On the Rollwave-Trains Appearing in the Water Flow on a Steep Slope Surface

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Synopsis. When we carefully observe a thin sheet flow with a suitable depth on a surface or in a channel of which the slope is greater than about 2%, we can see small continuous wave-trains moving to the down-stream with a uniform wave-length. For instance, we often find them on a road surface with a steep slope in a heavy rain, and the wave-trains in a thin sheet flow of rain-water are called "rain wave-trains". These wave-trains are rollwaves with a wave-height of several times the mean water depth and it is said that they have a close connection with the soil erosion, but researches on such wave-trains are few and unsatisfactory.

In this paper, in order to obtain a foothold necessary to study the relations between the soil erosion and these rollwave-trains from the point of view of civil and agricultural engineering, the hydraulic properties of the wave-trains are researched theoretically and experimentally by the wooden flume with a smooth bed. At first, a criterion in which the rollwave-trains begin to appear is determined, and then the various properties of the wave-trains, i. e. period, wave-length and wave-velocity etc. are ascertained by using an electromagnetic oscillograph or a cathoderay oscilloscope and recording the wave-profile.

1. Introduction

When we carefully observe a thin sheet flow on a surface or in a channel with a steep slope, we can see that the flow is usually not uniform, there existing small rollwave-trains moving to the down-stream with an almost uniform wave-length when the slope is steeper than 2% and the water depth is suitable. For instance, we often find this phenomenon in a flow with a tolerable depth on the ground of a steep slope in a heavy rain or in a flow of water washing the road surface or the deck of a ship etc., and the wave-trains due to rainfall as in the former case are called " rain wave-trains ".

Now, if we consider a uniform flow without wave-trains, the depth of this flow on the ground is generally from several mm to several cm, even in the case of a very heavy rain, so that the velocity and the force of erosion may be considered not so large. But actually the soil on an inclined surface is considerably eroded and even large stones are transported from the mountain-side in the case of a flood, and it seems reasonable to consider that the above mentioned facts are caused by the erosion and transportation by the flow on the ground due to suddenly changing its type and becoming a wave-train or something similar. At present, there are yet no satisfactory researches on these rollwave-trains and only a few reports have been made on them. Therefore in this paper, the hydraulic properties of these rollwave-trains are researched theoretically and experimentally by the wooden flume with a smooth bed, and applying these results to the problem of the stability of slopes and road surfaces, it is attempted to contribute to prevent these surface from erosion.

2. Criterion in which Rollwave-trains Begin to Appear

It is interesting to clarify the condition in which the rollwave-trains begin to appear and also important from the view of preventing erosion. Assuming that the flow is turbulent completely and the frictional resistance is proportional to the second power of the mean velocity, H. Jeffreys[1] derived mathematically the relation $u_{m0} \geqq 2\sqrt{gh_m}$ as a criterion for the formation of wave-trains, using the condition that a small disturbance given on the free surface of a flow does not vanish, where u_{m0} and h_m are the mean velocity and the water depth in the critical condition respectively. But judging from our experimental fact that the rollwave-trains begin to appear in the laminar region, the above theory is clearly unreasonable because it is based on the assumption that the frictional resistance is proportional to the second power of the mean velocity. On the other hand, calculating the distance from the watershed to the position where the rollwave-trains begin to appear when it rains on an inclined surface, R. E. Horton[2] ascertained that the distance by H. Jeffreys' equation of criterion becomes too large and that by Belanger's equation of criterion $u_{m0} = \sqrt{gh_m}$ for a bore is more suitable. Moreover, G. H. Keulegan and G. W. Patterson[3] derived the equation of criterion $u_{m0} = (3/2)\sqrt{gh_m}$, using Boussinesq's equation of the propagation velocity of a volume element in the disturbing wave and Manning's mean velocity formula, and mentioned that their equation coincides with H. Jeffreys' when Chézy's formula is employed instead of Manning's formula, but this theory is also derived under the assumption differing from the experimental fact. Prof. M. Kurihara and T. Tsubaki[4] derived the critical condition by the same method of small oscillation as H. Jeffreys' and the authors', using the equations of motion and continuity containing the velocity perpendicular to the bottom surface, but their results do not coincide with the authors' experimental fact because their calculation was carried out for the case of a turbulent flow.

Thus there exists yet no settled theory, but the following facts were made clear from the authors' experimental results, that is, (1) as the wave-trains develop satisfactorily in the laminar region, the criterion in which the wave-trains begin to appear exists in the region where Chézy's or Manning's formula is clearly not applicable, (2) the wave-trains appear under values smaller than the criterion given by H. Jeffreys or Keulegan and Patterson, and (3) contrary to R. E. Horton's report that wave trains were found even in the case of gentle slope of 1~2%, the formation of wave-trains on a surface gentler than 2% was not recognized in our laboratory. According to the experimental facts above mentioned, the criterion in which wave-trains begin to appear is obtained as follows by the method of small oscillation using the momentum equation.

Now, taking x–axis in the down-stream direction along the bottom surface and z–axis vertically upward, we have the equation by the law of momentum,

$$\int_0^h \rho\frac{\partial u}{\partial t}dz+\frac{\partial}{\partial x}\int_0^h \rho u^2 dz=-\tau_0+\rho g h\sin\alpha-\rho g h\cos\alpha\frac{\partial h}{\partial x}, \quad \text{.............}(1)$$

where t is the time, u the velocity, h the water depth, α the slope angle of the bottom surface, ρ the density of water, τ_0 the frictional stress on the bottom surface and g the gravity acceleration. Using the mean velocity u_m, the equation of continuity is

$$\frac{\partial h}{\partial t}+\frac{\partial}{\partial x}(u_m h)=0. \quad \text{...}(2)$$

Since the wave-trains appear in the laminar region, we apply the following equation as the velocity distribution[5)6)].

$$u/u^*=(u^*z/\nu)\{1-(z/2h)\}, \quad \text{.....................................}(3)$$

where $u^*=\sqrt{\tau_0/\rho}$. From Eq. (1) and (3), we have

$$\frac{3\nu u_m}{h}=gh\sin\alpha-\left(gh\cos\alpha+\frac{6}{5}u_m^2\right)\frac{\partial h}{\partial x}-u_m\frac{\partial h}{\partial t}-\frac{12}{5}u_m h\frac{\partial u_m}{\partial x}-h\frac{\partial u_m}{\partial t}. \qquad (4)$$

If the small variations of the mean velocity and the water depth caused by a small disturbance are expressed by u_m' and h' respectively,

$$u_m=u_{m0}+u_m', \quad h=h_m+h'.$$

Introducing these equations into Eq. (2) and (4) and neglecting the terms smaller than the second power of small quantities and also eliminating u_m', we have

$$\left(gh_m\cos\alpha-\frac{6}{5}u_{m0}^2\right)\frac{\partial^2 h'}{\partial x^2}-\frac{12}{5}u_{m0}\frac{\partial^2 h'}{\partial x\partial t}-\frac{\partial^2 h'}{\partial t^2}$$

$$-\left(\frac{6\nu u_{m0}}{h_m^2}+g\sin\alpha\right)\frac{\partial h'}{\partial x}-\frac{3\nu}{h_m^2}\frac{\partial h'}{\partial t}=0. \quad \cdots\cdots(5)$$

Now, assuming that the given small disturbance is expressed by

$$h'=A\exp(\gamma t+i\beta x) \quad \cdots\cdots(6)$$

and solving Eq. (5) for γ into which h' above given is introduced,

$$\gamma=\frac{6}{5}u_{m0}i\beta+\frac{3\nu}{2h_m^2}\pm\left\{-\beta^2\left(gh_m\cos\alpha+\frac{6}{25}u_{m0}^2\right)-i\beta\left(g\sin\alpha+\frac{12\nu u_{m0}}{5h_m^2}\right)+\left(\frac{3\nu}{2h_m^2}\right)^2\right\}^{\frac{1}{2}}.$$

Since the real part of γ must be zero at the criterion in which the given small disturbance does not vanish but keeps stable, we can derive the following relation when $\cos\alpha \fallingdotseq 1$.

$$u_{m0}=\sqrt{gh_m\cos\alpha/3}\fallingdotseq 0.577\sqrt{gh_m}. \quad \cdots\cdots(7)$$

Eq. (7) gives the criterion in which the wave-trains begin to appear, and now we will compare it with our experimental results.

In Fig. 1 which represents the relation between the velocity u_{m0} and the hydraulic mean depth $R(\fallingdotseq h_m)$, the black circles correspond to the cases in which the wave-trains were observed and contain the region of $\sin\alpha=0.222\sim0.020$. As shown in the figure, the criterion discovered experimentally in which the wave-trains begin to appear is $u_{m0}=\sqrt{gh_m}$ or a little smaller, and exists near the criterion between the ordinary flow and the jet flow.

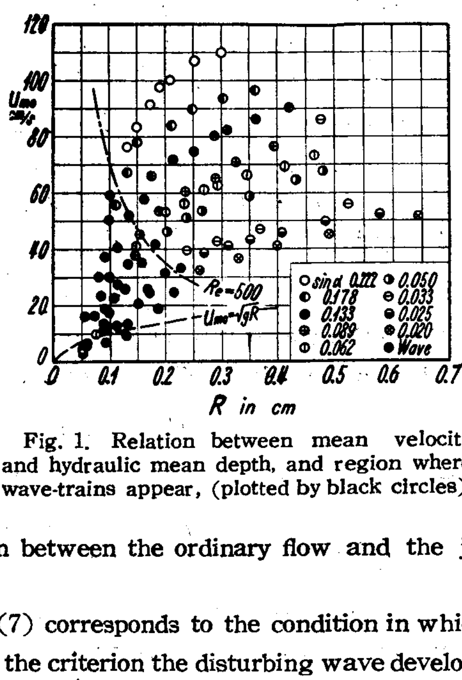

Fig. 1. Relation between mean velocity and hydraulic mean depth, and region where wave-trains appear, (plotted by black circles).

Since the critical value given by Eq. (7) corresponds to the condition in which the given disturbance should be stable, near the criterion the disturbing wave develops slowly. Therefore it being difficult to confirm experimentally the criterion in which the wave-trains begin to appear, we presume that the critical value confirmed experimentally was taken as $u_{m0}=\sqrt{gh_m}$ or a little smaller, because a mean velocity a little larger than the critical value obtained theoretically is necessary in order to confirm the wave-trains clearly. Moreover, it is considered in common sense and also observed experimentally that these rollwave-trains don't appear when the water depth is comparatively deep, and V. Cornish reported that he has never seen a stream with a uniform depth of more than 4 inches adopt spontaneously the intermittent flow in a series of roll-waves. In order to obtain the upper limit of the criterion, it is necessary to experiment with a very long channel in which the wave-trains can develop completely. As presumed roughly from Fig. 1, the upper limit of the criterion seems to be a function of Reynolds' number, but this relation is not yet made clear theoretically.

3. Properties of Rollwave-trains as a Wave

(*A*) *Experiment.* In order to reveal the properties of the rollwave-trains as a wave, i. e. wave profile, wave velocity, period and wave lenght etc., we experimented by an apparatus as shown in Fig. 2. As the experimental flume, we used a planed wood flume having a breadth of 20 cm, a depth of 10 cm, a length of about 5 m of which the effective length 3.8 m and a slope variable from 0 to 1/4. The water depth was measured at a position 40 cm downstream from the entrance of the channel where the wave-trains hardly develop, because it is almost impossible to measure the depth at the position where the wave-trains appear. The wave-trains were measured by pick-ups made of safety razor blade as shown in Fig. 2, by converting the variation of the water level into that of electric current or voltage and recording the wave profile of wave-trains by an electromagnetic oscillograph or a cathoderay oscilloscope, and from these records we computed the wave velocity, period and wave length etc.

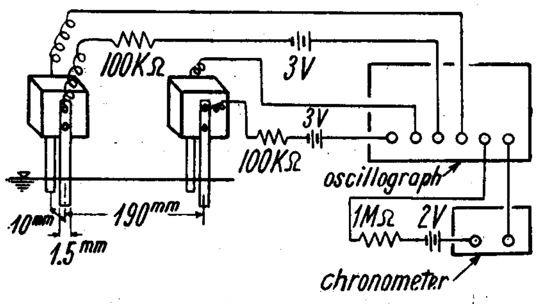

Fig. 2. Apparatus by which wave-trains were measured.

As shown in Fig. 3 which represents an example of wave profiles recorded as above mentioned, the wave profile recorded by an electromagnetic oscillograph becomes a very uneven form, differing to some extent from the actual form, which is a fact most likely due to the effect of the inertia of the galvanometer mirror. The wave profile recorded by a cathoderay oscilloscope becomes a smooth form, because the effect of the inertia vanishes in this case, as shown in the figure.

(*B*) *Wave profile of rollwave-trains.* Since the rollwave-trains develop only in the case of a very small water depth of less than several mm in our laboratory experiment, it is difficult to record the wave profile accurately but its rough profile can be obtained as shown in Fig. 3 by a cathoderay oscilloscope. In this way, we can see that the slope of the wave front moving to the downstream is very steep, becoming almost vertical, and that the larger the depth and the slope are, the more remarkable the tendency is. On the contrary, the slope of the wave back is not so remarkable as the front, but the water depth upstream from the wave crest decreases at a great rate and then flattens out and becomes more shallow at the front of the next wave. The wave height seems to reach twice or several times the water depth.

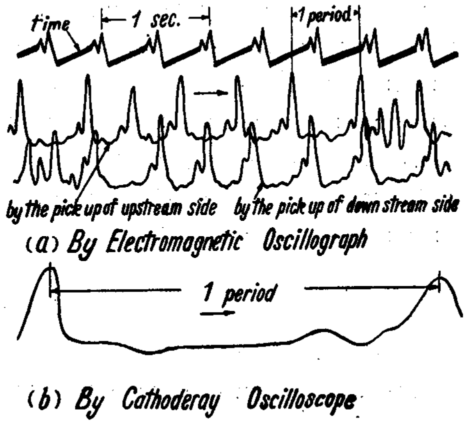

Fig. 3. Wave profiles of wave-trains recorded by an electromagnetic oscillograph and a cathoderay oscilloscope.

(C) *Wave velocity.* Now we can obtain the propagation velocity of the wave-trains in the case of a laminar flow, assuming that the wave height is small compared with the water depth, that is, neglecting the last two terms in Eq. (5) because these terms are small compared with the other terms, we have

$$\left(gh_m \cos\alpha - \frac{6}{5}u_{m0}^2\right)\frac{\partial^2 h'}{\partial x^2} - \frac{12}{5}u_{m0}\frac{\partial^2 h'}{\partial x \partial t} - \frac{\partial^2 h'}{\partial t^2} = 0 . \qquad (8)$$

The general solution of this equation is given by $h' = F(mx - nt)$ and the propagation velocity becomes $\omega = n/m$, so that introducing this general solution into Eq. (8), we have the following equation when $\cos\alpha \doteqdot 1$.

$$\omega = n/m = (6/5)u_{m0} \pm \sqrt{gh_m + (6/25)u_{m0}^2} . \qquad (9)$$

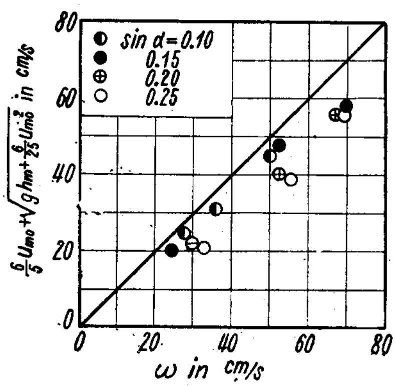

Fig. 4. Wave velocity.

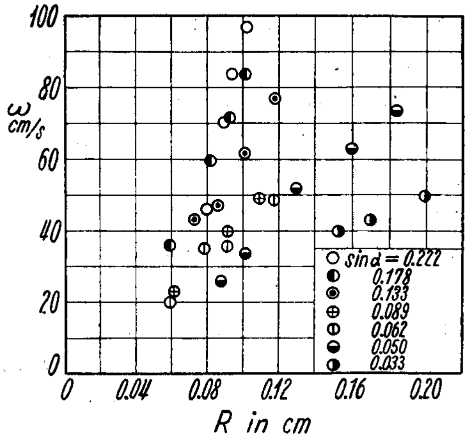

Fig. 5. Relation between the wave velocity and the hydraulic mean depth.

The larger the propagation velocity ω becomes, the further the experimental results deviate from the straight line corresponding to Eq. (9) as shown in Fig. 4, but this fact is of course due to the assumption that the wave height is small compared with the water depth, and we can see that the experimental results coincide with Eq. (9) more satisfactorily than with the equation $\omega = u_{m0} + \sqrt{gh_m}$.

Refering to Fig. 5, the relation between the wave velocity ω and the hydraulic mean depth R seems to be almost linear.

(*D*) *Period and wave length.* If we observe the completely developed wave-trains, it is ascertained that the variation of the wave length and the period of each train is comparatively large in spite of the small difference in each wave velocity. V. Cornish reported that the periods of wave-trains measured at the exit of a conduit varied from 1 sec. to 10 sec. and the period of 4 sec. occured most frequently. Taking the average of these periods measured in our experiments and expressing it by T, we represent the relation between T, R and the slope in Fig. 6. From this figure, a clear co-relation is not found, but we can see as a rough tendency that the period decreases as the slope increases in the same hydraulic mean depth. Moreover, in the range of our experiments, $T = 0.3 \sim 0.6$ sec., among which $0.4 \sim 0.5$ sec. is most frequent, and these facts almost coincide with the results observed at the entrance of a conduit by V. Cornish. Next, if we consider the wave length λ as the product of the mean period and the wave velocity, the wave length increases proportionally to the mean velocity independently of the slope as shown in Fig. 7, where the proportional constant is 1 in c. g. s. unit.

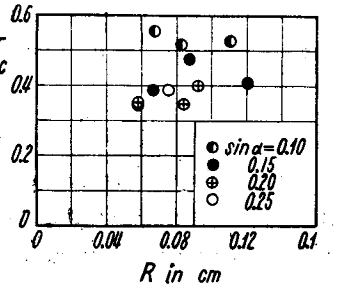

Fig. 6. Relation between the period and the hydraulic mean depth.

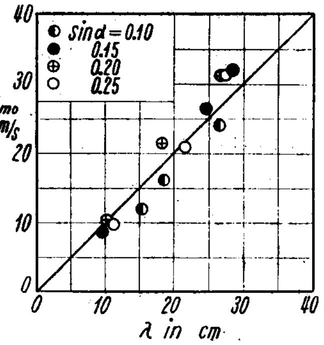

Fig. 7. Relation between the wave length and the mean velocity.

4. Distance from the Watershed to the Place where the Rollwave-trains Begin to Appar on a Slope in Rainy Weather

In order to reveal the counter measure to prevent the soil surface from erosion, it is interesting to obtain the distance from the watershed to the place where the rollwave-trains begin to appear on an inclined surface in rainy weather. Taking the origin on the watershed, x-axis in the direction of the downstream along the inclined surface and z-axis vertically upward, and also assuming that $q = (q_1 - q_2) \times \cos\alpha = \text{const.}$, where q_1 is the rainfall intensity and q_2 the infiltration capacity, we have the following equation from the momentum equation, the equation of continuity and Eq. (3) expressing the velocity distribution in a laminar flow[6].

$$\frac{dh_m}{dx} = F_1(h_m, x)/F_2(h_m, x), \qquad (10)$$

where

$$F_1(h_m, x) = \left(\frac{h_m\sqrt{gh_m \sin\alpha}}{\nu}\right)^2 - \frac{12}{5}\frac{q^2 h_m x}{\nu^2} - \frac{3qx}{\nu},$$

$$F_2(h_m, x) = \left(\frac{h_m\sqrt{gh_m}}{\nu}\right)^2 \cos\alpha - \frac{6}{5}\frac{q^2x^2}{\nu^2}.$$

Since this equation can not be integrated directly, carring out the numerical integration of this equation, the profile of the water surface is approximately given by the following equation, the error being about 4% except in the region of the small values of x.

$$h_m = \sqrt[3]{3q\nu x/g \sin\alpha}. \qquad (11)$$

Now, if the criterion where the wave-trains begin to appear is given by $u_{m0} = \beta\sqrt{gh_m \cos\alpha}$, using the relation $u_{m0} = qx/h_m$ and Eq. (11), the distance x_w from the watershed to the point where the wave-trains begin to appear on an inclined surface becomes as follows.

$$x_w = 3\nu\beta^2/gJ, \quad \text{where} \quad J = \tan\alpha \qquad (12)$$

Since the criterion where the wave-trains begin to appear is theoretically given by $u_{m0} = \sqrt{gh_m \cos\alpha/3}$ and the practical criterion observed in the experiments exists nearly in the neighbourhood of the criterion of ordinary and jet flows as mentioned above, taking $\sqrt{1/3}$ and $\sqrt{5/6}$ respectively as the value of β and also $\nu = 0.01\ \text{cm}^2/\text{s}$, we can represent Eq. (12) as in Fig. 8, where q is considered as a parameter.

When the rainfall intensity, the infiltration capacity and the slope are given, we can obtain the distance from the watershed to the place where the wave-trains begin to appear, refering to Fig. 8. For example, for the case of q=0.0002 cm/s and J=0.1, this distance becomes 5 m for the theoretical condition (represented by broken lines) and 12.5 m for the

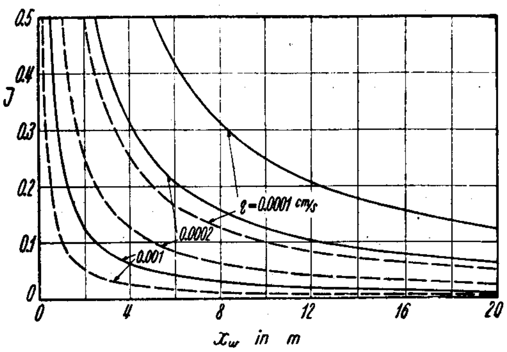

Fig. 8. Distance from the watershed to the place where wave-trains begin to appear.

visible i. e. experimental condition (represented by real lines).

5. Conclusion

As for the force of erosion of rollwave-trains, R. E. Horton said that it becomes about 5 times the force when the wave-trains do not appear, owing to the increase of tractive force due to an increase of water depth, which is based on du Boys' idea. Now if we drop powders of aluminium on the surface between the wave-crests of wave-trains, the greater part of the powders move acceleratedly to the down-stream wavecrest and the powders gathering together near the crest take the motion to be rolled in, that is, the vortices having the axis parallel to the bottom surface and perpendicular to the stream seem to exist near the wave-crests. If we observe carefully the wave-trains appearing in a muddy flow in the field, most of the silty sands exist in the wave-crests. From these states of motion, a diffusion or lift due to the turbulent eddies existing in the parts of the wave-crests is considered, together with which the increases of tractive force due to an increase of water depth seem to explain the fact that the wave-trains give an important effect on the soil erosion. In this paper, we made clear the hydraulic properties of wave-trains for the case of smooth beds, and these results in addition to the researches for the cases of rough and movable beds which we are now undertaking will contribute to reveal the relation between the wave-trains and soil erosion quantitatively.

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