 d\xi+w_2a_2\int_0^h\eta_2^2 dx_2+W\left(\eta_2\right)^2_{x_2=h}\right]} \\ T_r &= \frac{2\pi}{n} \qquad T_r' = \frac{2\pi}{\sqrt{n^2-\varepsilon^2}} = \frac{2\pi}{\sigma} \end{aligned}\right\} \quad (6)$$

Thus the period of free vibration T of the individual pier is obtained from eq. (5) and (6).

Next, if the case when all the piers are considered to vibrate as one structure is, for the sake of convenience considered as shown in Fig. 2a, $f=\infty$ in eq. (1), but as the top end of the pier is considered as hinged as shown in the figure, a horizontal force P_r due to P_s is distributed on the top surface of all the piers as

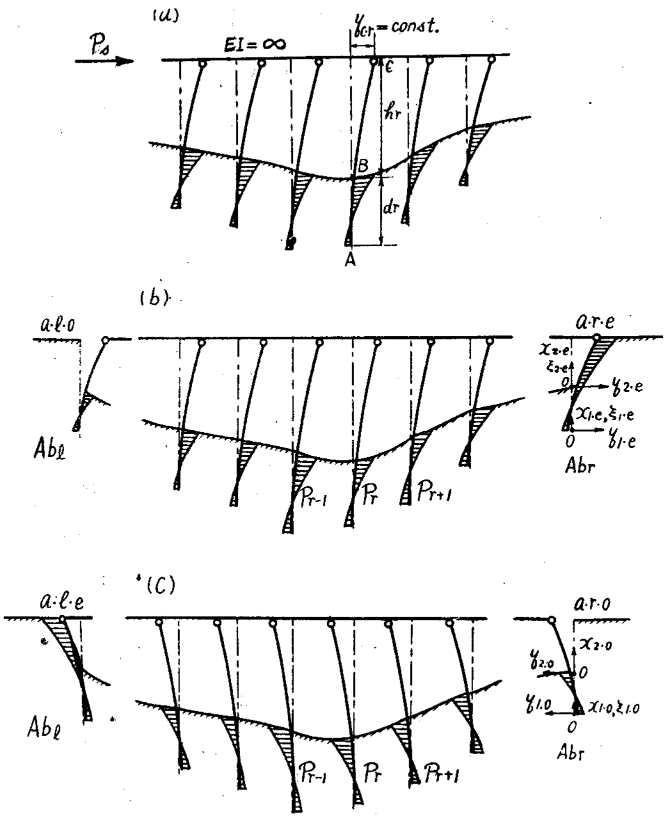


Fig. 2

$$P_r = \beta_r\eta_{c\cdot r} = \beta_r\eta_c, \qquad \beta_r = \eta_c^{-1}P_r, \qquad \eta_{c\cdot r} = \text{const.} \tag{7}$$

To make it simple, if eq. (6) for the case of each pier as an individual body is expressed as

$$n_r^2 = \frac{\lambda_r}{\mu_r}, \qquad \varepsilon = \frac{\nu_r}{2\mu_r}$$

then the case for all the piers as a whole is

$$\left.\begin{aligned} n_s^2 &= \frac{\sum_{r=1}^{n} \lambda_r}{\sum_{r=1}^{n} \mu_r} = \frac{\sum_{r=1}^{n} \lambda_r' P_r^2}{\sum_{r=1}^{n} \mu_r' P_r^2} = \frac{\sum_{r=1}^{n} \beta_r^2 \lambda_r'}{\sum_{r=1}^{n} \beta_r^2 \mu_r'} \\ \varepsilon_s &= \frac{\sum_{r=1}^{n} \nu_r}{2\sum_{r=1}^{n} \mu_r} = \frac{\sum_{r=1}^{n} \nu_r' P_r^2}{2\sum_{r=1}^{n} \mu_r' P_r^2} = \frac{\sum_{r=1}^{n} \beta_r^2 \nu_r'}{2\sum_{r=1}^{n} \beta_r^2 \mu_r'} \\ T_s' &= \frac{2\pi}{\sqrt{n_s^2 - \varepsilon_s^2}}, \qquad T_s = \frac{2\pi}{n_s} \end{aligned}\right\} \quad (8)$$

As for the vibration of all the piers as a whole, it is sufficient only to add suffix s to each letter in eq. (5). β_r, therefore, expresses the degree of influence the individual pier has on the vibration of all the piers as a whole, namely, influence factor. Moreover, the following relation exists between T_r and T_s.

$$\min T_r < T_s < \max T_r$$

(3) Free vibration of bridge as a whole.

First, for the sake of convenience, the vibration of the abutment will be considered as shown in Fig. 2b and 2c. Let Ab_l represent the left abutment, Ab_r the right abutment, suffix $a{\cdot}o$ the case when the abutment displaces towards the river center and suffix $a{\cdot}e$ when the abutment displaces towards the bankside. Then, $n_{a\cdot o}$ is equal to n of the bridge pier and $n_{a\cdot e}$ is obtained from the case when the elastic ground is assumed as attaining the same level as the top of pier. Although $n_{a\cdot o}$ is in the form of n in eq. (6), the terms of the numerator and denominator shall respectively be represented by λ and μ for the sake of simplicity. Then the period of the free vibration of an individual abutment can be calculated as follows.

$$\left.\begin{aligned} n_{a\cdot o}^2 &= \frac{\lambda_{1\cdot o} + \lambda_{2\cdot o} + \lambda_{3\cdot o}}{\mu_{1\cdot o} + \mu_{2\cdot o} + \mu_{3\cdot o}} = \frac{\lambda_{a\cdot o}}{\mu_{a\cdot o}}, \\ n_{a\cdot e}^2 &= \frac{\lambda_{1\cdot e} + \lambda_{2\cdot e} + \lambda_{3\cdot e} + \frac{b_a K m_a}{8}\int_0^{\alpha_2}\left(\frac{d^2\eta_2}{d\xi_2^2}\right)^2 d\xi_2 + \frac{b_a K m_a}{2}\int_0^{\alpha_2}\eta_2^2\, d\xi_2}{\mu_{1\cdot e} + \frac{w_a a_a m_a}{2g}\int_0^{\alpha_2}\eta_2^2\, d\xi_2 + \frac{W}{2g}\left(\eta_2^2\right)_{\xi_2=\alpha_2}} = \frac{\lambda_{a\cdot e}}{\mu_{a\cdot e}} \\ T_a &= \pi\left(\frac{1}{n_{a\cdot o}} + \frac{1}{n_{a\cdot e}}\right) \end{aligned}\right\} \quad (9)$$

Next, in order to calculate the free vibration of the bridge as a whole consisting of piers and abutments, it requires only to combine the above results. That is, as shown in Fig. 2a and 2b, if suffix s_R is attached for the case when the whole bridge deforms to the right and suffix s_L when it deforms to the left, then

$$\left.\begin{aligned} n_{sR}^2 &= \frac{\beta_{a\cdot l\cdot o}^2\lambda_{a\cdot l\cdot o}+\sum_{r=1}^{n}\beta_{pr}^2\lambda_{pr}+\beta_{a\cdot r\cdot e}^2\lambda_{a\cdot r\cdot e}}{\beta_{a\cdot l\cdot o}^2\mu_{a\cdot l\cdot o}+\sum_{r=1}^{n}\beta_{pr}^2\mu_{pr}+\beta_{a\cdot r\cdot e}^2\mu_{a\cdot r\cdot e}} \\ n_{sL}^2 &= \frac{\beta_{a\cdot l\cdot e}^2\lambda_{a\cdot l\cdot e}+\sum_{r=1}^{n}\beta_{pr}^2\lambda_{pr}+\beta_{a\cdot r\cdot o}^2\lambda_{a\cdot r\cdot o}}{\beta_{a\cdot l\cdot e}^2\mu_{a\cdot l\cdot e}+\sum_{r=1}^{n}\beta_{pr}^2\mu_{pr}+\beta_{a\cdot r\cdot o}^2\mu_{a\cdot r\cdot o}} \\ T_s &= \pi\left(\frac{1}{n_{sR}}+\frac{1}{n_{sL}}\right) \end{aligned}\right\} \quad (10)$$

The following relation also exists in this case.

$$\min\,(T_r \text{ or } T_a) < T_s < \max\,(T_r \text{ or } T_a)$$

When there is a traffic load, this case may be dealt with by adding the reaction of the live load to weight W of the superstructure corresponding to the piers and abutments on which the traffic load acts directly.

(4) Calculation formulas proposed hithertofore.

As the elasticity of the ground is not considered in the calculation formulas proposed in the past, they correspond to the case of $K=\infty$ in this paper and can be expressed in the following form.

$$T_{K=\infty} = 2\pi\sqrt{\frac{33w_2a_2h+140W}{420gE_2\frac{I_2}{h^3}}}, \qquad (T_s)_{K=\infty} = 2\pi\sqrt{\frac{\sum_{r=1}^{n}(33w_{2r}a_{2r}h_r+140W_r)}{420g\sum_{r=1}^{n}E_{2\cdot r}\frac{I_{2r}}{h_r^3}}} \quad (11)$$

Next, if Dr. Mononobe's calculation formula is written in the same form it becomes as follows.

$$\left.\begin{aligned} T_e &= 2\pi\sqrt{\frac{33wa(h+d)+140W}{420gE\frac{I}{h^3}(1+C)}}, \quad C = \frac{bK(h+d)^4}{12EI}\gamma^5\left(\frac{9}{5}-\gamma+\frac{\gamma^2}{7}\right), \quad \gamma = \frac{d}{h} \\ T_{se} &= 2\pi\sqrt{\frac{\sum_{r=1}^{n}\left\{33w_ra_r(h_r+d_r)+140W_r\right\}}{420g\sum_{r=1}^{n}E_r\frac{I_r}{h_r^3}(1+C_r)}} \end{aligned}\right\} \quad (12)$$

$$C_r = \frac{b_r K (h_r+d_r)^4}{12E_rI_r}\gamma_r^5\left(\frac{9}{5}-\gamma_r+\frac{\gamma_r^2}{7}\right), \quad \gamma_r = \frac{d_r}{h_r}$$

3. Forced Vibration due to Seismic Motion

(1) Fundamental equation of forced vibration.

Although seismic motion is a complicated function of time, it can be expressed as follows considering only its horizontal component.

$$e(t) = \sum_i A_i \sin\left(\frac{2\pi}{T_i}t+\frac{2\pi}{\lambda_i}r\right) = \sum_i e_i \sin(p_i t+\phi_i) \tag{13}$$

where A_i : half amplitude of seismic motion,

T_i : period, λ_i : wave length,

r : distance from seismic center.

Referring to Fig. 3, when a seismic motion $e(t)$ occurs in the part of the pier in the ground, then if origin O is taken on the vibrating coordinates, the part of the bridge pier under the ground and the part above the ground respectively receive $-\left(\frac{w_1a_1}{g}\right)\ddot{e}(t)$ and $-\left(\frac{w_2a_2}{g}\right)\ddot{e}(t)$ per unit length and the top of the pier receives $-\left(\frac{W}{g}\right)\ddot{e}(t)$. Therefore, the forced vibration of the displacement due to the seismic motion expressed by eq. (13) is, in the case of the pier, equal to the forced vibration of the force expressed by the following equation.

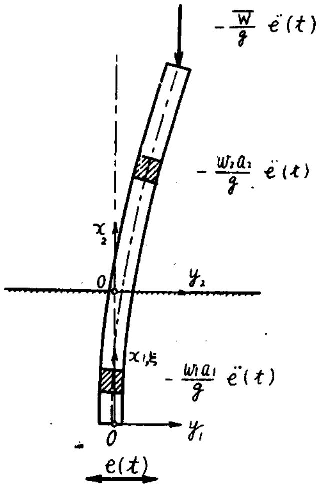


Fig. 3

$$-\ddot{e}(t)\left\{\int_0^a \frac{w_1a_1}{g}md\xi+\int_0^h \frac{w_2a_2}{g}dx_2+\left(\frac{W}{g}\right)_{x_2=h}\right\}$$

From this the generalized force is

$$Q = \left\{w_1a_1m\int_0^a \eta_1 d\xi+w_2a_2\int_0^h \eta_2 dx_2+W(\eta_2)_{x_2=h}\right\}\sum_i k_i \sin(p_i t+\phi_i) \tag{14}$$

where

$$k_i = \frac{4\pi^2 e_i}{gT_i^2}$$

k_i expressing the so-called seismic intensity. If the calculation is carried on by substituting the generalized force of eq. (14) into Lagrange's equation of motion

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_t}\right)-\frac{\partial T}{\partial q_t}+\frac{\partial F}{\partial \dot{q}_t} = -\frac{\partial V}{\partial q_t}+Q$$

the next fundamental equation is obtained.

$$\ddot{q}_t+2\varepsilon\dot{q}_t+n^2q_t = \gamma\sum_i k_i\sin(p_it+\phi_i) \tag{15}$$

where n^2, ε are the values given by eq. (6) and

$$\gamma = \frac{1}{2\mu}\left\{w_1a_1m\int_0^a \eta_1 d\xi + w_2a_2\int_0^h \eta_2 dx_2 + W(\eta_2)_{x_2=h}\right\}$$

(2) Forced vibration of an individual pier.

When each pier is considered to vibrate individually, eq. (15) is to be solved. According to the method of variation of constants the solution of eq. (15) is as follows if C_1, C_2 are integration constants determined from the initial conditions.

$$\left.\begin{aligned} q_t &= C_1q_{1\cdot t}+C_2q_{2\cdot t}-\gamma q_{1\cdot t}\int\frac{q_{2\cdot t}\sum_i k_i\sin(p_it+\phi_i)}{q_{1\cdot t}\dot{q}_{2\cdot t}-q_{2\cdot t}\dot{q}_{1\cdot t}}dt \\ &+\gamma q_{2\cdot t}\int\frac{q_{1\cdot t}\sum_i k_i\sin(p_it+\phi_i)}{q_{1\cdot t}\dot{q}_{2\cdot t}-q_{2\cdot t}\dot{q}_{1\cdot t}}dt \end{aligned}\right\} \tag{16}$$

In the case of a pier, $n>\varepsilon$, so, if the integration variable and the lower limit of the integration are respectively chosen at $t=\tau$ and $\tau=t_0=0$, then eq. (16) becomes

$$q_t = e^{-\varepsilon t}(C_1\cos\sigma t+C_2\sin\sigma t)+\frac{\gamma e^{-\varepsilon t}}{\sigma}\sum_i k_i\int_0^t \sin(p_i\tau+\phi_i)e^{\varepsilon\tau}\sin\sigma(t-\tau)\,d\tau \tag{17}$$

Using the relation of eq. (2), assuming $y=y_0=\eta_0$, $\dot{y}=\dot{y}_0=\dot{\eta}_0$ when $t=0$, eq. (17) can be solved and the deflection vibration curve can be obtained as follows.

$$\left.\begin{aligned} y_j &= e^{-\varepsilon t}\left\{\eta_{0\cdot j}\cos\sigma t+\frac{1}{\sigma}(\varepsilon\eta_{0\cdot j}+\dot{\eta}_{0\cdot j})\sin\sigma t\right\} \\ &+\eta_j\gamma\sum_i\frac{k_i}{\sqrt{(n^2-p_i^2)^2+4p_i^2\varepsilon^2}}\sin\left(p_it+\phi_i-\tan^{-1}\frac{2p_i\varepsilon}{n^2-p_i^2}\right) \\ &-\eta_j\gamma e^{-\varepsilon t}\sum_i\frac{k_i}{(n^2-p_i^2)^2+4p_i^2\varepsilon^2}(\delta_i\cos\sigma t+\delta_i'\sin\sigma t) \\ \text{where} & \\ \delta_i &= (n^2-p_i^2)\sin\phi_i-2p_i\varepsilon\cos\phi_i, \\ \delta_i' &= \frac{1}{\sigma}\left\{(n^2+p_i^2)\varepsilon\sin\phi_i+(n^2-p_i^2-2\varepsilon^2)p_i\cos\phi_i\right\} \end{aligned}\right\} \tag{18}$$

The first term of eq. (18) expresses the free vibration of the pier determined from the initial conditions, the second term, the forced vibration due to seismic motion and the third term, the free vibration due to seismic motion.

(3) Forced vibration of all the piers as a whole.

When all the piers are considered as vibrating as one structure this case may be dealt with in the same manner as the case of the free vibration, the generalized force being

$$\left.\begin{aligned} Q_s &= \eta_e \sum_{r=1}^{n} \sum_{t} \beta_r \left\{ w_{1\cdot r} a_{1\cdot r} m_r \int_0^{\alpha_r} \eta'_{1\cdot r} d\xi_r + w_{2\cdot r} a_{2\cdot r} \int_0^{h_r} \eta'_{2\cdot r} dx_{2\cdot r} \right. \\ &\quad \left. + W_r \left(\eta'_{2\cdot r} \right)_{x_{2\cdot r}=h_r} \right\} k_t \sin(p_t t + \phi_{r\cdot t}) \end{aligned}\right\} \quad (19)$$

The fundamental equation corresponding to eq. (15) is

$$\ddot{q}_{t\cdot s} + 2\varepsilon_s \dot{q}_{t\cdot s} + n_s^2 q_{t\cdot s} = \sum_{r=1}^{n} \sum_{t} \gamma_r k_t \sin(p_t t + \phi_{r\cdot t}) \quad (20)$$

and corresponding to eq. (17) is

$$\left.\begin{aligned} q_{t\cdot s} &= e^{-\varepsilon_s t} (C_{1\cdot s} \cos \sigma_s t + C_{2\cdot s} \sin \sigma_s t) \\ &\quad + \frac{e^{-\varepsilon_s t}}{\sigma_s} \sum_{r=1}^{n} \sum_{t} \gamma_r k_t \int_0^t \sin(p_t \tau + \phi_{r\cdot t}) e^{-\varepsilon \tau} \sin \sigma_s (t-\tau)\, d\tau \end{aligned}\right\} \quad (21)$$

From this the flexural vibration curve of pier No. r can be calculated as follows.

$$\left.\begin{aligned} y_{j\cdot r} &= e^{-\varepsilon_s t} \left\{ \eta_{0\cdot j} \cos \sigma_s t + \frac{1}{\sigma_s} (\varepsilon_s \eta_{0\cdot j} + \dot{\eta}_{0\cdot j}) \sin \sigma_s t \right\} \\ &\quad + \eta_{j\cdot r} \sum_{r=1}^{n} \sum_{t} \frac{\gamma_r k_t}{\sqrt{(n_s^2 - p_t^2)^2 + 4p_t^2 \varepsilon_s^2}} \sin\left(p_t t + \phi_{r\cdot t} - \tan^{-1} \frac{2p_t \varepsilon_s}{n_s^2 - p_t^2} \right) \\ &\quad - \eta_{j\cdot r} e^{-\varepsilon_s t} \sum_{r=1}^{n} \sum_{t} \frac{\gamma_r k_t}{(n_s^2 - p_t^2)^2 + 4p_t^2 \varepsilon_s^2} (\delta_{r\cdot t} \cos \sigma_s t + \delta'_{r\cdot t} \sin \sigma_s t) \\ &\text{where} \\ \delta_{r\cdot t} &= (n_s^2 - p_t^2) \sin \phi_{r\cdot t} - 2p_t \varepsilon_s \cos \phi_{r\cdot t} \\ \delta'_{r\cdot t} &= \frac{1}{\sigma_s} \left\{ (n_s^2 + p_t^2) \varepsilon_s \sin \phi_{r\cdot t} + (n_s^2 - p_t^2 - 2\varepsilon_s^2) p_t \cos \phi_{r\cdot t} \right\} \end{aligned}\right\} \quad (22)$$

The forced vibration of the pier can be calculated from the above equations.

Earthquake damages of piers are mostly the results of the falling of the superstructure due to the displacement of the pier head and the failure of the substructure due to bending moment and shearing force. In this paper as the vibration curve is considered as shown in Fig. 1, the maximum amplitude y_{max} occurs at the top of the pier, maximum bending moment M_{max} near the ground surface B and maximum shearing force S_{max} at part B~C above the ground. Representing these respectively as $y_{C\cdot max}$, $M_{B\cdot max}$, and $S_{B-C\cdot max}$, they can be obtained from eq. (18) and (22), and if these are respectively represented as $y_{C\cdot f\cdot max}$, $M_{B\cdot f\cdot max}$ and $S_{B-C\cdot f\cdot max}$ for the case when the term of the forced vibration, namely the second term which occupies the greater part of the vibration in eq. (18) and (22) becomes largest in accordance with the various periods of the seismic motion, then

$$\left.\begin{aligned} y_{C\cdot f\cdot \max} &= \left(\eta_2\right)_{x_2=h}\cdot\left(q_{t\cdot f}\right)_{\sin(\ \ )=1} = \eta_c\frac{\gamma k_t}{\sqrt{(n^2-p_i^2)^2+4p_i^2\varepsilon^2}} \\ M_{B\cdot f\cdot \max} &= -E_2I_2\left(\frac{d^2\eta_2}{dx_2^2}\right)_{x_2=0}\cdot\left(q_{t\cdot f}\right)_{\sin(\ \ )=1} = -b_2hP\frac{\gamma k_t}{\sqrt{(n^2-p_i^2)^2+4p_i^2\varepsilon^2}} \\ S_{B-C\cdot f\cdot \max} &= -E_2I_2\left(\frac{d^3\eta_2}{dx_2^3}\right)\cdot\left(q_{t\cdot f}\right)_{\sin(\ \ )=1} = b_2P\frac{\gamma k_t}{\sqrt{(n^2-p_i^2)^2+4p_i^2\varepsilon^2}} \end{aligned}\right\} \quad (23)$$

Similarly with the case of all the piers as a whole,

$$\left.\begin{aligned} y_{C\cdot f\cdot \max} &= \left(\eta'_{2\cdot r}\right)_{x_{2\cdot r}=h_r}\cdot\beta_r\eta_c\left(q_{t\cdot s\cdot f}\right)_{\sin(\ \ )=1} = \eta_c\sum_{r=1}^{n}\frac{\gamma_r k_t}{\sqrt{(n_s^2-p_i^2)^2+4p_i^2\varepsilon_s^2}} \\ M_{B\cdot f\cdot \max} &= -E_{2\cdot r}I_{2\cdot r}\beta_r\eta_c\left(\frac{d^2\eta'_{2\cdot r}}{dx_{2\cdot r}^2}\right)_{x_{2\cdot r}=0}\cdot\left(q_{t\cdot s\cdot f}\right)_{\sin(\ \ )=1} \\ &= -b_2h_r\beta_r\eta_c\sum_{r=1}^{n}\frac{\gamma_r k_t}{\sqrt{(n_s^2-p_i^2)^2+4p_i^2\varepsilon_s^2}} \\ S_{B-C\cdot f\cdot \max} &= -E_{2\cdot r}I_{2\cdot r}\beta_r\eta_c\frac{d^3\eta'_{2\cdot r}}{dx_{2\cdot r}^3}\left(q_{t\cdot s\cdot f}\right)_{\sin(\ \ )=1} \\ &= b_2\beta_r\eta_c\sum_{r=1}^{n}\frac{\gamma_r k_t}{\sqrt{(n_s^2-p_i^2)^2+4p_i^2\varepsilon_s^2}} \end{aligned}\right\} \quad (24)$$

Stresses can be calculate from these values and earthquake-proof calculation is possible.

4. Vibration Test of the Nakatsuno Bridge Pier and Example of Numerical Calculation

(1) Nakatsuno Bridge of the Keifuku Electric Railway Company which spans Kuzuryu River is a multi-span simple steel deck plate girder bridge of the Mikuni-Awara line.

The bridge suffered damages as shown in Fig. 4 due to the Fukui Earthquake of June 1948. Vibration tests were performed on this Nakatsuno Bridge three times and the mechanism of the earthquake damage thoroughly studied by doing a detailed calculation applying the theoretical equation introduced above.

The first test was performed from the 14 th to the 25 th of Sept. 1948 when the bridge was in the state of temporary repair, using one electric horizontal vibroscope (T_0=0.55 sec, magnification=133) and 5 Sasa type model C electric vibroscopes. The second test was performed from the 20 th to the 30 th of Nov. 1948 when the reinforcement of the piers were practically completed, using two electric horizontal vibroscopes together with Takahashi type horizontal vibroscope employing mechanical magnification and smoke cylinder recording (T_0=0.50 sec, magnification=200). The third test was performed after the piers were reinforced completely from the 11 th to the 18 th of Sept. 1949 in order to determine whether it is safe or not to return electric cars to the usual speed using one Takahashi type vibroscope.

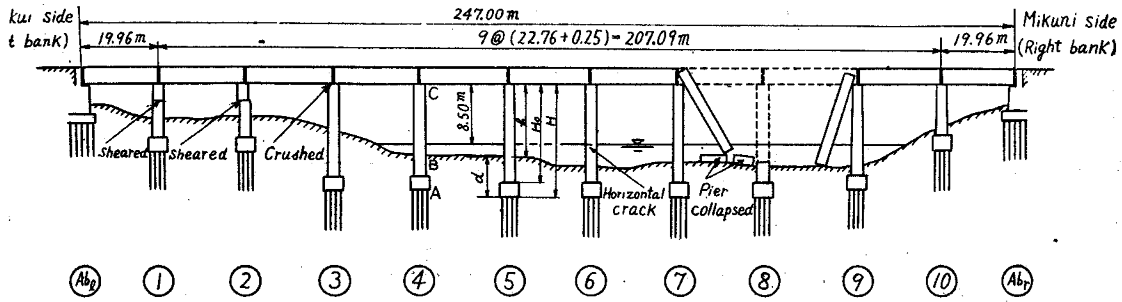


Fig. 4. General View of the Earthquake Damages of the Nakatsuno Bridge
(Scale : —Vertical : Horizontal=2 : 1)

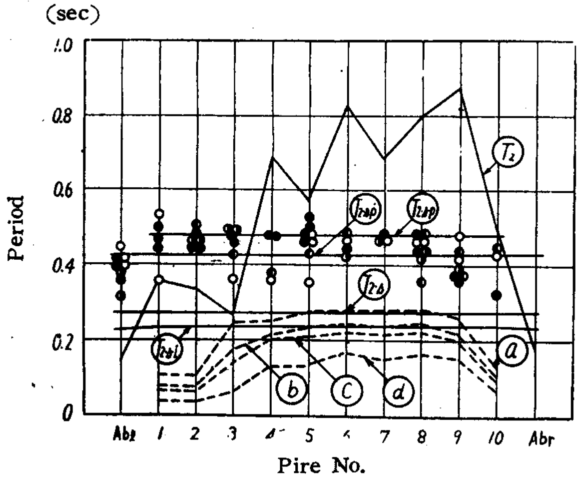
The first test was performed on piers No. 1, 2, 3, 5, 6 and 7 vibrating the bridge by running electric cars, horizontal oscillator and hammering. The second test was performed on the same piers giving the vibration by running electric cars only. The third test was performed on piers No. 1, 2, 5 and 6 giving the vibration by running electric cars.

(2) Results of the tests.

1) Period of vibration. The observed periods were as shown in Fig. 5 and the fact that there is no great difference in the observed periods of the piers in spite of the fact that the parts of the piers above and below the ground differ as shown in Fig. 4 is an attractive feature. There is no distinct relation between the observed periods for the various cases of different number of electric cars, the running direction and the speed and also no numerical relation between the period in the direction of the bridge axis and that perpendicular to the bridge axis. No remarkable difference can be recognized between the periods of piers No. 1 and 2 observed in the first and second tests and between the periods of pier No. 6 before and after the reinforcement observed in the first, second and third tests.

2) Amplitude and phase of the vibration. There is no regular change in the maximum amplitude due to speed in the first and second tests when the speed of the running car is within the limits of 8~18 km/hr, but in the third test when

Fig. 5.1. Observed and Theoretical Values of Vibration Period of Bridg Piers and Abutments. (Two electric cars are loaded approx. above the top of bridge pier or abutment.)



T_2 calculated from eq. (6), $T_{2 \cdot sp}$ from eq. (8), $T_{2 \cdot sp}'$ from eq. (8) (abutment considered as a bridge pier), $T_{2 \cdot s}$ from eq. (10) (all piers and abutments are loaded), $T_{2 \cdot si}$ from eq. (10) (only pier No. i is loaded).
(a) from eq. (12), (b), (c), (d) from eq. (11) for the respective cases when the bottom third, lower half and entire part of the rooted length is perfectly fixed.
Test results; parallel to bridge axis ○: electric car for Fukui, ●: electric car for Mikuni. perpendicular to bridge axis: ◐: for Fukui, ◑: for Mikuni.

the speed is 35~40 km/hr the maximum amplitude in the direction of the bridge axis increases slightly with the increase of speed while in the direction perpendicular to the bridge axis the maximum amplitude increases almost linearly. The damping of the amplitude is linear in the case of piers No. 1, 2 and 6, exponential in the case of piers No. 3 and 5 and in the case of other piers it exists between the above two cases. It can be considered that piers which show a linear damping state are those whose bases are not sufficiently fixed or which have been damaged. Actually in the case of piers No. 1 and 2, as the part in the ground is not very deep and the base not sufficiently fixed, no difference was recognized in the damping before and after the reinforcement, but with pier No. 6, as the part in the ground is deep and the base fixed sufficiently, it is seen that the linear damping changed to an exponential damping and thus the effectiveness of the reinforcement near the water surface is acknowledged. As for the phase of vibration, adjacent piers show exactly the same phase in the direction of the bridge axis, but this tendency cannot be seen in the direction perpendicular to the bridge axis.

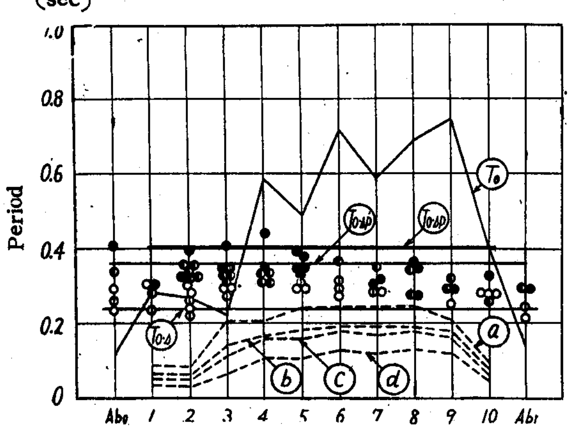
3) Other vibrations. Other vibrations are observed as follows. The period of the vibration given to the piers by electric cars passing over rail joints is 0.04

sec. and the amplitude, 0.1~0.07 mm. As for the vibration of the abutment produced by 2 electric cars, the period is 0.31~0.45 sec. and the maximum amplitude, 0.025 mm. As for the vibration of the base ground of the abutment, the maximum period in the direction of the bridge axis is 0.13 sec. and the maximum amplitude, 0.14 mm, while in the direction perpendicular to the axis the maximum period is 0.08 sec. and the maximum amplitude, 0.07 mm.

(3) Comparison between the theoretically calculated values and experimentally observed values of the period of the free vibration.

As it was found that as for the ground reaction coefficient of the site of the bridge $K=6.0\,kg/cm^3$ should be adopted from the result of the seismic prospecting performed at the site of the bridge, the values calculated by the various theoretical equations given in this paper considering the elasticity of the ground are as shown by the full lines in Fig. 5. In comparing the observed values with the calculated values in these figures, it is seen that the abutments and piers vibrats almost the same as the case when the whole bridge is considered to vibrate as one structure. Fig. 5. 2. is the case when the car load does not cause reaction on the piers directly, particularily when the car is not on the bridge, namely the value with the ○, ⦶ mark in Fig. 5. 2. should without doubt express the free vibration of the bridge pier. As can be seen from the figure, the observed values are found between $T_{0\cdot sp'}$ and $T_{0\cdot s}$, so that it can be said that the theoretical

Fig. 5. 2. (When the electric car load has no direct influence on the pier and abutment considered.)



T_0 calculated from eq. (6), $T_{C\cdot sp}$ from eq. (8), $T_{C\cdot sp'}$ from eq. (8), $T_{0\cdot s}$ from eq. (10).
(a), (b), (c) and (d) are the same as Fig. 5. 1.
Test results; ●: parallel to bridge axis, ◐: perpendicular to bridge axis when electric cars have no direct reaction on the pier considered. ○: parallel to bridge axis, ⦶: perpendicular to bridge axis when there is no electric car on bridge,

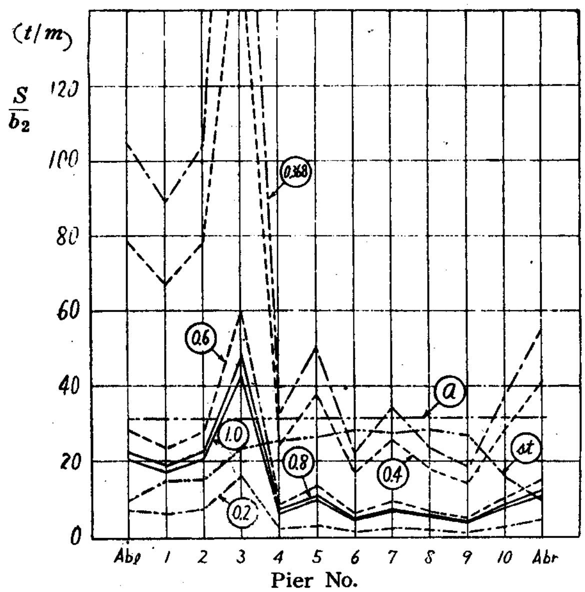
equations in this paper give a considerable accurate explanation of the free vibration of the Nakatsuno Bridge. On the other hand from the fact the observed values are quite larger than $T_{2\cdot sp'} \sim T_{2\cdot s}$, T_{2st} as shown in Fig. 5. 1. it can be considered that it is strictly insufficient to merely substitute running electric car load with the equivalent statical loading. From the viewpoint of car dynamics it can be considered that the up and down free vibration of the running electric car greatly governs the nature of the vibration of the bridge pier and this was also confirmed in the actual observation of this bridge.

The values calculated by the former equations for the case of independent piers as shown by the dotted lines a, b, c and d in Fig. 5. 1. and 5. 2. are all considerably smaller than the observed values. Upon obtaining the values for the case of the bridge as a whole by assuming $K=\infty$ they were all smaller than 0.1 sec, with the result that it is quite impossible to explain the observed values of this experiment by any of the former equation.

(4) Calculation of forced vibration due to seismic motion.

Similar to the case of free vibration, a numerical calculation was performed based upon the most practical data of the site of the Nakatsuno Bridge. The values

Fig. 6. Calculated Maximum Shear Values of Pier above Ground Surface; eq. $(24)_3$

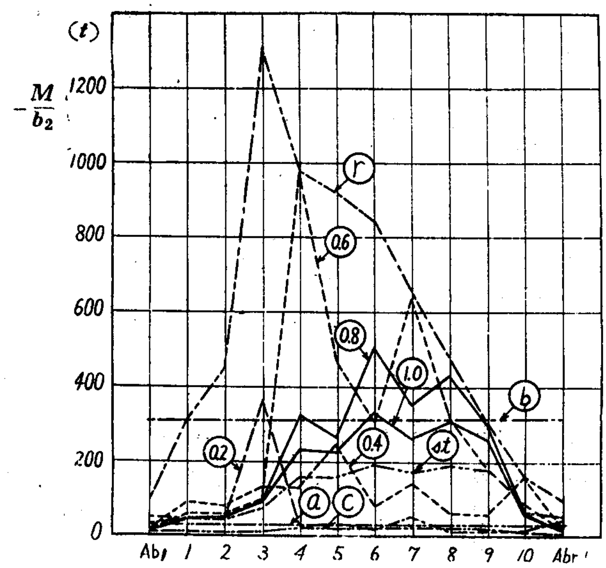


Line indicated (0.2) represents the values of max. shear S/b_2 corresponding to period of seismic vibration, 0.2 sec. Lines (0.4), ⋯, (1.0) are interpreted as above. (0.368): the same for the period of resonance 0.369 sec. (st); result of statical calculation, (a); value corresponding to allowable shearing stress 4.5 kg/cm².

employed in the calculation were $a_1=a_2=10.20\,m^2$, $b_1=b_2=4.84\,m$, $I_1=I_2=4.19\,m^4$, $E_1=E_2=2.1\times10^6\,t/m^2$, $w_1=w_2=2.3\,t/m^3$, $W=25.4\,t$ (pier No. 2~9), 22.3 t (pier No. 1 and 10), 9.6 t (abutments), $K=6\,kg/cm^3$, and ε was determined from experimental records. Furthermore upon using seismic intensity $k_t=0.52$ by Dr. R. Takahashi and propagation velocity of P wave $v_p=1000$ m/sec. by Dr. K. Sasa, the numerical calculation of the theoretical equation in this paper was performed.

Numerical calculation, shows that a seismic motion of 0.8~1.0 sec. was the principal cause of the damages of this bridge. To explain this concretely, piers No. 1, 2 and 3 being near the abutment are not easily vibrated, with the result that at the beginning of the seismic motion they receive a vibration almost similar to that of the bridge as a whole due to a seismic motion of about 0.3 sec., and so pier No. 1 and 2 are damaged by shearing force at the comparatively weak concrete placing joint and pier No. 3 is crushed and damaged at the top (Fig. 6). Piers No. 6 and 8 are located where the bridge as a whole vibrates easily, and so it is considered that they were damaged near the ground surface where the bending moment becomes maximum, as the weak point between the super and sub-structure was damaged at the beginning of the earthquake causing

Fig. 7. Calculated Values of Maximum Bending Moment at Ground Surface; eq. $(23)_2$



Pier No.

Line indicated (r) represents the values of max. bending moment $-M/b_2$ corresponding to period of resonance, (a); value of $-M/b_2$ for the case when the extreme fibre stress equals allowable tensile stress $3.5\,kg/cm^2$, (b); value corresponding to breaking tensile stress $35\,kg/cm^2$, (c); value when resultant load coincide with the core of pier cross section. As otherwise noted see Fig. 6.

the piers to vibrate almost individually (Fig. 7). Also it can be explained by the numerical calculation why such a big displacement that would make the superstructure fall in this bridge did not occur.

5. Dynamical Consideration on Earthquake Damage

In studying earthquake damages of piers and bridges, it is always necessary to consider the problem dynamically. That is, the vibration of a bridge must be considered as a transition phenomenon of the free and forced vibrations when a quasi periodical or impulsive irregular seismic force acts. Also in dealing with the vibration of the bridge as a whole it is necessary to consider the condition how the bridge beams, piers and abutments are connected. Accordingly, although the study of this problem must be considered and investigated from various points of view, the following can be concluded from what has been discussed in this paper.

(1) Falling of bridge beam.

When the part connecting the pier and the superstructure is weak, the piers have different periods of free vibration as aforementioned and the vibration of the piers as individual structures appears strongly. In this case bridge beams vibrate on the top of piers during the earthquake and finally slip off the pier and drop.

(2) Damages of piers.

In the connection between the piers and superstructure is rigid, the vibration of the bridge as a whole is predominant, resulting in the pier with the larger value of β_r in this paper being subjected to a large shearing force. When the vibration of the pier as an individual structure is strong, it is evident that those with parts higher above the ground are more easily bent and damaged.

(3) Subsequent studies.

From the results of the third test, it became clear that vibration perpendicular to the bridge axis is also of considerable importance. Therefore, in this first report the vibration in the direction of the bridge axis was discussed, but in the succeeding reports it will be attempted to make a theoretical study on the free vibration perpendicular to the bridge axis and the forced vibration due to seismic motion. Two vibration tests (first test in May 1950, second test in Oct. 1950) were performed on the Kuzuryu Bridge of the National Highway No. 12 spanning the Kuzuryu River. Furthermore vibration tests were performed on the Seta River Bridge of the National Railway Tokaido Line.

6. Conclusion

In short, in this paper a theoretical consideration was made on the free vibration and forced vibration of structures rooted in an elastic ground using the energy method in order to make clear the nature of the vibration and the earthquake-proof features of bridge piers and the bridge as a whole which have a close relation with the ground, and a comparison was made between the theoretically obtained results and experimental results to find out the appropriateness of the theoretical consideration.

A further reasonable consideration will be made in the future on the assumption of the distribution of the ground reaction and the seismic motion.

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