Heat Transfer on the Surface of a Flat Plate in the Forced Flow

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Synopsis

It is an important problem in the field of the heat exchange to make clear the surface heat transfer. Many studies have been performed regarding this problem. However, because heat transfer is influenced by numerous factors, there are many things remaining unknown. This research was carried out to make clear the effects of some of these factors on heat transfer on the surface of a flat plate in the forced flow. Especially, we wish to know the effects of the surface roughness systematically. As the first step, we experimented with the smooth flat plate which was heated or cooled while measurement was carried on. And by comparing the results with theoretical analyses, we found that our experimental method was reliable. As the next step, we experimented with the roughened plate and made clear the effects of surface roughness. The results of this experiment are: in a range of larger numbers than a certain Reynolds' number, there is formed a special turblent boundary layer which is influenced by the surface roughness, and in this range, the local heat transfer coefficient is independent of the position, and its value becomes the largest when the ratio of the height to pitch of the surface roughness is 0.055.

1. Introduction.

Surface heat transfer is the phenomenon of heat exchange between the solid body and the sorrounding fluid. In other words, it is the heat transmission through a boundary layer existing in contact with the solid surface. So all factors that have influence on the state of the boundary layer have the effect upon heat transfer. Accordingly, in order to research on heart transfer, we must make clear the effects of all factors systematically. Much efforts made in this field have been reported by many previous investigators. Especially, the case of the laminar boundary layer was made clear experimentally and theoretically. But in the case of the turbulent boundary layer, there are few reliable results which are made comparatively clear, namely results merely pertaining to a smooth flat plate and fluid flow which contains very little turbulence. With an actual apparatus, however, it is generally expected that the surface can not be smooth and the flowing fluid contains considerable turbulence. So we consider that it is necessary henceforce to research systematically in this field.

This research is performed mainly to make clear systematically the relationship between the surface roughness and heat transfer. We chose a flat plate for the solid body to be able to know easily and exactly the state of the boundary layer. As the first step, we experimented with a smooth flat plate, and on the other hand, we made theoretical analyses to examine the accuracy of our experiment. As the next step, we experimented with a plate having artificial roughness on its surface and we acertained the effects of surface roughness, by comparing the results with these in the case of the smooth plate..

2. Experimental Method.

The experiment was carried out by the following method. A flat plate was placed in the air flow which was produced by a blower, and the measurement was done while the plate was heated or cooled. We calculated the heat transfer coefficient from the measurement. Namely, as shown in Fig. 1, we take the distance x from

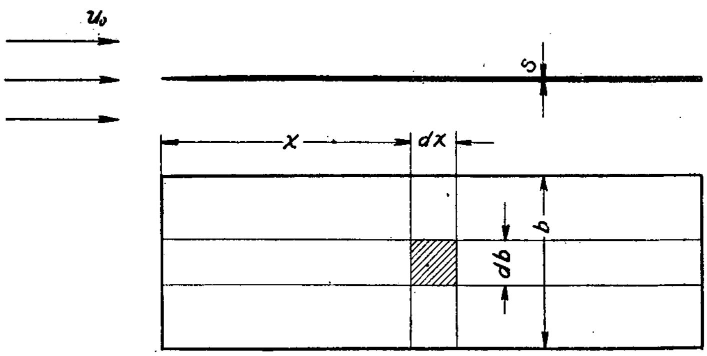

Fig. 1.

the front edge of the plate in the flowing direction, and consider the heat balance of the infinitesimal part like the hatching part whose length is dx and breadth db. But it is not necessary to consider the heat conduction in the breadth direction, because it was negligible in our experiment. So the next formula can be set:

$$\lambda s\cdot db\cdot\frac{\partial^2\theta}{\partial x^2}\cdot dt-2\alpha\cdot db\cdot dx\cdot\theta\cdot dt=s\cdot db\cdot dx\cdot\gamma c\frac{\partial\theta}{\partial t}dt \tag{1}$$

where θ: the temperature difference between the plate surface and the fluid,

α: the coefficient of local surface heat transfer,

λ, γ, c : thermal conductivity, specific weight and specific heat of the plate respectively,

t : time, and

s : thickness of the plate.

The first term of the left side of this equation represents the difference between heat quantity conducted in to this small part and heat quantity conducted out from this part in x direction. The second term is heat quantity which is transmitted to the sorrounding fluid by surface heat transfer. And the right side term is heat quantity necessary for raising the temperature of this small part. If we put

$$a = \lambda / c\gamma, \quad B = 2\alpha / c\gamma s, \tag{2}$$

eq. (1) can be rewritten as follows:

$$\frac{\partial \theta}{\partial t} = a \frac{\partial^2 \theta}{\partial x^2} - B\theta \tag{3}$$

In such an infinitesimally small part of dx, the temperature gradient can be regarded as linear in the x direction, so we set $\partial^2\theta / \partial x^2 = 0$, in comparison with other two term. Then.

$$\frac{d\theta}{dt} = -B\theta \tag{4}$$

The general solution of this equation is as follows:

$$\theta = Ae^{-Bt} \tag{5}$$

Namely, if we measure the time rate of the change of θ experimentally, we can obtain the value of B in eq. (5) and so the value of α by eq. (2).

General explanation of the main parts of our experimental apparatus is as follows: the flowing air is heated directly by the nichrom wire which is spreaded like a net inside a duct of a wind channel. The turbulence reduced by this nichrom wire is damped by a fine net which is settled in the down stream, and so the damping region behind the fine net. Moreover, to laminarize and accelerate, a throttle duct follows it. A measuring duct is settled following this throttle. Its inside sectional dimension is 100×100 mm. A plate for measuring is set in a groove of the measuring duct and fixed with gypsum. The plate is 400 mm in length, 110 mm in breadth and about 1 mm in thickness. To measure the surface temperature of the plate, copper-constantan thermocouples are fitted with at 10~13 points on this plate. As the first step, measurement was carried out on a flat smooth plate, the surface of which was finished in enough smoothness and flatness from engineering's stand-point. And we researched on the influence of the shape of fhe front edge, the difference produced by the heating and the cooling of the plate, and the difference between the case of setting the plate in vertical position and the case in horizontal position. As the next seep, we experimented with plates with rough surfaces, which have grooves or projections of a constant pitch. And by these experiments, we made clear the effects of the surface roughness.

The measurement of surface heat transfer coefficient is carried out by the method described hereafter. In the case of cooling the plate, the plate is previously heated by the hot wire up to a fixed temperature, then we cut off the heating current of hot wire, and immediately begin the measurement of temperature both of the plate and of the air flow outside the boundary layer, at regular intervals of time, with two millivolt meters. Thus, we find the time rate of the change of the temperature difference between them. In the case heating the plate, we send cold flow to the plate constantly and then the same measurement as above-mentioned is carried out, as soon as the hot wire circuit is switched on.

Aside from this experiment, the velocity distribution is measured in the non-heated flow, to find out the state of the boundary layer. The Pitot tube used for this measurement is made of a tube of 0.5 mm in outside diameter, and its tip is pressed to form an oval section.

3. Experimental Results and the Consideration thereof.

a. A Smooth Flat Plate.

Using a smooth flat plate, we made clear the influence of the fundamental factors: namely, the effect of the shape of the front edge, the effect of the setting position of the plate, and the difference between the cooling and the heating of the plate, etc.

(i) Heat Transfer in the Laminar Boundary Layer.

In the case of the sharp front edge and the isothermal flow, the boundary layer was kept in the laminar state till Reynolds' number got to about 2×10^5. An example of the velocity distribution in this case is shown in Fig. 2, where δ is the thickness of boundary layer. The full line shows Blasius' theoretical solution, and our results nearly agree with it. By using Nusselt's number Nu and Reynolds' number, Re, results of the measurement of heat transfer coefficient are shown in Fig. 3, where, $Nu=\alpha x/\lambda$, $Re=u_0x/\nu$, α is the coefficient of local heat transfer, λ & ν are the thermal conductivity and the coefficient of kinematic viscosity of air, and

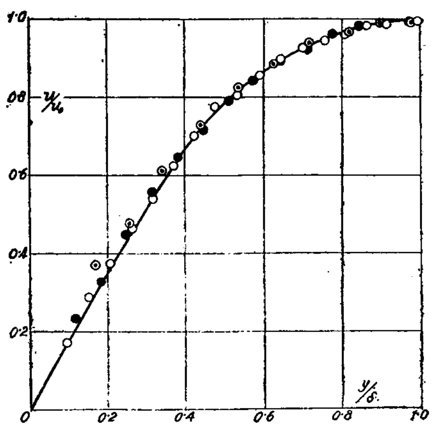

Fig. 2.

● $x=188$ mm $u_0 \doteqdot 7.0$ m/s
○ $x=332$ mm $u_0 \doteqdot 7.0$ m/s
⊙ $x=188$ mm $u_0 \doteqdot 14.0$ m/s

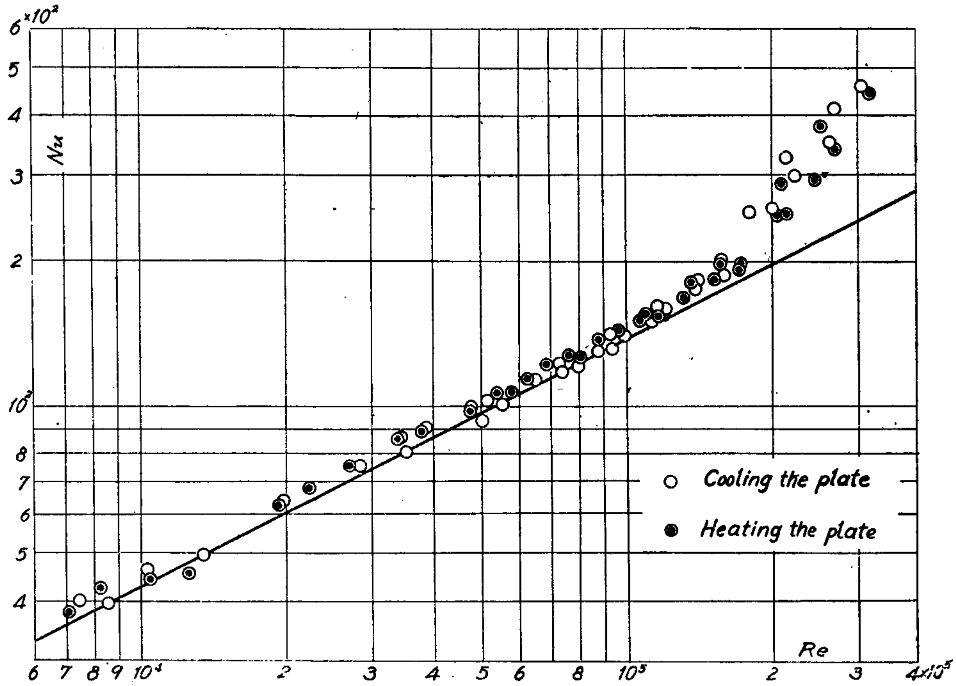

Fig. 3. Laminar Heat Tranfer.

u_0 is air velocity outside the boundary layer. In this figure, the results can be regarded almost linear till Re is about 1.6×10^5.

The mean temperature difference between the plate and air is nearly 20°C, so there cannot be observed any great difference between cooling and heating of the plate, but in the case of heating, Nusselt's number is slightly larger than that in the case of cooling. In other words, number is slightly larger than that in the case of cooling. In other words, heat transfer is slightly better in the case of heating. These results are considerably larger than results of theoretical research in the case of the laminar boundary layer, which were carried out with an assumption that the surface temperature was constant. This is ascribable to the effect of the temperature gradient on the surface of the plate.

The theoretical solution, in case the temperature gradient exists on the plate surface, can be obtained as follows:

In the case of two dimentional steady flow, the differencial equation of heat balance in the boundary layer can be written as follows:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \kappa\frac{\partial^2 T}{\partial y^2}, \qquad (6)$$

where y is the distance from the plate surface, u & v are velocity components in the x & y directions, T is temperature, and $\kappa=\lambda/\rho c_p$, ρ and c_p are the density and the specific heat of air under constant pressure. The theoretical solution of velocity distribution for the plate in the laminar boundary layer has been given by Blasius[1] and Töpfer[2] as follows:

$$u=\frac{1}{2}u_0 f$$

$$v=\frac{1}{2}\sqrt{\frac{u_0\nu}{x}}(\eta f'-f), \tag{7}$$

where

$$\eta=\frac{1}{2}\sqrt{\frac{u_0}{\nu x}}\,y$$

$$f=\frac{a}{2!}\eta^2-\frac{a^2}{5!}\eta^5+\frac{11}{8!}a^3\eta^8-\dots \tag{8}$$

$$a=1.32824.$$

If we put

$$g=\frac{T_1-T}{T_1-T_0} \tag{9}$$

and

$$T_1-T_0=Ax^m, \tag{10}$$

where T_0 is the temperature of the plate, and T_1 that of air, then by using equations (7) & (9), eq. (6) is

$$2\frac{\partial T_0}{\partial x}f'g+\frac{(T_1-T_0)}{x}fg'+\frac{1}{\sigma}\frac{(T_1-T_0)}{x}g''=0, \tag{11}$$

where σ is Prandtl's number and equal to ν/κ. Therefore, we may write, by applying eq. (10) to eq. (11),

$$g''+\sigma fg'-2m\sigma f'g=0. \tag{12}$$

This is a linear differential equation of the second order, which we can solve, assuming g to be the following power series:

$$g=b_0+b_1\eta+b_2\eta^2+b_3\eta^3+\dots=\sum_0^\infty b_s\eta^s. \tag{13}$$

For convenience's sake, we express eq. (8) as follows:

$$f=\sum_0^\infty a_s\eta^s$$

and

$$a_{3l}=0,\quad a_{3l+1}=0. \tag{14}$$

By putting eqs. (13) and (14) into eq. (12), and arranging it, we gain

$$b_{3l+2} = 0$$

$$b_{3l+3} = \frac{\sigma}{(3l+3)(3l+2)} \sum_{j=0}^{l} \{2(3\overline{l-j}+2)m - 3j\} b_{3j} a_{3(l-j)+2} \tag{15}$$

$$b_{3l+4} = \frac{\sigma}{(3l+4)(3l+3)} \sum_{j=0}^{l} \{2(3\overline{l-j}+2)m - (3j+1)\} b_{3j+1} a_{3(l-j)+2}$$

The boundary conditions are:— on the plate surface $T = T_0$, that is, $g = 1$ for $\eta = 0$, and the other condition is, $T = T_1$, that is $g = 0$ for $\eta = \infty$. By the first condition,

$$b_0 = 1,$$

so that the group of b_{3l+3} may be determined by the values of m and σ. And the group of b_{3l+4} may be written as follows:

$$b_{3l+4} = B_{3l+4} b_1,$$

where B_{3l+4} is a function of m and σ. And so b_1 can be determined so as to satisfy the second condition. Namely, it can be determined by the convergent value of $\sum b_{3l}\eta^{3l}/\sum B_{3l+1}\eta^{3l+1}$ according to η getting larger than 0. Therefore we come to know the value of g and accordingly the temperature distribution in the boundary layer.

On the other hand, in our experiments, the value of m was about 1/2 in the range of the laminar boundary layer. So we calculated: $m = 0.5$ and $\sigma = 0.73$, and therefore,

$$b_1 = -0.8286$$

Considering the other side, α, the local heat transfer coefficient, is given as

$$\alpha = \frac{\lambda}{(T_1 - T_0)} \frac{\partial T}{\partial y}\bigg|_{y=0},$$

that is,

$$\alpha = -\frac{b_1}{2} \lambda \sqrt{\frac{u_0}{\nu x}}.$$

Therefore,

$$Nu = -\frac{b_1}{2} Re^{0.5} = 0.414\ Re^{0.5} \tag{16}$$

The relation obtained in our experimental results is

$$Nu = 0.425\ Re^{0.5} \tag{17}$$

The difference between these two equations is merely 3% and so we can recognize their agreement. The following is our result when $m = 0$, that is, when the surface temperature is constant, which is presented here for comparison:

$$Nu = 0.297\ Re^{0.5} \quad \text{for} \quad \sigma = 0.73.$$

From the abov-mentioned results, we may consider that our experimental method is very much reliable.

Next, the comparison of two sets of results obtained by setting the plate vertically and horizontally is shown in Fig. 4. We cannot recognize any difference between them, so we may consider that there is no influence of free convection in our experiments.

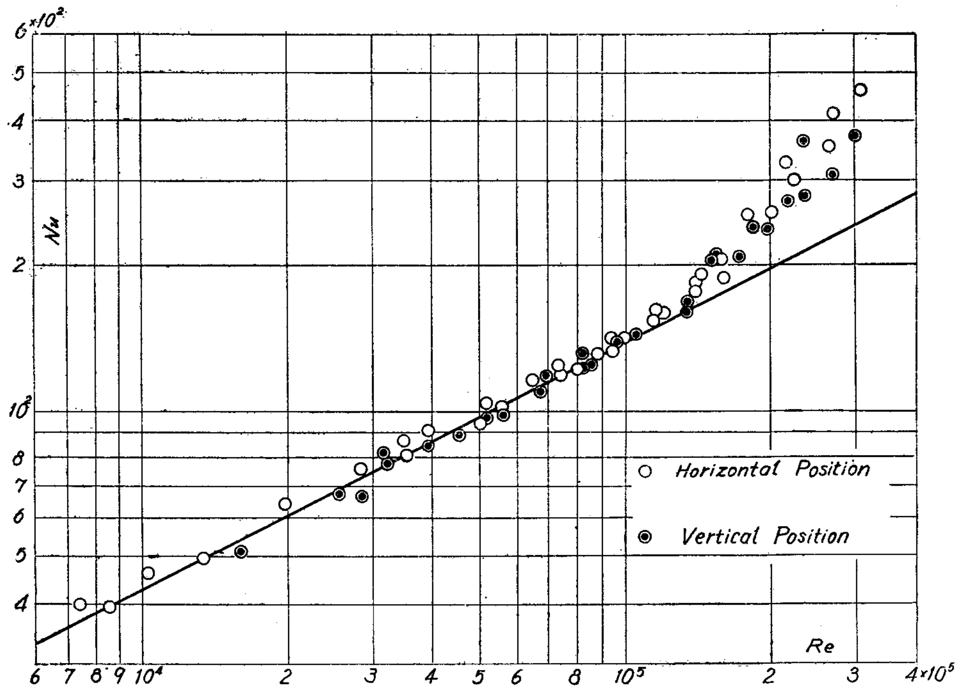

Fig. 4.

(ii) Heat Transfer in the Turbulent Boundary Layer.

In Fig. 3, the results in the range of Re larger than about 1.6×10^5 are these in the transitional range, but we could not experiment with the larger Re range, because of our inadequate apparatus. The turbulent boundary layer cannot be completed as far as the plate front edge is sharp. Therefore, we flattened the front edge to cause the transition occur near the front edge, and the results of this are shown in Fig. 5. In this figure, the transitional point advances near $Re = 2\times10^4$, and after this point, the results seem to be of a linear relation as far as our experiments are concerned. This linear relation is shown as follows:

$$Nu = 0.0194\ Re^{0.8} \tag{18}$$

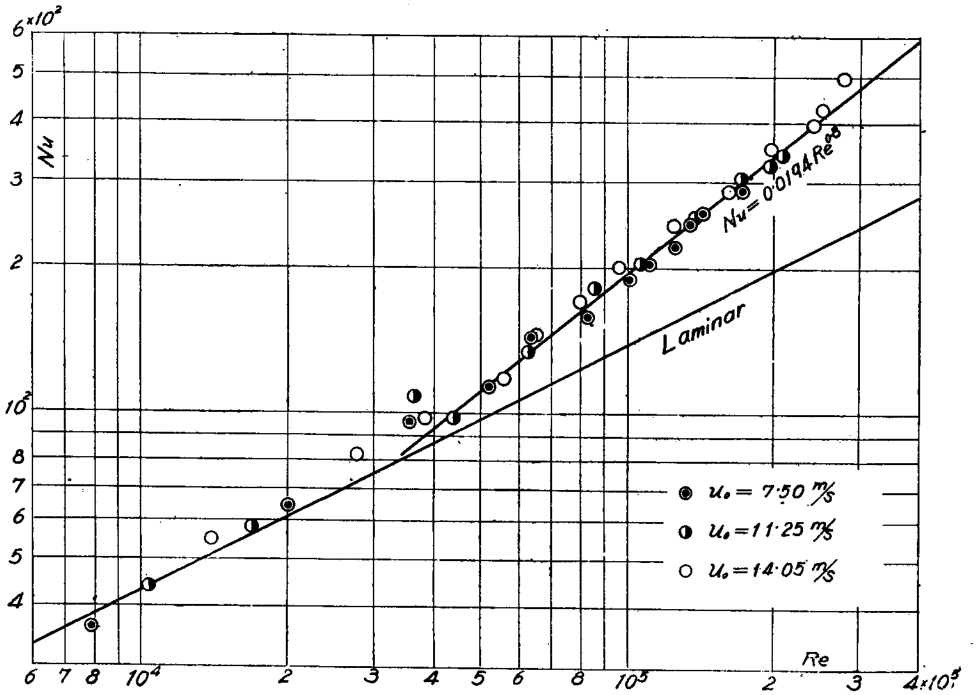

Fig. 5. Turbulent Heat Transfer I.

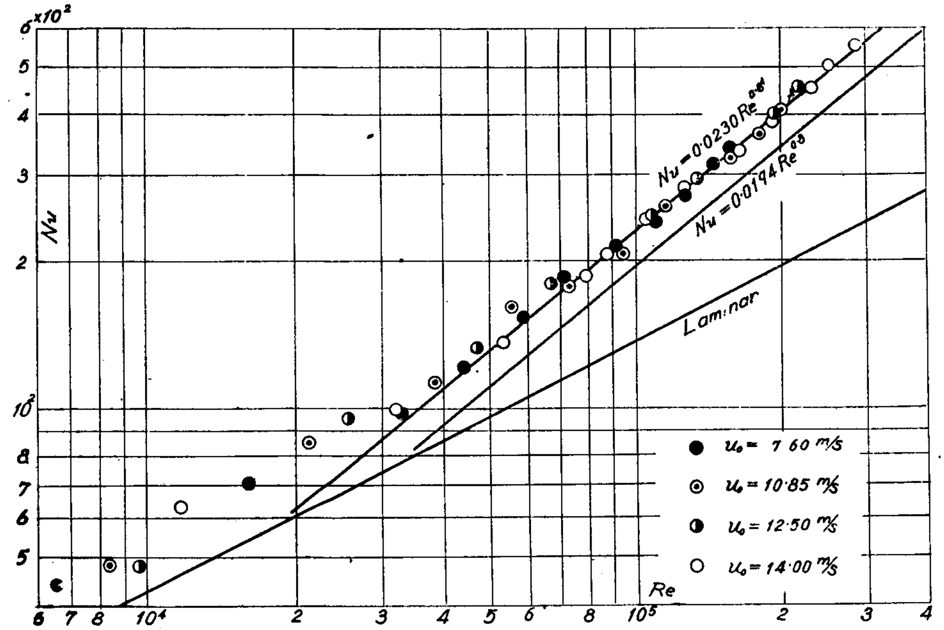

Fig. 6 Turbulent Heat Transter II.

Even in this condition, there is a certain range where the laminar boundary layer exist. To let the turbulent boundary layer form immediately from the front edge, we used a turbulent screen and increased the turbulence by about 4% in the air flow.[4] The result of this is shown in Fig. 6. In this case the boundary layer can be regarded as turbulent immediately from the front edge, and its relation is expressed in the following formula:

$$Nu = 0.0230\ Re^{0.8} \tag{19}$$

On the other hand, according to Blasius the coefficient of the local surface friction in the turbulent boundary layer c_f is

$$\frac{c_f}{2} = 0.0296\ Re^{-0.2}\ .$$

If we put the non-dimentional heat transfer coefficient as $k_h = a/(\rho c_p u_0)$, then,

$$Nu = k_h \cdot \sigma \cdot Re\ .$$

By T. v. Kármán,[3]

$$\frac{1}{k_h} = \frac{g(\sigma)}{\sqrt{c_f}} + \frac{2}{c_f}$$

$$g(\sigma) = 5[(\sigma-1) + l_n\{1+5/6(\sigma-1)\}]$$

If we assume the cooling factor $\xi = 0.93$,[5] the following relation is obtained for about 10^5 of Re and $\sigma = 0.73$:

$$Nu = 0.0234\ Re^{0.8} \tag{20}$$

This relation is for the case of the turbulent boundary layer begining from the front edge. The result of eq. (18) agrees with index of Re, as compared with that of eq. (20), but the value is considerably smaller. And the result of eq. (19) agrees well with that of eq. (20). As above-mentioned the transitional point from the laminar state of the boundary layer to the turbulent state moves considerably according to the shape of the front edge, and the value of the turbulent heat transfer increases according to the growth of turbulence in the main stream. The mean turbulence in the main stream is supposed to be 1~1.5% when the transition point and other factors are considered,[6] in the condition of Fig. 5. The comparison of the results of cooling the plate with these of heating the plate in the turbulent heat transfer is shown in Fig. 7. The results of heating the plate are slightly larger, but the temperature difference is about 20°C as in the former experiments, so the difference is not so remarkable. In the case of heating the plate, we considered that the heat loss from the measuring duct may influence the results a little, so, thereafter, the measurement was carried out only in the case of cooling the plate.

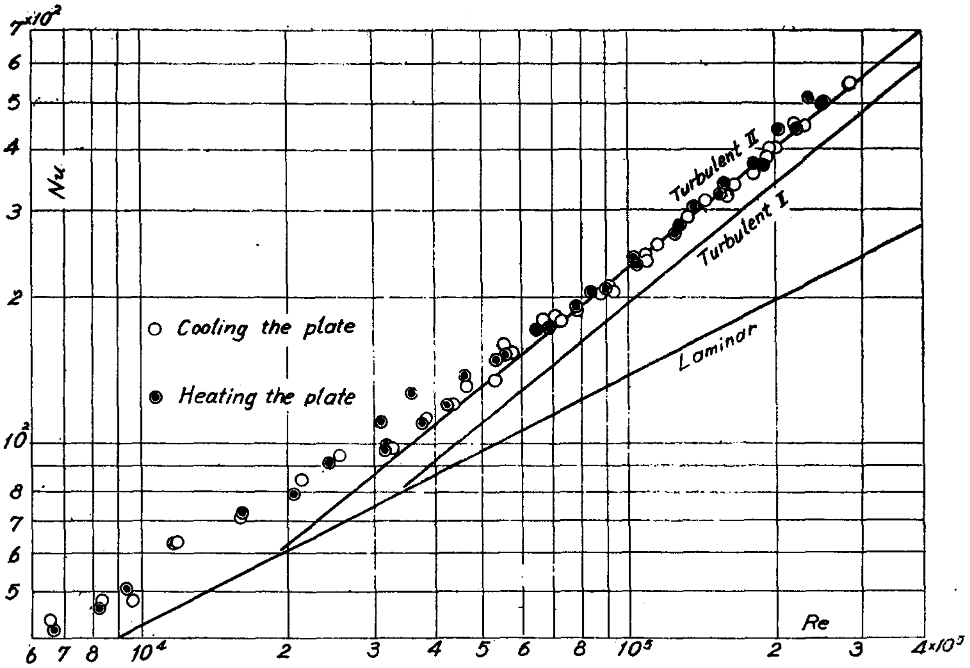

Fig. 7.

b. A Rough-Surfaced Flat Plate.

In giving roughness to a plate surface, if we use sand or other materials to be stuck on it, many difficulties will follow in thermal experiments, so we created artificial roughness by cutting grooves or projections of a fixed pitch on the surface. And by changing the pitch, depth of grooves or height of projections, we researched on the effect of the surface roughness systematically.

(i) A Rough Plate with Grooves on its Surface.

The sectional views of these rough plates with grooves of a constant pitch on their surfaces are shown in Fig. 8. The depth of grooves is always 0.5 mm. The experimental results relative to these plates are as follows:

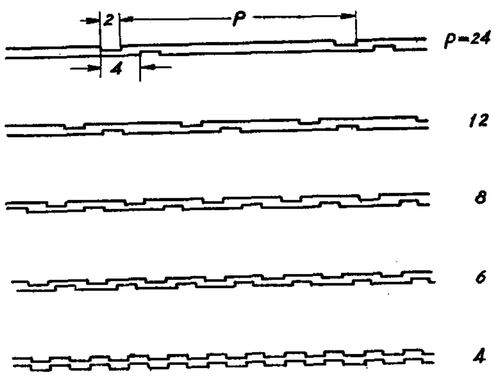

Fig. 8.

In Figs. 9 and 10, we show two examples of the velocity distribution in the boundary layer. They differ considerably from the velocity distribution in the boundary layer of a smooth plate.

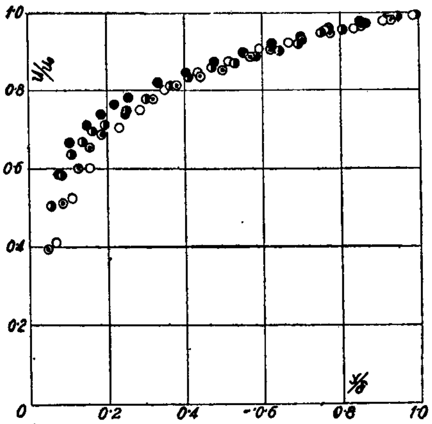

Fig. 9. Velocity Distribution. Rough plate with grooves. p = 12 mm.
○ $x = 207$ mm $u_0 \doteqdot 7.0$ m/s
● $x = 207$ mm $u_0 \doteqdot 14.0$ m/s
⊙ $x = 366$ mm $u_0 \doteqdot 7.0$ m/s
◐ $x = 366$ mm $u_0 = 14.0$ m/s

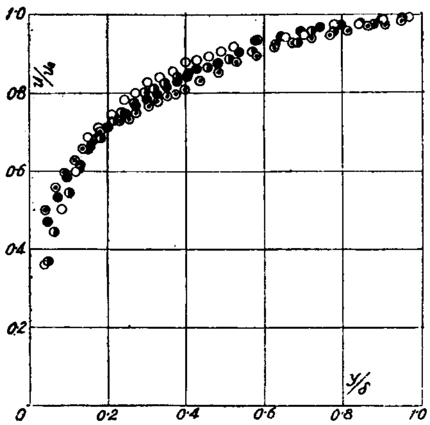

Fig. 10. Velocity Distribution. Rough plate with grooves. p = 8 mm.
○ $x = 208$ mm $u_0 \doteqdot 8.0$ m/s
● $x = 208$ mm $u_0 \doteqdot 14.0$ m/s
◐ $x = 343$ mm $u_0 \doteqdot 8.0$ m/s
⊙ $x = 343$ mm $u_0 \doteqdot 14.0$ m/s

We call such turbulent boundary layer which is influenced by surface roughness the turbulent-rough boundary layer for convenience sake. In the turbulent-rough boundary layer, α was independent of the position and was almost constant in our experiments, so we show the results with Nu/Re and Re. Fig. 11 shows the results when the pitch of grooves p was 6 mm, and Fig. 12 when $p = 24$ mm. By these figures, Nu/Re decreases slightly, according as Re increases, but we consider that

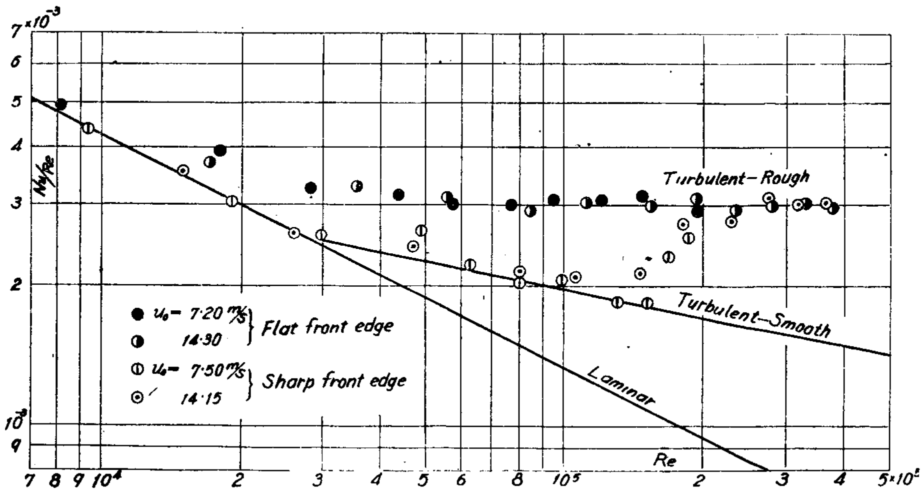

Fig. 11. Turbulent-Rough Heat Trnsfer. Rough plate with grooves. p = 6 mm.

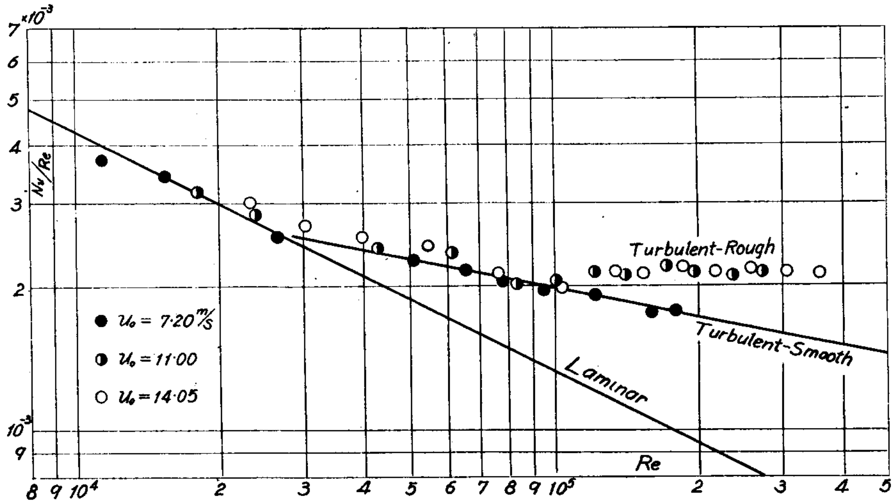

Fig. 12. Turbulent-Rough Heat Transfer. Rough plate with grooves. p = 24 mm.

Nu/Re is almost constant, and so we put the relation* showing the results of each pitch as the following formula:

$$Nu = K\,Re, \tag{21}$$

where the relation between K and p is as follows:

$p =$	4	6	8	12	24
$K =$	0.00209	0.00303	0.00262	0.00252	0.00216

(ii) A Rough Plate with Projections on its Surface.

Next, we produced artificial roughness by carving projections of a constant pitch as shown in Fig. 13. In this case, we experimented, by changing the height of a projection, with 3 kinds of plates ... that is, plates of 0.3, 0.5 and 1.0 mm. We show the velocity distributions of the turbulent-rough boundary layer for these plates in Fig. 14 and Fig. 15. The higher the projections become,

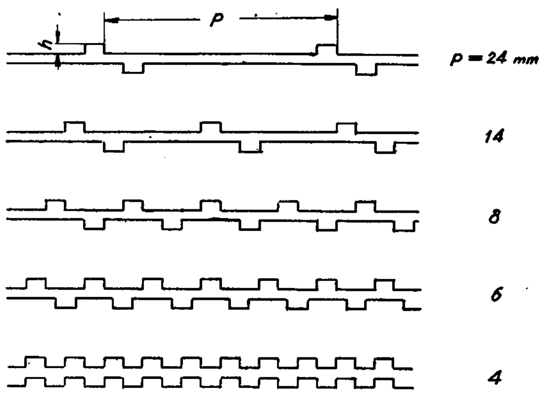

Fig. 13

* We are researching on this relation theoretically.

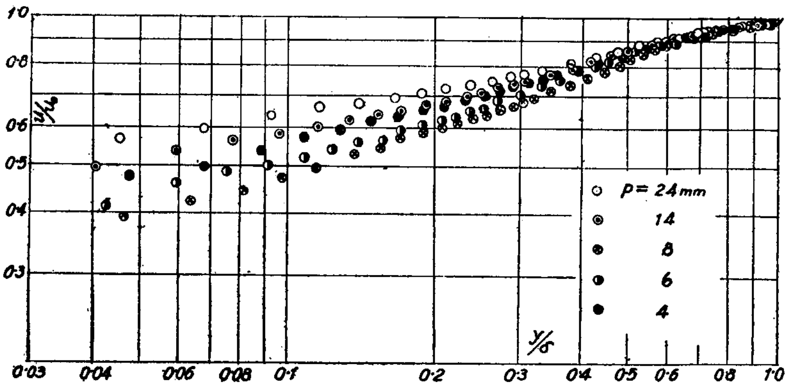

Fig. 14. Velocity Distribution. Rough plate with projections. h = 0.5 mm.

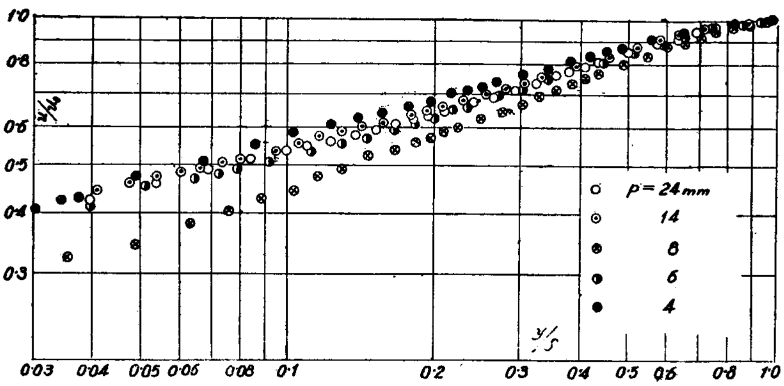

Fig. 15. Velocity Distribution. Rough plate with projections. h = 1.0 mm.

the sharper the inclination of the distribution curve becomes. Figs. 16 to 18 show a few examples of the results regarding the heat transfer. Their tendencies are the same as in the case of (i). Namely, a turbulent-rough boundary layer is formed at larger Re than a certain Re, and in this region the results almost satisfy the relation of eq. (21). The value of K are as follows, (h is the height of a projection):

$h = 0.3$ mm					
	$p =$ 4	6	8	14	24
	$K =$ 0.00257	0.00284	0.00281	0.00250	0.00223
$h = 0.5$ mm					
	$K =$ 0.00209	0.00305	0.00280	0.00258	0.00233
$h = 1.0$ mm					
	$K =$ 0.00249	0.00308	0.00354	0.00359	0.00332

By comparing the case of the plate having grooves with the case of the plate having projections, it is noted that when the height or the depth is 0.5 mm, the relation between p and K is almost the same (Fig. 17.), and in the case of the larger p the result of the case with projections is slightly larger (Fig. 16.), but the difference is not so remarkable. So we may consider that with such kinds of rough surfaces, only the pitch and the height and depth of projections and grooves have effect on heat transfer.

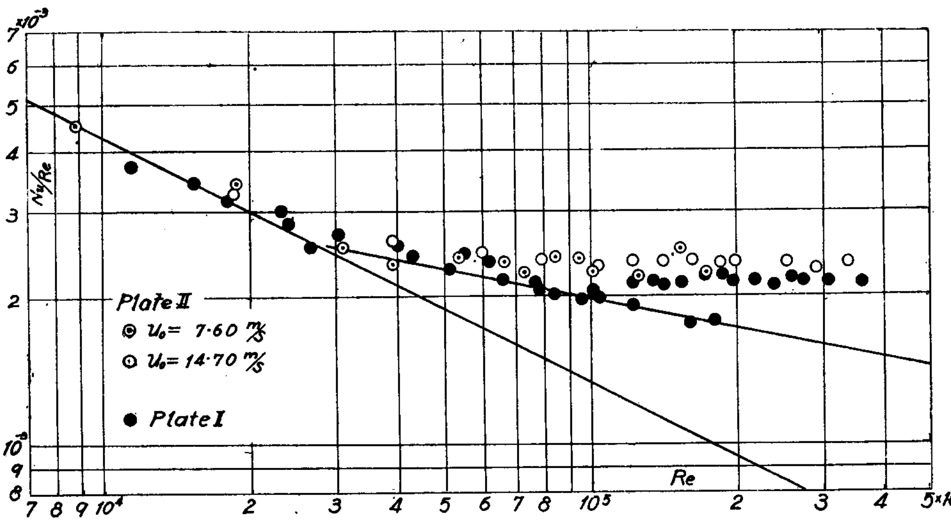

Fig. 16. Turbulent-Rough Plate with projections. p = 24 mm. h = 0.5 mm. Plate I is a rough plate with grooves.

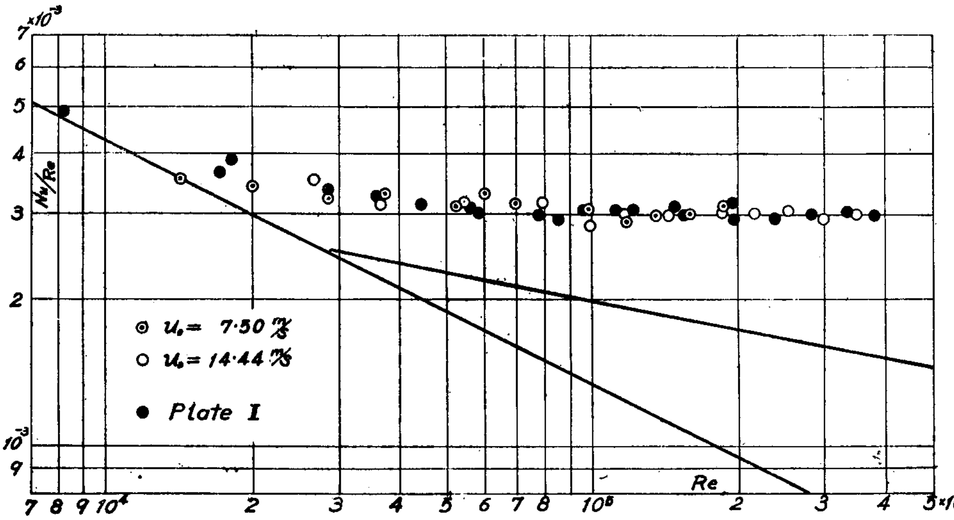

Fig. 17. Turbulent-Rough Heat Tranfer. p = 6 mm. h = 0.5 mm.

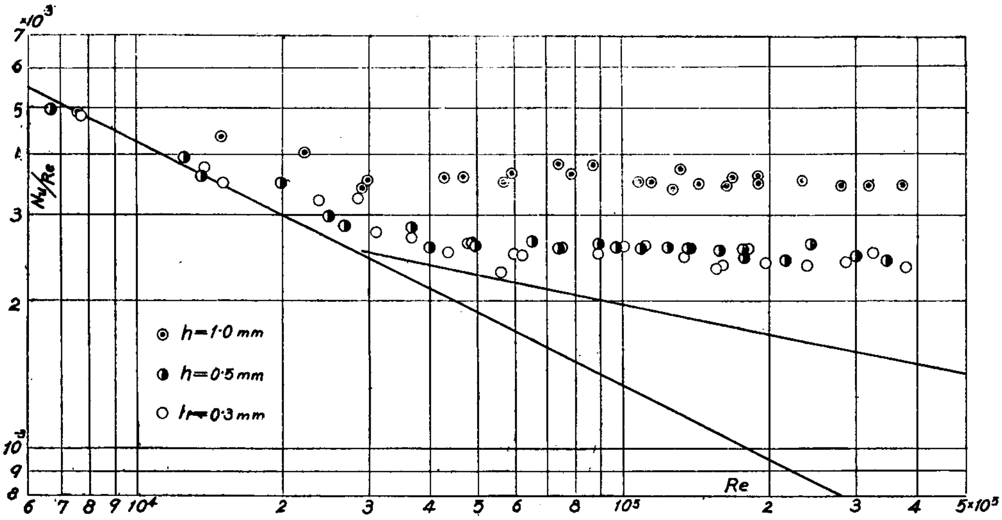

Fig. 18. p = 14 mm.

In the relation between the pitch and K, K becomes the largest at a certain pitch, which means the best heat transfer. Thus in a range where the pitch has effect on heat transfer, we must arrange experimental results not with the apparent heat transfer coefficient obtained from the above-mentioned experiments but with the heat transfer coefficient in the calculation of which the increment of the surface area by surface roughness is taken into account. We take the increment of the surface area f as

$$f = F'/F \,;$$

then

$$\alpha' = \alpha/f \,,$$

where F' and α' are the surface area and the heat transfer coefficient when the increment of the surface area is reckoned with, and F is the surface area when that increment is not considered. By using this α', the factor K in eq. (21) becomes as follows:

$$K' = K/f$$

In Fig. 19, the relation between K' and h/p is shown, in which h is the height or the depth of projections or grooves on the surface. The value of K' becomes maximum at

$$h/p = 0.055 \,.$$

We considered that in the range of $h/p > 0.055$, a vortex caused by a projection is influenced and damped by the following projection, so heat transfer does not become better. On the other hand, in the range of $h/p > 0.055$, the ratio of the part where

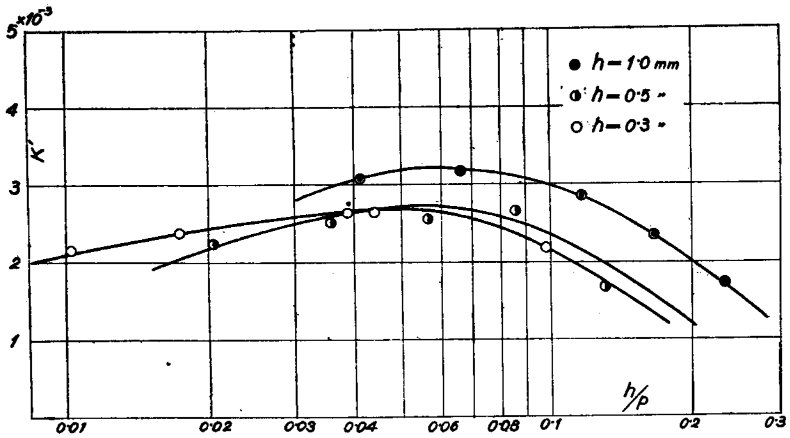

Fig. 19.

heat transfer becomes better because of a vortex, caused by a projection to the part where heat taansfer is not remarkably influenced by vortex becomes smaller, so as a whole, heat transfer does not ameliorate.

4. Summary.

As the first step, we carried out the experiments on the laminar heat transfer and turbulent heat transfer of a smooth flat plate, of which few reliable experimental results could be found. Then, we experimented with plates having the artificial surface roughness created by grooves or projections of a constant pitch whittled on the surface. The results of these experiments are:— in a range of larger numbers than a certain Reynolds' number, the turbulent boundary layer of rough surface is formed and in this range, the local heat transfer coefficient is independent of the position. From these results, we obtained the relation of eq. (21). On a rough surface such as used in in our experiments, heat transfer becomes the best when the ratio of the height of the projections to pitch is about 0.055.

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