Finding Air Quantities in the Mine Ventilation Networks by Making Use of the Calculation of an Electric Circuit

By

Yoshio HIRAMATSU

Department of Mining

(Received November, 1952)

Introduction

To foresee air quantity in every airway of ventilation networks is indispensable for reasonable planning of mine ventilation, but as it involves so troublesome mathematical treatment that rational design of improving ventilation of such mines as suffer from a lack of effective air quantity and necessitate an increase in it has been, until recent years, hardly possible.

Being interested in this respect, the author and Y. Oka, one of my collaborators, made inquiries to find some practicable ways of the solution, i.e. the ways to calculate each air quantity in the given networks, and obtained one of the methods which is essentially a trial and error method and has a characteristic of finding errors mathematically by applying the Taylor's expansion.[1] About the same time, Scott and Hinsley published, apart from us, another idea of a solution similar in principle to ours.[2] W. Maas proposed an experimental solution that enables to find air quantities by measuring currents on an electric circuit composed on purpose similar to the ventilation networks to be solved.[3] S. Batzel and W. Schmidt also wrote on this method.[4]

The former two methods at least succeeded in solving those complicated problems which had been thought impossible, yet they need considerably troublesome calculations and some intuition, so that they necessitate the specially trained operators who must work carefully for long hours, while the latter needs an apparatus of large scale and seems to give less accurate results. Under these circumstances, the writer has intended to find a possible method easier than those ever found.

The Proposition of a Term ρ, the Product of R and Q

The fundamental relations to be taken into account in solving ventilation networks are, without doubt, that the algebraic sum of air quantities in airways at any junction is zero, and that the algebraic sum of the pressure drops over any circuit of airways is equal to the ventilating pressure, i.e. the sum of the fan pressure and the natural ventilating pressure. These relations are like the Kirchhoff's law regarding electric circuits.

In general it is admitted that for any flow the relation among the pressure drop h, the quantity of flow Q and the resistance of passage R may be written as

$$h_i = R_i Q_i^n, \tag{1}$$

where i, being a representative of 1, 2, 3, ..., indicates the number of each passage, and n is a constant determined by the kind and state of the flow. For the underground air flow, it will not be a serious error to take n being equal to 2. In case of an electric circuit, $n=1$ because Ohm's law holds. Now for air flow, we can allow Q and h to have either plus or minus sign according to the direction of the air current, but we want to accept R to be always plus value, so that eq. (1) may be rewritten as:

$$h_i = R_i |Q_i| Q_i . \tag{2}$$

Putting

$$\rho_i = R_i |Q_i| , \tag{3}$$

we get

$$h_i = \rho_i Q_i . \tag{4}$$

Of course ρ is a term always positive as well as R. In an electric circuit, ρ coincides with the electric resistance itself, but in an air flow ρ is a new term proposed by the author of this paper for the use in the solution of ventilation networks.

The dimension of ρ for air flow is (M/L^4T). Provided that the specific resistances of all the airways and the ventilating pressure are known, ρ for every airway is determined, because R is a constant and Q is fixed for every airways, but the value of ρ is left unknown until the networks are solved.

The Proposed Method of Solution

This solution consists in finding air quantity and the value of ρ for each airway by successive approximation. At first assume values of ρ for all the airways by a manner described afterwards. Let them be ρ_i'. Then calculate air quantities of all the airways, each of them having the assumed values of ρ_i, ρ_i', supposing the true ventilating pressure is applied. This calculation is practised in the same manner as that of an electric circuit, because in this case the Kirchhoff's law for electric circuits holds also for this air circuit, if ρ_i are looked upon as electric resistances and the ventilating pressure as source voltage. Let air quantities obtained from this calculation be Q_i'. If all the values of ρ_i' were assumed correctly, Q_i' would be exact, but generally they are not, as

ρ_i' are nothing but assumed values. Therefore we proceed to the second stage of calculation. As to the secondary assumed values ρ, ρ'', adopt the values given by the equation below,

$$\rho_i'' = \tfrac{1}{2}(\rho_i' + R_i|Q_i'|). \qquad (5)$$

Of course the true ventilating pressure must be used in these calculation. Let air quantities from this calculation be Q_i''. In general Q_i' are nearer to the true values than Q_i' as explained later. Again for the tertiarily assumed values of ρ_i, ρ_i''', take:—

$$\rho''' = \tfrac{1}{2}(\rho_i'' + R_i|Q_i''|). \qquad (6)$$

With ρ_i''' carry out the same procedure. Such a calculation is repeated until each value of Q_i converges to a definite value, which is the true value required.

As for ρ_i', the primarily assumed values of ρ, though they may be taken at random, a nice selection is preferable since it affords rapid convergence in air quantities calculated. The ways proposed by the author are that, assuming at first the values of ρ_i are equal to the specific resistances of airways, air quantities q_i are found in the same manner as above, and then the products of R_i and $\sqrt{q_i}$ are taken for ρ_i'.

A Discussion on Convergence

The reason why the air quantity of each airway converges to each definite value in this method of solution will be treated here briefly. If ρ_c' is assumed slightly larger than the true value, the calculated Q_c' will be slightly smaller than the true value, so far as the pressure drop is true; here c means a certain number. Therefore the product $R_c|Q_c'|$ is slightly smaller than the true value of ρ. Thus the mean value of ρ_c' and $R_c|Q_c'|$ is supposed to be nearer to the true value than ρ_c'.

Strictly speaking, if dQ_c' is the error of air quantity of a certain airway due to $d\rho_c'$, the error of ρ_c' of this airway, and a true pressure drop appears on this airway, then from eq. (4) we get, in general form,

$$dh = 0 = \rho_c dQ_c + Q_c d\rho_c,$$

or

$$dQ_c = -\frac{Q_c}{\rho_c} d\rho_c.$$

Considering eq. (3),

$$\frac{Q_c}{\rho_c} = \pm R_c.$$

Here and hereafter the upper of the double signs is in case of $Q_c > 0$, while the lower is in case of $Q_c < 0$.

Thus

$$dQ_c = \mp \frac{1}{R_c} d\rho_c.$$

Since dQ_c' corresponds to $d\rho_c'$

$$dQ_c' = \mp\frac{1}{R_c}d\rho_c' . \tag{7}$$

Now from eq. (5) $$d\rho_c'' = \frac{1}{2}(R_c d|Q_c'| + d\rho_c'),$$

or $$d\rho_c'' = \frac{1}{2}(\pm R_c dQ_c' + d\rho_c').$$

Considering eq. (7), $$d\rho_c'' = \pm\left\{\pm R_c\left(\mp\frac{1}{R_c}d\rho_c'\right) + d\rho_c'\right\} = 0 .$$

Hence it is seen that ρ_c'' is the true value on the assumption that $d\rho_c'$ is small and the pressure drop is true. But as a matter of fact the pressure drop of each airway is not always expected to distribute correctly over each airway, although the true total pressure is given to the networks. Besides $d\rho_c'$ is not always small. Under these circumstances the mean value of ρ_c' and $R_c|Q_c'|$ can not be the true value of ρ_c, but it is not difficult to suppose that both ρ_c and Q_c will converge. The following example illustrates the process of solution by this method as well as how each air quantity converges as compared with Scott and Hinsley's method, and what is the influence of the selection of ρ_i' in the accuracy of this method of solution.

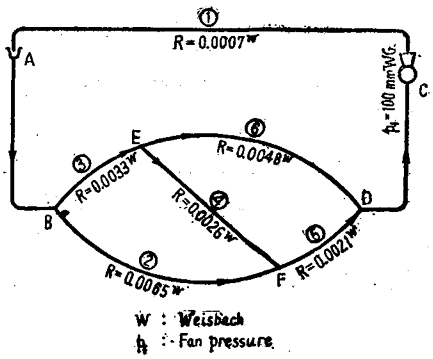

Fig. 1

The networks shown in Fig. 1 are the example adopted. *AB* is an intake airway, *CD* a return airway. At *C* a fan operates with a fan pressure of 100 mm water gauge. The process of calculation by this method is shown in Table 1. The first column shows the numbers of airways, the second column, their specific resistances. The third column shows q_i, the air quantities obtained assuming that R_i is equal to ρ_i; the fourth column $\sqrt{q_i}$; the fifth column Q_i'', and so on. The twelfth column shows air quantities obtained from the fourth stage of calculation. It is observed that Q_i'' are accurate enough. Q_i'''' are accurate to two places of decimals. Fig. 2 shows the variation of each calculated air quantity. For comparison, the variation of each air quantity calculated by Scott and Hinsley's method is plotted on Fig. 3. It is obviously seen that the author's solution gives much quicker convergence than the latter method. Fig. 4 illustrates the convergence of each air quantity in the author's method in case that the first assumed values ρ_i' are not taken as the products of R_i and $\sqrt{q_i}$ but they are all taken as a unity. Even in this extremely poor

Table 1. Solution of the ventilation networks shown in Fig. 1 by the author's method

weis.=weisbach

1	2	3	4	5	6	7	8	9	10	11	12
No. of airway	R (weis.)	q $(m^3/s)^2$	$\sqrt{q}$ (m^3/s)	$\rho' = R\sqrt{q}$ $(m^3$ weis./s)	Q' (m^3/s)	ρ'' $(m^3$ weis./s)	Q'' (m^3/s)	ρ''' $(m^3$ weis./s)	Q''' (m^3/s)	ρ'''' $(m^3$ weis./s)	Q'''' (m^3/s)
1	0.0007	21.960	148	0.104	199.0	0.121	196.08	0.129	196.11	0.133	196.11
2	0.0065	8.668	93	0.604	83.4	0.573	83.03	0.556	83.11	0.548	83.12
3	0.0033	13.293	115	0.380	115.7	0.381	113.09	0.377	113.00	0.374	112.99
4	0.0026	4.800	69	0.180	35.5	0.136	32.96	0.110	32.71	0.098	32.69
5	0.0021	13.468	116	0.244	118.9	0.247	115.96	0.245	115.83	0.244	115.82
6	0.0048	8.492	92	0.442	80.1	0.413	80.13	0.399	80.28	0.392	80.30

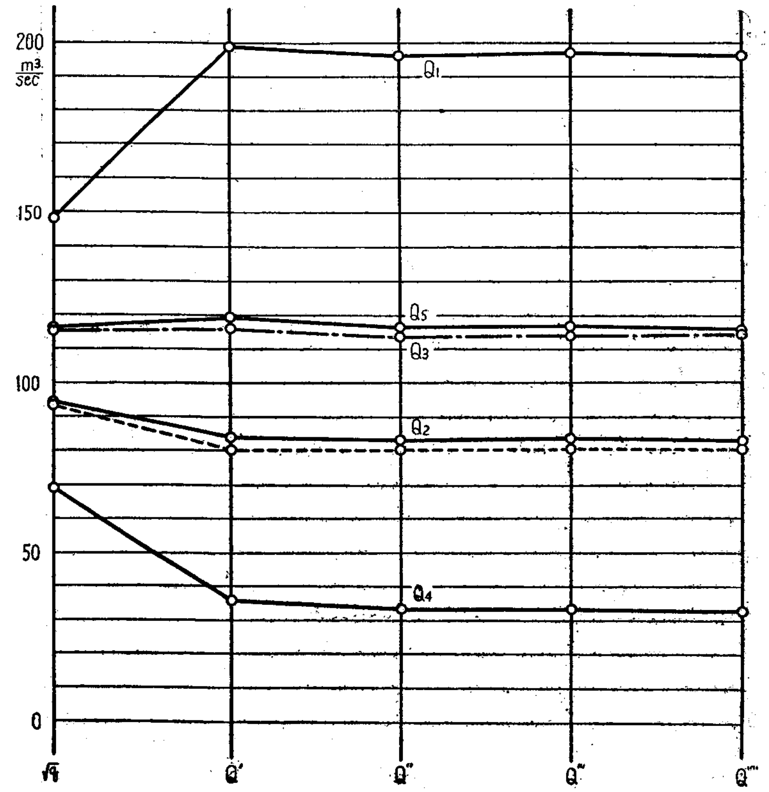

Fig. 2 Diagram showing convergence of air quantities in the author's method.

case the air quantities positively converge with appreciable speed, and that two or three times more stages of calculation will be enough to attain the same accuracy.

The Utilization of Electric Circuit

The main part of this solution is in finding air quantity of each airway, knowing all the values of ρ and the ventilating pressure. Since this calculation is just the same as the calculation of an electric circuit, an electric circuit, constituted similar to the ventilation networks, can be used to find each air

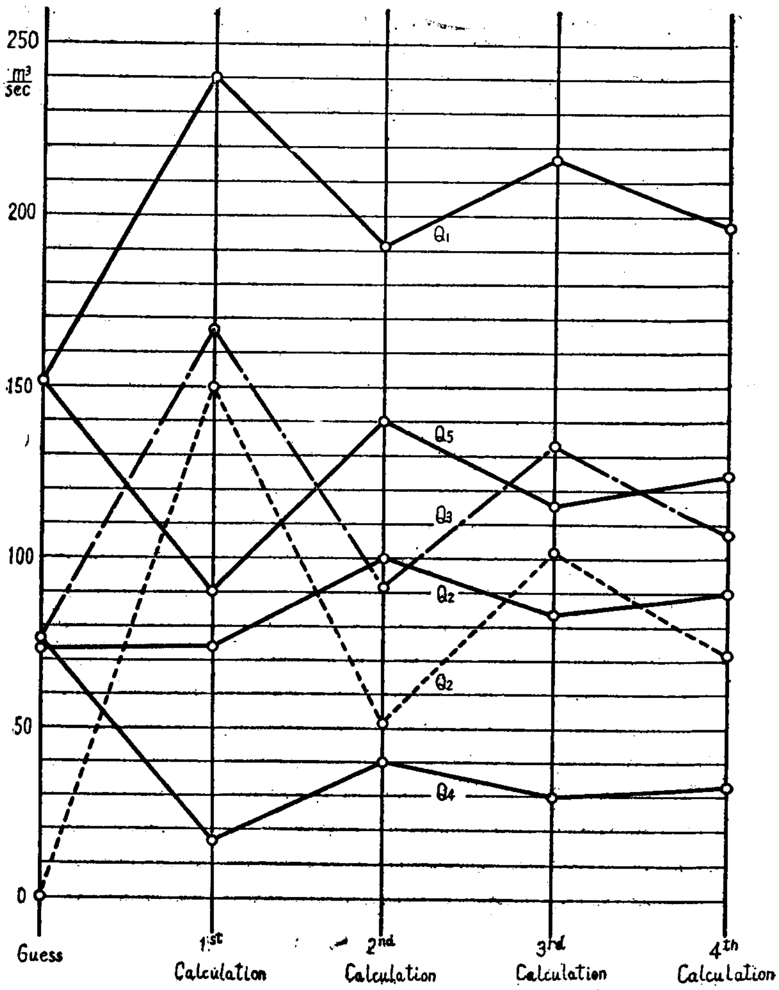

Fig. 3 Diagram showing convergence of air quantities in Scott and Hinsley's method.

quantity by measuring electric current experimentally. By this means, any calculation, however complicated it may be, can be easily carried out. This is the greatest advantage of the solution described above. ρ, Q and h are replaced by electric resistance, current and voltage drop respectively. A fan is represented by an electric source. The characteristic of the fan is applied in the following manner. At first give a random voltage to the electric source representing a fan, and measure the current and voltage. Translate them into the air quantity and fan pressure, and examine whether they fall on the characteristic curve. If not, adjust the voltage and again try the examination. Repeating these processes two or three times, the voltage and current of the source will be adjusted to a state corresponding to the fan performance.

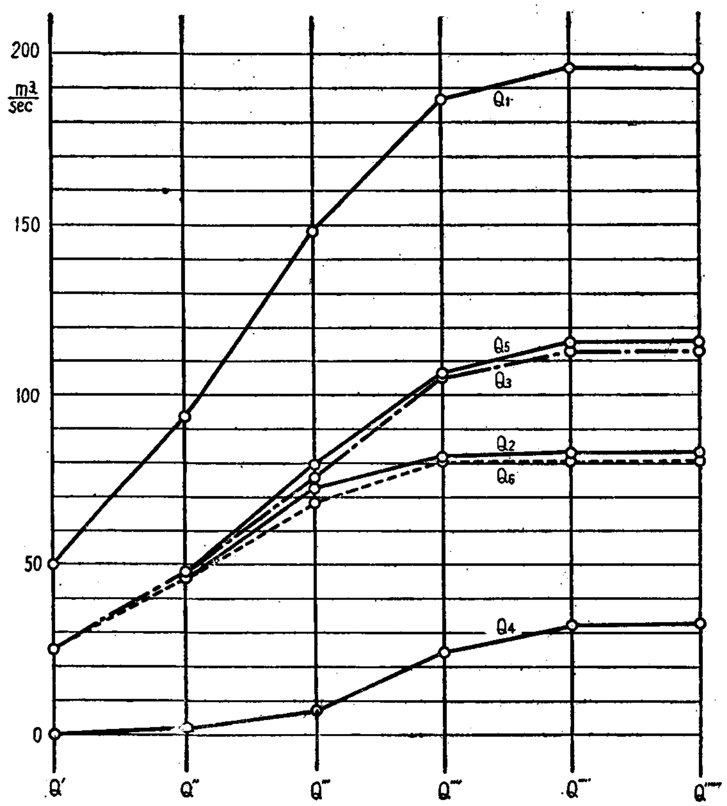

Fig. 4 Diagram showing convergence of air quantities in the author's method when ρ_i' are taken as unity.

Conclusion

The author has proposed a new method of solution of ventilation networks utilizing the calculation of an electric circuit. It has been noticed that the method affords the results converging rapidly, that the operation is easier than other methods ever published, for this method allows an experimental calculation with an electric circuit. A rational design of ventilation can only be attained by comparing several plans, solving each networks, so that the method of solution is desired to be as easy as possible. It might be concluded that the method described in this article would suit to this purpose to some extent.

References

1) Y. Hiramatsu and Y. Oka: Journal of the Mining Institute of Japan, 68, No. 763, (1952), 25
2) D. R. Scott and F. B. Hinsley: Colliery Engineering, 28, No. 324, 326, 328, 334, (1951).
3) W. Maas: Geologie en Mijnbouw, 12, (1950), 117.
4) S. Batzel and W. Schmidt: Glückauf, 88 (1952), 415.