On the Solution of Ventilation Networks

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Introduction

Large mines in Japan, perhaps in other countries, too, frequently have such complicated ventilation networks that we feel great difficulty to solve them.

The formula giving the resultant resistance of splitted mine air currents is applicable only to the case in which all the currents splitted at a point meet at a certain other point. Some investigations were carried out concerning a possible solution of such networks[1], but none has succeeded in finding convenient methods practicable.

We studied this problem and obtained a method of solution which enables the determination of air quantity and pressure drop in each air way with sufficient accuracy and without too much troublesome calculation. we should like to report, in this article, the outline of the method to solve the ventilation networks.

Assumptions and Fundamental Relations

In this solution, it is assumed that among specific resistance R, pressure drop h and air quantity Q, there exists a relation,

$$h=RQ^2, \qquad (1)$$

(in fact it seems to hold considerably well). In ventilation networks, it is admitted that the algebraic sum of the air quantities in the currents which meet at a point is zero, and that the sum of the pressure drops along any closed circuit is equal to zero or to the applied pressure on the circuit, such as the fan pressure or the natural venitilation pressure. These facts resemble the electric circuits.

Q can take any possitive or negative value according to the direction of the air flow. In order to express that the pressure drops take place in the direction of the air flow, equation (1) may be written as

$$h = RQ|Q|. \quad (1')$$

Let our problem be to find air quantity or pressure drop for any air current in a given network, with the knowledge of specific resistance of each air way which constitutes the network. Each suffix placed behind R, Q and h represents the airway. The units for R, Q and h used in the examples shown in Figs. 1 and 2 are not specific. They can be any units that hold the relation,

(unit of h)=(unit of R)×(unit of Q)^2

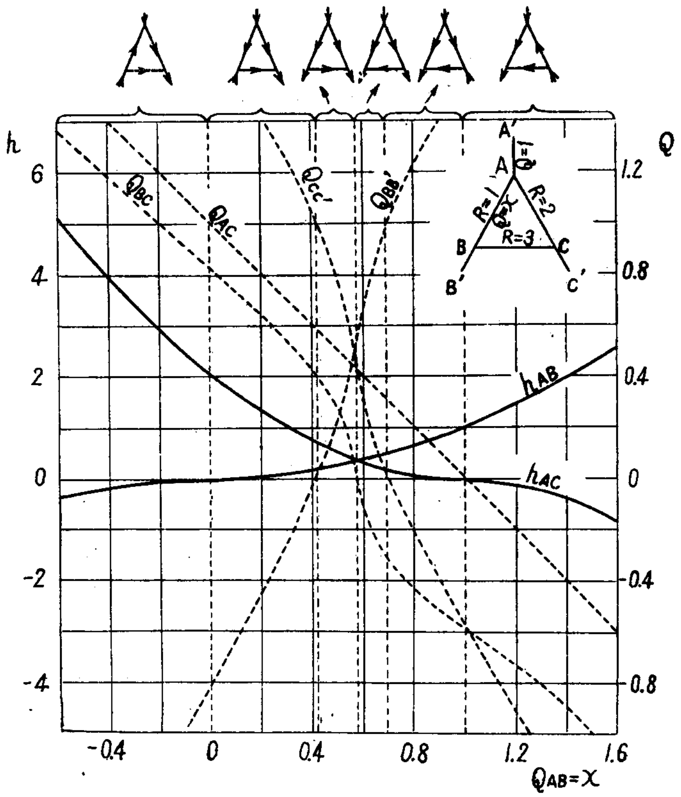

Fig. 1. Characteristics of a Triangular Circuit.

Triangular Circuit

If there is a triangular circuit ABC in a given network, as shown in Fig. 1, assuming $Q_{A'A}$ is a unit air quantity, Q_{AB}an unknown x, we get

$$Q_{AC}=1-x \quad (2)$$

$$h_{AB}=R_{AB}\,x|x|, \qquad h_{AC}=R_{AC}(1-x)|1-x| \quad (3)$$

$$h_{BC}=h_{AC}-h_{AB} \quad (4)$$

or

$$h_{BC}=R_{BC}Q_{BC}|Q_{BC}|,$$

$$\therefore \quad R_{BC}Q_{BC}|Q_{BC}|=R_{AC}(1-x)|1-x|-R_{AB}x|x| \quad (5)$$

$$Q_{BB'}=x-Q_{BC}, \qquad Q_{CC'}=1-x+Q_{BC}. \quad (6)$$

Therefore, all the values of Q and h are represented by x. The value of x can take any positive or negative value. All the values of Q and h vary with x, as shown in Fig. 1.

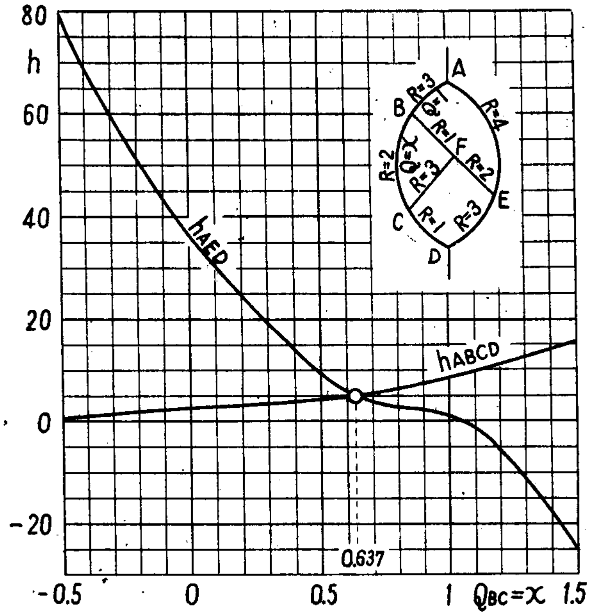

Fig. 2. An Example of Ventilation Network Solved with a Single Unknown.

Solution of Simple Networks

We will now explain the methods to solve simple networks treated with one unknown on an example shown in Fig. 2.

Assuming Q_{AB} is a unit air quantity, and Q_{BC} is an unknown, x, the values of Q and h in the air ways AB, BC, BF, FC, CD and FE can be represented by x as described above. Accordingly, Q and h of ED and AE are also represented by x. The relation of h_{ABCD} or h_{AED} to x is shown in the figure. Because of the equality of the both pressure drops, we can obtain $x=0.637$ from the abscissa of intersection of the two curves. In this way this problem is solved.

Solution of Complicated Networks

If it is impossible to represent all the values of Q and h by a single unknown, a necessary and sufficient number of other unknowns may be selected. Then the sums of pressure drops along as many closed air circuits as the number of unknowns are equated to zero or to the applied pressure on the circuits, such as the fan pressure or the natural ventilation pressure. The selection of unknown must be such that simultaneous equations composed of the same number of equations as unknowns are obtainable. Then the equations are solved either graphically or by trial and error method or by the application of Taylor's expansion. The details of the last methods are explained below.

In the case of two unknowns, x and y, assume that x_1 and y_1 are the first estimation in trial and error method, that the differences between the estimated values x_1, y_1 and the true values x, y are Δx and Δy respectively, namely $x=x_1+\Delta x$, $y=y_1+\Delta y$, and that the sums of the pressure drops along two closed circuits are $F_1(x, y)$, $F_2(x, y)$, which are independent functions to each other,

then
$$\left.\begin{aligned} F_1(x,y) &\doteqdot F_1(x_1,y_1)+\left(\frac{\partial F_1}{\partial x}\right)_{\substack{x=x_1\\ y=y_1}}\Delta x+\left(\frac{\partial F_1}{\partial y}\right)_{\substack{x=x_1\\ y=y_1}}\Delta y \\ F_2(x,y) &\doteqdot F_2(x_1,y_1)+\left(\frac{\partial F_2}{\partial x}\right)_{\substack{x=x_1\\ y=y_1}}\Delta x+\left(\frac{\partial F_2}{\partial y}\right)_{\substack{x=x_1\\ y=y_1}}\Delta y \end{aligned}\right\}. \qquad (7)$$

But as $F_1(x,y)=0, \quad F_2(x, y)=0,$

we can put approximately
$$\left.\begin{aligned} F_1(x_1,y_1)+\left(\frac{\partial F_1}{\partial x}\right)_{\substack{x=x_1\\ y=y_1}}\Delta x+\left(\frac{\partial F_1}{\partial y}\right)_{\substack{x=x_1\\ y=y_1}}\Delta y=0 \\ F_2(x_1,y_1)+\left(\frac{\partial F_2}{\partial x}\right)_{\substack{x=x_1\\ y=y_1}}\Delta x+\left(\frac{\partial F_2}{\partial y}\right)_{\substack{x=x_1\\ y=y_1}}\Delta y=0 \end{aligned}\right\}. \qquad (8)$$

As the partial differential coefficients in the equations (8) are known functions of x and y, as well as $F_1(x,y)$ and $F_2(x,y)$, the values of $\left(\frac{\partial F_1}{\partial x}\right)_{\substack{x=x_1\\ y=y_1}}$, $\left(\frac{\partial F_1}{\partial y}\right)_{\substack{x=x_1\\ y=y_1}}$, $\left(\frac{\partial F_2}{\partial x}\right)_{\substack{x=x_1\\ y=y_1}}$, and $\left(\frac{\partial F_2}{\partial y}\right)_{\substack{x=x_1\\ y=y_1}}$ can be calculated. By solving them, Δx and Δy, accordingly, x and y are roughly determined. By presuming these values are the second estimation x_2, y_2, and by repeating the similar process, more accurate values of x and y are obtainable. This process is repeated untill sufficiently accurate results are reached.

The selection of air quantities which are taken as unknowns has considerable influence upon the calculation. To simplify the calculation as much as possible, it is preferable to follow the advice described below.

(1) If a triangular circuit is found in a given network, the unit and the unknown air quantity are selected in those of air currents which meet at a vertex of the triangle.

(2) If the directions of the air currents all or some of them, are known, the process of calculation is ceased when it is found that in any air way the air flows in the reversed direction. The calculation must be carried on with other trial values.

(3) If the directions of air currents at a certain junction are known, and if it is necessary to select an unknown air quantity in that of the air current, it is preferable to select the unknown in the quantity of one of the splits, the air quantity of the main airway at the junction being the unit.

Fig. 3 shows two examples of networks solved with two unknowns.
The air quantities assumed as a unit or unknowns in the calculation are shown in the figure. It took about twenty hours to accomplish the calculation of the

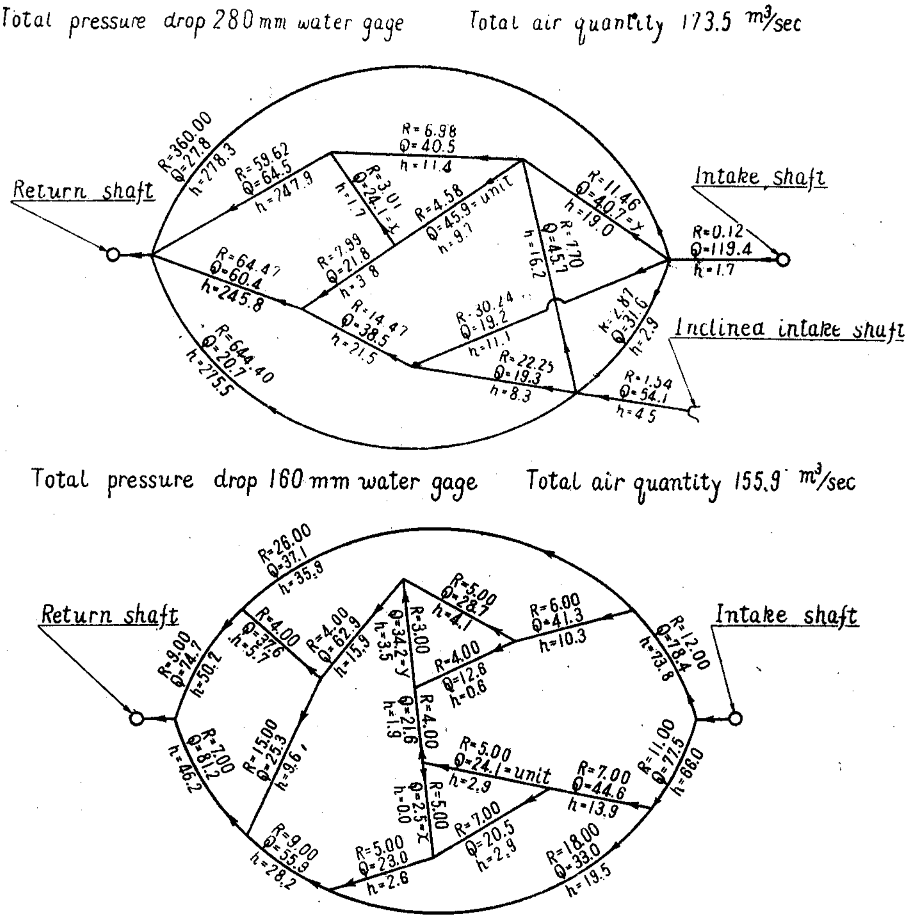

Fig. 3. Examples of Ventilation Networks Solved with Two Unknowns.
R: Specific resistance (murgue), Q: Air quantity (m^3/sec),
h: Pressure drop (mm water gage).

both problems. The solution of networks containing natural ventilation or auxiliary fans will be described in the second report.

Practical Use

The solution described above has been employed since last summer in the investigation on the improvement of ventilation of Sakito Coal Mine, Miike Coal Mine, Kyushu, and Ooyubari Coal Mine, Hokkaido, Japan, and so on.

References

1) H. Czeczott: Berechnung der Grubenbewetterrung, Zeitschrift des Oberschlesischen Berg-und Hüttenmännischen Vereins, 1931.

Y. Kumazawa: Journal of the Mininng Institute of Hokkaido (Hokkaido Kozangakkai-Shi), Vol. 1, No. 5, May, 1942 p. 203.