On the Laws of Resistance to Turbulent Flow in Open Smooth Channels

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Synopsis It is generally considered that differing from the pipe flow, the flow in open channels has a free surface and waves appearing on water surface relate to some extent with the laws of resistance on turbulent flow in open channels. According to the above opinion, instability of flow is connected with the mixing length of turbulence, and computing the velocity distribution by Prandtl's equation which expresses the flow near the wall, Froude Number is introduced with Reynolds Number in the laws of reistance on turbulent flow in open channels. Applying this theory, experimental results by authors, Dr. Matuo and R. W. Powell with smooth open channels can be explained within the region of sub- and supercritical flow, and, therefore, the difference between the laws of resistance to turbulent flow in pipes and that in open channels can be made clear.

1. Introduction

Since Prandtl and Kármán proposed the logarithmic law as the velocity distribution of turbulent flow in circular pipes, many authorities have attempted to explain the results of the experiment with turbulent flow in open channels by applying the same logarithmic law. It is especially worthy of note that G. H. Keulegen[1] added $-\bar{\varepsilon}\, u_m/u_R{}^*$ as the effect of the free surface and the nonuniformity of the shear force on the wall and β/κ as the effect of the shape of the cross section to the equation of logarithmic law for turbulent flow in circular pipes and expressed the characteristics as an open channel. However, expressing the effect of the free surface in this form is intuitive and is actually neglected as being a small value. Later when R. W. Powell[2] analysed the result of his experiment using the equation expressed by Keulegan, he also put $\bar{\varepsilon}=0$. In the paper[3,4] he published thereafter, he assumed $\bar{\varepsilon}=-0.208$. According to Keulegan the additional term for the shape of the cross section is

$\beta=\log_e\{1+(2h/B_0)\}$ when a rectangular section. For example, as $\beta=0.098$ when the ratio of the hydraulic mean depth R, to the width of the channel B_0 $R/B_0=0.1$, then $\beta/\kappa=0.245$ if $\kappa=0.4$, which is negligibly small compared with the accuracy of the experiment. The value of $\bar{\varepsilon}$, however, cannot be theoretically appreciated as β can be done. In this paper the instability of the flow is considered as the effect of the free surface and connecting it with the mixing length of turbulence, the velocity distribution is calculated by the same method as J. Rotta[5], W. Szablewski[6] and an attempt is made to introduce Froude Number into the equation of the logarithmic law by using Prandtl's equation expressing the flow near the wall.

2. Mixing Length and Logarithmic Law

L. Prandtl and Th. v. Kármán[7] assumed that near the wall mixing length l is proportional to distance y from the wall surface in the case of a turbulent flow in smooth circular pipes and putting $l=\kappa y$, deduced the following logarithmic law of the velocity distribution.

$$\frac{u}{u^*}=A_s+\frac{1}{\kappa}\log_e\frac{u^*y}{\nu}, \qquad (1)$$

where u is the velocity at distance y from wall surface, ν the kinematic viscosity of fluid, u^* the friction velocity which is equal to $\sqrt{\tau_0/\rho}$, τ_0 the frictional stress on wall surface and ρ the density of fluid. It was found experimentally that the value of A_s in Eq. (1) is a constant value 5.5 for turbulent flow in smooth circular pipes. A_s, however, is not a constant value for turbulent flow in smooth open channels, it being clarified by experiments[8] by the authors that it decreases with the increase of the slope of the channel. The fact that the value of A_s in the case of supercritical flow changes remarkably in comparison with the case of subcritical flow is well recognized in the experiments by Powell[2] and Homma[9,10]. It is considered that this fact is the radical difference between the flow in circular pipes without free surface and that in open channels with free surface.

Now it will be assumed that due to the effect of the free surface the mixing length increases or decreases as follow when compared with that of the turbulent flow in smooth circular pipes. When it increases, it is as shown by the real line in Fig. 1, that is;

for $\delta_L<y<100\nu/u^*$:

$$l=\kappa'(y-\delta_L), \qquad (2)$$

where $$\kappa' = \kappa + \left(\frac{u^* l_w}{\nu}\Big/ 100 - \frac{u^*\delta_L}{\nu}\right).$$

for $100\nu/u^* < y$:

$$l = \kappa(y-\delta_L) + l_w, \tag{3}$$

where l_w, which is constant in y-direction, is the amount of increase of the mixing length due to the effect of free surface in the region $y > 100\nu/u^*$, and δ_L is the thickness of laminar sublayer. On the other hand, when it decreases, as

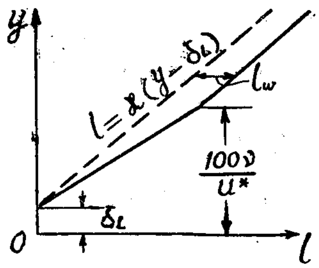

Fig. 1

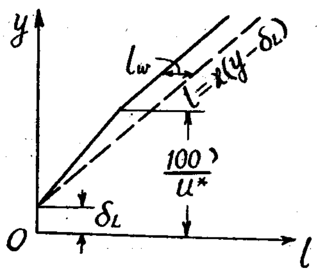

Fig. 2

shown by the real line in Fig. 2, the values of l_w in Eqs. (2) and (3) are put negative.

From the momentum transfer theory proposed by Prandtl, frictional stress is expressed as

$$\tau = \rho\left(\nu + l^2\left|\frac{du}{dy}\right|\right)\frac{du}{dy}. \tag{4}$$

If τ is put equal to frictional stress τ_0 on the wall, Eq. (4) becomes

$$\frac{du}{dy} = \frac{\sqrt{4l^2u^{*2}+\nu^2}}{2l^2} + \frac{\nu}{2l^2}. \tag{5}$$

If the above equation is integrated using Eq. (2) and the integral constant determined from the condition of $u = u^{*2}\delta_L/\nu$ at $y=\delta_L$, then the following is obtained for $\delta_L < y < 100\nu/u^*$.

$$\frac{u}{u^*} = \frac{1}{\kappa'}\log_e\left(2\xi + 2\sqrt{\xi^2+\frac{1}{4}}\right) + \frac{1}{\kappa'\xi}\left(\frac{1}{2} - \sqrt{\xi^2+\frac{1}{4}}\right) + \frac{u^*\delta_L}{\nu}, \quad \xi = \frac{u^* l}{\nu}. \tag{6}$$

If Eq. (5) is integrated using Eq. (3) and the integral constant determined from the condition $\xi = \xi_0$ and

$$\frac{u}{u^*} = \frac{1}{\kappa'}\log_e\left(2\xi_0 + 2\sqrt{\xi_0^2+\frac{1}{4}}\right) + \frac{1}{\kappa'\xi_0}\left(\frac{1}{2} - \sqrt{\xi_0^2+\frac{1}{4}}\right) + \frac{u^*\delta_L}{\nu}$$

at $y = 100\nu/u^*$, then

$$\frac{u}{u^*} = \frac{1}{\kappa}\log_e\frac{\xi + \sqrt{\xi^2+(1/4)}}{\xi_0 + \sqrt{\xi_0^2+(1/4)}} + \frac{1}{\kappa\xi}\left(\frac{1}{2} - \sqrt{\xi^2+\frac{1}{4}}\right) + \frac{1}{\kappa'}\log_e\left(2\xi_0 + 2\sqrt{\xi_0^2+\frac{1}{4}}\right)$$
$$-\left(\frac{1}{\kappa} - \frac{1}{\kappa'}\right)\frac{1}{\xi_0}\left(\frac{1}{2} - \sqrt{\xi_0^2+\frac{1}{4}}\right) + \frac{u^*\delta_L}{\nu}, \tag{7}$$

where $$\xi_0 = 100\kappa + \xi_w - \frac{\kappa u^* \delta_L}{\nu}, \quad \xi_w = \frac{u^* l_w}{\nu}.$$

For the value of large y, the following is derived as A_s in Eq. (1) from Eq. (7),

$$A_s = \frac{1}{\kappa}(\log_e 4\kappa - 1) - \left(\frac{1}{\kappa} - \frac{1}{\kappa'}\right)\log_e\left(2\xi_0 + 2\sqrt{\xi_0^2 + \frac{1}{4}}\right)$$
$$-\left(\frac{1}{\kappa} - \frac{1}{\kappa'}\right)\frac{1}{\xi_0}\left(\frac{1}{2} - \sqrt{\xi_0^2 + \frac{1}{4}}\right) + \frac{u^*\delta_L}{\nu}. \qquad (8)$$

As $\kappa = \kappa'$ when there is no effect of the free surface, i.e. when $l_w = 0$, Eq. (8) becomes the same as the following equation deduced by J. Rotta.[5]

$$A_s = \frac{1}{\kappa}(\log_e 4\kappa - 1) + \frac{u^*\delta_L}{\nu}.$$

If experimental values in pipe flow $A_s = 5.5$ and $\kappa = 0.4$ are taken, the value of $u^*\delta_L/\nu = 6.83$ is obtained. Therefore, Eq. (8) becomes

$$A_s = 5.5 - \left(\frac{1}{\kappa} - \frac{1}{\kappa'}\right)\log_e\left(2\xi_0 + 2\sqrt{\xi_0^2 + \frac{1}{4}}\right)$$
$$-\left(\frac{1}{\kappa} - \frac{1}{\kappa'}\right)\frac{1}{\xi_0}\left(\frac{1}{2} - \sqrt{\xi_0^2 + \frac{1}{4}}\right), \qquad (8)'$$

and ξ_0 and κ' become respectively

$$\left.\begin{aligned} \xi_0 &= \xi_w + 37.27, \\ \kappa' &= 0.4 + 0.01073\xi_w. \end{aligned}\right\} \qquad (9)$$

Regarding the value of κ in open channels, $\kappa = 0.4$, the same as in pipe flow, may be used as can be understood from the experiments by the authors[8] and by Hosoi[11] and from Fig. 7.

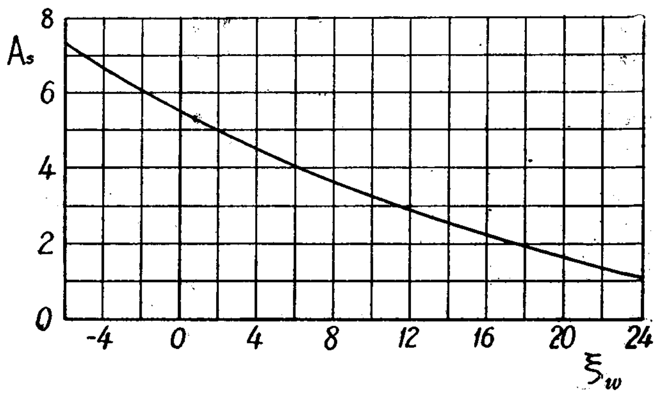

Fig. 3 Relation between A_s and ξ_w

As is clear from Eqs. (8)' and (9), A_s is a function of ξ_w only. This relation is shown in Fig. 3, from which the relation between A_s and the amount of increase or decrease of the mixing length due to the effect of the free surface can be understood.

3. Instability of Flow and Mixing Length

Now consider a two-dimensional flow on a slope surface. Take x-axis in the downstream direction along the bottom surface, y-axis vertically upwards from the bottom surface and put depth: h, time; t, slope angle: α, mean velocity: u_m, gravity acceleration: g and $\alpha_m=(1/h)\int_{\delta_L\to 0}^{h}(u/u_m)^2dy$. Then generally the momentum equation becomes[12]

$$u^{*2}=gh\sin\alpha-gh\cos\alpha\frac{\partial h}{\partial x}-\alpha_m\frac{\partial}{\partial x}(u_m{}^2h)-\frac{\partial}{\partial t}(u_mh)\,.$$

Putting $u_m/u^*=C_m$, the above equation is expressed as follows.

$$\frac{u_m{}^2}{C_m{}^2}=gh\sin\alpha-gh\cos\alpha\frac{\partial h}{\partial x}-\alpha_m\frac{\partial}{\partial x}(u_m{}^2h)-\frac{\partial}{\partial t}(u_mh)\,. \tag{10}$$

The equation of continuity is

$$\frac{\partial h}{\partial t}+\frac{\partial}{\partial x}(u_mh)=0\,. \tag{11}$$

In order to simplify the treatment of the equation, assume C_m=const. and put $u_m=u_{m0}+u_m'$, $h=h_m+h'$ when u_m' and h' respectively represent the small variations in the mean velocity and water depth caused by small disturbance. Then the following equation is derived from Eqs. (10) and (11) by neglecting the small terms.

$$\begin{aligned}(gh_m\cos\alpha-\alpha_mu_{m0}^2)\frac{\partial^2h'}{\partial x^2}-2\alpha_mu_{m0}\frac{\partial^2h'}{\partial x\partial t}-\frac{\partial^2h'}{\partial t^2}\\ -\left(g\sin\alpha+\frac{2u_{m0}^2}{C_m{}^2h_m}\right)\frac{\partial h'}{\partial x}-\frac{2u_{m0}}{C_m{}^2h_m}\frac{\partial h'}{\partial t}=0\,.\end{aligned} \tag{12}$$

Since Eq. (12) is always a hyperbolic form, it can be solved by means of Riemann's integral. An attempt has also been made to solve it by operational method[13]. In this paper, for the purpose of simplifying the treatment, if Eq. (13)

$$h'=A_0\exp(\gamma t-i\beta x) \tag{13}$$

expressing a small disturbance of which the initial condition is a simple harmonic form is considered, Eq. (13) satisfies Eq. (12). Putting $\gamma=\gamma_1+i\gamma_2$, when γ is divided into real and imaginary parts, the following equation for γ_1 is obtained from Eqs. (12) and (13).

$$4\gamma_1^4+8a\gamma_1^3+(5a^2+b^2+4c)\gamma_1^2+(a^3+ab^2+4ac)\gamma_1+a^2c+abd-d^2=0, \qquad (14)$$

where $a=2u_{m0}/C_m^2h_m$, $b=2\alpha_m u_{m0}\beta$, $c=\beta^2(gh_m\cos\alpha-\alpha_m u_{m0}^2)$, $d=3u_{m0}^2\beta/C_m^2h_m$. In the following u_{m0}, h_m will be denoted as u_m, h, respectively. If the orders of the coefficients of the terms in Eq. (14) are compared, assuming $u_m=200cm/s$, $C_m=15$, $h=1$cm, $\alpha_m=1$, $\beta=1\text{cm}^{-1}$, $\cos\alpha=1$ and $g=980\text{cm/s}^2$, the coefficients of the 1st, 2nd, 3rd, 4th and 5th terms respectively become 4, $8a\doteqdot1.4\times10$, $5a^2+b^2+4c\doteqdot3.9\times10^3$, $a^3+ab^2+4ac\doteqdot7\times10^3$ and $a^2c+abd-d^2\doteqdot-1.2\times10^5$. Therefore, for $|\gamma_1|<5$, an approximate calculation is possible within an error of several percent, neglecting the 1st and 2nd terms of Eq. (14). As understood below, this condition is satisfied when Froude Numder is less than 10. Thus γ_1 becomes as follows, neglecting the 1st and 2nd terms of Eq. (14).

$$\gamma_1=\frac{u_m}{C_m^2h}\left\{\sqrt{1+\frac{(9-8\alpha_m)F^2-1}{\alpha_m(\alpha_m-1)F^2+1}}-1\right\}, \qquad (15)$$

where $F=u_m/\sqrt{gh\cos\alpha}$.

If α_m is computed using Eq. (1),

$$\alpha_m=1+(1/\kappa)^2(u^*/u_m)^2=1+6.25(u^*/u_m)^2.$$

Introducing this into Eq. (15) gives,

$$\gamma_1=\frac{u_m}{C_m^2h}\left[\sqrt{1+\frac{\{1+50(u^*/u_m)^2\}F^2-1}{6.25(u^*/u_m)^2\;F^2+1}}-1\right]. \qquad (15)'$$

According to this theory, when a disturbance is caused, the wave height of the disturbance increases with the time if γ_1 is positive and dampens if γ_1 is negative. If the wave height becomes large in comparison with the depth, this theory no longer applies. The wave height, however, actually neither becomes infinitely large nor zero, but becomes stable after attaining a certain magnitud. Rollwavetrains can be mentioned as one of the most distinguished examples[12].

If t_0 is the time required for the wave height to become constant after a disturbance is produced, t_0 may be considered as being proportional to water depth h and contrarily proportional to u^*. Since $h/u^*=\sqrt{h/g\sin\alpha}$, t_0 becomes small when the water depth is small and the slope steep. This is a fact obviously in the experiment on rollwave-trains.

Next, regarding $\xi_w=u^*l_w/\nu$, it may be considered that $l_w=0$, viz. $\xi_w=0$ as follows, when $\gamma_1=0$. $\gamma_1=0$ means the criterion which the height of the given disturbing wave theoretically neither dampens nor increases. That is, in this case in order to maintain this wave height, no energy is given to the wave and no energy which the wave hold decreases, and it is considered that the free surface has absolutely no effect on the mixing length.

Since the dimension of γ_1 is $[T]^{-1}$, $\gamma_1 t_0$ becomes dimesionless, and from the correspondence of $\gamma_1=0$ and $\xi_w=0$, ξ_w will be assumed as being proportional to $\gamma_1 t_0$. Furthermore, as mentioned above, if t_0 is assumed as proportional to h/u^*, the following Eq. (16) and (17) are obtained from Eq. (15)′, taking K as proportional constant.

$$\xi_w = K\Phi(u_m/u^*, F), \tag{16}$$

$$\Phi(u_m/u^*, F)=\frac{u^*}{u_m}\left[\sqrt{1+\frac{\{1-50(u^*/u_m)^2\}F^2-1}{6.25(u^*/u_m)^2\ F^2+1}}-1\right]. \tag{17}$$

The reletion between Φ and F, taking u_m/u^* as the parameter, is shown in Fig. 4. Therefore, if K is determined from experimental results, the relation between A_s and F and u_m/u^* can be obtained from Eqs. (8)′ and (16).

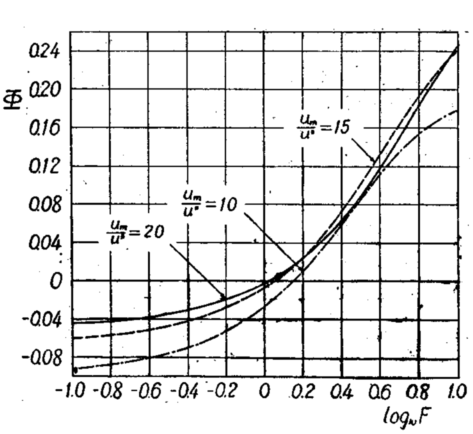

Fig. 4 Reletion between Φ and F and u_m/u^*

4. Determination of K from Experimental Results and Laws of Resistance

The authors[8] once performed experiments on the thin sheet flow for slopes between 0.0021 and 0.024, using a rectangular section planed wood flume 40 cm wide, 19 cm deep and about 18 m long (effective length 10m), but further experiments* were performed for slopes ranging from 0.025 to 0.222, using a rectangular section planed wood flume 20 cm wide, 10 cm deep and about 5 m long (effective length 3.8 m).

Fig. 5 shows an example of the water surface profile measured along the center of flume, the water depth becoming nearly constant from a distance of

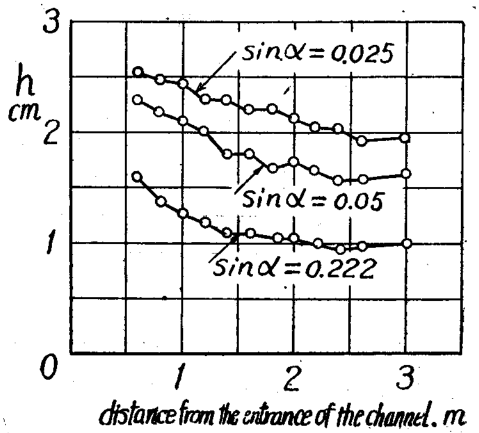

Fig. 5 An example of the water surface profile

* The expeiments were mainly performed by Mr. Y. Ishihara, Postgraduate Student.

about 2.5 m downstream from the entrance of the channel. As this depth is considered the normal depth of uniform flow, the water depth and velocity distribution are measured at a distance 3 m downstream from the entrance of the channel. Since the experiment is three-dimensional, if $u_R^*=\sqrt{gR\sin\alpha}$ using hydraulic mean depth instead of water depth h is taken as the friction velocity and $F_R=u_m/\sqrt{gR\cos\alpha}$(Boussinesq Number[14]) taken as Froude Number for the sake of convenience, Eq. (17) obtained as two-dimension, becomes

$$\Phi(u_m/u_R^*,F_R)=\frac{u_R^*}{u_m}\left[\sqrt{1+\frac{\{1-50(u_R^*/u_m)^2\}F_R^2-1}{6.25(u_R^*/u_m)^2\ F_R^2+1}}-1\right]. \qquad (17)'$$

The values of A_s and Φ in Eq. (17)′ computed for $u_R^*R/\nu>100$ from the results of experiments by authors, Dr. H. Matsuo[15] and R. W. Powell[2] are plotted in Fig. 6. The real line in this figure represents the curve obtained from Eqs. (8)′ and (17)′ for $K=90$. It is difficult to determine a suitable value for K, because for the reason explained below the experimental values are scattered widely, but the value of 90 may be considered suitable judging from the experimental results. In Fig. 6 the results of experiments by Dr. Matsuo are distinguished as (1), (2) and (3), which respectively correspond to the results obtained using flumes 70 cm, 35 cm and 20 cm wide.

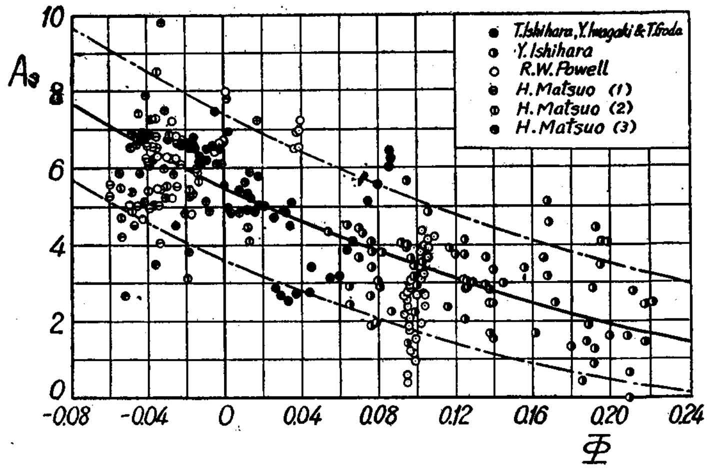

Fig. 6 Relation between A_s and Φ.

Now, as the value of K is determined, the value of ξ_w is computable from Eqs. (16) and (17) if the values of u_m/u^* and F are given. Thus, as the values of ξ_0 and κ' are determined from Eq. (9), the velocity distribution is obtained from Eqs. (6) and (7). Velocity distribution curves computed by assuming the value of ξ_w properly and the experimental results obtained by the authors are

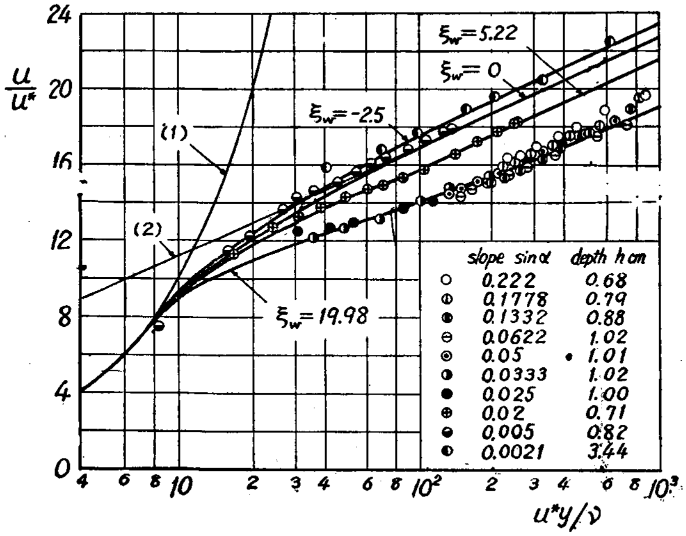

Fig. 7 An example of the velocity distribution

shown in Fig. 7. Although it will be understood from the accuracy of experiments explained below, values of ξ_w obtained experimentally shown in the figure generally do not coincide with the values of ξ_w used in the computation. However, the manner in which the change takes place from the turbulent region through the transition region to the velocity distribution curve in the region of laminar sublayer agrees quite well, substantiating the fact that the assumption of Eqs. (2) and (3) is almost correct. (1) and (2) in the figure respectively represent curve $u/u^*=u^*y/\nu$ and straight line $u/u^*=5.5+5.75\log_{10}u^*y/\nu$.

As is clear from Fig. 4, the values of Φ for cases $u_m/u^*=15$ and 20 show no great difference except when F is small. In Fig. 8, the results of experiments and computation are shown taking the logarithm of F_R along the abscissa and A_s along the ordinate. For $F_R>1$, the curves of $u_m/u_R^*=15$ and 20 almost coincide. From the results of computation shown in Fig. 8, the experimental formulas will be expressed simply as

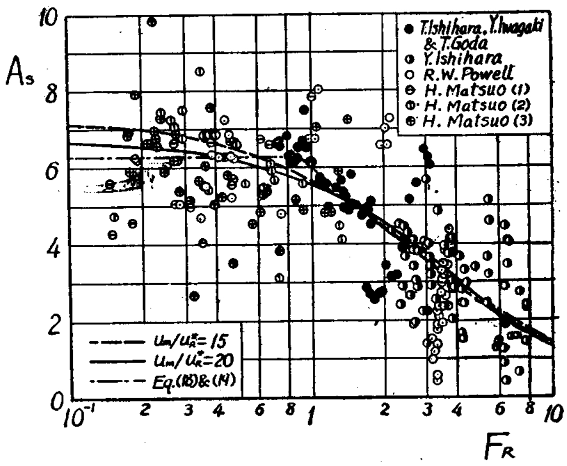

Fig. 8 Relation between A_s and F_R

$$A_s = 6.3 \tag{18}$$

for $F_R \leqq 0.89$, and

$$A_s = 6.0 - 5.75 \log_{10} F_R + 1.2(\log_{10} F_R)^2 \tag{19}$$

for $F_R \geqq 0.89$. The chain line in Fig. 8 obtained by Eqs. (18) and (19) agrees with the theoretical curve except in the region of small values of F_R. If Eq. (19) is transformed,

$$A_s = 6.0 - 5.75 \log_{10} \frac{u_m}{u_R^*} - 5.75 \log_{10} \sqrt{J} + 1.2(\log_{10} \frac{u_m}{u_R^*} \cdot \sqrt{J})^2, \tag{20}$$

where $J = \sin \alpha$. Thus A_s is expressed as a function of u_m/u_R^* and slope J. Fig. 9 shows the relation between A_s and J for the cases of $u_m/u_R^* = 15$ and 20. The chain line in the figure represents

$$A_s = -1.1 - 5.75 \log_{10} \sqrt{J}. \tag{21}$$

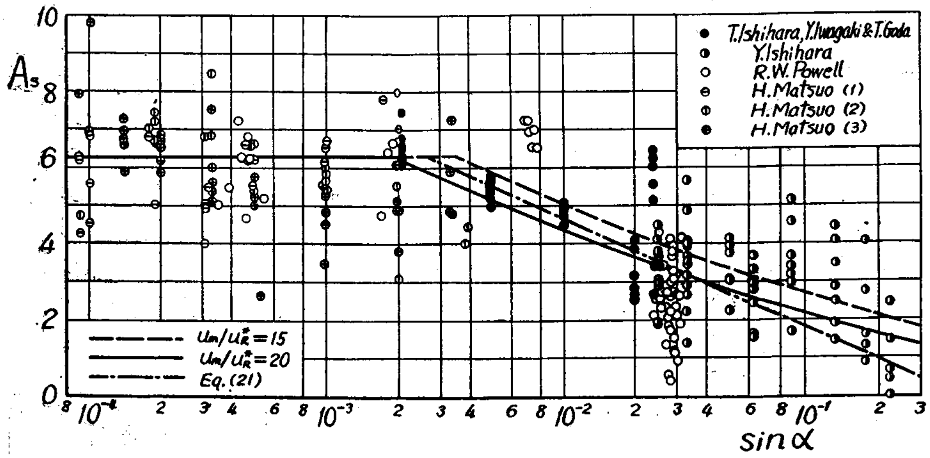

Fig. 9 Reletion between A_s and J

This represents the experimental formula proposed in the previous paper[8] and corresponds to the case of $u_m/u_R^* = 17.17$ when the last term of the right side of Eq. (20) is neglected.

Furthermore, if λ is the coefficient of frictional resistance and $\tau_0 = \lambda \rho u_m^2/2$, then $\lambda = 2\ (u_R^*/u_m)^2$, so from Eqs. (18) and (19) and the equation of mean velocity derived from Eq. (1), the following equation is obtained.

$$F_R \leqq 0.89;\ 1/\sqrt{\lambda} = 2.07 + 4.07 \log_{10}(R_e \cdot \sqrt{\lambda}), \tag{22}$$

$$F_R \geqq 0.89;\ 1/\sqrt{\lambda} = 1.861 - 4.07 \log_{10} F_R + 0.849(\log_{10} F_R)^2 + 4.07 \log_{10}(R_e \cdot \sqrt{\lambda}). \tag{23}$$

Fig. 10 shows the relation between F_R and λ, the coefficient of frictional resistance due to Eqs. (22) and (23), using Reynolds Number $R_e = u_m R/\nu$ as the parameter.

R. W. Powell[15] used the experimental data on ultra-rapid flow from the experiment he performed and proposed an experimental formula containing Vedernikov Number, a parameter with which the instability of flow is appreciated, and Froude Number. His method of introducing these two factors is not analytical, but his idea is very interesting in comparison with the author's analysed results.

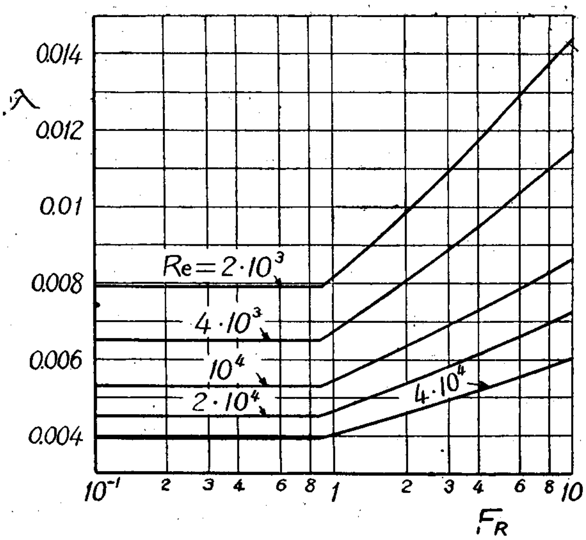

Fig. 10 Relation between λ and F_R

5. Accuracy of Experiment

As is clear from Figs. 6, 8 and 9, experimental results are scattered widely. This reason will be considered from the point of view of accuracy of experiment.

To make it simple, the two dimensional case will be discussed. Putting the discharge per unit width as Q, the equation of mean velocity is expressed as follows.

$$\frac{Q/h}{\sqrt{ghJ}} = A_s - \frac{1}{\kappa} + \frac{1}{\kappa}\log_e\frac{\sqrt{ghJ}h}{\nu}. \tag{24}$$

Differentiating the above equation assuming Q and J as constant gives,

$$\frac{dA_s}{A_s} = -\frac{3}{2}\left(1+\frac{1}{A_s}\frac{1}{\kappa}\log_e\frac{u^*h}{\nu}\right)\frac{dh}{h}. \tag{25}$$

Assuming $(1/\kappa)\log_e u^*h/\nu = 20$ and $dh/h = \pm 0.05$, that is an error of 5% exists in the water depth, Eq. (25) becomes

$$dA_s = \mp(0.075A_s + 1.5). \tag{26}$$

The chain lines in Fig. 6 represent Eq. (26).

If Eq. (25) is differentiated assuming Q and h as constant, it becomes

$$\frac{dA_s}{A_s} = -\frac{1}{2}\left(1+\frac{1}{A_s}\frac{1}{\kappa}\log_e\frac{u^*h}{\nu}\right)\frac{dJ}{J}. \tag{27}$$

Now consider the case of a gentle slope and put $(1/\kappa)\log_e u^*h/\nu=14$, $A_s=7$, then Eq. (27) becomes

$$dA_s/A_s = -1.5\,dJ/J\,, \tag{28}$$

and the error of A_s becomes one and a half times the error of the slope J.

When the slope is steep, namely F_R is large, an error of 5% is very likely to occur in the water depth due to the remarkable variation of the free surface and the difficulty of obtaining the normal depth of uniform flow, and when the slope is gentle, namely F_R is small, it may be expected that the error of the slope is comparatively large and from Eq. (28) the error of A_s is considerably large. Although all experimental results are more or less widely scattered, this can be explained quite satisfactorily with the above consideration on accuracy of experiment.

Furthermore, as the bottom surface of the flume is not perfectly smooth, there arises doubt that due to the effect of the roughness, the values of A_s become small and get scattered. The maximum value of u_R^*/ν in the above mentioned experiments is 1465, so if the average height of unevenness is k and $k \lesssim 0.02$ mm, it can be regarded as being hydraulically smooth. Although the roughness of the flume made of planed Japanese cypress used in the experiments by the authors is not clear, it is thought that the condition $k \lesssim 0.02$ mm is probably almost satisfied. In the velocity distribution shown in Fig. 7, the condition of change in the transition region between the turbulent and the laminar sub-layer region is different from that of the case of rough surface[5]. It can also be conjectured from this fact that the change in A_s is not the result of the effect of the roughness of the bottom surface. Experiment by R. W. Powell is performed with a flume with a painted bottom surface, concerning which there is no question.

6. Conclusion

Connecting the instability of flow with the mixing length of turbulence has made it possible to introduce both Reynolds Number and Froude Number into the laws of resistance to turbulent flow in open channels. As mentioned at the beginning the effect of the shape of the cross section is very small compared with the accuracy of the experiments, and also a comparison of experimental results obtained with channels of various widths shown in the figure will show that the effect of the nonuniformity of the shear force on the wall is so small that the difference is difficult to be distinguished when compared with the accuracy of the experiment. Therefore, only the effect of the free surface produces a remarkable difference between the law of resistance of pipe and that of open channel. This is the reason why the effect of the free surface is chiefly discussed in this paper.

Concerning the instability of flow, investigation by graphical method[17] has also been made besides many other interesting researches. There are many problems regarding details of the relation between instability of flow and mixing length of turbulence. It is considered that together with the very problem of instability of flow, these are problems for future study.

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