On the Yielding of Twist Steel Bar

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Synopsis

The yielding of steel proceeds in successive appearance of flow layers. Having been taken account of it, the yielding phenomena of bending steel bar has been fully explained by the authors. In the present paper the yielding phenomena of twist steel bar is discussed as an another example of yielding of steel under uneven distribution of stress. In the first part are exhibited the results of measuring the stress existing in a yielding bar subjected to pure torsion. It leads to the conclusion that the twisted bar yields by the same mechanism as that of the yielding of bent steel beam. Then the problems of elasto-plastic torsion may generally be treated analytically in the same fashion as in the case of bending.

I. Introduction

On bending a mild steel beam with rectangular cross-section, the moment -deflection curve is nearly straight until the applied moment increases as high as 1.5 times of M_s, where M_s denotes the moment at which the stress of outer-most fiber reaches the equivalent yielding stress σ_s in tension test. It seems the yield stress of bent steel bar rises as high as 1.5 times of σ_s owing to the unevenness of stress distribution. On measuring the surface stress of yielding beam by means of X-ray, however, the stress exceeding σ_s was not found. As a matter of fact, the steel is distinguished by such a behaviour of yielding as having the horizontal part in stress-strain diagram of tension or compression test and as deforming not uniformly along its length but sporadic as is noticed by the appearance of Lüder's lines. Taking account of these facts, the authors discussed the yielding of bending steel beam and induced formulae applicable to elasto-plastic bending of any kinds of steel with any forms of cross-section.[1] This theory will serve to give the full explanation of the above mentioned views apparently contradictory in simple show.

As an another case of yielding of steel under uneven distribution of stress, a number of results of torsion test are found.[2)] Twisting a mild steel bar of circular cross-section, the relation of the moment M to the specific angle θ of torsion is nearly proportional up to as high magnitude of moment as 4/3 times of M_s, where M_s means the moment by which the shear stress of the outer-most fiber reaches τ_s. τ_s is the yielding shear stress corresponding to the yielding stress σ_s and is chosen one half of the latter in the present research. In the analogous way as in the case of bending, the yielding stress of twisted mild steel bar might be taken to rise as high as 4/3 times of τ_s. Although the surface stresses of bent beam have been investigated experimentally by a few researchers, no experimental work is found which has been performed on the stress existing in a yielding steel bar under torsion. From this point of view, the authors measured the surface stress of mild steel bars subjected to pure torsion of various stage of moment by means of X-ray. Besides the distribution of stress in the cross-section of yielding bar were investigated utilizing the residual stress measurement by corrosion method. In the first half of the present paper the results of the experiments are exhibited. On surveying the results, such a conclusion will be given that the twisted steel bar yields by the same mechanism as that of bent beam. In the latter half, the analogous mode of analytical treatment of the yielding as was carried out in case of bending is applied for the case of torsion of round steel bar. The $M-\theta$ relations are determined by calculation with regard to various kinds of steel and its comparison with the experimental results will be exhibited.

II. Experiments

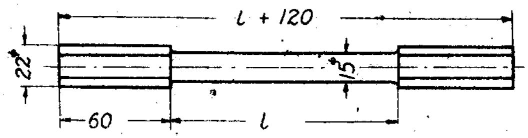
i) Specimens:

The materials of specimens used are illustrated in Table 1, being the same as those used for the study of bending. After having undergone machining into

Table 1

	C %	Mn %	Si %	P %	S %	heat treatment
0.1% C steel annealed	0.10	0.62	0.16	0.040	0.044	annealed at 900°C for one hour
0.6% C steel annealed	0.44	0.60	0.20	0.027	0.020	annealed at 780°C for one hour
0.6% C steel annealed	0.65	0.62	0.20	0.031	0.020	annealed at 750°C for one hour
0.4% C steel quenched	0.44	0.60	0.20	0.027	0.020	770°C oil quench, 150°C temper

the form as is shown in Fig. 1, the specimens were heat-treated in vacuum furnace under the condition indicated in the last column of the table. The A type specimens of 0.1% C steel were used for the surface stress measurement by X-ray and the B type of the same material were for the observation of the distribution of residual stress utilizing the corrosion method. The $M-\theta$ relations were determined by experiment for each material using the C type specimens.



Type	l mm
A	100
B	250
C	400

Fig. 1

ii) The stress investigation by means of X-ray.

As to the X-ray stress measurement the detailed description is here spared.[3] On twisting specimens, the surface stresses were measured at several steps of the moment applied. At each step the moment was released and the residual stresses existing on the surface were investigated. The results obtained are exhibited in Fig. 2, where the ordinate τ_x denotes the measured value of stress and the abscissa τ^* the one calculated from the value of applied moment using the simple formula due to elasticity. The yielding shear stress τ_s of the material had been determined as 12 kg/mm^2. The figure shows that, as far as the applied moment is less than the magnitude

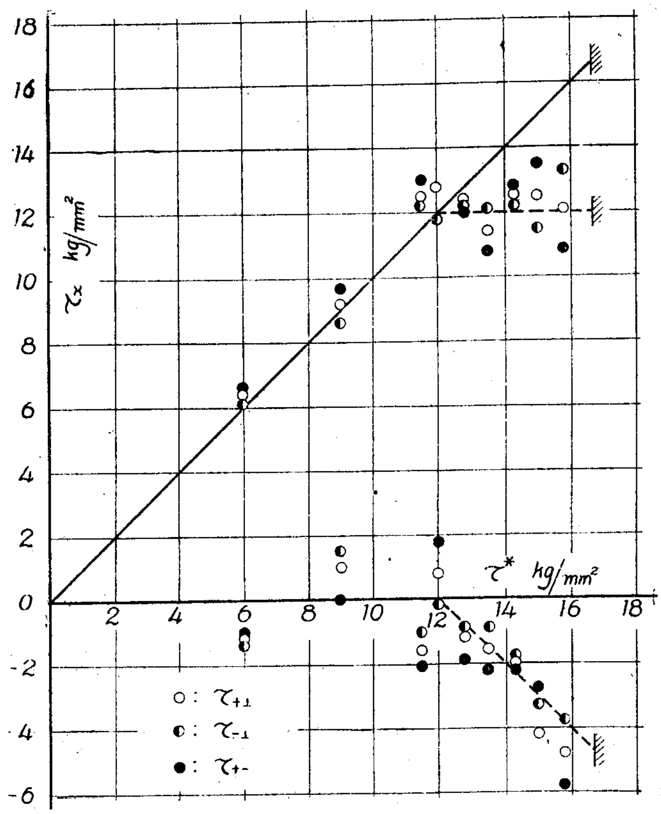


Fig. 2

of M_s, τ_x rises proportionally to τ^* and no residual stress is found when the torque applied is released. The applied moment exceeding M_s, the increase of τ_x is not proportionate to that of τ^* but scarecely rises beyond τ_s and, removing the applied moment, residual stress is found on the surface. The result of the X-ray investigation shows that the surface stress of twisted bar tends to behave likewise as that of bent steel beam.

iii) The stress investigation by the corrosion method.

In a bar subjected to a torsional moment exceeding M_s plastic deformation is supposed to have broken out, as above. The X-ray investigation, however, gives only the behaviour of superficial stress. For the purpose to observe the distribution of stress within the cross-section of twisted bar, the residual stress measurement by corrosion method was utilized. Having been subjected to a moment beyond M_s and then removed from the testing machine, the specimen T was set into the corrosion vessel V, filled with 30% solution of nitric acid as corrosive agent, as is shown in Fig. 3. A mirror M is attached to the upper end of the specimen. For the purpose to moderate the temperature of the solution, cooling water is circulated through the glass pipe G and the solution is stirred by the agitator P. Thus by removing laminar portion from the surface of the specimen, which has residual stress caused by severe twisting beyond elastic range, the mirror rotates around the axis of the specimen. Through the telescope the rotating angle of the mirror M is measured by reading the reflected image of the scale 2500 mm distant from the mirror. The specimen is put out of the vessel from time to time and the decrement of the diameter is measured. Thus the relation of the rotation angle of the mirror to the depth of corrosion is

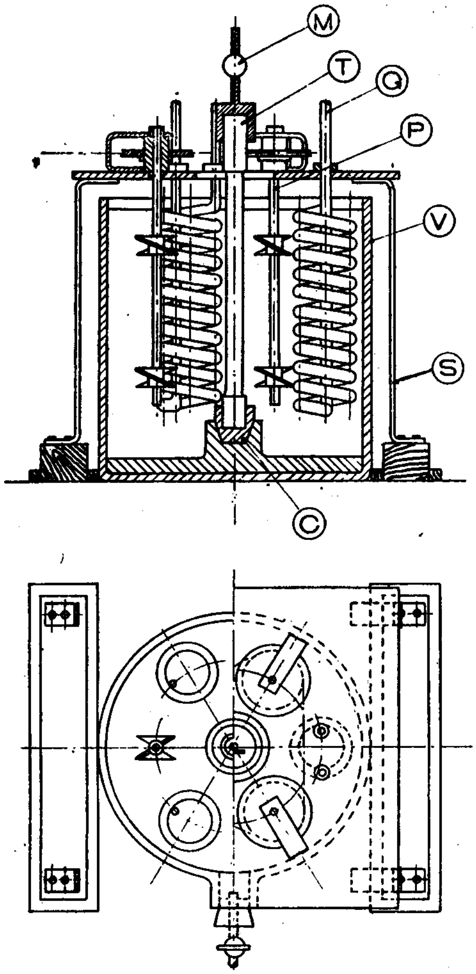


Fig. 3

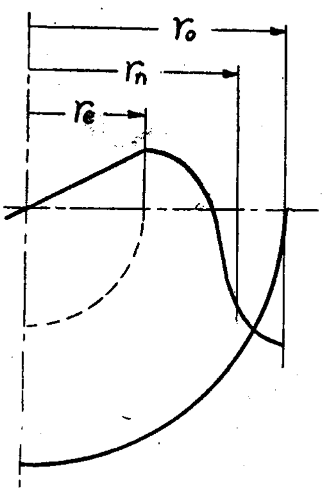


Fig. 4

obtained. Consider a bar of a diameter $2r_0$, in which there exists a residual stress as is shown in Fig. 4. Let $\Delta\theta$ be the rotating angle of mirror per unit length of bar, i.e. the specific angle of rotation, corresponding to the decrement $2(r_0-r_n)$ of diameter, the following relation is given from the condition of equilibrium of moment around the axis of the bar before corrosion.

$$-GI_n\Delta\theta=2\pi\int_{r_n}^{r_0}\tau(r)r^2dr\,, \qquad (1)$$

where

G : the shear modulus

I_n : the polar moment of inertia of the area after corrosion

$\tau(r)$: the residual stress distribution

Dividing the portion between radius r_0 and r_n into n layers and denoting the radius of each dividing circles as $r_1, r_2, r_3, \ldots\ldots, r_{n-1}$ in the order of length, the continuous distribution of residual stress of the portion is represented approximately by a stepped one, of which each step corresponds to the mean stress of the thin layer divided. Let the change of the specific angle of rotation corresponding to the removal of the stepwise decreasing stress $\tau_1, \tau_2, \tau_3, \ldots\ldots, \tau_n$, be $\Delta\theta_1$, $\Delta\theta_2$, $\Delta\theta_3$, $\ldots\ldots$, $\Delta\theta_n$, and we get

$$-GI_n\sum_{n=1}^{n}\Delta\theta_n=\frac{2\pi}{3}\sum_{n=1}^{n}\tau_n(r_{n-1}^3-r_n^3) \qquad (2)$$

from equation (1). Obtaining $\Delta\theta_n$ corresponding to r_{n-1} as well as r_n by experiment and utilizing (2), $\tau_1, \tau_2, \tau_3, \ldots\ldots, \tau_n$ are determined in succession as follows:

$$\left.\begin{aligned}
\tau_1&=\frac{3}{4}G\frac{-\Delta\theta_1 r_1^4}{r_0^3-r_1^3}\\
\tau_2&=\frac{3}{4}G\frac{\Delta\theta_1 r_1^4-(\Delta\theta_1+\Delta\theta_2)r_2^4}{r_1^3-r_2^3}\\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots,\\
&\text{in general,}\\
\tau_n&=\frac{3}{4}G\frac{r_{n-1}^4\sum_{n=1}^{n-1}\Delta\theta_n-r_n^4\sum_{n=1}^{n}\Delta\theta_n}{r_{n-1}^3-r_n^3}
\end{aligned}\right\}. \qquad (3)$$

Denoting the distance between the mirror M and the scale as a, length of corroded portion of bar as l and the change of reading of the scale as $\Delta\Theta$, the mean stresses of any section are calculated by (4).

$$\tau_n = A \frac{\left(\frac{d_{n-1}}{d_0}\right)^4 \sum_{n=1}^{n-1} \Delta\Theta_n - \left(\frac{d_n}{d_0}\right)^4 \sum_{n=1}^{n} \Delta\Theta_n}{\left(\frac{d_{n-1}}{d_0}\right)^3 - \left(\frac{d_n}{d_0}\right)^3}, \tag{4}$$

where

$$A = \frac{3Gd_0}{16al} \text{ and } d_n = 2r_n, \quad (n=0,\ 1.2\ldots n).$$

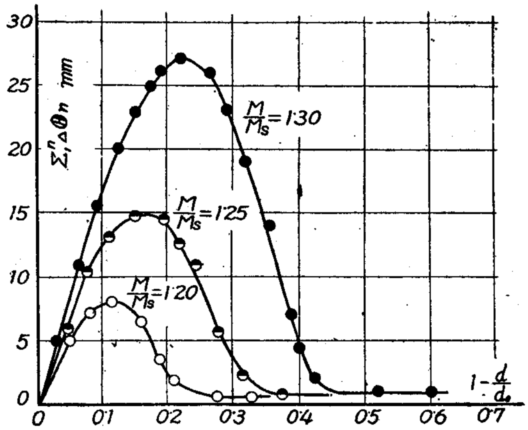


Fig. 5

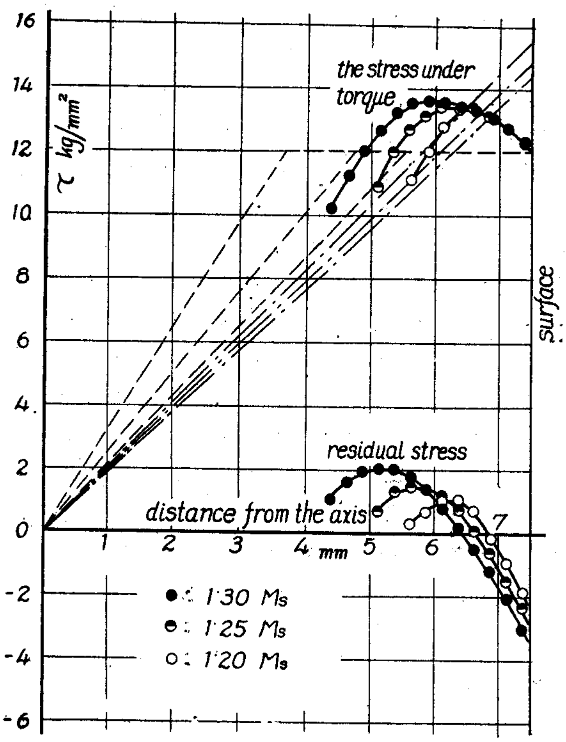


Fig. 6

Fig. 5 demonstrates the experimental relation of the change of reading to the decrement of diameter on the specimens subjected to the moment of 1.20 M_s, 1.25 M_s and 1.30 M_s. In the inner portion where the mirror no longer rotated with proceeding corrosion, the distribution of residual stress is linear. Fig. 6 shows the obtained distribution of residual stress, led from Fig. 5 utilizing the equation (4). Assuming the material to behave elastically during the course of unloading, the stress distribution under twisted condition is obtained using the residual stress distribution. Thus determined distributions of stress are exhibited in the same figure. The stress in the inner portion tends to be measured slightly higher because of the inevitable defect of the corrosion method. The chain and the dotted lines in the figure mean the assumed distribution of stress of elastic and perfectly plastic state respectively, derived from the applied moment. The measured distributions are different from both of assumed ones but, near the surface, the stress is rather lower, yielding to near τ_s on the surface, whereas in the inner portion it increases and then decreases again. Here, the fact attracts our attention that the experimentally determined distribution of stress in the section of twisted steel bar is surprisingly similar to that of bent steel beam.

Surveying the results of experiments we could introduce the conclusion that the twisted steel bar yields by the same mechanism as that of bent steel beam.

III. Yielding mechanism of steel bar under torsion

Standing on the view point that the yielding of steel bar under torsion is the same phenomenon as the one under bending, the former may easily be explained by applying the theory concerning the yielding of steel under bending moment, which has been developed by the authors. As far as the applied moment is below the magnitude of M_s, the bar deforms elastically. When the moment just exceeds M_s local plastic deformation occurs here and there near the surface. With increase of moment, the plastic deformation prevails lengthwise as well as thicknesswise. In a bar subjected to a moment exceeding M_s, therefore, two portions are considered to coexist, the one elastic and the another plastic. In the discussion of the yielding of bent steel beam, at the occurence of local slip the strain of slipped portion near the surface was considered to increase instantaniously from ε_e to ε'', where ε_e means the elastic strain corresponsing to the yield stress σ_s and ε'' the value of total strain at the end point of horizontal line in the stress-strain diagram of tension test. In the present case, the same may be taken into account, where we consider the shearing strains γ_e and γ'' in place of ε_e and ε'' respectively. That is to say, near the surface of severely twisted bar the excessive strain of plastic portion alleviates the strain of elastic portion and so the stress near the surface of the latter decreases and yields in average to the value near τ_s. On the contrary, the stress of the inner portion is obliged to rise, as was shown in Fig. 6, so as to sustain the applied moment. Such figure of the yielding of steel under torsion may be treated analytically in the same manner as in the explanation of the yielding phenomena of steel under bending.

Let us consider a round steel bar, subjected to a moment beyond M_s, whose unit length on the surface is composed of φ of plastic portion and $(1-\varphi)$ of elastic one. Putting

$$\varphi=\frac{k}{1+k}\,, \tag{5}$$

k means the ratio of the length of local plastic portion to that of elastic one.

Similarly as the case of bending, k is represented by

$$k=\frac{\gamma^*-\gamma_e}{\gamma''-\gamma}\,, \tag{6}$$

where γ^* and γ denote the strain given by assuming the stress distribution to be elastic and perfectly plastic respectively, required to sustain the same moment. In formula (6) the values of γ_e as well as γ'' are inherent to materials and can be determined by experiments.

$$\gamma_e=\frac{\tau_s}{G}\,,\ \text{where } G\text{: the shear modulus,} \tag{7}$$

$$\gamma''=\varepsilon''\,. \tag{8}$$

γ^* is given simply by assuming the linear stress distribution sustaining M as follows:

$$\gamma^*=\frac{16}{\pi d^3}\frac{M}{G}\,,\ \text{where } d\text{: the diameter,} \tag{9}$$

and γ is given from the perfectly plastic state assumed as is shown in Fig. 7.

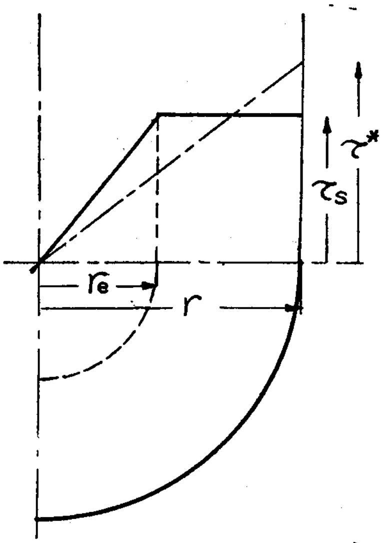


Fig. 7

$$\gamma=\gamma_e\frac{r}{r_e}$$

$$M=\frac{3}{4}\pi d^3\tau_s\left[4-\left(\frac{r_e}{r}\right)^3\right]\,, \tag{10}$$

where r_e denotes the radius of the elastic region in the assumed state of perfect plasticity. Substituting (7)~(10) for (6), we have

$$k=\frac{\dfrac{M}{M_s}-1}{\dfrac{\gamma''}{\gamma_e}-\dfrac{1}{\sqrt[3]{4-3\dfrac{M}{M_s}}}}\,, \tag{11}$$

For the moment equal to M_s, we have $k=0$ from (11) or $\varphi=0$ from (5) and it means the elastic state. For the materials as $\gamma''\gg\gamma_e$, e.g. annealed mild steel, the value of φ yields to unity at the moment nearly equal to $4/3\ M_s$, when the plastic deformation spreads all over the length. The condition that the plastic portion prevails through the whole length is given by putting the denominators of (11) to be zero. Denoting the moment under the condition as M_y, we have

$$\frac{M_y}{M_s}=\frac{4}{3}\left[1-\frac{1}{4}\left(\frac{\gamma_e}{\gamma''}\right)^3\right]\,, \tag{12}$$

Fig. 8 is the theoretical relation of M_y/M_s to γ_e/γ'', where M_y/M_s values

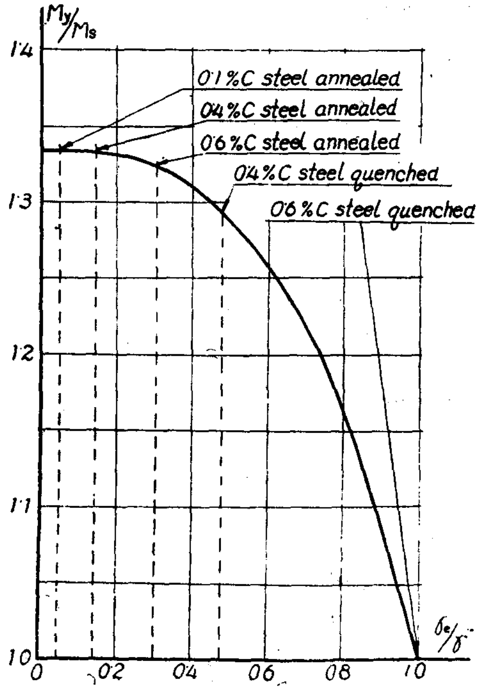


Fig. 8

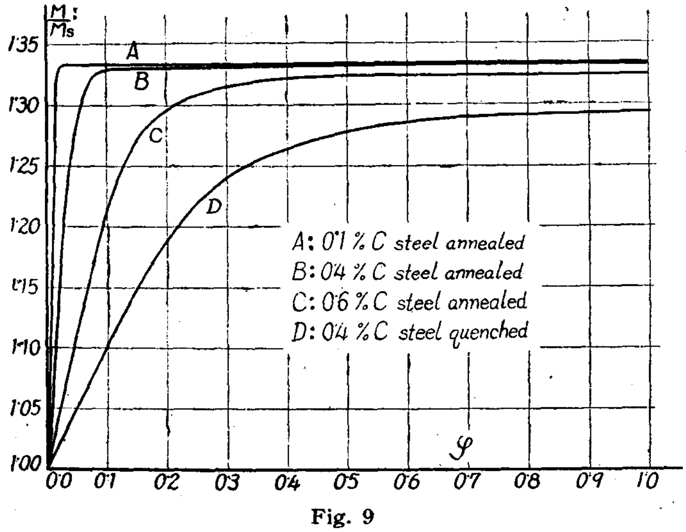


Fig. 9

of a few kinds of steel are remarked, induced from the experimental data of tension test.

Analytical view of the growth of the plastic portion with increase of moment is obtained using the formulae (5)~(11), provided the values of γ_e and γ'' are given. Fig. 9 shows the $\varphi - M/M_s$ relations for the specimens made of the materials indicated in Table 1, obtained by calculation using the values of γ_e and γ'' as are shown in Table 2 induced from the results of tension test. The feature of each diagram is quite similar to those shown at the bending of various kinds of steel beam. That is to say, as soon as the applied moment exceeds M_s, local plastic deformation appears and with subsequent increase of moment the plastic portion spreads lengthwise in different feature according to the kind of materials. The harder the material is, the more increases the ratio of plastic portion and vice versa.

Table 2

	γ_e %	γ'' %
0.1% C steel annealed	0.148	3.14
0.4% C steel annealed	0.179	1.31
0.6% C steel annealed	0.182	0.60
0.4% C steel quenched	0.337	0.70

IV. $M-\theta$ relation

While the applied moment is less than M_s, the $M-\theta$ relation is determined by the formula from the elasticity. It is reasonable to consider that in a bar subjected to a moment exceeding M_s plastic portion distributes uniformly at any portion along the axis. The resultant shear strain γ' of the extreme fiber containig φ of plastic portion is, therefore, uniform along the axis and represented by

$$\gamma' = \varphi\gamma'' + (1-\varphi)\gamma_e \,. \tag{13}$$

From the fundamental conception of the present problem, looking at the portion of infinitesimal length the radii of the bar, having been a straight line in free state, are curved. On the other hand, considering a portion of unit length, it is necessary that the initially straight radii are not distorted but kept to be straight. Hence, the specific angle θ of torsion is represented by

$$\theta = r\gamma' \,. \tag{14}$$

Combining (5), (6), (13) and (14) the $M-\theta$ relation is easily given. The full lines in Fig. 10 a, b and c illustrate the $M-\theta$ relations obtained by calculation for the materials as indicated in Table 1, using the values of γ_e and γ'' exhibited in Table 2. In the figures the small circles represent the experimental results. We see the curves agree very well with the experimental results. It proves the

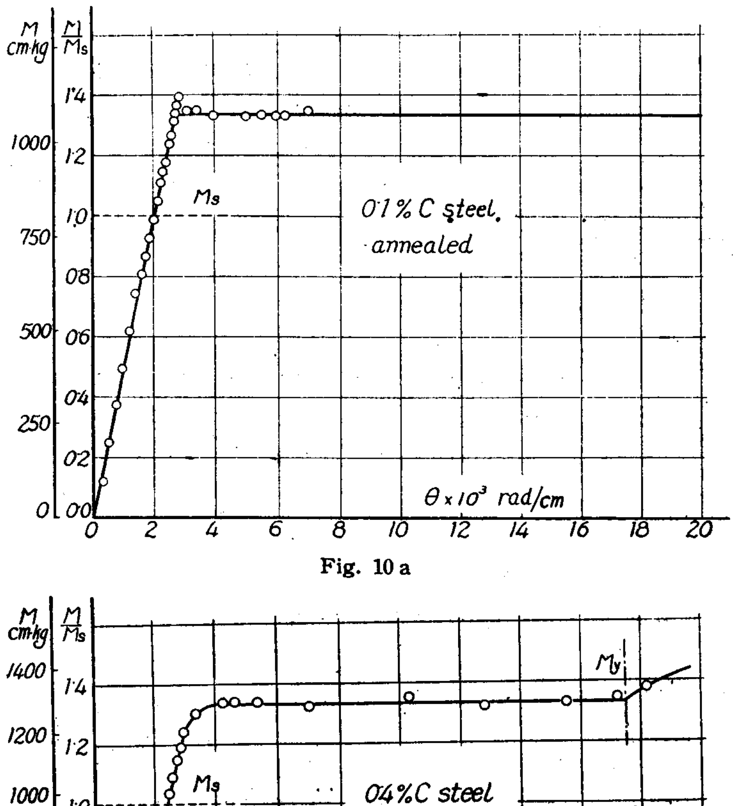


Fig. 10 a

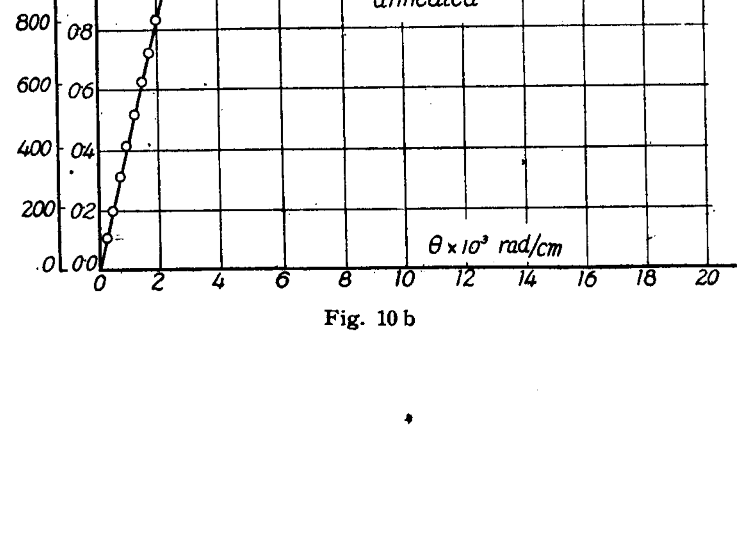


Fig. 10 b

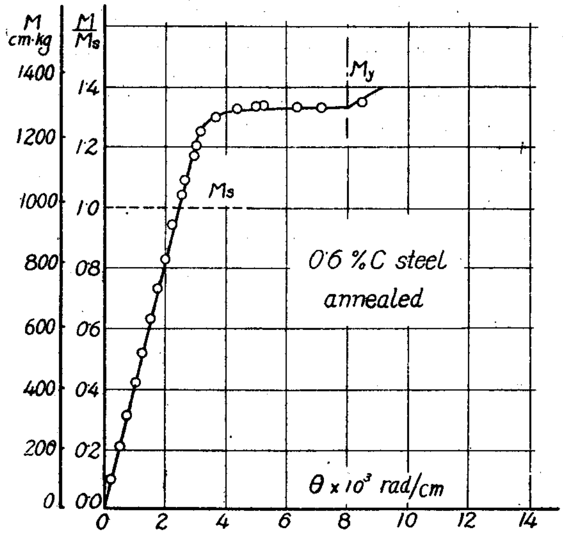


Fig. 10 c

validity of applying the authors idea concerning the yielding of steel, which has been developed in case of bending, to the present problem of elasto-plastic deformation under torsion.

V. Summary

In the present paper, the authors proved the mechanism of the yielding of twisted steel bar is quite similar to that of bending. Hence, it is natural the both yielding phenomena should be explained by the same analytical treatment. As a concluding remark, the authors intend to emphasize that the local yielding and the occurence of relaxation near the surface are the actual figure of the yielding phenomenon of steel and without taking them into consideration the yielding phenomena of steel under other conditions of uneven distribution of stress can not be explained thoroughly.

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