

Studies on Perforated Plate Fractionating Column

By

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Introduction

Many researches have been made on a bubble cap column and a packed column. As to the perforated plate column, however, studies made are few. Besides, this type of column is not used but in a limited case such as separation of gas. The perforated plate fractionating column, however, is simple in construction, workable with small plate spacing, and, moreover, its efficiency is very high in a certain range of gas or vapor rate.

Because of its excellent characteristics, it is worthwhile to inquire in detail into the performance of this type of column.

Part 1

Effect of Entrainment

Introduction

In a fractionating column, the mass transfer rate between liquid and vapor increases, generally speaking, as the velocity of vapor increases. On the other hand the increment of vapor velocity causes entrainment and fractionation becomes less efficient.

The perforated plate column has little amount of entrainment as compared with the bubble cap column, because the rising vapor passes uniformly through liquid layer on the perforated plate. Therefore a shorter spacing will do in case of the perforated plate column than on a bubble cap column.

Theoretical Consideration

Many reports¹⁻⁵⁾ have been made public on the study of the influences of entrainment on plate efficiency, but their conclusion are very complicated and

are inconvenient to use for the design of a fractionating column. Authors have theoretically studied the influences of entrainment phenomena on the efficiency of column, and found a convenient method for the determination of number of plates.

In analysing the phenomena of entrainment the following three assumptions were established.

- 1 There is no entrainment from the top plate to the condenser.
- 2 Concentration of liquid entrainment is equal to that of liquid on the plate from which it spurts.
- 3 Decrease of plate efficiency caused by entrainment is due to the decrease of the average concentration of the rising vapor caused by liquid particles which accompany the vapor.

The average composition of wet vapor which rises from the n th plate is given by the following equation,

$$y_n = \frac{100 - \epsilon}{100} y_n' + \frac{\epsilon}{100} x_n, \quad (1)$$

where

x_n = the concentration of the more volatile component in the liquid leaving the n th plate.

y_n' = the actual concentration of the more volatile component in the dry vapor rising from the n th plate, excluding entrainment and not in equilibrium with x_n .

y_n = the actual average concentration of the more volatile component in the wet vapor leaving the n th plate, including entrainment and not in equilibrium with x_n .

ϵ = mol percent of liquid particles which accompany the wet vapor.

In Fig. 2,

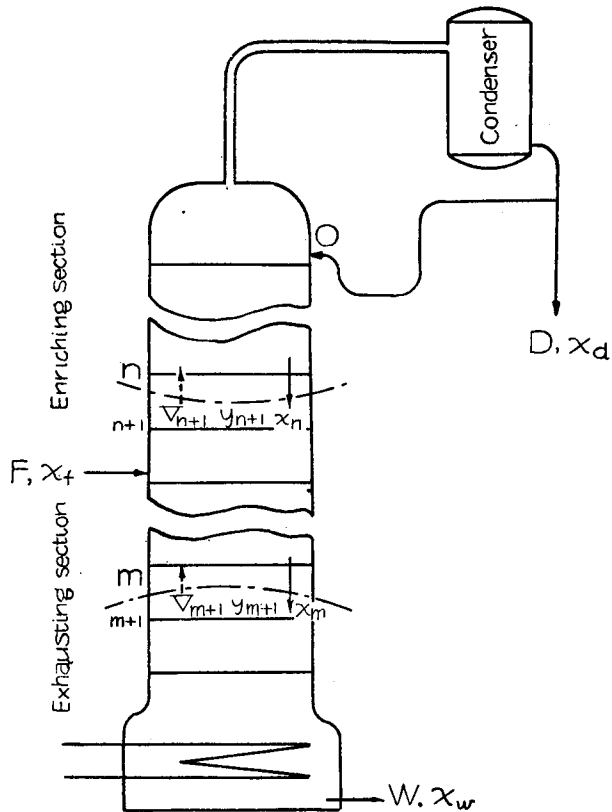


Fig. 1 Schematic diagram of fractionating Column.

$$\begin{aligned} a_F &= y_n^* - y_{n+1} \\ b &= y_n' - y_{n+1} \\ b^* &= y_n - y_{n+1} \end{aligned} \quad (2)$$

In such cases, the percentage of the decrease of efficiency caused by entrainment f_n is written as:

$$\begin{aligned} f_n &= \frac{b/a_F - b^*/a_F}{b/a_F} \times 100 \\ &= \frac{b - b^*}{b} \times 100 \end{aligned} \quad (3)$$

From equations (1) and (2)

$$b^* = b - \frac{\epsilon}{100}(y_n' - x_n) \quad (4)$$

In Fig. 1, the following equations are obtained from the material balance on the n th plate.

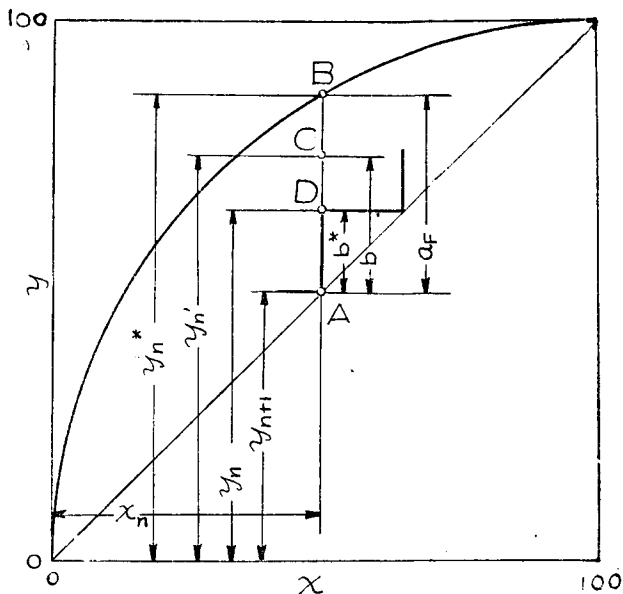


Fig. 2. x - y Equilibrium Curve.

$$\left(V + \frac{\epsilon V}{100 - \epsilon}\right) y_{n+1} = \left(R + \frac{\epsilon V}{100 - \epsilon}\right) x_n + D x_d \quad (5)$$

$$\left(V + \frac{\epsilon V}{100 - \epsilon}\right) = \left(R + \frac{\epsilon V}{100 - \epsilon}\right) + D \quad (6)$$

In case the entrainment is not considered, the equation of enriching line will be:

$$y_{n+1}^* = \frac{r}{1+r} x_n + \frac{1}{1+r} x_d,$$

where

$$r = R/D.$$

In case the entrainment is considered, the enriching line is obtained by the following equation, based on the equations (5) and (6)

$$y_{n+1} = \frac{r'}{1+r'} x_n + \frac{1}{1+r'} x_d, \quad (7)$$

where

$$\left(\frac{\epsilon V}{100 - \epsilon} + R\right) / D = r' \text{ and } \left(\frac{\epsilon V}{100 - \epsilon} + R\right) \text{ is moles of the overflow.}$$

From equation (7), (4) and (3), gain the following equation for the enriching section.

$$f_n = \epsilon + \frac{\epsilon}{br'}(x_a - y_{n+1}) \tag{8}$$

In the case of total reflux, i.e. $r' = \infty$

$$f_n = \epsilon \tag{9}$$

Using equation (2), equation (8) becomes

$$f_n = \epsilon + \frac{\epsilon}{r'} \cdot \frac{1}{E_{MV}} \left(\frac{x_a - y_{n+1}}{y_n^* - y_{n+1}} \right), \tag{10}$$

where

E_{MV} = Murphree Vapor Efficiency

$$= \frac{y_n' - y_{n+1}}{y_n^* - y_{n+1}}$$

$(x_a - y_{n+1}) / (y_n^* - y_{n+1})$ is the ratio of total coconcentration change which is likely to be attained in the whole upper part of tower above the n th plate to ideal increment of concentration at the n th plate. Hence, we named it the "theoretical enriching fraction" and denoted it by α_n . This value can be calculated for any degree of concentration in the column if the equilibrium curve and the operating line are decided. Consequently, the equation (10) becomes

$$f_n = \epsilon \left(1 + \frac{\alpha_n}{r' \cdot E_{MV}} \right). \tag{10'}$$

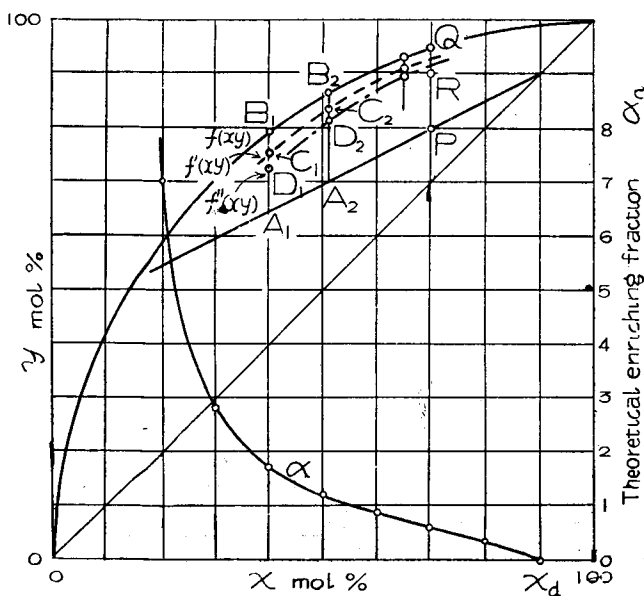


Fig. 3. Theoretical Enriching Fraction,

Fig. 3 is an example of calculation of u_n , where x_a is 90%. At $x = 70\%$, we find $u_n = PR/PQ = 0.6$

The correlation between r and r' is shown by the following equation:

$$r' = \frac{100r + \epsilon}{100 - \epsilon}. \quad (11)$$

From equations (10') and (11), one obtain (12)

$$f_n = \epsilon \left(1 + \frac{100 - \epsilon}{100r + \epsilon} \cdot \frac{u_n}{E_{MV}} \right).$$

From this relation, we can calculate the percentage of decrease of plate efficiency caused by entrainment.

In the exhausting section, the following equation is obtained by similar method.

$$f_m = \epsilon \left(1 + \frac{100 - \epsilon}{100\lambda + \epsilon} \cdot \frac{\beta_m}{E_{MV}} \right), \quad (13)$$

where

$$\beta_m = \frac{y_{m+1} - x_w}{y_m^* - y_{m+1}}, \quad \lambda = \frac{R + F}{W}.$$

β_m is the ratio of total concentration change which is likely to be attained in the whole lower part of column below the m th plate to ideal concentration decrease at the m th plate. Hence, we term it the "theoretical exhausting fraction".

Experiments

For the purpose of investigating what variables give influences upon entrainment ϵ , we have carried out experiments using air and aqueous solution of NaCl.

(1) Apparatus and operation

Fig. 4 shows the schema of the experimental apparatus. Water from tank B flows across

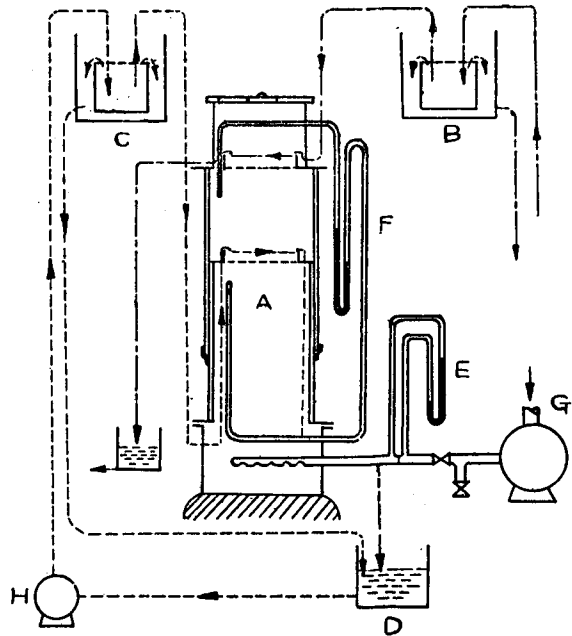


Fig. 4.

Schematic diagram of the experimental apparatus.

- A: Perforated plate column
- B: Constant head water tank
- C: Constant head NaCl sol. tank
- D: Receiver for NaCl solution
- E: Maekawa's orifice meter for air
- F: Manometer to measure the static pressure in column
- G: Roots blower
- H: Centrifugal pump to circulate NaCl solution

the upper perforated plate of column *A* and is exhausted. On the other hand, NaCl solution from tank *C* streams across the lower plate and is circulated by the pump *H*. Air is blown into the column by the blower *G*. We used a column, a glass cylinder of 12.7 cm inside diameter, and a perforated plate as shown in Fig. 5.

The amount of entrainment could be calculated from the increase of Cl ion in the upper plate solution measured by the caustic silver method.

(2) State of the liquid layer on the plate

It was observed that the condition of liquid layer on the plate varied as the velocity of vapor changed as shown in the photographs a, b, c, and d of Fig. 6.

When the vapor rate is very slow, there is a clear liquid layer on the plate such as in Photograph a. In this case, the air passes sporadically through some of the plate holes, and these holes, move constantly just like in case of water boiling at the bottom of a container. The liquid surface is covered with a single layer of foams and the foam is almost all semi-spherical in shape. And these foams are constantly formed and vanished. Hence, the driving force of entrainment depends, in this case, only upon the breaking of foams.

As the velocity of air increases, the clear liquid layer decreases abruptly and changes into a thick foam layer. The more intense the air velocity, the higher the foam layer becomes which at last, reaches the maximum height as shown in photograph b. The driving force of entrainment is the same as in case of Photograph a.

When the air velocity is increased beyond this limit, the height of foam layer gradually begins to lower and becomes unstable, and develops a turbulent state as observed in Photograph c. In this state, the flow of air homogenizes the liquid on its surface, form the liquid particles, and then gives ascending power to the particles as entrainment. At the highest air velocity, there occurs splashing of whole liquid mass into fine particles as shown in Photograph d.

It is a very intricate problem which state of liquid would be the most suitable for the operation of the fractionating column.

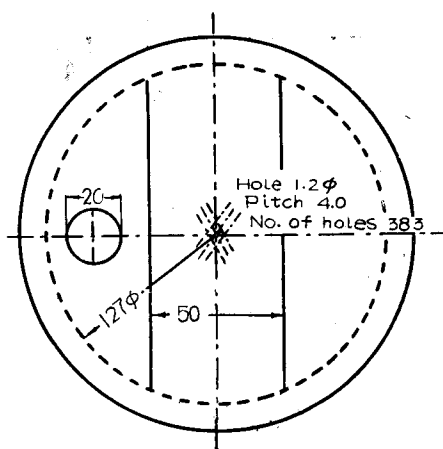
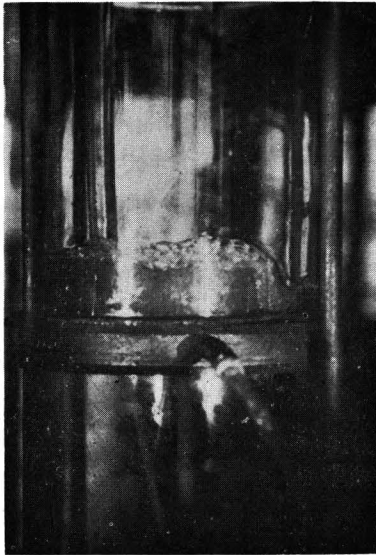
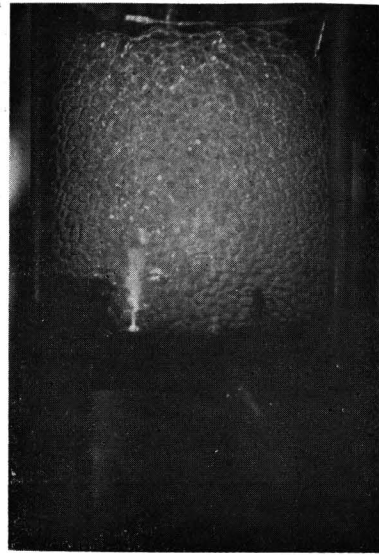


Fig. 5. Perforated plate used in experiments (Dimension in mm)



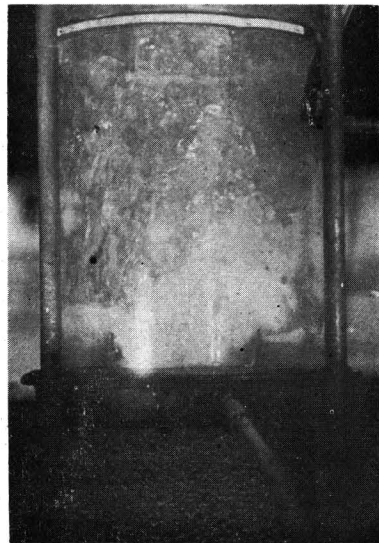
(a)



(b)



(c)



(d)

Fig. 6. State of the liquid layer on the plate.

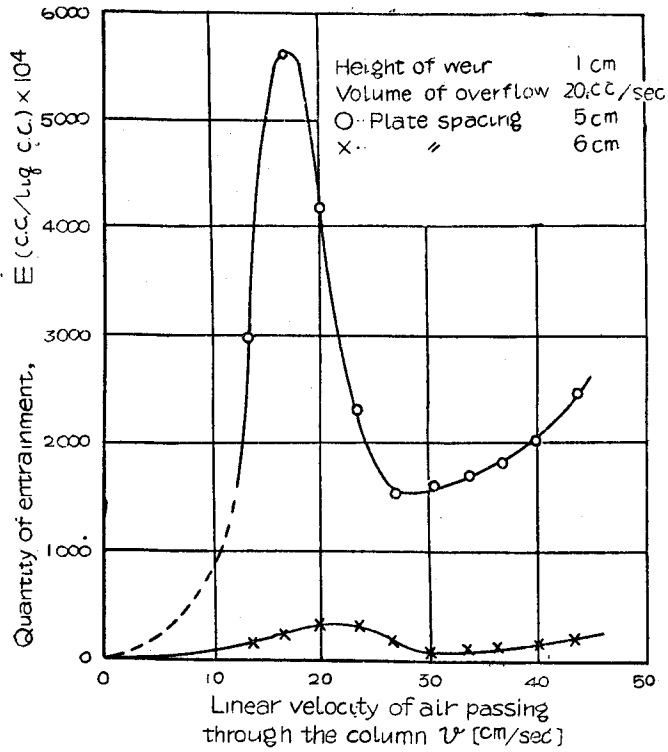


Fig. 7. Amount of entrainment at mall plate spacing.

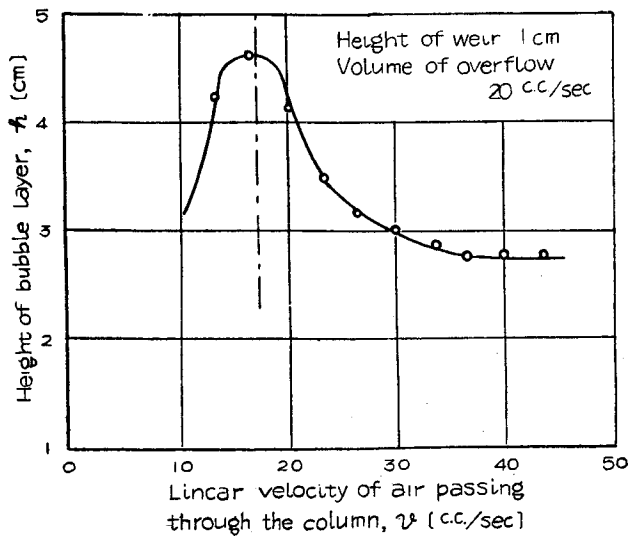


Fig. 8. Effect of air velocity on the foaming level.

(3) Influences of vapor velocity and plate spacing on entrainment

(i) When the plate spacing is small: Fig. 7 shows the plots of the amount of entrainment against the air velocity at plate spacing of 5 and 6 cm. At small plate spacing the amount of entrainment has a maximum value at the air velocity of 17–18 cm/sec. This is explained as follows. The foam layer on the plate increases its height with the increase of the air velocity. Consequently, the vapor space clearance between the liquid surface and the upper plate decreases. But this clearance increases again as the height of foam becomes lower at a higher air velocity. The amount of entrainment becomes maximum at a certain air velocity which causes the minimum vapor space as shown in Figs. 7 and 8.

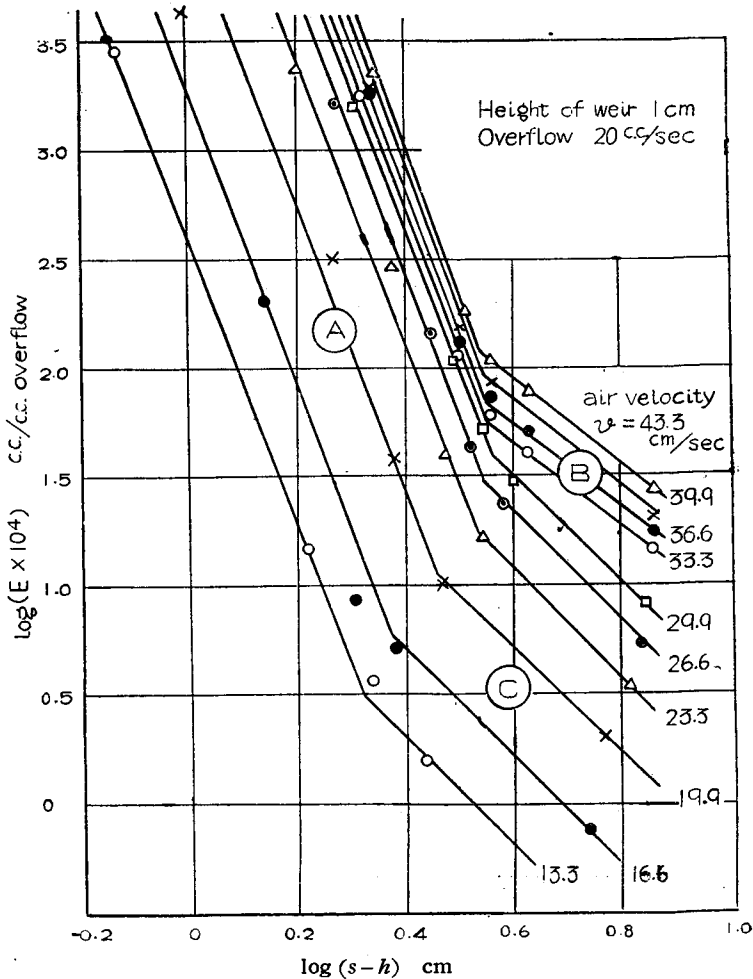


Fig. 9. Amount of entrainment vs. vapor space clearance at small vapor space clearance.

It is observed that the liquid level is constant in a turbulent range, and within this range the amount of entrainment increases with the increase of the air velocity. In the case of the large plate spacing, however, the influence on entrainment of the changes in height of the foam layer is not so great as the amount of entrainment attains its maximum value.

Fig. 9 shows the plots of the amount of entrainment E versus the vapor space clearance ($s-h$) with logarithmic coordinates, here, s is the plate spacing and h , the height of foam layer. At part A in Fig. 9, the clearance between the liquid surface and the upper plate is less than 3 cm, and it is conceivable that unstable liquid particles fly. Parts B and C corresponds to the turbulent stage and the foam forming stage respectively, and the liquid particles are carried up in steady condition by the ascending air currents.

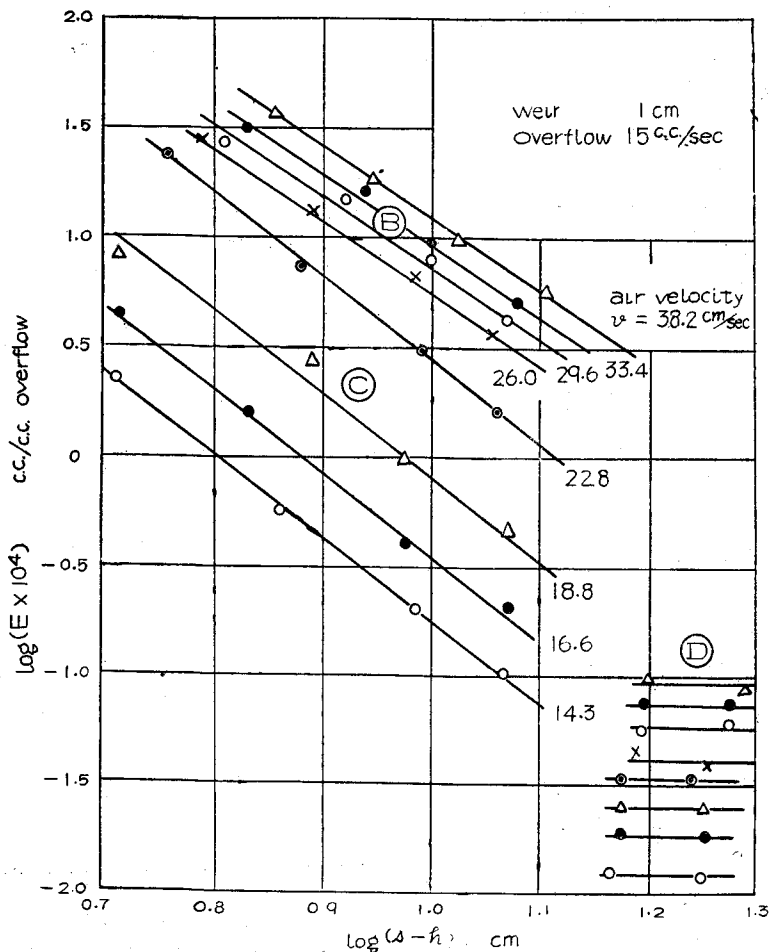


Fig. 10. Amount of entrainment vs. vapor space clearance at large vapor space clearance.

(ii) When the plate spacing is large: When the plate spacing is more than 10 cm and where the height of foam no longer has a direct influence upon the amount of entrainment, the vapor phase, in which liquid particles are flying, can be divided into two parts; the lower and the upper. The liquid particles found in the lower part, are the ones that are falling back to the plate and the larger the particle, the lower the position from which it falls. In the upper part, all the floating particles are carried away by the air current. Fig. 10 shows the relation between E and $(s-h)$ in this case, and B, C and D correspond respectively to foam forming stage, turbulent stage, and particles flying stage.

(4) Influence on entrainment of overflow rate and height of weir

One example of the experimental results is shown in table 1. The table shows that the amount of entrainment is almost constant regardless of the varied rates of overflow. However, the influences of certain degree can not be avoided because of the decreased clearance of vapor space caused by the increase of overflow.

Similarly, it can be anticipated that the decrease of the vapor clearance caused by the increase of the height of weir will increase the amount of entrainment. In Fig. 11 is shown the relation between the amount of entrainment and vapor spacing. It is noted that Fig. 10 and Fig. 11 are exactly equal. Consequently, both the rate of overflow and the height of weir will not have any direct connection with the driving force of entrainment.

(5) Examination of the experimental results

It was worthy of notice that the states of the liquid layer on the perforated plate are varied with the change of the vapor velocity and the overflow rate, and that they can be divided into the foam forming range and the turbulent range. It is obvious from this experiment that the driving forces of

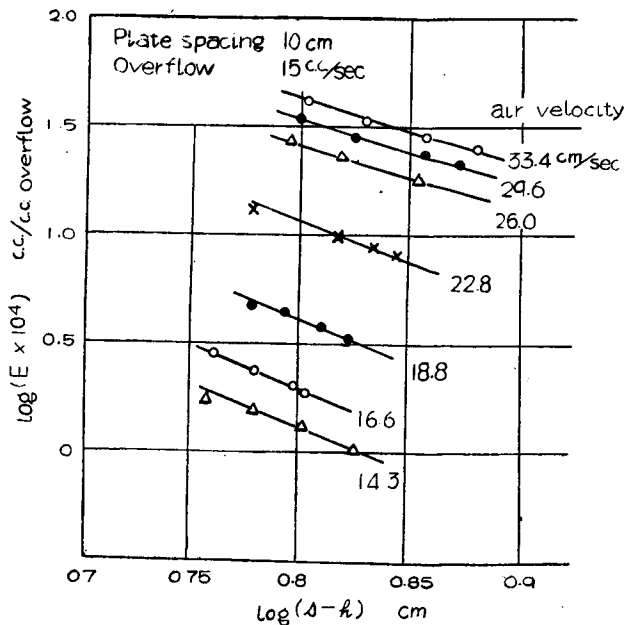


Fig. 11. Amount of entrainment vs. vapor space clearance at change of weir height.

entrainment are different at both ranges.

In case of small plate spacing the foam layer reaches the upper plate at certain air velocity and the amount of entrainment becomes greater. Therefore, in determining the plate spacing of fractionating column, a value must be taken at which the foam layer will not reach the upper plate.

It should be further noticed that foam height and vapor clearance are varied with the changes of overflow rate and weir height, therefore the amounts of entrainment are influenced by these two variable factors, though indirectly.

In fractionating substances that have different characteristics in producing foams depending upon the degree of concentration it is rational not to arrange the plate spacings of column equally throughout the whole column.

Table 1. Effect of overflow on amount of entrainment.

$\begin{matrix} q \\ v \end{matrix}$	4.55	8.0	10.5	14.5	17.0	20.0	23.0
16.5 :	24.2	25.2	27.1	27.2	24.0	22.0	19.5
	25.3	26.4	27.5	28.0	24.6	22.8	21.1
18.6 :	30.8	33.1	32.8	36.0	35.7	33.0	31.3
	33.5	34.4	36.4	36.8	37.2	34.4	32.2
22.3 :	64.0	65.8	70.7	72.5	70.0	70.5	67.2
	65.0	71.0	73.2	74.2	73.0	76.5	72.5
29.4 :	80.0	85.0	87.4	91.5	93.5	92.5	95.5
	81.0	87.2	90.0	92.7	95.5	94.5	96.8

q : Volume of overflow c.c/sec,

v : Linear velocity of air passing through the column cm/sec

The amount of entrainment is shown in (c.c/sec) $\times 10^4$

Plate spacing: 12 cm, Weir height: 1 cm.

Example of Calculation for benzene-toluene system

In the continuous fractionating column a solution containing 40 wt % of benzene and 60 wt % of toluene is treated. The distillate is to contain 97% of benzene, and the bottoms, 98% of toluene. The reflux ratio is 3.5, the molal latent heats of vaporization of both components are equal, and the liquid feed enters the column at its boiling point. Enriching line is given by the following equation.

$$y_{n+1} = 0.777x_n + 0.216$$

The result of the experiment is: the amount of entrainment is 0.6 c.c./c.c overflow or $\epsilon = 80$ mol% of rising vapor (at overflow rate is 20/c.c./sec, air velocity

is 18cm/sec and plate spacing is 5 cm) when a foam layer is in a state at which it reaches the upper plate. When the plate spacing is 5 cm and the state of liquid layer is in the turbulent range, this amount ϵ is about 50 mol%. At the plate spacing of 6 cm ϵ is about 10 mol%. If these results can be applied to the above illustrated fractionation of benzene-toluene system, the following results can be calculated; at $\epsilon=80$ mol%, r' would be 21.5 and $\lambda'=25.5$; the values of α , β , and f are given in Table 2; then 86 number of plates are required. At $\epsilon=50$ mol% and $\epsilon=10$ mol% 32 and 20 respectively of plates are required as shown in Fig. 12. The decrease of plate efficiency caused by entrainment is negligible small in case of $\epsilon=10$ mol% or less.

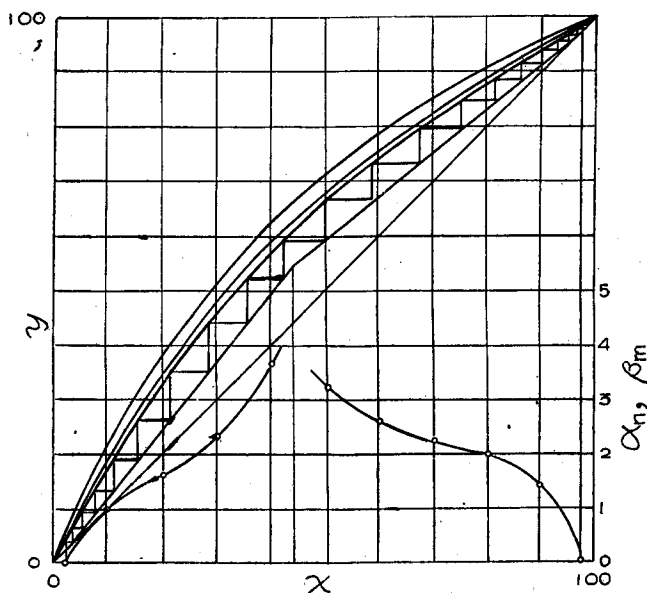


Fig. 12. Graphical solution for Benzene-Toluene system.
Feed 44 mol% Product 97.4 mol% Bottoms 2.34 mol%
 $E_{MV}=0.7$ $\epsilon=10$ mol%

Table 2 Percentage of decrease of efficiency caused by 80 mol % entrainment

$x\%$	97.4	90	80	70	60	50	40	30	20	10	2.34
α_n, β_m	0	1.5	1.9	1.95	2.18	2.35	1.9	1.4	1.02	0.7	0
f	80.0	87.95	90.0	90.03	91.55	92.5	88.7	86.5	84.6	83.2	80.0

It seems that if the foam layer does not touch the upper plate in the operation of fractionating column, the entrainment scarcely influences on plate efficiency.

Part 2

Theoretical Consideration of Plate Efficiency

introduction

The degree of approach to the state of mutual equilibrium between average composition of liquid and vapor leaving a plate is called the "plate efficiency", and the Murphree vapor efficiency⁶⁾ is defined by the following equation

$$E_{MV} = \frac{y_n - y_{n+1}}{y_n^* - y_{n+1}},$$

where

E_{MV} = Murphree vapor efficiency,

y_n = The average composition of vapor which rises from the n th plate,

y_{n+1} = The average composition of vapor which rises from the $(n+1)$ th plate,

y_n^* = The vapor composition which is equilibrium with the average composition x_n of liquid that leaves the n th plate.

For the design of fractionating column of binary mixtures, the stepwise calculation method proposed by McCabe and Thiele⁹⁾ is widely used because of its simplicity, and in doing so Murphree efficiency is assumed to be 100%. But the value of Murphree efficiency varies with vapor velocity, rate of overflow, composition and degree of concentration of mixtures, etc, and it is difficult to foresee the value of Murphree efficiency for any particular condition.

If liquid and vapor on the plate were mixed completely, the Murphree efficiency would be the most appropriate to explain the plate performance. However, the mixture on the plate is actually incomplete. Liquid and vapor flow crosswise against each other on the plate, and it is likely that there exist a concentration gradient in both liquid and vapor in the direction of flow. Based on this point of view, Kirschbaum⁵⁾ and Lewis⁷⁾ studied the plate efficiency. They divided the liquid layer into small elements in the direction of flow, assuming the liquid concentration on the plate to be uniform in vertical direction. The efficiency in these small elements was called "local efficiency" and the correlation of this efficiency with Murphree efficiency was introduced by them. As the local efficiency is also a function of many variables, the estimation of it is very difficult. Gerster⁸⁾ suggested a method of calculation based on the concept of number of transfer unit that is commonly used for the packed tower, but it is still insufficient predict plate efficiencies at any given condition.

Authors assumed the crosscurrent flows of liquid and vapor on the plate, and introduced an equation from which we could calculate Murphree efficiency.

This equation is conceivable to be a function of gradient of equilibrium curve, travelling length of liquid, height of foam, mass-transfer coefficient, vapor rate and overflow rate.

Flow of liquid and vapor on the plate

At usual operating conditions of fractionating column the state of liquid on the plate is in the foam forming range.

According to the experimental observations, the flow of liquid on the plate is as schematically shown in Fig. 13. The liquid from the upper plate is assumed to be distributed uniformly to the level of foam and flows horizontally on the plate and falls down to the lower plate. On the other hand the vapor flows vertically, that is, crosswise to the liquid on the plate.

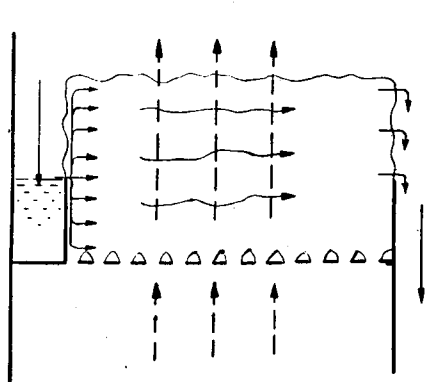


Fig. 13. State of flow of liquid and vapor on the plate.

Plate efficiency when the liquid film alone exists

When the mass-transfer between liquid and vapor is discussed, the double film theory is very useful. Authors treated the resistances of liquid film and the gas film individually, and then the case in which both of them exist together. At first we shall refer to the case in which the concentration of gas is constant in respect to the plate, and there is no resistance in the gas film but the liquid film resistance exists. Absorption of pure CO_2 gas by water belongs to this case.

At any point on the plate the concentration of liquid is given by

$$\frac{dx}{dl} = \frac{1}{L_x}(x^* - x) \frac{(1-x)}{(1-x)_r} \quad (14)$$

where x is the liquid concentration at any point on the plate, x^* is the liquid concentration which is in equilibrium with the gas concentration that is constant, $(1-x)_r$ is the logarithmic mean of $(1-x)$ and $(1-x^*)$, l is the travelling length of liquid, and L_x is the "length per transfer unit" which means the resistance of mass-transfer. When the change of liquid moles is negligibly small, Eq. (14) becomes approximately

$$\frac{dx}{dl} = \frac{1}{L_x}(x^* - x). \quad (15)$$

Integrating this equation between $x = x_{n-1}$ and x_n , correspondingly $l = 0$ and l ,

$$x_n = x^* \left(1 - e^{-\frac{l}{L_x}} \right) + x_{n-1} e^{-\frac{l}{L_x}}. \quad (16)$$

By definition of the Murphree efficiency, it is

$$E_{ML} = \frac{x_n - x_{n-1}}{x^* - x_{n-1}}. \quad (17)$$

From equations (16) and (17), the following equation is given.

$$E_{ML} = 1 - e^{-\frac{l}{L_x}}. \quad (18)$$

It is obvious from equation (15) that L_x is the coefficient which represents the resistance of mass transfer and has the dimension of length. This coefficient is called the length per transfer unit. L_x is a function of the liquid and vapor rates, and the characters of liquid and vapor. If L_x and l are known the Murphree efficiency can be calculated by equation (18).

Plate efficiency when the gas film alone exits

When the concentration of liquid is constant on the plate and only the gas film has a mass-transfer resistance, for example, in the case of the adiabatic humidification, the following equation can be given at any point on the plate.

$$\frac{dy}{dh} = \frac{1}{L_y} (y^* - y) \quad (19)$$

where h is the travelling length of gas or level of foam, y^* is the gas concentration that is in equilibrium with the constant liquid concentration.

Integrating the equation (19) between $y = y_{n+1}$ and y_n , correspondingly $h = 0$ and h ,

$$y_n = y^* \left(1 - e^{-\frac{h}{L_y}} \right) + y_{n+1} e^{-\frac{h}{L_y}}. \quad (20)$$

By definition of the Murphree efficiency, it is:

$$E_{MV} = \frac{y_n - y_{n+1}}{y^* - y_{n+1}}. \quad (21)$$

From equations (20) and (21),

$$E_{MV} = 1 - e^{-\frac{h}{L_y}} \quad (22)$$

If foam level h and L_y are known, the Murphree efficiency can be calculated, where L_y is the length per transfer unit of gas film. L_y is also a function of the liquid and vapor rates, and the characters of liquid and vapor.

The overall length per transfer unit

In the foregoing paragraph, L_x and L_y were defined separately, but it is usual that the mass-transfer resistance exists both in gas and liquid films. For example, in distillation, the transfer of low-boiling component from liquid to vapor is considered as shown in Fig. 14. x is the liquid composition in the main body and x_i is the liquid composition at the gas-liquid interface. y_i is the vapor composition at the interface which is in equilibrium with x_i , and y is the vapor composition in the main body.

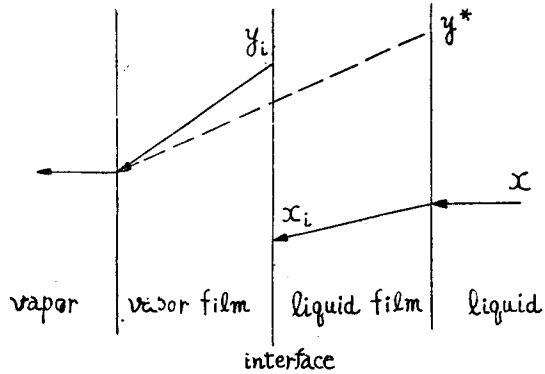


Fig. 14. Mass transfer resistance based on double film theory.

Based upon the double film theory, the following equation is obtained by using L_{oy} which means the overall mass-transfer resistance,

$$\frac{\partial h}{\partial y} = \frac{1}{L_{oy}}(y^* - y), \quad (23)$$

where y^* is the vapor composition which is in equilibrium with the liquid composition x .

Since the mass-transfer rate is equal in both films, the following equation can be obtained:

$$\frac{G}{L_{oy}}(y^* - y) = \frac{G}{L_y}(y_i - y) = \frac{L}{L_x}(x - x_i), \quad (24)$$

where G and L are the vapor and liquid rates at any point on the plate.

In Fig. 15 $y^* - y_i$ and $x - x_i$ correspond to BD and CD respectively. At the differential part near by A , the gradient of the equilibrium curve m is shown by the following equation.

$$m = \frac{BD}{CD} = \frac{y^* - y_i}{x - x_i}$$

or

$$x - x_i = \frac{1}{m}(y^* - y_i) \quad (25)$$

From equations (24) and (25),

$$\frac{y^* - y}{L_{oy}/G} = \frac{y_i - y}{L_v/G} = \frac{y^* - y_i}{L_x m/L},$$

$$\therefore L_{oy} = L_v + L_x \left(\frac{mG}{L} \right). \quad (26)$$

Equation (26) shows the relation between the overall length per transfer unit based on the change of vapor composition and the length per transfer unit of each film.

As for L_{ox} , the overall length per transfer unit based on the change of liquid composition, the following equation is gained,

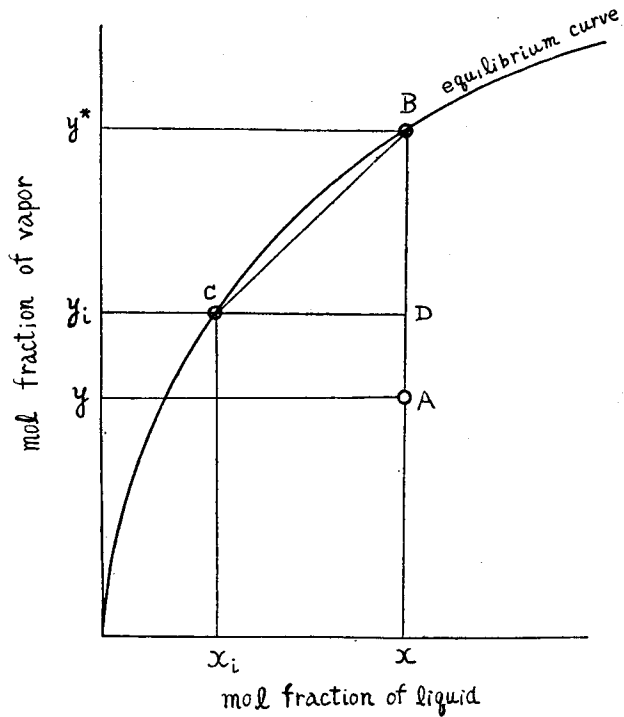


Fig. 15.

$$\frac{L}{L_{ox}}(x - x^*) = \frac{G}{L_v}(y_i - y) = \frac{L}{L_x}(x - x_i). \quad (27)$$

Assuming that the equilibrium curve is a straight line near this point, we obtain

$$y_i - y = m(x_i - x^*). \quad (28)$$

From equations (27) and (28)

$$L_{ox} = \frac{L_v}{(mG/L)} + L_x \quad (29)$$

This equation shows the relation between the overall length per transfer unit based on the liquid composition change and the length per transfer unit of each film.

From equations (26) and (29), the relation between L_{oy} and L_{ox} is shown by the following equation

$$L_{oy} = \left(\frac{mG}{L} \right) L_{ox} \quad (30)$$

Concentration gradient on the plate when the mass transfer resistance exits in both films

In distillation the mass-transfer resistance exists in both liquid and vapor films and the concentrations of liquid and vapor change with their positions on the plate.

In this case the following equation is given at any point on the plate:

$$\frac{\partial y}{\partial h} = \frac{1}{L_{oy}}(y^* - y) \quad (31)$$

$$-\frac{\partial x}{\partial l} = \frac{1}{L_{ox}}(x - x^*), \quad (32)$$

where

$$y^* = mx + c \quad (33)$$

$$x^* = (y - c)/m. \quad (34)$$

The value of m in the Eq. (33) is different from that in Eq. (34) in the strict meaning, but it can be safely assumed that they are equal so far as only one plate is being discussed.

If the equilibrium curve is extremely convex and the reflux ratio is very large, it is unavoidable that the above-mentioned assumption will bring about some discrepancy.

Combining Eqs. (33) and (34) with Eqs. (31) and (32), and using the relation of Eq. (30), we have:

$$\frac{\partial y}{\partial h} = -\frac{\partial x}{\partial l} \left(\frac{G}{L} \right) \quad (35)$$

This is an equation of continuity for the concentration on the plate. Consequently, the concentrations of the liquid and vapor on the plate are determined by the following three equations.

$$\frac{\partial y}{\partial h} = \frac{1}{L_{oy}}(y^* - y) \quad (31)$$

$$y^* = mx + c \quad (33)$$

$$\frac{\partial y}{\partial h} = -\frac{\partial x}{\partial l} \left(\frac{G}{L} \right) \quad (35)$$

In order to solve the above partial differential equations, the following boundary conditions are assumed. The liquid, having a composition of x_{n-1} , flows down from the upper plate to the n th plate, is distributed uniformly over vertical section of foam layer, and enters the effective part of plate. The composition of vapor is considered to take a constant value of y_{n+1} at $h=0$ all over the plate.

x_n and y_n are respectively the average compositions of liquid and vapor which leave the n th plate.

Therefore, at $l=0$

$$\left(\frac{\partial y}{\partial h}\right)_{l=0} = \frac{1}{L_{ov}}(y_{l=0}^* - y), \quad (36)$$

where $y_{l=0}^* = mx_{n-1} + c$.

Solving the above equation, we obtain:

$$y = y_{l=0}^* - (y_{l=0}^* - y_{n+1})e^{-\frac{h}{L_{ov}}} \quad (37)$$

for $l = 0$

$$\text{and } y = y_{n+1} \quad \text{for } h = 0 \quad (38)$$

Eqs. (37) and (38) represent the boundary conditions.

Combining three equations (31), (33), and (35), we have:

$$\frac{\partial^2 y}{\partial h \cdot \partial l} + \frac{(mG/L)}{L_{ov}} \cdot \frac{\partial y}{\partial h} + \frac{1}{L_{ov}} \cdot \frac{\partial y}{\partial l} = 0 \quad (39)$$

solving the above partial differential equation by using the Laplace transformation at the boundary conditions of Eqs. (37) and (38), we gain the vapor composition at any point on the plate as the follows.

$$y = (y_{l=0}^* - y_{n+1}) \left\{ 1 - e^{-\frac{mG/L + h}{L_{ov}}} \sum_0^{\infty} \left[\frac{\left(\frac{mG/L}{L_{ov}}\right)^n}{n!} \left(1 + \frac{h/L_{ov}}{1!} + \frac{(h/L_{ov})^2}{2!} + \dots + \frac{(h/L_{ov})^n}{n!}\right) \right] \right\} + y_{n+1} \quad (40)$$

Concerning with the liquid composition, the following equation is obtained by the similar method.

$$\frac{\partial^2 x}{\partial h \cdot \partial l} + \frac{\left(\frac{mG}{L}\right)}{L_{ov}} \cdot \frac{\partial x}{\partial h} + \frac{1}{L_{ov}} \cdot \frac{\partial x}{\partial l} = 0 \quad (41)$$

The boundary conditions are obtained by the following equations,

$$-\left(\frac{\partial x}{\partial l}\right)_{h=0} = \frac{1}{L_{ov}}(x - x_{h=0}^*)$$

where

$$x_{h=0}^* = (y_{n+1} - c)/m,$$

that is,

$$\text{at } l = 0, \quad x = x_{n-1} \quad (42)$$

$$\begin{aligned} \text{at } h = 0, \quad x &= x_{h=0}^* + (x_{n-1} - x_{h=0}^*)e^{-\frac{l}{L_{ov}}} \\ &= x_{h=0}^* + (x_{n-1} - x_{h=0}^*)e^{-\frac{mG}{L}l/L_{ov}} \end{aligned} \quad (43)$$

Solving the above equation, the result is:

$$x = x_{n-1} - (x_{n-1} - x_{h=0}^*) \left\{ 1 - e^{-\frac{m_G l + h}{L_{oy}}} \sum_0^\infty \left[\frac{(h/L_{oy})^n}{n!} \left(1 + \frac{m_G l / L_{oy}}{1!} + \frac{(m_G l / L_{oy})^2}{2!} + \dots + \frac{(m_G l / L_{oy})^n}{n!} \right) \right] \right\} \quad (44)$$

If the compositions of the liquid and vapor which enter certain plate and $m_G l / L_{oy}$ & h / L_{oy} at this plate are known, the liquid and vapor composition at any point on this plate can be calculated from Eqs. (40) and (44). Since these solutions are very complex, it is more convenient to make a diagram which shows these solutions. They are reformed as follows:

$$x = x_{n-1} - (x_{n-1} - x_{h=0}^*) (1 - S) \quad (45)$$

$$y = (y_{h=0}^* - y_{n+1}) (1 - S) + y_{n+1} \quad (46)$$

where

$$S = e^{-(A+B)} \sum_0^\infty \left[\frac{A^n}{n!} \left(1 + \frac{B}{1!} + \frac{B^2}{2!} + \dots + \frac{B^n}{n!} \right) \right] \quad (47)$$

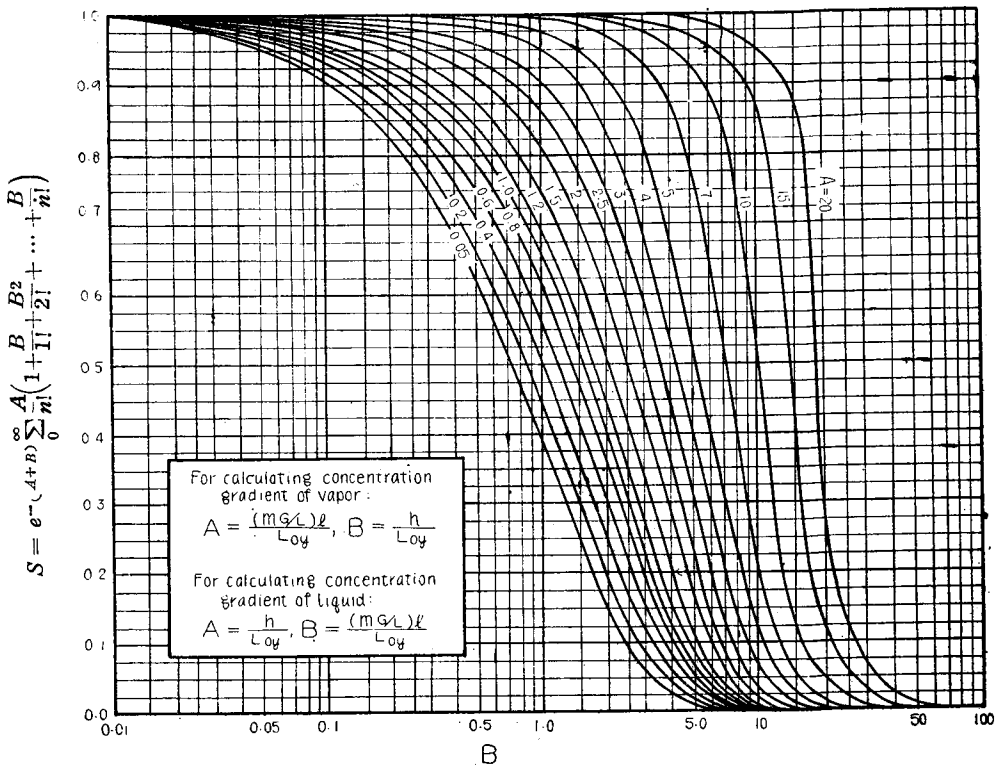


Fig. 16. Concentration gradient factor.

For calculation of liquid composition, $A = h/L_{ov}$, $B = m \frac{G}{L} l / L_{ov}$, for calculation of vapor composition, $A = m \frac{G}{L} l / L_{ov}$, $B = h/L_{ov}$, and S is called "concentration gradient factor". Fig. 16 shows the relation between h/L_{ov} , $m \frac{G}{L} l / L_{ov}$ and S .

For example, a mixture of methylalcohol and water is being rectified at the column of $l = 7.5$ cm. Assuming that x_{n-1} is 0.805, y_{n+1} is 0.614, and then $x_{h=0}^*$ is 0.23, we find that Eq. (45)

becomes $x = 0.805 - 0.575(1 - S)$. From this equation the values of x were plotted versus l with h as the parameter in Fig. 17, where $h = 4$ cm. $m \frac{G}{L} = 0.53$ and $L_{ov} = 2.66$ cm.

As the liquid flowing along the bottom of foam layer has a large driving-force, the mass-transfer rate is larger at the bottom of foam layer compared with other positions. The vapor which contacts the liquid that flows along the top of foam layer is enriched with more volatile component in the lower part. Therefore, the driving-force between liquid and vapor is smaller at the upper part and the mass-transfer rate is also smaller than that at the bottom. The concentration gradient of vapor can also be calculated by the similar method.

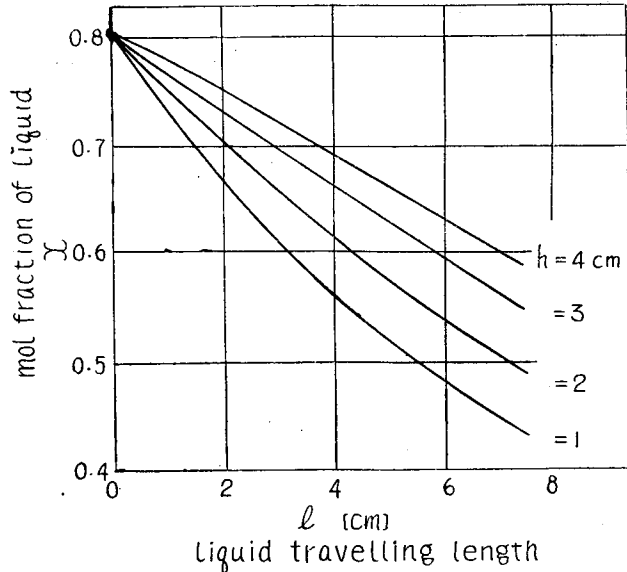


Fig. 17. Concentration gradient of liquid on the plate. methanol-water system, $x_1 = 0.805$ $y = 0.614$, $L_{ov} = 2.66$ cm, $h = 4$ cm, $mG/L = 0.53$

Plate efficiency when the mass transfer resistance exists in both films

Eqs. (40) and (44) show the vapor and liquid compositions at any point on the plate when the foam level is h and the liquid travelling length is l . therefore, the average composition x_n of the liquid which flows down to the lower plate is obtained by the following equation:

$$x_n = \frac{1}{h} \int_0^h x \, dh \tag{48}$$

Combining Eq. (48) with Eq. (44)

$$x_n = x_{n-1} - (x_{n-1} - x_{h=0}^*) + (x_{n-1} - x_{h=0}^*) \frac{L_{ov}}{h} \sum_0^{\infty} \left[e^{-\frac{mG}{L}l} \cdot P_n \left(\frac{mG}{L}l \right) \left\{ 1 - e^{-\frac{h}{L_{ov}}} \cdot Q_n \left(\frac{h}{L_{ov}} \right) \right\} \right], \quad (49)$$

where

$$P_n \left(\frac{mG}{L}l \right) = 1 + \frac{mG}{L}l / L_{ov} + \frac{\left(\frac{mG}{L}l / L_{ov} \right)^2}{2!} + \dots + \frac{\left(\frac{mG}{L}l / L_{ov} \right)^n}{n!}$$

$$Q_n \left(\frac{h}{L_{ov}} \right) = 1 + \frac{h/L_{ov}}{1!} + \frac{(h/L_{ov})^2}{2!} + \dots + \frac{(h/L_{ov})^n}{n!}$$

The average composition y_n of the vapor which rises from the n th plate is obtained by the following equation,

$$y_n = \frac{1}{l} \int_0^l y \, dl \quad (50)$$

Combining Eq. (50) with Eq. (40),

$$y = y_{n+1} + (y_{i=0}^* - y_{n+1}) - (y_{i=0}^* - y_{n+1}) \frac{L_{ov}}{mG} \sum_0^{\infty} \left[e^{-\frac{h}{L_{ov}}} \cdot Q_n \left(\frac{h}{L_{ov}} \right) \left\{ 1 - e^{-\frac{mG}{L}l} \cdot P_n \left(\frac{mG}{L}l \right) \right\} \right] \quad (51)$$

When the composition of liquid that enters the n th plate is x_{n-1} , and of the vapor is y_{n+1} , and the foam level is h and the liquid travelling length is l , Eqs. (49) and (51) respectively give the average compositions of the liquid and vapor which leave the n th plate.

By definition the Murphree vapor efficiency, it is

$$E_{Mv} = \frac{y_n - y_{n+1}}{(mx_n + c) - y_{n+1}} \quad (52)$$

From Eq. (49)

$$mx_n + c = mx_{h=0}^* + c + m \left\{ x_{n-1} + \frac{c}{m} - \left(x_{h=0}^* + \frac{c}{m} \right) \right\} \frac{L_{ov}}{h} \sum_0^{\infty} \left[e^{-\frac{mG}{L}l} \cdot P_n \left(\frac{mG}{L}l \right) \left\{ 1 - e^{-\frac{h}{L_{ov}}} \cdot Q_n \left(\frac{h}{L_{ov}} \right) \right\} \right]$$

and

$$mx_{n-1} + c = y_{i=0}^*, \quad mx_{h=0}^* + c = y_{n+1},$$

and then

$$(mx_n + c) - y_{n+1} = (y_{i=0}^* - y_{n+1}) \frac{L_{ov}}{h} \sum_0^{\infty} \left[e^{-\frac{mG}{L}l} \cdot P_n \left(\frac{mG}{L}l \right) \left\{ 1 - e^{-\frac{h}{L_{ov}}} \cdot Q_n \left(\frac{h}{L_{ov}} \right) \right\} \right]$$

$$y_n - y_{n+1} = (y_{-0}^* - y_{n+1}) - (y_{-0}^* - y_{n+1}) \frac{L_{ov}}{m \frac{G}{L} l} \sum_0^\infty \left[e^{-\frac{h}{L_{ov}} \cdot Q_n \left(\frac{h}{L_{ov}} \right)} \left\{ 1 - e^{-\frac{m \frac{G}{L} l}{L_{ov}} \cdot P_n \left(\frac{m \frac{G}{L} l}{L_{ov}} \right)} \right\} \right]$$

Therefore, from Eq. (52)

$$E_{MV} = \frac{1 - \frac{L_{ov}}{m \frac{G}{L} l} \sum_0^\infty \left[e^{-\frac{h}{L_{ov}} \cdot Q_n \left(\frac{h}{L_{ov}} \right)} \left\{ 1 - e^{-\frac{m \frac{G}{L} l}{L_{ov}} \cdot P_n \left(\frac{m \frac{G}{L} l}{L_{ov}} \right)} \right\} \right]}{\frac{L_{ov}}{h} \sum_0^\infty \left[e^{-\frac{m \frac{G}{L} l}{L_{ov}} \cdot P_n \left(\frac{m \frac{G}{L} l}{L_{ov}} \right)} \left\{ 1 - e^{-\frac{h}{L_{ov}} \cdot Q_n \left(\frac{h}{L_{ov}} \right)} \right\} \right]} \quad (53)$$

Eq. (53) is the general equation of the Murphree vapor efficiency. The Murphree vapor efficiency can be calculated from this equation, if $m \frac{G}{L} l / L_{ov}$ and h / L_{ov} are given.

The Murphree liquid efficiency is obtained by the following equation in the same way as in the vapor efficiency.

$$E_{ML} = \frac{x_{n-1} - x_n}{x_{n-1} - \left(\frac{y_n - c}{m} \right)} \quad (54)$$

From Eqs. (49), (51) and (54),

$$E_{ML} = \frac{1 - \frac{L_{ov}}{h} \sum_0^\infty \left[e^{-\frac{m \frac{G}{L} l}{L_{ov}} \cdot P_n \left(\frac{m \frac{G}{L} l}{L_{ov}} \right)} \left\{ 1 - e^{-\frac{h}{L_{ov}} \cdot Q_n \left(\frac{h}{L_{ov}} \right)} \right\} \right]}{\frac{L_{ov}}{m \frac{G}{L} l} \sum_0^\infty \left[e^{-\frac{h}{L_{ov}} \cdot Q_n \left(\frac{h}{L_{ov}} \right)} \left\{ 1 - e^{-\frac{m \frac{G}{L} l}{L_{ov}} \cdot P_n \left(\frac{m \frac{G}{L} l}{L_{ov}} \right)} \right\} \right]} \quad (55)$$

Eq. (55) is the general expression of the Murphree liquid efficiency.

Eqs. (53) and (55) are very complex, therefore the Murphree efficiency can not be easily calculated by these equations. Authors calculated the values of the Murphree efficiency at various values of $m \frac{G}{L} l / L_{ov}$ & h / L_{ov} and made the "plate efficiency diagram" Fig. 18 for the practical use.

In this figure, we took h / L_{ov} as abscissa and $m \frac{G}{L} l / L_{ov}$ as the parameter to calculate the Murphree vapor efficiency, and $m \frac{G}{L} l / L_{ov}$ as abscissa and h / L_{ov} as the parameter to calculate the Murphree liquid efficiency. The Murphree vapor efficiency approaches to 100% with increasing of h / L_{ov} and 0% with decrease of it and the Murphree liquid efficiency approaches to 100% with increase of $m \frac{G}{L} l / L_{ov}$

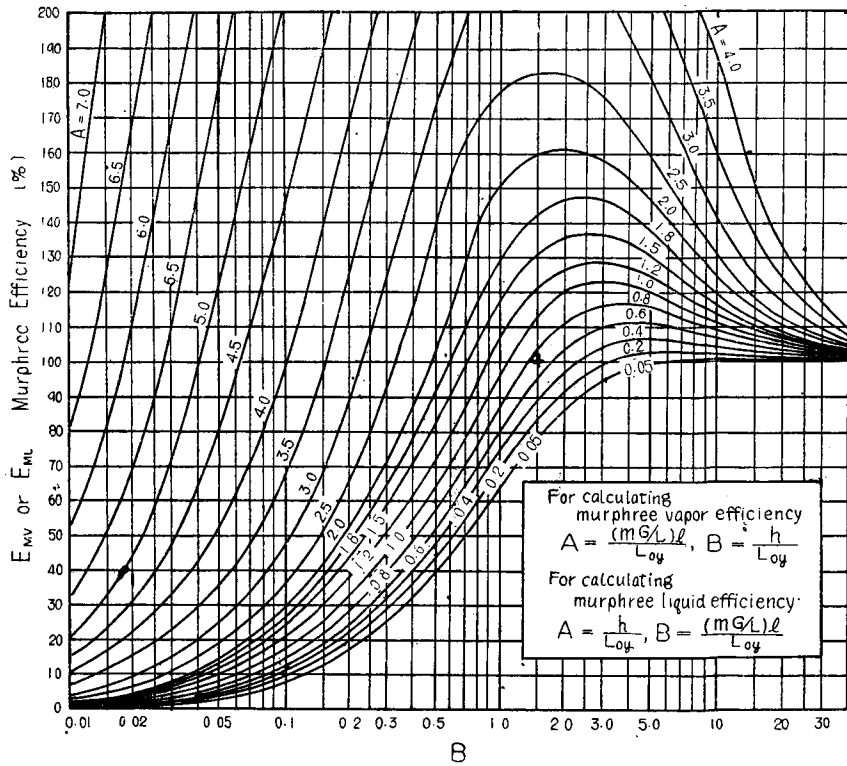


Fig. 18. Plate efficiency diagram.

and 0% with decrease of it. Both efficiencies assume the maximum values at certain value of h/L_{oy} and $m^G/L/L_{oy}$.

The Murphree vapor efficiency is unobtainable in Fig. 18, when $m^G/L/L_{oy}$ is larger than 7, but the Murphree liquid efficiency is obtainable. Therefore, one can correct the equilibrium curve and calculate the actual number of plates needed from the values of the Murphree liquid efficiency.

It has been reported by many investigators that the experimental value of the Murphree efficiency exceeds 100% in case of the perforated plate fractionating column.

The Murphree efficiency becomes more than 100% when the relation between $m^G/L/L_{oy}$ and h/L_{oy} satisfies the following equation :

$$\frac{L_{oy}}{h} \sum_0^{\infty} \left[e^{-\frac{m^G}{L_{oy}} \cdot P_n \left(\frac{m^G}{L_{oy}} \right) \left\{ 1 - e^{-\frac{h}{L_{oy}} \cdot Q_n \left(\frac{h}{L_{oy}} \right) \right\}} \right]$$

$$+ \frac{L_{ov}}{m \frac{G}{L}} \sum_0^{\infty} \left[e^{-\frac{h}{L_{ov}} \cdot Q_n \left(\frac{h}{L_{ov}} \right) \left\{ 1 - e^{-\frac{m \frac{G}{L} l}{L_{ov}} \cdot P_n} \right\}} \right] < 1 \quad (56)$$

The relation between the plate efficiency and the gradient of equilibrium curve *m*

Many investigators have reported that the experimental values of the Murphree efficiency vary with changes of the liquid composition on the plate in spite of all other conditions being equal. This phenomena has been explained that the heat and mass transfer rates decrease as the temperature difference between liquid and vapor decreases. Kirschbaum¹⁰⁾ explained it by both views of the temperature difference between liquid and vapor and the theoretical rate of heat transfer. For the temperature difference between liquid and vapor, he took the difference between the temperature of liquid leaving a plate and that of the vapor entering it. But it is questionable whether this temperature difference represents the true difference between liquid and vapor on the plate. Moreover, the state of flow on the plate should be considered when the plate efficiency is being considered. Taking the foam level, the liquid travelling length, and the liquid and vapor rates as constant in Eqs. (53) and (55), we considered how the Murphree efficiency will change with the change of the liquid composition on the plate which brings on the change of the value of *m*. It is assumed that the variations of *L_x* and *L_y* with the liquid and vapor composition are negligibly small. Line B in Fig. 19 shows the results of calculation for the Murphree

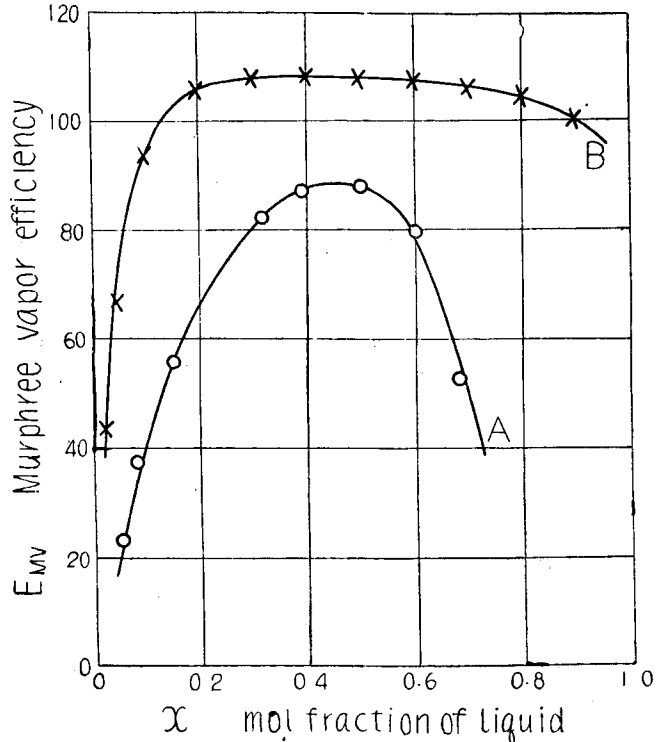


Fig. 19. Murphree efficiency of ethyl alcohol-water
h=4cm, *l*=10cm, *L_y*=2cm, *L_x*=4cm, *G/L*=1

efficiency E_{MV} for ethylalcohol-water system, where $L_y=2$ cm, $L_x=4$ cm, $l=10$ cm, $h=4$ cm, $G/L=1$. Line *A* represents the experimental results which were reported by Kirschbaum. Line *A* and *B* do not coincide completely, and it is quite possible since the values of l , h , L_y , L_x etc. are assumed optionally. But the tendency of the variation of these two lines are equal. If the fact that the foam level takes the maximum values at about 40 mol% for ethyl alcohol-water system is taken into consideration, the both lines will coincide well. Fig. 20 shows the results for the benzene-toluene system. In this case the both lines agree completely.

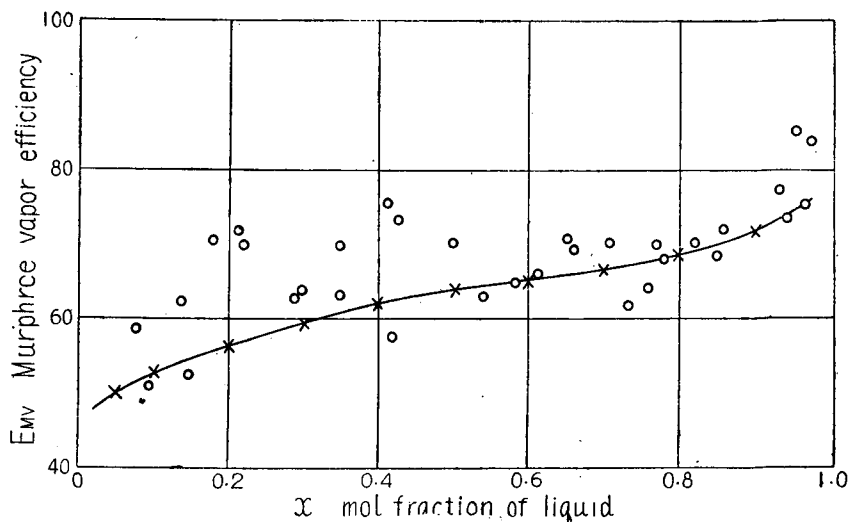


Fig. 20. Murphree efficiency of benzene toluene.
 $h=4$ cm, $l=10$ cm, $L_y=3$ cm, $L_x=6$ cm $G/L=1$

It is inferred from this fact that, in the case of benzene-toluene system, the foam level hardly changes with the change in composition.

It is concluded that the variation of the Murphree efficiency with the change of liquid composition on the plate depends neither upon the fact that the mass-transfer rate decreases with the decrease of the temperature difference, nor on the relation between the temperature difference and the theoretical rate of heat transfer, but the influence of m is the principal cause of it and it is inevitable phenomena in the cross-current of liquid and vapor.

Nomenclature

V = vapor rate	Kmol/hr
R = reflux rate	kmol/hr
D = distillate rate	kmol/hr

