# On the Rate of Combustion of Coke Briquettes in the Fuel Bed 

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(Received April 1951)

## Introduction

Undoubtedly the combustion on the grate is one of the oldest and simplest ways of using solid fuel. The physical and chemical changes in this case, however, seem to be rather complicated and many problems worth studying still remain untouched.

Generally the combustion of solid fuel in the fuel bed is explained by the scheme represented in Fig. 1. Namely, the oxidation of coke to carbon dioxide by primary air, which comes from under the grate, takes place in a layer immediately above the ash bed. Oxygen is thus consumed and the concentration of carbon dioxide increases, which, however, is again converted into carbon monoxide in a layer, the so called reducing zone, just above the oxidizing zone. Although this general view of reactions was worked out in the first decade of this century, the quantitative formulation of these phenomena had not been touched until quite recently. Mayers ${ }^{1)}$ was the first who presented a formula which gave the temperature distribution in the fuel


Fig. 1. Composition of the Gases within the Fuel Bed of a Coal-fired Furnace.
bed. Having based himself on experimental data, he also succeeded in expressing the rate of oxidation $\mu_{1}$ and reduction $\mu_{2}$ in such commoner terms as porosity of coke or the void of the packed bed. ${ }^{2)}$

In the present paper, the oxidizing and reducing reaction in a packed fuel bed will be treated in minuter detail, and carefull considerations will be given to the effects of physical and chemical changes occuring at and near the carbon surface,
and, further, the results calculated with our theoretical formula will be compared with experimental data.

## 1. Theoretical Consideration

For the purpose of simplifying the problem, let us confine our attention to the investigation of the steady state, taking into consideration only the reaction of coke with air, and excluding other reactions such as of coke with water vapour or the distillation of coke.

As the first step let us consider somewhat in detail the reactions which will take place in the oxidizing zone.

Oxygen comes into contact with and reacts with the fresh carbon surface after passing through the boundary and ash layers by diffusion.

The chemical reactions can be formulated as

$$
\begin{equation*}
2 \mathrm{C}+\mathrm{O}_{2} \rightarrow 2 \mathrm{CO}, \quad(1) \quad \mathrm{C}+\mathrm{O}_{2} \rightarrow \mathrm{CO}_{2} \tag{1}
\end{equation*}
$$

giving carbon mono-and dioxides as ultimate products.
However, the carbon dioxide thus produced may be reduced to carbon monoxide according to the reaction

$$
\begin{equation*}
\mathrm{CO}_{2}+\mathrm{C} \rightarrow 2 \mathrm{CO} \tag{3}
\end{equation*}
$$

Now, in the present case, all three reactions (1), (2) and (3) may occur simultaneously on the carbon surface, and, in a temperature range of $900^{\circ}-1400^{\circ} \mathrm{C}$, the rates of reactions are so high that the equilibrium will be established instantaneously. Furthermore, as the eqilibrium constants for the above reactions are known to be larger than $10^{2}$ at temperatures aforementioned, the concentration of carbon dioxide and oxygen at the carbon surface may be considered as practically zero. ${ }^{3}$

Carbon monoxide thus formed, however, may readily be oxidized to carbon dioxide by the reaction

$$
\begin{equation*}
2 \mathrm{CO}+\mathrm{O}_{2} \rightarrow 2 \mathrm{CO}_{2} \tag{4}
\end{equation*}
$$

at a point very close to the surface, because the oxygen necessary for the reaction can be supplied by diffusion through the layer of ash, and consequently the main components which pass through the ash and boundary layers may be regarded as composed of oxygen and carbon dioxide.

Hence, it is possible to derive a theoretical formula representing the rate of the oxidizing reaction of carbon by taking into account only the interdiffusion of oxygen and carbon dioxide through both layers in the opposite direction.

Usually the rate of reaction $K\left(\mathrm{~mol} / \mathrm{cm}^{2} . \mathrm{sec}\right)$ of coke with air is represented by the equation

$$
\begin{equation*}
K=k p_{s} \tag{5}
\end{equation*}
$$

where $k\left(\mathrm{~mol} / \mathrm{cm}^{2} . \mathrm{sec}\right)$ is the reaction constant, and $p_{s}$ the partial pressure of oxygen at the reaction surface.

On the other hand, the oxygen has to pass through, by interdiffusion, the boundary layer at the surface of coke and also the layer of ash before entering into the reaction. Hence, $K$ becomes

$$
\begin{equation*}
K=\frac{D_{1}\left(p-p_{s}\right)}{R T A_{s}\left[\frac{\left.\underline{(r} r_{0}-r\right)}{\sqrt{A_{s} \cdot A_{i}}}+\frac{F}{\sqrt{A_{i} \cdot A_{0}}}\right]} \tag{6}
\end{equation*}
$$

in which $D_{1}\left(\mathrm{~cm}^{2} / \mathrm{sec}\right)$ is the coefficient of interdiffusion of oxygen and carbon dioxide, $A_{s}, A_{i}$ and $A_{0}\left(\mathrm{~cm}^{2}\right)$ the area of reacting surface, the surface of coke, and the outer surface of boundary layer, respectively, $R\left(\mathrm{~cm}^{3} . \mathrm{atm} / \mathrm{mol} .{ }^{\circ} \mathrm{K}\right)$ the gas constant, $\boldsymbol{T}\left({ }^{\circ} \mathrm{K}\right)$ the temperature of diffusing zone, a the factor for the coefficient of diffusional resistance in the ash layer, $F(\mathrm{~cm})$ the thickness of the boundary layer, $p$ the partial pressure of oxygen at the outer surface of the boundary layer, $r_{0}(\mathrm{~cm})$ the radius of the briquette, and $r$ the radius of the part which still remains unaffected.

If we take " $a$ " for the shape factor, we can substitute the terms $A_{8}, A_{i}$, and $A_{0}$ by $a \pi r^{2}, a \pi r_{0}^{2}$, and $a \pi\left(r_{0}+F\right)^{2}$, respectively.

Putting the above relations in the equation (6), we have

$$
\begin{equation*}
K=\frac{D_{1}\left(p-p_{s}\right)}{R T\left[\frac{a r\left(r_{0}-r\right)}{r_{0}}+\frac{F r^{2}}{r_{0}\left(r_{0}+F\right)}\right]}, \tag{7}
\end{equation*}
$$

and by eliminating $p_{s}$ from (5) and (7), we obtain the equation

$$
\begin{equation*}
K=\frac{p}{1} \frac{R T}{k_{1}}+\frac{R T}{D_{1}}\left[\frac{\alpha r\left(r_{0}-r\right)}{r_{0}}+\frac{F \cdot r^{2}}{r_{0}\left(r_{0}+F\right)}\right] \tag{8}
\end{equation*}
$$

Let $N$ be the number of briquettes in the unit volume, and the rate of oxidizing reaction $\mu_{1}\left(\mathrm{~mol} / \mathrm{cm}^{3}\right.$. sec ) will be expressed as

$$
\begin{equation*}
\mu_{1}=N a \pi r^{2} K / p \tag{9}
\end{equation*}
$$

As it is possible to express $N$ with the void of the packed bed $v$ by making use of the relation

$$
\begin{equation*}
N=\frac{3(1-v)}{a \pi r_{0}^{3}}, \tag{10}
\end{equation*}
$$

we get the expression

$$
\begin{equation*}
\mu_{1}=\frac{3(1-v)}{\frac{r_{0}^{3}}{k_{1} \cdot r^{2}}+\frac{R T}{D_{1}}\left[\frac{\alpha\left(r_{0}-r\right) r_{0}^{2}}{r}+\frac{F \cdot r_{0}^{2}}{r_{0}+F}\right]}, \tag{11}
\end{equation*}
$$

by substituting the equations (8) and (10) into (9).
In the case of the oxidizing reaction, i. e., $\mathrm{C}+\mathrm{O}_{2} \rightarrow \mathrm{CO}_{2}$, the velocity of the chemical reaction at temperatures higher than $800^{\circ}$ is so great that $\mu_{1}$ is entirely governed by diffusion, ${ }^{4)}$ and therefore the first term in the denominator of the equation (11) can be neglected, so that the expression for $\mu_{1}$ becomes

$$
\begin{equation*}
\mu_{1}=\frac{3 D_{1}(1-v)}{R T\left[\frac{\underline{\alpha}\left(r_{0}-r\right) r_{0}^{2}}{r}+F \cdot r_{0}^{2}\right.} r_{0}+. \tag{12}
\end{equation*}
$$

This denotes that the rate of oxidizing reaction $\mu_{1}$ is proportional to ( $1-v$ ) and is inversely proportional to the square of the radius of the briquette $r_{0}$ as long as the values of $D_{1}, T$ and ( $r_{0} / r-1$ ) can be regarded as constant, the thickness of the boundary layer $F$ being neglected as it is much smaller as compared with the radius of the briquette $r_{0}$.

Obviously, in the reducing zone, the above equation (11) holds good also for the expression of the rate of the reaction $\mathrm{C}+\mathrm{CO}_{2} \rightarrow 2 \mathrm{CO} .{ }^{5)}$

Then we obtain the equation for the rate of reduction of carbon dioxide $\mu_{2}\left(\mathrm{~mol} / \mathrm{cm}^{3} . \mathrm{sec}\right)$

$$
\begin{equation*}
\mu_{2}=\frac{3(1-v)}{\frac{r_{0}^{3}}{k_{2} r^{2}}+\frac{R T}{D_{2}}\left[\frac{\underline{\mu}\left(r_{0}-r\right) r_{0}^{2}}{r}+\frac{F \cdot r_{0}^{2}}{r_{0}+F}\right]}, \tag{13}
\end{equation*}
$$

in which $k_{2}$ is the reaction constant and $D_{2}$ the coefficient of interdiffusion of carbon mono- and dioxides.

However, at the temperature as low as $1100^{\circ} \mathrm{C}$ the rate of this chemical reaction is so slow that the term for the diffusion can be omitted. Hence, the expression for $\mu_{2}$ becomes

$$
\begin{equation*}
\mu_{2}=\frac{3 k_{2}(1-v)}{r_{0} \frac{r_{0}^{2}}{r^{2}}}, \tag{14}
\end{equation*}
$$

which means that $\mu_{2}$ is closely correlated with the value of $\frac{(1-v)}{r_{0}}$. Basing his research on the experimental data, Mayers ${ }^{2)}$ comes to the same conclusion.

Having been able to express $\mu_{1}$ and $\mu_{2}$ in terms of chemical and diffusional resistance, we are now in a position to discuss problems of the composition of gases in a packed fuel bed.

Let us concentrate our attention on the following main reaction: $\mathrm{C}+\mathrm{O}_{2} \rightarrow \mathrm{CO}_{2}$, $\mathrm{C}+\mathrm{CO}_{2} \rightarrow 2 \mathrm{CO}$ and $2 \mathrm{CO}+\mathrm{O}_{2} \rightarrow 2 \mathrm{CO}_{2}$. These reactions proceed under partial pressures $p_{1}, p_{2}, p_{3}$ of oxygen, carbon dioxide, and carbon monoxide respectively, at any point in a fuel bed.

Although, usually, the reactions $\mathrm{C}+\mathrm{O}_{2} \rightarrow \mathrm{CO}_{2}$ and $\mathrm{C}+\mathrm{CO}_{2} \rightarrow 2 \mathrm{CO}$ are treated as those of the first order according to the stoichiometric relations, the reaction. $2 \mathrm{CO}+\mathrm{O}_{2} \rightarrow 2 \mathrm{CO}_{2}$ is known to be more complicated and proceed through complex chain mechanism. ${ }^{6)}$ However, the authors have assumed that these series of partly still unknown chain reactions can be represented by an ordinary second order reaction, because it is simpler and accurate enough for the present purpose.

Hence, as the rate of decrease of the concentration of oxygen along the axial distance of a packed bed $G \frac{d p_{1}}{d x}$ must be equal to the rate of oxygen consumption according to the first and last equations, we have

$$
\begin{equation*}
G \frac{d p_{1}}{d x}=-\left(\mu_{1} p_{1}+k_{3} p_{1} p_{3}\right) \tag{15}
\end{equation*}
$$

In the same way we have the relations

$$
\begin{equation*}
G \frac{d p_{2}}{d x}=-\mu_{2} p_{2}+\mu_{1} p_{1}+2 k_{3} p_{1} p_{3} \tag{16}
\end{equation*}
$$

and

$$
\begin{equation*}
G \frac{d p_{3}}{d x}=2 \mu_{2} p_{2}-k_{3} p_{1} p_{3} \tag{17}
\end{equation*}
$$

in which $G$ is the quantity of air supply, $x$ the thickness of the oxidizing layer measured upwards from the lower end, $k_{3}$ is the reaction constant of the reaction $2 \mathrm{CO}+\mathrm{O}_{2} \rightarrow 2 \mathrm{CO}_{2}$. Formally, we are able to get the values of $p_{1}, p_{2}$ and $p_{3}$ by integrating the above simultaneous equations, but the actual calculations can not be carried out so simply, because $\mu_{1}, \mu_{2}$ and $k_{3}$ may have different values at every point in the fuel bed.

Substituting the equations (11) and (13) in (15), (16) and (17), we obtain the relations

$$
\begin{align*}
G \frac{d p_{1}}{d x}= & -\frac{3(1-v) p_{1}}{\frac{R T}{D_{1}}\left[\frac{\alpha\left(r_{0}-r\right) r_{0}^{2}}{r}+\frac{F \cdot r_{0}^{2}}{r_{0}+F}\right]+\frac{r_{0}^{3}}{k_{1} r^{2}}}-k_{3} p_{1} p_{3},  \tag{18}\\
G \cdot \frac{d p_{2}}{d x}= & \frac{3(1-v) p_{1}}{\frac{R T}{D_{1}}\left[\frac{\alpha\left(r_{0}-r\right) r_{0}^{2}}{r}+\frac{F \cdot r_{0}^{2}}{r_{0}+F}\right]+\frac{r_{0}^{3}}{k_{1} \cdot r^{2}}} \\
& -\frac{3(1-v) p_{2}}{\frac{R T}{D_{2}}\left[\frac{\alpha\left(r_{0}-r\right) r_{0}^{2}}{r}+\frac{F \cdot r_{0}^{2}}{r_{0}+F}\right]+\frac{r_{0}^{3}}{k_{2} \cdot r^{2}}}+2 k_{3} p_{1} p_{2}, \tag{19}
\end{align*}
$$

and

$$
\begin{equation*}
G \frac{d p_{3}}{d x}=2 \frac{3(1-v) p_{2}}{\frac{R T}{D_{2}}\left[\frac{\ddot{\left(2\left(r_{0}-r\right) r_{0}^{2}\right.}}{r}+\frac{F r_{0}^{2}}{r_{0}+F}\right]+\frac{r_{0}^{3}}{k_{2} \cdot r^{2}}}-k_{3} p_{1} p_{2} \tag{20}
\end{equation*}
$$

By this means, we can eliminate the unknown $\mu_{1}$ and $\mu_{2}$.
On the other hand, as it is clearly seen from the equations $\mathrm{C}+\mathrm{O}_{2} \rightarrow \mathrm{CO}$, $\mathrm{C}+\mathrm{CO}_{2} \rightarrow 2 \mathrm{CO}$, one molecule of oxygen or carbon dioxide reacts upon one atom of carbon so that the rate of the decrease of carbon $\frac{d r}{d x}$, in the present case, can be expressed by the equation
whose first and the second terms in the blacket correspond to the carbon consumption by the oxidizing and the reducing reactions, respectively. In the equation, $f$ represents the content of fixed carbon, $\rho$ the apparent density, and $u$ the rate of translational movements ( $\mathrm{cm} / \mathrm{sec}$ ) of coke briquettes.

The thickness of the boundary film $F$ can be obtained from the relation

$$
\begin{equation*}
F=\frac{D_{m} \cdot P}{R \cdot T \cdot k_{g} \cdot p_{f}} \tag{22}
\end{equation*}
$$

in which $D_{m}$ is, for the present case, the mean value of coefficient of interdiffusion in the systems $\mathrm{CO}_{2}-\mathrm{O}_{2}$ and $\mathrm{CO}_{2}-\mathrm{CO}$, $p_{r}$ the logarismic mean of the concentration of gas at the inner- and outer surfaces of the boundary film, ${ }^{7)} P$ the total pressure of gases, and $R$ and $T$ are, as usual, gas constant and absolute temperature, respectively. The mass transfer coefficient of the gas film $k_{g}$ in a packed bed, which appears in the above equation, can be calculated in the following manner: ${ }^{8)}$

The mass-transfer factor $j_{a}$ is usually expressed by the equation

$$
\begin{equation*}
j_{a}=\left(\frac{k_{g} \cdot M \cdot p_{f}}{G}\right)\left(\frac{\mu}{d \cdot D_{1}^{\prime}}\right)^{2 / 3}, \tag{23}
\end{equation*}
$$

and for $\left(D_{p} G / \mu\right)<350$

$$
\begin{equation*}
j_{a}=1.82\left(\frac{D_{p} \cdot G}{\mu}\right)^{-0.41} \tag{24}
\end{equation*}
$$

therefore, we can calculate $k_{g}$, using $j_{a}$ as an intermediate, if other values in the equations are ready to hand.

The figures, such as $M$, the mean molecular weight of air, $G$, the quantity of air supply, $\mu$, the viscosity of gases, $d$, the density of gases, and $D_{p}$, the effective diameter of the briquette can be regarded as known. Hence, from the known
value of the modified Reynolds number $\left(\frac{D_{p} \cdot G}{\mu}\right)$ we can get the value of $j_{a}$ and, by substituting this in the equation (23), the necessary value $k_{g}$ can readily be obtained.

By solving the equations (18), (19), (20) and (21) simultaneously by the method of numerical integration, we obtain the values of partial pressure of oxygen and carbon dioxide at any point of the fuel bed. The following experiments have been carried out in the hope of finding out how the calculated values agree to the experimental data.

## 2. Experiments

Coke and clay powder, which had passed through the 60 meshes of the standard sieve, were mixed in various proportions and kneaded well with a proper amount of water. Then the mixture was made into cylindrical briquettes of 2 cm in height and in diameter by means of a small auger machine. The technical analysis of coke gave the results; water 2.2, volatile matter 7.1, fixed carbon 80.9 , ash $9.8 \%$.

In Fig. 2 is shown a sketch of the apparatus used for the present experiments. A small cylindrical furnace, $F$, of 60 cm high and 13 cm in inner diameter was charged with about 700 pieces of briquettes, and after they had been ignited on the grate $B$, air was supplied from under the grate so that the combustion proceeded upwards. The temperature of the central axis of the packed fuel bed, which changes according to the displacement of the combustion zone, was measured every five minutes with $\mathrm{Pt}-\mathrm{Pt} \cdot \mathrm{Rh}$ thermo-couples fixed at the position (1) (4) and (6).


Fig. 2. Diagram of Experi. mental Apparatus.
B; Grate, G; Gas Sampling Tube, $T$; Thermocouple, F; Furnace.

For the purpose of measuring the radial temperature distribution, the thermocouple (5) was moved in the protecting tube and the reading was taken at the five fixed points. The protecting tube of the thermo-couple (4) was furnished by the necessary equipments so that it can also be used as a suction pyrometer for the measurements of gas-temperature.

The furnace was provided with three sampling holes (3)', (4) and (5)', from which the gases in the fuel bed were drawn out every ten or twenty minutes according to the state of combustion and analysed by the method of Hempel.

The conditions of the experiments are tabulated in Table 1.
Table 1.

| Experiment <br> No. | Fixed carbon <br> in briqutte <br> $\%$ | Apparent <br> density of <br> briquette | Rate of air <br> flow, <br> $\mathrm{kg} / \mathrm{m}^{2} . \mathrm{hr}$ | Volume of <br> voids in bed |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 66.0 | 1.34 | 190 | 0.43 |
| 2 | 60.0 | 1.32 | 190 | 0.43 |
| 3 | 52.0 | 1.35 | 190 | 0.43 |
| 4 | 60.0 | 1.32 | 313 | 0.43 |
| 5 | 60.0 | 1.32 | 246 | 0.43 |

The series of curves in Fig. 3, (a), (b) and (c) illustrate typically the general feature of the progress of temperature change when the burning zone passes

(a) Exp. No. 2.

(b) Exp. No. 4.

(c) Exp. No. 5.

Fig. 3. Experimental Temperature-time Curves.
through successively the points from (2) to (5). Also a curve for the gas temperature at the point (4) is drawn in dotted line, which may serve as a reference to estimating the difference of solid and gas-temperature. The relative positions of the curves will indicate the rate of "burning up" of briquettes and also thickness of the burning zone.

The results of gas analysis are represented graphically in Fig. 4, which disclose the change of CO -,


Radial Distance from Furnace Axis, cm.

Fig 5. Temperature Distribution at the Plane (5).



Fig. 4. Composition of Gases in the Coke Bed.
$\mathrm{CO}_{2-}$ and $\mathrm{O}_{2}$ content of the gases in the fuel bed during the burning up of briquettes.

The radial temperature distribution at the plane (5) is given in Fig. 5, in which each curve represents the distribution at the time $t(\mathrm{hr})$ indicated on the curve.

The temperature range of the reducing and oxidizing zones of the fuel bed can be estimated by comparing the curves in Figs. 3 and 4.

It can be pointed out that the oxidizing reaction
begins to predominate at the point where the temperature is still a little lower than the maximum, and ends at the point of about $500^{\circ} \mathrm{C}$ where the oxygen content amounts to the normal value of $21 \%$. On the other hand, in the present cases, the reducing reaction seems to begin at about $950^{\circ} \mathrm{C}$ corresponding to the point of maximum of CO content. These relations have already been given in Fig. 1 example.

The maximum temperature and the time necessary for completing the reaction at serveral positions in the furnace under different experimental conditions are summarized in Table 2.

Table 2.

| $\begin{gathered} \text { Exp. } \\ \text { No. } \end{gathered}$ | Maximum temperature in ${ }^{\circ} \mathrm{C}$ |  |  | Time necessary for completing the reaction in min |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Oxidizing | reaction | Reduding | reaction |
|  | Measured at the position | (4) |  | $(4)^{\prime}$ | $(5)^{\prime}$ | $(4)^{\prime}$ | $(5)^{\prime}$ |
| 1 | 1230 | 1250 | 1320 | 190 | 150 | 140 | 150 |
| 2 | 1290 | 1320 | 1350 | 170 | 170 | 140 | 140 |
| 3 | 1220 | 1320 | 1360 | 150 | 160 | 140 | 130 |
| 4 | 1390 | 1460 | 1480 | 130 | 120 | 90 | 80 |
| 5 | 1430 | 1450 | 1350 | 160 | 160 | 110 | 100 |

The increase of air blast accelerates the burning velocity and consequently the larger number of briquettes enter simultaneously into the reaction which results in the building up of a thicker reaction zone. The richer in carbon content are the briquettes, the longer will be the time necessary for their burning out, which fact means that the velocity of burning up of the reaction zone is slow.

From the curves in Fig. 3, it can be found out that the temperature of gas is about $50^{\circ} \mathrm{C}$ higher than that of solid when the green briquettes enter into the reaction. This difference broadens, gradually up to the maximum value of $150^{\circ} \mathrm{C}$, and then narrow until finally the curves representing gas- and solid-temperatures cut each other at some point before the solid-temperature reaches the maximum and then both curves run almost pararell until the briquettes completely burn out.

The radial temperature gradient is confirmed to be comparatively small as long as the reaction between solid and gas phases goes on.

## 3. Comparison of Calculated Values with Experimental Data

As all measurements have been carried out with the furnace of the fixed fuel bed having no continous charge and discharge system, the steady state required
in our simplified calculations can not be obtained, and consequently both results, the experimental and theoretical, are not to be compared as they stand. However, if we assume the charge and discharge are going on constantly with the velocity equal to the mean velocity $u(\mathrm{~cm} / \mathrm{sec})$ of the burning up of briquettes calculated from the experimental data, it would be possible to realize the steady state in which all other figures are practically the same as in the present experiments.

The mean velocities, which act as intermediate between steady and non-steady states, are as follows:

| Experiment No. | 1 | 2 | 3 | 4 | 5 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $u \times 10^{3}(\mathrm{~cm} / \mathrm{sec})$ | 1.14 | 1.35 | 1.6 | 3.0 | 2.2 |

The coefficient of diffusion $D$ and velocity constnat $k$ of chemical reaction which appear in the equations (18), (19) and (20) can be expressed as $D=D_{0}\left(\frac{T}{273}\right)^{n}, k=\frac{c^{\prime}}{\sqrt{T}} e^{-\frac{B}{R T}}$, respectively, in which $D$ and $D_{0}$ are the coefficient of interdiffusion at $T^{\circ}$ and $273^{\circ} \mathrm{K}, n$ is a constant having the values between $1.5-2.0, R$ the gas constant, $E$ the activation energy, and $c^{\prime}$ a constant.

The numerical values of $D_{0}, E, c^{\prime}$ and $n$ for the reactions $\mathrm{C}+\mathrm{O}_{2} \rightarrow \mathrm{CO}, \mathrm{C}+\mathrm{CO}_{2}$ $\rightarrow 2 \mathrm{CO}$ and $2 \mathrm{CO}+\mathrm{O}_{2} \rightarrow 2 \mathrm{CO}_{2}$ are given in Table 3.

Table 3.

|  | Reactions |  |  |
| :--- | :---: | :---: | :---: |
|  | $\mathrm{C}+\mathrm{O}_{2} \rightarrow \mathrm{CO}_{2}$ | $\mathrm{C}+\mathrm{CO}_{2 \rightarrow 2} \rightarrow 2 \mathrm{CO}$ | $2 \mathrm{CO}+\mathrm{O}_{2} \rightarrow 2 \mathrm{CO}_{2}$ |
| $D_{0}\left(\mathrm{~cm}^{2} / \mathrm{sec}\right)$ | 0.168 | 0.137 | - |
| $E(\mathrm{Kcal})$ | 20 | 53 | 20 |
| $c^{\prime}$ | $1.04 \times 106$ | $8.9 \times 105$ | 10 |
| $n$ | 1.6 | 1.5 | - |

Obviously, $u=\frac{d x}{d t}$. Therefore, by means of the relations given by (18), (19), and (20), we can calculate the values of the partial pressure of gases in the fuel bed at any moment during the experiment and thus compare them with the experimental data. In our calculations, we used the following numerical values: coefficient of diffusional resistance $\alpha$, thickness of gas film $F$, cinder loss $L$, partial pressure of carbon dioxide $p_{2}$ at the end of the reducing zone, modified Reynolds number, and mass-transfer factor $j_{a}$ as shown in Table 4.

Table 4.

| Exp. No. | $\alpha$ | $F(\mathrm{~cm})$ | $L(\%)$ | $p_{2}$ | $R_{e}$ | $j_{a}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 1.73 | 0.24 | 0.8 | 0.04 | 26.4 | 0.34 |
| 4 | 1.73 | 0.20 | 0.8 | 0.06 | 43.0 | 0.25 |

Needless to say, both $D$ and $k$ are the functions of temperature. The numerical integrations, however, can be carried out without difficults, when we assume that the temperature was constant during the short period of $\Delta t=2000 \mathrm{sec}$. and would be the same as the mean value of the corresponding temperatures of solid phase obtained by direct measurements (Fig. 3).

In Fig. 6, the values calculated by the method of Runge-Kutta are compared with the results of the measurement. It will be seen that the agreement of both values is good enough. Hence, the basic equations (18), (19), and (20) can be regarded as expressing cor-


Fig. 6. Composition of Gas in the Combustion of Coke. rectly the physical and chemical changes occuring in the fuel bed, although a great drawback in practical use is that the temperature at every point be known beforehand. However, the materials contained in this paper will throw light on the analysis of the complicated phenomena which must be taken into account for discussing these reactions.

1 The problems of temperature distributions in the fuel bed will be treated in our papers which will appear in succession.

## Summary

The results of this investigation may be summarized as follows:
(1) A quantitative formulation of the reaction rate of coke upon air in the packed bed has been presented under the conditions, under which both diffusion and chemical reaction may affect the reaction rate.
(2) By using the theoretical formulas for the rate of reactions, $\mathrm{C}+\mathrm{O}_{2} \rightarrow \mathrm{CO}_{2}$ and $\mathrm{CO}_{2}+\mathrm{C} \rightarrow 2 \mathrm{CO}$, we could calculate the percentages of $\mathrm{CO}_{2}, \mathrm{CO}$ and $\mathrm{O}_{2}$ in the packed bed. Comparing the calculated values with the experimental results, we found the agreement satisfactory.

## Acknowledgment

The authors are indebted to the financial support of the Ministry of Eduction granted in support of the present investigation, and also to T. Morishita for his assistance in these experiments.

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