A Field Determination of Permeability*

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(Received July, 17, 1952)

Synopsis: In order to determine the permeability of the undisturbed field ground, a new method using a simple pool is proposed instead of the conventional unreasonable assumption or the troublesome method proposed hitherto. The theoretical formulae on which this method depends are derived, and a discussion is presented on the result obtained by this method and its verification.

1. Introduction.

The coefficient of soil permeability as defined in soil engineering is founded on Darcy's law, and the methods of measuring it include the laboratory method using a permeameter and the field method performed without disturbing the layer condition of the ground.

With the laboratory method performed with a small quantity of a sample, the collected sample being disturbed, the layer condition which affects the permeability usually differs from that of the field ground, making it very difficult to determine quantitatively the corresponding relation of them even with the penetration test and others. Furthermore, it is difficult to avoid the undesirable effect of the space in between the sample and the face of the measuring apparatus.

As the above defects are eliminated in the field method which has the merit of not disturbing the layer condition of the ground, various methods have been studied by many scholars[1]. For instance, the method of measuring the traveling time of a high electrolyte[2] and another method of measuring the hydraulic gradient of water flowing from a channel by boring several holes near the measuring channel[3] have been proposed. In either case, however, as a considerable amount of preparation is necessary in order to carry out measurement and as water pouring in forms a three dimensional flow, the theoretical analysis

* This paper was published in Japanese on Feb. 15, 1952 in the Journal of the Japan Soc. of Civil Engrs.

is difficult and, in most cases, the reliability of the obtained results is considered low, as they are based on unreasonable assumptions.

In our field method, the three dimensional flow will be changed into the two dimensional and the theoretical treatment will be simplified in a suitable way. The theoretical formulae for this case will be deduced so that the coefficient of permeability of the field ground can be calculated easily. These are the main characteristics of this method.

2. Fundamental theoretical equations.

(1) Gravity flow and the free water surface.

Generally the condition of water seeping through soil can be expressed by using the following complex conjugate function:

$$\omega = \varphi + i\psi = f(z) = f(x+iy), \tag{1}$$

where φ and ψ which form a flow net perpendicular to each other represent the function of the velocity potential and the flow function of the two dimensional flow, respectively. The most important factor governing the condition of water seeping through a dam or leaking from a channel is obviously gravity. If such a gravity flow is considered a two dimensional flow, potential function φ and presure function p both satisfy Laplace's equation, and φ can be expressed in the following form:

$$\nabla^2\varphi = \nabla^2 p = 0, \quad \varphi = \frac{k_0}{\mu}(p \pm \gamma g y), \tag{2}$$

where μ: coefficient of viscosity of the fluid, γ: weight of the unit volume of the fluid, g: gravity acceleration, k_0: coefficient of permeability, and the double sign corresponding to the upper and lower directions of y-axis, respectively. If the atmospheric pressure is 0, if the coefficient of effective permeability defined as $k = \frac{k_0 \gamma g}{\mu}$ is taken instead of k_0, and if the downward direction of y-axis is assumed to be positive, then φ in Eq. (2) becomes as follows:

$$\varphi = -ky. \tag{2'}$$

The gravity flow problem differs from other cases in that a free water surface appears in a part of the boundary limiting the flow. Physically, the free water surface is defined as a streamline along which the pressure is uniform. When it exists, its shape is not given as a boundary from the beginning, but must be determined together with the pressure distribution of the flow. Therefore, in the case of the three dimensional flow it is very difficult or almost impossible to solve them, and even in the case of the two dimensional flow, it can be said that there is no method of the mathematical analysis besides using the conjugate function stated above under a special boundary condition.

(2) Derivation of the solution of the flow from a channel.

The representation of the complex function is given by the following equation which is applicable to the case where water seeping from a channel having a section shown in Fig. 1 has a free water surface in the case of the two dimensional flow:

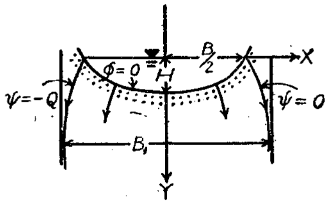

Fig. 1 The seepage out of a ditch with free surfaces bounded by vertical asymptotes.

$$z=\frac{1}{k}\left(-Hke^{\frac{\pi\omega}{Q}}-i\omega+\frac{Q}{2}\right), \tag{3}$$

i.e.

$$\left.\begin{aligned} x&=\frac{1}{k}\left(-Hke^{\frac{\pi\varphi}{Q}}\cos\frac{\pi\psi}{Q}+\psi+\frac{Q}{2}\right)\\ y&=\frac{1}{k}\left(-Hke^{\frac{\pi\varphi}{Q}}\sin\frac{\pi\psi}{Q}-\varphi\right), \end{aligned}\right\} \tag{3'}$$

where H: depth of water in the center of the cross section of the channel, Q: discharge of water from unit horizontal depth. The two streamlines, $\psi=0,\ -Q$, are free-surface streamlines. If Eq. (2′) is introduced into the above equations, the streamlines become curves symmetrical in relation to y-axis, represented by the following equations:

$$x=\frac{1}{k}\left(-Hke^{-\frac{\pi ky}{Q}}+\frac{Q}{2}\right),\quad x=\frac{1}{k}\left(Hke^{-\frac{\pi ky}{Q}}-\frac{Q}{2}\right). \tag{4}$$

If $\varphi=0$ is substituted into Eq. (3) to obtain the shape of a channel to which Eq. (3) is applicable, the following equation is derived:

$$\left\{x+\frac{1}{k}\left(\frac{Q}{\pi}\sin^{-1}\frac{y}{H}-\frac{Q}{2}\right)\right\}^2+y^2=H^2. \tag{5}$$

If B is the width of the channel section at the water surface and $x=\frac{B}{2}$ is substituted into this equation at $y=0$,

$$B\doteqdot\frac{Q}{k}-2H,\quad B_1=2|x|_{y=\infty}=\frac{Q}{k}=B+2H. \tag{6}$$

Thus B_1, the maximum width of the sheet of water seeping down at infinite depth, is knowable.

The above simple results will, of course, strictly be only for the case where the shape of the channel is close to that given by Eq. (5) and where the uniform soil layer is of a great depth, so that water can maintain indefinitely its vertical downward seepage. However, among these boundary conditions, it has been stated that the former assumption is not so important compared with the latter.[4] Therefore it can be said that the above formulae are applicable only to the case where an underground water table exists deep in the earth. In many actual situations, however, water seeping down from a channel will reach the normal ground water level at a relatively shallow depth, thus forcing the streamlines to assume a horizontal rather than a vertical trend. Fig. 2 represents such a case and the complex function is given by the following equation:

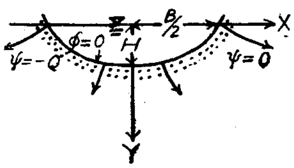

Fig. 2 The seepage out of a ditch with radially spreading free surfaces.

$$z=\frac{1}{k}\left(Hke^{-\frac{\pi\omega}{Q}}-i\omega+\frac{Q}{2}\right), \tag{7}$$

i. e.

$$\left.\begin{aligned} x&=\frac{1}{k}\left(Hke^{-\frac{\pi\varphi}{Q}}\cos\frac{\pi\psi}{Q}+\psi+\frac{Q}{2}\right)\\ y&=\frac{1}{k}\left(-Hke^{-\frac{\pi\varphi}{Q}}\sin\frac{\pi\psi}{Q}-\varphi\right). \end{aligned}\right\} \tag{7'}$$

The two symmetrical free-surface streamlines, $\psi=0,\ -Q$, are given as before by the following equations:

$$x=\frac{1}{k}\left(Hke^{\frac{\pi ky}{Q}}+\frac{Q}{2}\right),\quad x=\frac{1}{k}\left(-Hke^{\frac{\pi ky}{Q}}-\frac{Q}{2}\right). \tag{8}$$

A comparison of Eq. (4) with Eq. (8) reveals that the former has vertical asymptotes at the distance $\frac{Q}{2k}$ from the center of the channel which the free water surface approaches deep in the ground, while with the latter the depth of the streamlines increases logarithmically with increasing distance from the channel.

Substituting $\varphi=0$ so that the shape of the channel in this case satisfies Eq. (7),

$$\left\{x+\frac{1}{k}\left(\frac{Q}{\pi}\sin^{-1}\frac{y}{H}-\frac{Q}{2}\right)\right\}^2+y^2=H^2. \tag{9}$$

If $x=\frac{B}{2}$ is substituted when $y=0$,

$$B=\frac{Q}{k}+2H \tag{10}$$

is obtained.

It should be noted that while Eq. (9) is formally identical with Eq. (5), one must take the positive radical $\sqrt{H^2-y^2}$ in solving Eq. (9) for x, whereas in Eq. (5) the negative values of that radical must be used in solving it for x.

(3) Solution for the case where the shape of the channel consists of straight lines.

Fig. 3 shows the seepage flow from a channel formed with straight lines when an underground water table exists at a great depth. Here, too, a consistent set of potential and streamline distributions and the shapes of the free surfaces are found rigorously by applying a succession of complex variable transformations, but the exact profile of the channel to which they correspond is found only at the end of the solution. The solution for this case has been given by Wedernikow[5], and the result is represented by the following equation corresponding to Eq. (6):

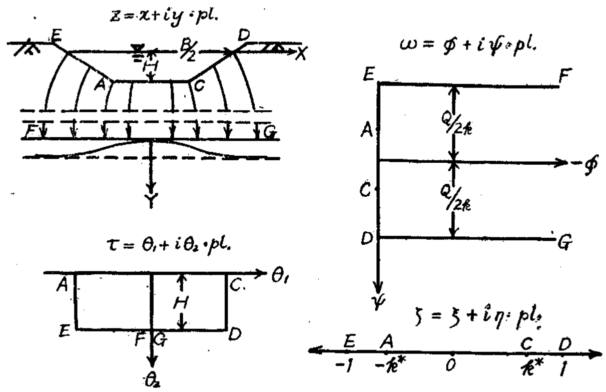

Fig. 3 The seepage out of a ditch through the ground with a deeplying water table and its transformation.

$$B = \frac{Q}{k} - 2H\frac{K}{K'},$$

$$B_1 = 2|x|_{y=\infty} = \frac{Q}{k} = B + 2H\frac{K}{K'}, \tag{11}$$

where K, K' are the complete elliptic integrals of the first kind with moduli $k^*, \sqrt{1-k^{*2}}$. When $K=K'$, namely, $k^* = \pm\frac{1}{\sqrt{2}}$, Eq. (11) is identical with Eq. (6).

3. Test measurements and results.

(1) Method of measurement.

Two test pools A, B with cross sections as shown in Fig. 4 were dug in the Kantō loam at the site of the Asamizo depositing reservoir, Kana-

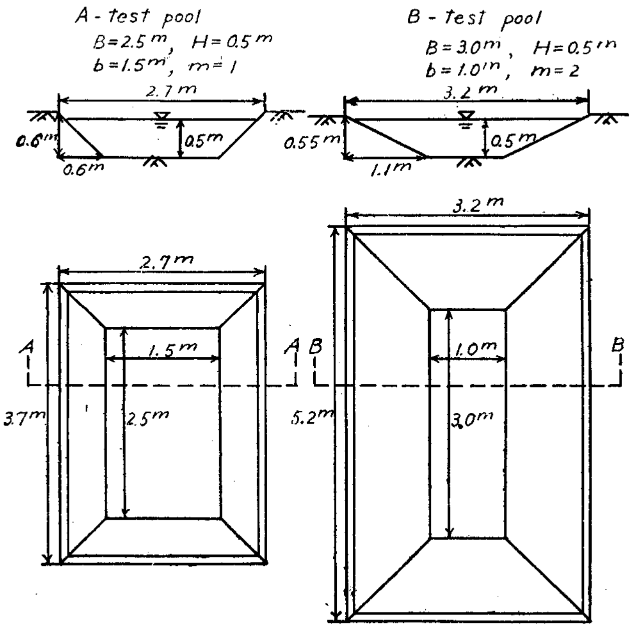

Fig. 4 Dimensions of test pools.

gawa Prefecture. Water was continuously poured in to maintain the water level of the pools always at 50 cm. The quantity of water seeping from the pools was measured in the following way: a bucket with a capacity of 10 liters was used for the measurement and the time required to fill it was taken. The seepage flow in this case is a three dimensional flow and diffuses on all sides from the pools. Next the pools were enlarged by digging several times in a direction perpendicular to the cross section, and by substracting the volume of water seeping from the original test pool from that of the enlarged one, the effect of the flow near the both ends of the section was eliminated. In this manner, Q, the volume of water flowing out from the section and the unit length of horizontal depth, was obtained. The flow of which the effect of the ends is elimenated in this way can be considered a two dimensional flow, and consequently the formulae derived from the fundamental theoretical equations stated in **2.** are applicable. The extent and the number of times of the enlargement of the pools were 2m twice for A and 4m once for B, and the difference in each case was taken.

(2) Results of measurement.

a) A test pool $\left(B=2.5\text{ m},\ H=0.5\text{ m},\ b=1.5\text{ m},\ m=\frac{B-b}{2H}=1\right)$

Average seepage discharge of the original pool	19.5 cc/sec	difference
		19.4 cc/sec,
When enlarged once	38.9 cc/sec	
		18.2 cc/sec,
When enlarged twice	57.1 cc/sec	
		average 18.8 cc/sec.

Average discharge from section A–A per unit horizontal depth in cm,

$$Q_A = 18.8 \div 200 = 0.094 \text{ cc/sec.}$$

b) B test pool $\left(B=3.0\text{ m},\ H=0.5\text{ m},\ b=1.0\text{ m},\ m=\frac{B-b}{2H}=2\right)$

Average seepage discharge of the original pool	112.2 cc/sec	difference
When enlarged	149.2 cc/sec	37.0 cc/sec.

Average discharge from section B–B per unit horizontal depth in cm,

$$Q_B = 37.0 \div 400 = 0.092 \text{ cc/sec.}$$

(3) Application of theoretical equations.

Strictly speaking, as the test pools A, B used for this measurement are, for the convenience of digging, both of a simple straight line section, the method explained in **2.** (3) must be applied, but in view of a proposition that a slight change in the shape of the pool does not obstruct the application of the formulae from which the simple result given in **2.** (2) is obtained, the coefficient of permeability k will be calculated adversely from Eq. (6) or (10). In this case, the selection between the two equations depends upon the direction of diffusion of the seepage flow governed by the depth of the underground water table. In this test measurement, the depth of the loam layer in which the test pools were dug is about 15–16 meters judging from the result of boring test, and it is known that the underground water level exists in gravel below the loam. As it clear when observed in a side trench built under the pool bottom, it is confirmed that the seepage water flows perpendicularly downwards but not horizontally, but to make sure, k will be computed using Eqs. (6) and (10), both giving the width B of the test pools. Thus from Eq. (6),

$$k_1 = \frac{Q}{B+2H}, \tag{6'}$$

and from Eq. (10),

$$k_2 = \frac{Q}{B-2H}. \tag{10'}$$

Applying the result obtained by measurement to the above equations, from Eq. (6′),

$$k_{1A} = \frac{Q_A}{B+2H} = \frac{0.094}{350} = 2.68\times10^{-4}\,\text{cm/sec},$$

$$k_{1B} = \frac{Q_B}{B+2H} = \frac{0.092}{400} = 2.30\times10^{-4}\,\text{cm/sec}, \quad \text{average } k_1 = 2.49\times10^{-4}\,\text{cm/sec},$$

and from Eq. (10′),

$$k_{2A} = \frac{Q_A}{B-2H} = \frac{0.094}{150} = 6.26\times10^{-4}\,\text{cm/sec},$$

$$k_{2B} = \frac{Q_B}{B-2H} = \frac{0.092}{200} = 4.60\times10^{-4}\,\text{cm/sec}, \quad \text{average } k_3 = 5.43\times10^{-4}\,\text{cm/sec}.$$

Therefore, it is found that the coefficient of permeability k of the Kantō loam layer with which the measurement was done is within the following values:

$$2.49\times10^{-4}\,\text{cm/sec} \leqq k \leqq 5.43\times10^{-4}\,\text{cm/sec}.$$

4. Consideration on the results found by measurement.

(1) Verification of the order of the coefficient of permeability.

Judging from the formation of the Kantō loam in this district, it has been expected that the coefficient ot permeability lies between "permeable" and "impermeable", namely, the order of 10^{-4} cm/sec. The result obtained by the permeability computation method based upon the rate of water level depression with the elapse of time gives $k=4.94\times10^{-4}$ cm/sec, which is clearly within the range of the abovementioned k.

(2) Observation of the width of the sheet of a seepage flow and its change with time at a great depth below the bottom of the pool.

In order to examine how water seeping from the test pool C in Fig. 5 diffused at a great depth underground and how the diffusion changed with time, the width of diffusion after water had been poured into the pool (including diffusion due to capillarity) was observed in a horizontal side trench connected with a vertical shaft 8 m deep. The result was as stated above, namely, it was found that as the Kantō loam was thick and the underground water level was

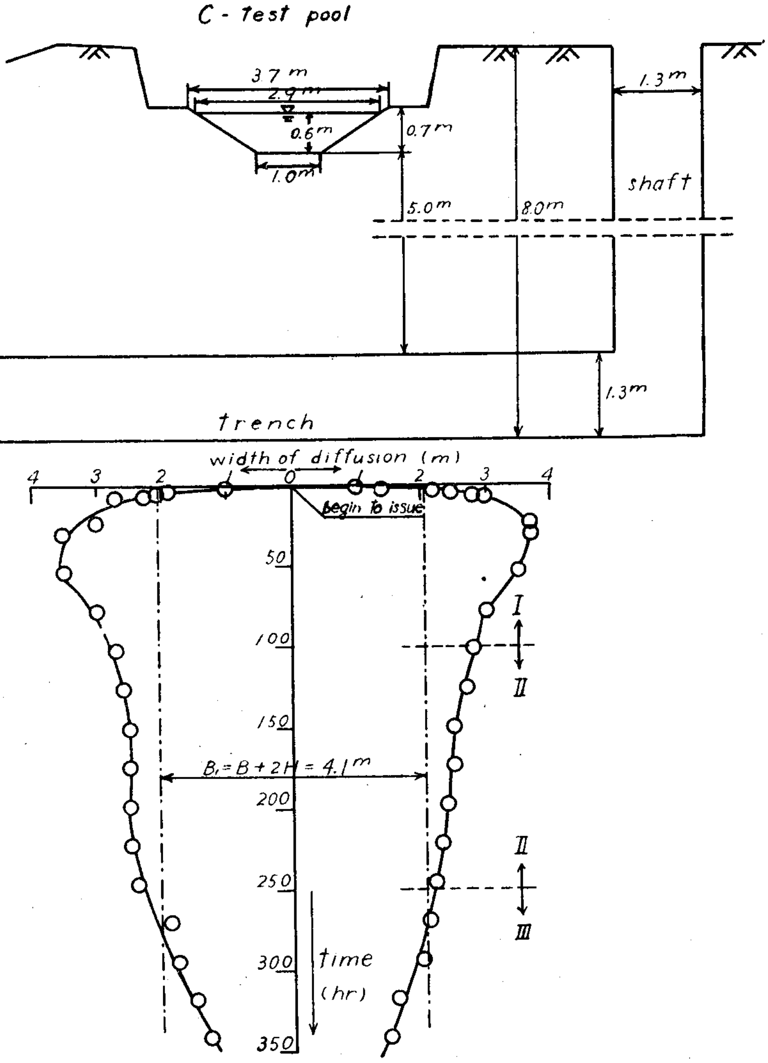

Fig. 5 Verification of the width of the sheet of the seepage and its change with time.

quite deep at this place, the free stream lines of water seeping from the pool had vertical asymptotes.

As is clear from Fig. 5, the change in the width of the sheet of a seepage flow with the elapse of time can be divided roughly into three steps. Namely, from the time when water begins to issue into the trench till about 100 hours later (I), the seepage water trends to diffuse horizontally due to the capillarity of the ground. During the next 150 hours (II), it continues to flow vertically downwards, becoming almost a stationary flow. From then on (III), the flow concentrates in the center of the pool bottom due to the primary piping action, that is, the width of the sheet diminishes with the elapse of time.

The theoretical width of the seepage calculated by Eq. (6) which is applicable to the step (II) corresponds to the range shown by the chain line in Fig. 5. This differs slightly from the result of actual measurement. It is considered that the discrepancy is due to the capillarity of the ground and the difference in the shape of the cross section of the test pool; if these differences are eliminated, the theoretical and the experimental results were be closer to each other, and it may be admitted that the accuracy of this method of measurement is high.

5. Conclusion.

In this paper, in order to measure the coefficient of permeability of loam or other soil which has a comparatively medium permeability, a new method has been proposed: a very simple field pool is made and enlarged at the site of an earth dam or an ordinary dam for the purpose of eliminating many great defects accompanying the conventional measuring method used in the laboratory using a small quantity of a sample. Thus, a rational method of finding the coefficient of permeability of soil has been obtained through the hydraulic anylysis formerly considered difficult. In order to verify the propriety of this method, the coefficient of permeability of the Kantō loam layer at the Asamizo depositing reservoir of the Yokohama Municipal Water Supply Bureau has been found, and considering the verification of its order and the adaptability of the fundamental theoretical formulae, an accurate result has been obtained.

This research was projected by Matsuo and carried out by the both. In connection with this research, their gratitude must be expressed to the Grant in Aid for Fundamental Scientific Research and to the efforts of Mr. Ichirō Ikeda, member of the Yokohama Municipal Water Supply Bureau, who has cooperated with them in this field experiment.

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