Plastic Flow of Reinforced Concrete

By

Shizuo BAN and Kiyoshi OKADA
Department of Architecture

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Synopsis

We present herein the theoretical analysis of plastic flow of reinforced concrete and the test results obtained during about fourty-six weeks of sustained loading on twenty-three reinforced concrete columns. Two conditions of reinforcement were investigated for the columns of same size. This research program was intended to examine the effects of the time of loading on the plastic flow of reinforced concrete. By the results obtained, we found that the Whitney's Law pertaining to the plastic flow of concrete can also be applied to the reinforced concrete when the ratio of reinforcement is comparatively low, but in case of high percentage in the ratio of reinfocement, due consideration must be given.

1. Introduction

It is well known that the shrinkage and plastic flow of concrete have a great effect on the properties of reinforced concrete beam and column. A wide range of experiments on the plastic flow of concrete have been carried out by many researchers in Europe and America since around 1920 and they are still being continued to-day. In reinforced concrete, the re-distribution of stress between concrete and reinforcement is done by the shrinkage and plastic flow of concrete and this theory was clear by W. H. Glanville (1) and Fr. Dischinger (2). In our research we have attempted to examine whether or not the Whiney's Ideal Curve (3) concerning the plastic flow of concrete can be applied to that of reinforced concrete.

2. Plastic Flow of Concrete

As to the terms used in the following discussion, we define; the term "immediate elastic" refers to the characteristic which appears at the time of loading, and the term "plastic flow" signifies the strain caused by sustained loading. This is the difference between all the strains occuring subsequent to the immediate elastic behavior and strain caused by shrinkage and reasons.

The two foundamental properties pertaining to the plastic flow of concrete are as follows:

(1) The amount of plstic flow is in direct proportion to stress and the proportion constant is equal both for compression and for tension. (Davis-Glanville's Law)

(2) The speed rate of unit plastic flow is constant for the same concerte. (Whitney's Law)

The first is the Hooke's Law on the plastic flow and has been proved by R. E. Davis up to comparatively high compressive stress of 80 kg/cm² and by W. H. Glanville experimentally for tensile plastic flow. The second was given by C. S. Whitney and shows the following facts. In Fig. 1, "A" denotes the unit plastic flow curve, the ordinate of which is f_1 at time t_1 and f_2 at time t_2. If the same concrete as "A" begins to flow from time t_1, the total amount of unit flow is equal to (f_2-f_1) at time t_2 as shown by the curve "B" in Fig. 1.

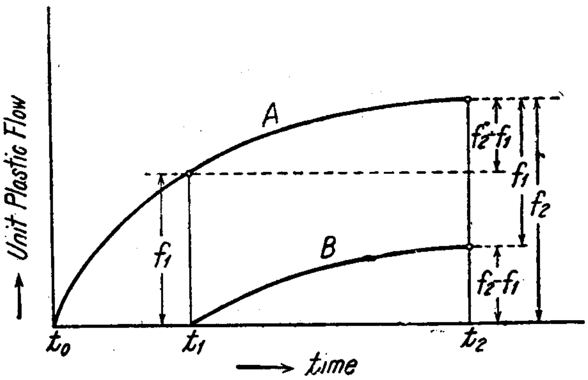


Fig. 1—Whitney's Ideal Curve

These are the curves which are called the Whitney's Ideal Curves. By these two foundamental laws, the total strain (the sum of immediate elastic strain and plastic flow strain) at time t when loanding begins from a given time $t=t_a$ will be given in the followings:

(1) When the modulus of elasticity, E_c, of concrete is constant.

$$\delta = \varepsilon + f_t - f_a = \varepsilon\left(1+\frac{f_t-f_a}{\varepsilon}\right) = \varepsilon(1+\varphi_t-\varphi_a) \quad\cdots\cdots\cdots(1)$$

under constant sustained stress σ

$$\delta = \varepsilon_t + \int_{t_a}^{t} \varepsilon_t \frac{d\varphi_t}{dt}dt \quad\cdots\cdots\cdots(2)$$

under time-variable sustained stress σ_t

where

$\varepsilon = \frac{\sigma}{E_c}$, $\varepsilon_t = \frac{\sigma_t}{E_c}$: immediate elastic strain at time $t=0$ and $t=t$

f_t, f_a : plastic flow strain at time $t=t$ and $t=t_a$

φ_t : plastic flow characteristics that is a function of time t

φ_a : value of plastic flow characteristics at $t=t_a$

(2) In case where the modulus of elasticity, E_{ct}, of concrete is variable with the increase in age of concrete.

It is assumed that the modulus of elasticity, E_{ct}, varies according to Eq. (3).

$$E_{ct} = E_{co}\frac{1}{1-q_t} \qquad (3)$$

Then, the total strain is

$$\begin{aligned}\delta &= \frac{\sigma}{E_{c0}}(1-q_a+\varphi_t-\varphi_a)\\ &= \varepsilon_0(1-q_a+\varphi_t-\varphi_a) \qquad (4)\end{aligned}$$

under constant sustained stress σ

$$\begin{aligned}\delta &= \frac{\sigma_t}{E_{c0}}(1-q_t)+\int_{t_a}^{t}\frac{\sigma_t}{E_{c0}}\frac{d\varphi_t}{dt}dt+\int_{t_a}^{t}\frac{\sigma_t}{E_{c0}}\frac{dq_t}{dt}dt\\ &= \varepsilon_{t0}(1-q_t)+\int_{t_a}^{t}\varepsilon_{t0}\frac{d\varphi_t}{dt}dt+\int_{t_a}^{t}\varepsilon_{t0}\frac{dq_t}{dt}dt \qquad (5)\end{aligned}$$

under time-variable sustained stress σ_t

where

E_{ct} : the modulus of elasticity at time $t=t$

E_{co} : the modulus of elasticity at time $t=0$

q_t : a function of time t

3. The Theoretical Analysis of the Plastic Flow of Reinforced Concrete.

The plastic flow of reinforced concrete must be studied for each case of direct forces, bending moment, and sometimes of those combined, because the characteristics of plastic flow in each of them are different. However we have taken up here the plastic flow for compression of the reinforced concrete and, on the basis of the foundamental properties pertaining to the plastic flow of concrete, we have attempted to carry out our theoretical discussions.

Firstly, we compute the immediate elastic behavior of reinforced concrete column which is subjected to the axial force P. In general, it is when the reinforced concrete is subjected to sustained loading that the problem of plastic flow is brought up. Under this condition, the stresses of concrete and reinforcing steel caused by the force P are less than the allowable stress for each of them. Therefore, they can be obtaind by the common computing method.

Notes:

E_c, E_s : modulus of elasticity of concrete and steel
A_c, A_s : area of cross section of concrete and steel
$p=A_s/A_c$: ratio of reinforcement
$n=E_s/E_c$: ratio of modulus of elasticity of steel and concrete
$D_c=A_cE_c$: compressive stiffness of concrete part
$D_s=A_sE_s$: compressive stiffness of reinforcement
P : load on column
P_c, P_s : partial load which is borne by concrete part and reinforcement respectively.

Then P_c and P_s are obtained as follows:

$$P_c = \frac{D_c}{D_c+D_s}P = \frac{1}{1+r}P \quad \cdots\cdots (6)$$

$$P_s = \frac{D_s}{D_c+D_s}P = \frac{r}{1+r}P = \alpha P \quad \cdots\cdots (7)$$

where

$$\left.\begin{aligned} r &= \frac{D_s}{D_c} = np \\ \alpha &= \frac{r}{1+r} = \frac{np}{1+np} \end{aligned}\right\} \quad \cdots\cdots (8)$$

The immediate elastic strain of column is equal to

$$\varepsilon = \frac{P}{D_c+D_s} = \frac{P_s}{D_s} = \frac{P_c}{D_c} \quad \cdots\cdots (9)$$

Assuming that, in course of time, the modulus of elasticity of concrete changes accordingly to Eq. (3), the change of compressive stiffness of concrete will be as shown below.

$$D_{ct} = \frac{D_{co}}{1-q_t} \quad \cdots\cdots (10)$$

At the same time, the strains caused by the shrinkage of concrete and other causes are produced, but we will consider theoretically of only the effect of shrinkage from hereon. As it is shown by many tests, a shrinkage-time curve of concrete is very similar to a plastic flow-time curve and it may be assumed that the process of shrinkage of concrete is similar to the plastic flow characteristics φ_t. Therefore, the amount of shrinkage strain S_t at the time $t=t$ can be expressed by Eq. (11)

$$S_t = K\varphi_t \quad \cdots\cdots (11)$$

where K : constant.

When concrete begins to flow and shrink after loading, a portion of the load sustaind by the concrete part decreases gradually and is transmitted to the reinforcement. If this partial load transmitted from the concrete part to the reinforcement is expressed by P_t, we can obtain the following balancing equation under the condition that the deformation of concrete is always equal to that of reinforcement at any time after loading.

$$K\varphi_t+\frac{P_c}{D_c}\varphi_t-\left[\frac{P_t}{D_c}(1-q_t)+\int_0^t\frac{P_t}{D_c}\frac{d\varphi_t}{dt}dt+\int_0^t\frac{P_t}{D_c}\frac{dq_t}{dt}dt\right]=\frac{P_t}{D_s} \quad\cdots\cdots\cdots(12)$$

This equation can be solved easily when we take into consideration the initial condition of $P_t=0$ at $t=0$ or $\varphi_t=0$.

(1) When E_c is constant.

$$q_t=0$$

$$\therefore\ P_t=(KD_c+P_c)(1-e^{-\alpha\varphi_t}) \quad\cdots\cdots\cdots(13)$$

Therefore, the loads that are sustained by concrete part and reinfocement are given by Eq. (14).

$$\left.\begin{aligned}P_{ct}&=P_c-P_t=P_c-(KD_c+P_c)(1-e^{-\alpha\varphi_t})\\P_{st}&=P_s+P_t=P_s+(KD_c+P_c)(1-e^{-\alpha\varphi_t})\end{aligned}\right\}\quad\cdots\cdots\cdots(14)$$

The total strain, δ_t, of reinforced concrete at the time $t=t$, since it is equal to that of reinforcing steel, is shown below:

$$\delta_t=\delta_{st}=\frac{P_{st}}{D_s}=\frac{P_s}{D_s}\left\{1+\frac{P_c}{P_s}(1-e^{-\alpha\varphi_t})\right\}+\frac{D_c}{D_s}K(1-e^{-\alpha\varphi_t}) \quad\cdots\cdots\cdots(15)$$

Equation (15) can be written in the same form as in the case of plain concrete column. Thus

$$\begin{aligned}\delta_t&=\varepsilon\left\{1+\frac{1}{r}(1-e^{-\alpha\varphi_t})\right\}+\frac{K}{r}(1-e^{-\alpha\varphi_t})\\&=\varepsilon(1+\varphi_{rt})+K\varphi_{rt}\end{aligned} \quad\cdots\cdots\cdots(16)$$

where

$$\left.\begin{aligned}\varphi_{rt}&=\frac{1}{r}(1-e^{-\alpha\varphi_t})\\\varepsilon&=\frac{P_s}{D_s}=\frac{P_c}{D_c}=\frac{P}{D_c+D_s}\end{aligned}\right\}\quad\cdots\cdots\cdots(16a)$$

In Eq. (16), φ_{rt} is nothing other than the plastic flow characteristics for compression of reinforced concrete.

(2) When E_{ct} is variable.

The similar solution as that shown by Eq. (16) is obtained for the case in which the modulus of elasticity of concrete varies with time t, but a little change in the form of φ_{rt} will serve the purpose. If the change of E_{ct} is given by Eq. (3) and q_t is assumed to be similar to φ_t, the plastic flow characteristics φ_{rt} given by Eq.*(17a) is shown by Eq. (17).

$$\varphi_{rt} = \frac{1}{r}(1-e^{-\overline{\alpha\varphi_t}}) \quad \cdots\cdots(17)$$

where

$$\left.\begin{aligned} \overline{\alpha\varphi_t} &= \frac{-1}{k} ln(1-\alpha k\varphi_t) \\ q_t &= k\varphi_t \quad k: \text{constant} \end{aligned}\right\} \quad \cdots\cdots(17a)$$

By Eq. (17), it is shown that when E_{ct} is variable, we can generally substitute $\overline{\alpha\varphi_t}$ for $\alpha\varphi_t$ in Eqs. (13)~(16).

Now, in order to check the applicability of Whitney's Law to reinforced concrete, we will try to solve Eq. (12) under the following conditions.

"The column was subjected to sustained loading from a given time $t=t_a$, (then $\varphi_t=\varphi_a$), and P_t had been affected only by shrinkage of concrete up to that time t_a".

In this case, corresponding to Eq. (13) in which E_c is constant, P_t is obtained by Eq. (18).

$$P_t = KD_c(1-e^{-\alpha\varphi_t})+P_c(1-e^{-\alpha\varphi_t+\alpha\varphi_a}) \quad \cdots\cdots(18)$$

Consequently P_{ct}, P_{st} and δ_t in correspondence to Eqs. (14) and (15) are shown in the same manner.

$$\left.\begin{aligned} P_{ct} &= P_c-KD_c(1-e^{-\alpha\varphi_t})-P_c(1-e^{-\alpha\varphi_t+\alpha\varphi_a}) \\ P_{st} &= P_s+KD_c(1-e^{-\alpha\varphi_t})+P_c(1-e^{-\alpha\varphi_t+\alpha\varphi_a}) \end{aligned}\right\} \quad \cdots\cdots(19)$$

$$\delta_t = \varepsilon\left\{1+\frac{1}{r}(1-e^{-\alpha\varphi_t+\alpha\varphi_a})\right\}+K\varphi_{rt} \quad \cdots\cdots(20)$$

In this case it is necessary to use the following value as the plasic flow characteristics for compression, ϕ_{rt}, of reinforced concrete:

$$\phi_{rt} = \frac{1}{r}(1-e^{-\alpha\varphi_t+\alpha\varphi_a}) \quad \cdots\cdots(21)$$

for sustained loading

$$\phi_{rt} = \frac{1}{r}(1-e^{-\alpha\varphi_t}) = \varphi_{rt} \text{ of Eq. (16a)} \quad \cdots\cdots(21a)$$

for shrinkage

When E_{ct} is variable as explained previously, $\overline{\alpha\varphi_t}$ or $\overline{\alpha\varphi_a}$ must be used in Eq. (18)~(21a) in place of $\alpha\varphi_t$ or $\alpha\varphi_a$.

$$\phi_{rt} = \frac{1}{r}(1-e^{-\overline{\alpha\varphi_t}+\overline{\alpha\varphi_a}}) \quad \cdots\cdots(22)$$

for sustained loading

$$\phi_{rt} = \frac{1}{r}(1-e^{-\alpha\varphi_t}) = \varphi_{rt} \text{ of Eq. (17)} \quad \cdots\cdots(22a)$$

for shrinkage

Hence, if $e^{-\alpha\varphi_t+\alpha\varphi_a}$ is assumed nearly equal to $(e^{-\alpha\varphi_t}+1-e^{-\alpha\varphi_a})$, the following relation is obtained. (4)

$$e^{-\alpha\varphi_t+\alpha\varphi_a} \doteqdot e^{-\alpha\varphi_t}+1-e^{-\alpha\varphi_a}$$

$$\therefore \quad \frac{1}{r}(1-e^{-\alpha\varphi_t+\alpha\varphi_a}) \doteqdot \frac{1}{r}\left\{(1-e^{-\alpha\varphi_t})-(1-e^{-\alpha\varphi_a})\right\} = \varphi_{rt}-\varphi_{ra}.$$

Then, Eqs. (20) and (21) can be written as follows.

$$\delta_t \doteqdot \varepsilon(1+\varphi_{rt}-\varphi_{ra})+K\varphi_{rt} \quad \cdots\cdots(23)$$

$$\phi_{rt} \doteqdot \varphi_{rt}-\varphi_{ra} \quad \cdots\cdots(24)$$

Eqs. (23) and (24) can be applied also in the case where E_{ct} is variable. This shows the Whitney's Law concerning the plasic flow for compression of reinforced concrete.

The errors which would be produced by using the approximate value ϕ_{rt} shown by Eq. (24) in substitute for the exact value ϕ_{rt} shown by Eq. (21) are illustrated in Table 1. However in this case, it is assumed that the ultimate value of φ_t is 4 and E_c is considered constant.

The value of $\alpha=\frac{1}{4}$, in case of n=10, is correspondent to the ratio of reinforcement p=3.3%, $\alpha=\frac{1}{8}$ to p=1.4% and $\alpha=\frac{1}{12}$ to p=0.9%.

Table 2 shows the similar errors as above when the modulus of elasticity of concrete is variable accoding to Eq. (3) and Eq. (17a). Also in this case, k is assumed to be 0.1 and the other coefficients are same as used in Table 1.

Table 1. Theoretical Errors (%) Produced by Equalizing (ϕ_{rt}) appr. to (ϕ_{rt}) exct. in case of $\varphi_{t=\infty}$=4 and constant E_c.

$\varphi_a : \varphi_{t=\infty}$	0	0.2	0.4	0.6	0.8	1.0
$\alpha=\frac{1}{4}$	0	18.2	32.9	45.1	55.1	0
$\alpha=\frac{1}{8}$	0	9.5	18.0	25.8	33.0	0
$\alpha=\frac{1}{12}$	0	6.5	12.5	18.1	23.7	0

Table 2. Theoretical errors (%) produced by equalizig (ϕ_{rt}) appr. to (ϕ_{rt}) exact. in case of $\varphi_{t=\infty}$=4 and variable $E_{ct}(k$=0.1)

$\varphi_a : \varphi_{t=\infty}$	0	0.2	0.4	0.6	0.8	1.0
$\alpha=\frac{1}{4}$	0	18.3	33.5	46.1	56.5	0
$\alpha=\frac{1}{8}$	0	9.5	18.7	26.2	33.6	0
$\alpha=\frac{1}{12}$	0	6.5	12.6	18.3	23.7	0

It is shown from these tables that the Whitney's Law can be applied to the plastic flow of reinforced concrete with less than 10% error only in the case where the percentage of reinforcement is low and loading is done at an earlier age of concrete, but there is a danger of producing considerably large error in case the percentage of reinforcement is high and loading is done at an older age of reinforced concrete.

4. The Expriment on the Plastic Flow of Reinforced Concrete Columns

The test was carried out to examine the effect of the time of loading and of the amount of reinforcement on the plastic flow of reinforced concrete column and to compare the results with the theoretical analysis described before.

Fig. 2 and Table 3 indicate the size of columns, the condition of reinforcement and the time of loading. The cross section was chosen to give concrete stress of about 30 kg/cm². Six plain concrete columns were cast to determine the plastic flow, four of which were used as controls for the other two columns. Nine columns reinforced with four bars of 9mm diameter and eight columns with four bars of 16mm diameter were cast, some of these were to be used as controls for the others.

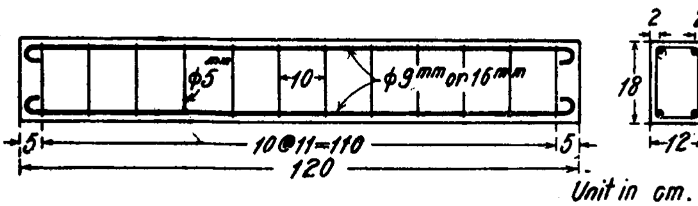


Fig. 2—Test Column Details

Table 3. Column Details

	Column No.	Width cm	Height cm	Length cm	Reinforcement A_s	Reinforcement p	Age at Loading week	Note
A	1, 7	12	18	120	4φ 9 mm (2.545 cm²)	1.18%	4	
	2, 8	12	18	120	" (")	"	14	
	3, 9	12	18	120	" (")	"	27	
	4, 10, 11	12	18	120	" (")	"	—	Control Specimens
	5	12	18	120	—	0	4	
	6	12	18	120	—	0	—	Control Specimen
B	12, 18	12	18	120	4φ 16 mm (8.04 cm²)	3.72%	4	
	13, 19	12	18	120	" (")	"	14	
	14, 20	12	18	120	" (")	"	27	
	15, 21	12	18	120	" (")	"	—	Control Specimens
	16	12	18	120	—	0	4	
	17, 22, 23	12	18	120	—	0	—	Control Specimens

The time of loading were chosen to be 4, 14 and 27 weeks of age after moulding.

Materials.

Aggregate and Cement: All columns were made with the concrete containing the Kamo River sand and gravel. The aggregate was hard and well rounded. The sand was considerably coarse and had a fineness modulus of 3.1; the gravel had a maximum size of 25mm. The cement used for columns was ASANO normal portland cement and had a strengh of 334 kg/cm² at 28 days. Steel: The properties of steel used in this experiment were as given below.

Table 4.

Size (mm)	Yield Point (kg/m²)	Ultimate Tensile Strength (kg/cm²)	Elongation in 8D (percent)
9	3300	4560	32.7
16	3270	4520	28.9

Concrete: Concrete was designed for a compressive strength of 200 kg/cm² at the age of 28 days and to possess a slump of 5 cm. The proportion of mixture actually apllied was 1:3:3.6, (by weight) and the water-cement ratio, w/c, was 62%. Mixing and compaction were done by hand.

Making, Curing and Testing.

The concrete was placed in about three layers by hand. Brass plugs with gauge mark were cast into the front and back side of each column along the longitudinal center line. Strains were measured on 10 in. gauge length by using a Whittemore Strain Gauge.

On the second day the side forms were removed and the columns were cured by fog for four weeks after moulding and they were all kept in the storage room after the curing was done.

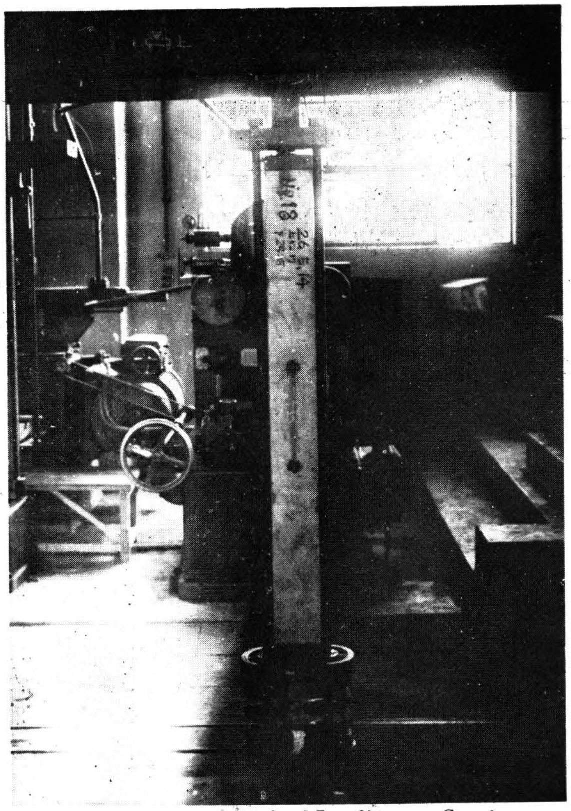
Two plain concrete columns were subjected to the sustained loading of 6 tons immediately after the curing, that is, at the age of 28 days.

These were termed the 4-week loaded specimens. The two reinforced concrete columns each of Series A and B were subjected to the sustained loading of 6 tons from the age of 4, 14 and 27 weeks. They were termed respectively 4-, 14- and 27-week loaded specimens.

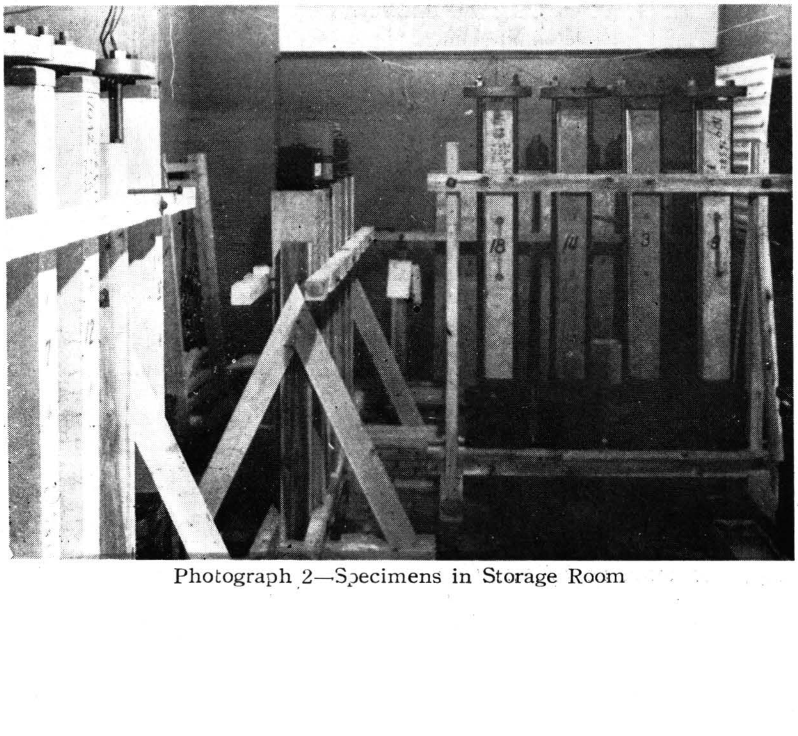
Constant load was given by pressing two springs of 3 tons capacity against the specimen which was held in a frame consisting of bearing plates and tie rods. (see Photograph 1)

The method of giving specimen the desired sustained load of 6 tons was as follows: The center position of the devices which hold the specimen was adjusted to the center of the press-plate of Amsler's pipe testing machine, then the specimen and the springs were compressed at the same time by the testing machine. When the dial of the testing machine precisely indicated the given load of 6 tons, the tie rods were tightened by screwing the nuts and the load on the testing machine was released, the same deformation on springs being produced.

Whether or not the given load was correctly maintained by the springs could be determined by measuring the strains on the specimen before and after releasing of the load



Photograph 1—Method of Loading on Specimen



Photograph 2—Specimens in Storage Room

on the testing machine and comparing them if they were equal.

The test stress maintained in the concrete column was 27.8 kg/cm². All loaded specimens were tested several times before the sustained loading was applied. The readings of strain were taken at certain intervals during the 46-weeks period of sustained loading.

During the 46-weeks of loading period, the temperature in the storage room varied from about 18 to 22°C, except in the first 8 weeks when it had gone up to 28°C on account of the cooler being out of order.

The relative humidity varied from about 65 to 90%. (see Photograph 2)

Test Results

The results of compression tests of control cylindens are shown in Table 5.

Table 5. Properties of Concrete Cylinders

Columns Represented	Ultimate Compressive Strength (kg/cm²)			Modulus of Elasticity* (kg/cm²)		
	4-week	14-week	27-week	4-week	14-week	27-week
A-1, 2, 3, 4, 5, 6	198	206	212	24.9×10^{4}	26.9×10^{4}	27.2×10^{4}
A-7, 8, 9, 10, 11	203	246	254	27.1 "	29.2 "	31.2 "
B-12, 13, 14, 15, 16, 17	187	203	215	24.6 "	26.9 "	28.3 "
B-18, 19, 20, 21, 22, 23	191	207	211	24.7 "	27.5 "	28.9 "
mean	195	218	221	25.3 "	27.6 "	28.9 "

Each result is the average of two tests.
* Secant value at 28.3 kg/cm².

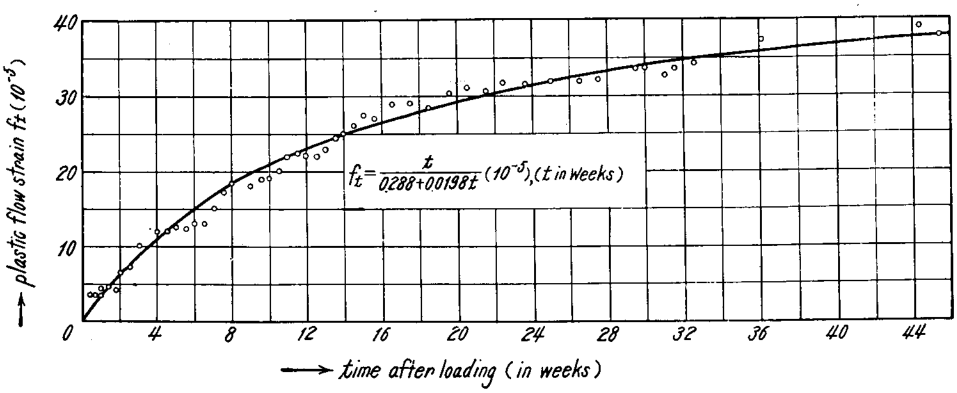


Fig. 3—Plastie Flow Strain of Plain Concrete Coluum

The modulus of elasticity, E_c, of concrete is the one determined by the secant method at the stress of 28.3 kg/cm².

The plastic flow of plain concrete column is shown in Fig. 3. One of the writers has shown that the plastic flow-time curve could be shown by a hyperbolic equation such as follows (5):

$$f=\frac{t}{a+bt}$$

where

f : plastic flow strain (10^{-5})
t : time in weeks after loading
a, b: constant determined by test

Using this equation, the plastic flow strain of concrete in this test is given by

$$f_t=\frac{t}{0.288+0.0198t} \quad (10^{-5}) \quad [t \text{ in weeks after loading}]$$

As the immediate elastic strain ε_t is 15×10^{-5}, the plastic flow characteristics φ_t is given by

$$\varphi_t=\frac{f_t}{\varepsilon_c}=\frac{t}{4.317+0.297t}$$

These equations show that the ultimate value of the plastic flow strain f_t and of the characteristics φ_t will become 50.5×10^{-5} and 3.367, respectively. As the change of modulus of elasticity, E_c, is shown in Table 5, the value of constant k in Eq. (17a) can be determined, for example, by use of least square and k=0.06 is obtained. Fig. 4 shows the relation between the plastic flow strain of reinforced concrete columns having four bars of 9mm dia. and the period of sustained loading; and Fig. 5 shows the same relation when the bars of 16mm dia. are used. Each result shows the average value of two test.

In the figures are given also the hyperbolic equations showing the relation between the plastic flow strain and the duration of load. They are all determined from the experimental values by the least square method.

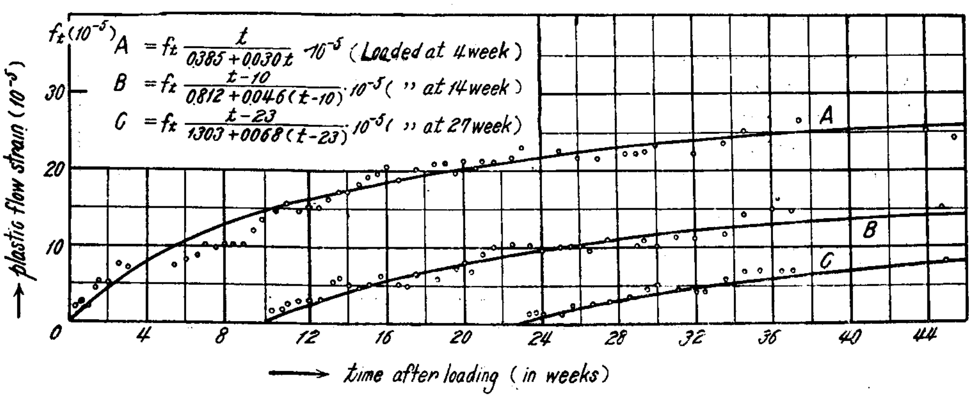


Fig. 4—Plastic Flow Strain of Reinforced Concrete Columns with p=1.18%

Fig. 6 and Fig. 7 indicate the experimental and theoretical values of the plastic flow characteristics, and the theoretical values are computed from Eq. (22) taking into consideration the variation of E_c.

In the figures are also drawn the theoretical "Ideal Curves" which are obtained by Eq. (24).

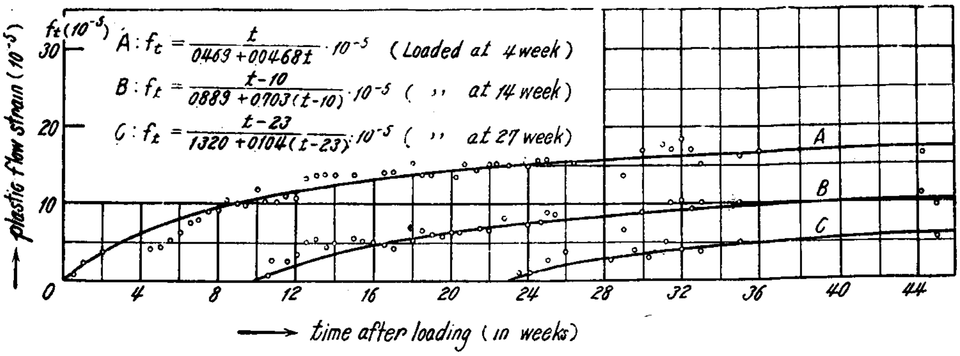


Fig. 5—Plastic Flow Strain of Reinforced Concrete Columns with p=3.72%

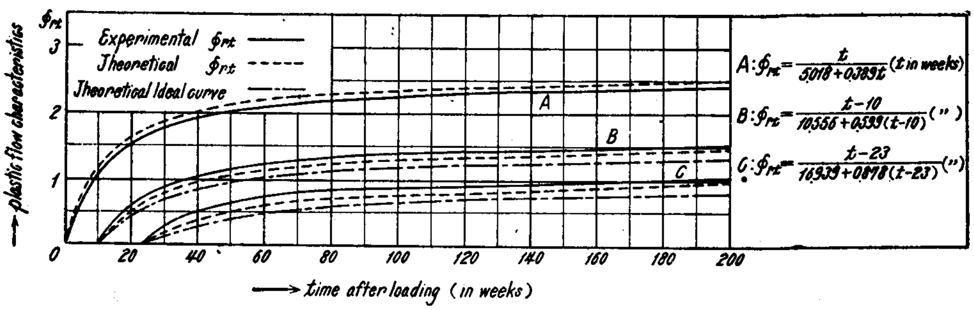


Fig. 6—Plastic Flow Characteristics of Reinforced Concrete Columns with p=1.18%

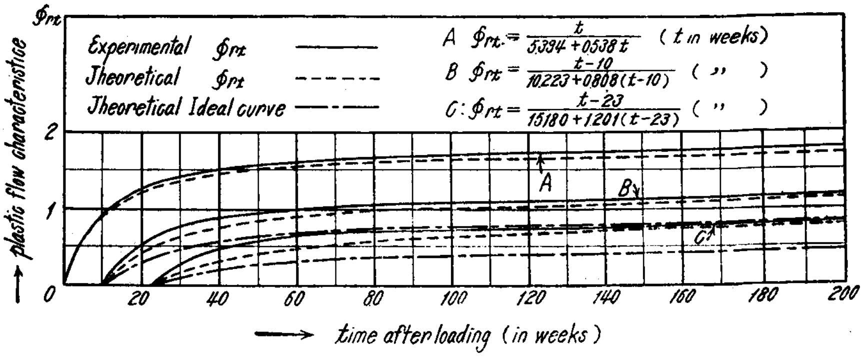


Fig. 7—Plastic Flow Characteristics of Reinforced Concrete Columns with p=3.72%

Table 6. Ultimate Values of ϕ_{rt}

(a) $p=1.18\%$ ($A_s=4\phi$ 9 mm)

Age at loading	Experimental $\phi_{rt.exp.}$	From Eq. (22)*		From Eq. (24)*	
		ϕ_{rt}	$\phi_{rt}/\phi_{rt.exp.}$	ϕ_{rt}	$\phi_{rt}/\phi_{rt.exp.}$
4-week	2.571	2.668	1.043	2.668	1.043
14-week	1.669	1.683	1.008	1.448	0.889
27-week	1.139	1.145	1.006	0.952	0.836

(b) $p=3.72\%$ ($A_s=4\phi$ 16 mm)

Age at loading	Experimental $\phi_{rt.exp.}$	From Eq. (22)*		From Eq. (24)*	
		ϕ_{rt}	$\phi_{rt}/\phi_{rt.exp.}$	ϕ_{rt}	$\phi_{rt}/\phi_{rt.exp.}$
4-week	1.859	1.803	0.971	1.803	0.973
14-week	1.238	1.248	1.008	0.900	0.727
27-week	0.883	0.893	1.072	0.546	0.655

* In calculation the change of E_c is considered.

Table 7. Ultimate Value of ϕ_{rt} Not considered the Change of E_c.

Age at loading	From Eq. (21)		From Eq. (24)	
	$p=1.18\%$	$p=3.72\%$	$p=1.18\%$	$p=3.72\%$
4-week	2.648	1.774	2.648	1.774
14-week	1.664	1.215	1.473	0.879
27-week	1.129	0.863	0.941	0.531

As to the ultimate values of ϕ_{rt}, a comparison is made between the experimental values and the theoretical ones, and is shown in Table 6 (a) and (b).

If the change of E_c is not considered, the ultimate theoretical ϕ_{rt} can be computed as shown in Table 7.

The effect of the variation of E_c on the theoretical value of ϕ_{rt} is insignificant.

As shown in Table 6, the test results are in good conformity with the theoretical analysis; and the assumption of "Ideal Curves" for the plastic flow of reinforced concrete gives less value than that of the experimental results as already indicated theoretically.

However, the experimental values of ϕ_{rt}, especially at the earlier stage of a case in which the sustained loading is given at the older age, are generally larger than the theoretical values as shown in Fig. 6 and Fig. 7. The cause for this may be due to the fact that the various factors, such as the seepage of colloidal liquid in the cement, the visco-elastic behavior of cenment, and the slippage or dislocation between the crystals of cement particles, give greater upon the specimen of the 14– or 27–week load at their early age than upon the specimen of the 4–week load.

Fig. 8 shows the relation between the shrinkage and plastic flow characteristics φ_{rt} of plain concrete specimens. It shows that the assumed Eq. (II) made in the theoretical analysis is not entirely wrong in spite of considerable variations in the temperature and the relative humidity in the storage room.

Consequently, the value of constant K in Eq. (II) becomes 15.5×10^{-5} and the straight line in the figure represents $S=15.5\varphi_t\times10^{-5}$.

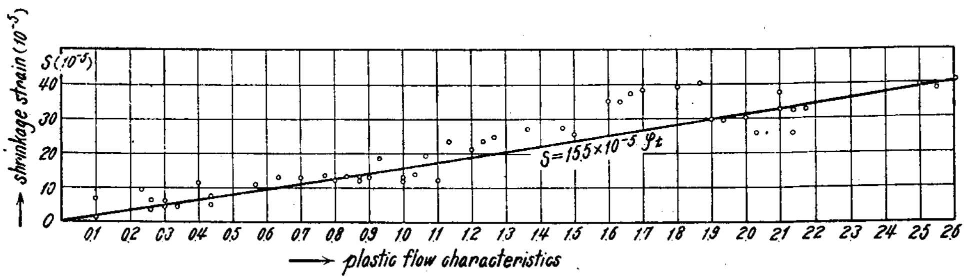


Fig. 8—Relation between Shrinkage and Plastic Flow Characteristics of Plain Concrete Columns

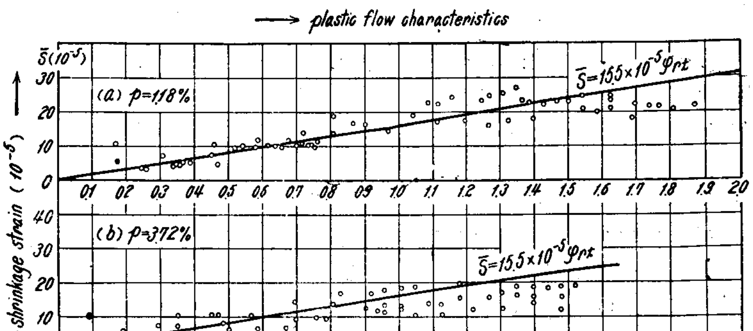


Fig. 9—Relation between Shrinkage and Plastic Flow of Reinforced Concrete

Using the value of $K=15.5\times10^{-5}$, the shrinkage $\bar{S}$ of reinforced concrete specimens can be given theoretically from Eq. (20) as follows.

$$\bar{S}=15.5\varphi_{rt}\times10^{-5}.$$

Fig. 9 (a) and (b) show the same relations obtained from the test results on the reinforced concrete specimens.

The theoretical straight lines agree fairly well with the experimental values, although the temperature and the humidity in the storage room were not perfectly constant.

5. Summary

C. S. Whitney proposed an important assumption as for the properties of plastic flow of plain concrete, which enabled us easily to estimate the plastic flow of reinforced concrete. However, as has been theoretically analysed, it is not right to assume that the idea of the "Ideal Curve" by Whitney can also applied be to the plastic flow of reinforced concrete.

According to the theoretical analysis, the error produced by using the ideal curve is generally small when the percentage of reinforcement is low, but the error becomes considerably large when the percentage of reinforcemeet is high and modulus of elasticity, E_c, of concrete is large.

This research was aimed to examine precisely the characteristics of the plastic flow of reinforced concrete and to prove the theoretical analysis by experiments.

The test results are in good conformity with the theoretical analysis; that is to say, when the assumption of the "Ideal Curve" for the plastic flow of reinforced concrete is used, there is a tendency to under-estimate the flow strain, the error at times exceeding 40%.

Therefore, we should make proper allowance for this erroneous values when we use the concept of "Ideal Curves" in computing the plastic flow problems of reinforced concrete structures.

Speaking, further, on the shrinkage of plain and reinforced concrete, it suffices to assume in the calculation that the shrinkage is proportional to the plastic flow characteristics.

This research was projected by Ban and carried out by both of us.

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References

1) W. H. Glanville: The creep or flow of concrete under load. Building Research Technical Paper No. 12, 1928.

2) Fr. Dischinger: Elastische und plastische Verformung der Eisenbetontragwerke. Bauing. 1939.

3) C. S. Whitney: Plain and reinforced concrete arch. Journal of A. C. I. March, 1932.

4) S. Ban: Stress-Redistribution in Statically Indeterminate Rigid Frame Caused by the Creep of Concrete. The Japan Science Review, Engineering Science, Vol. 2, No. 2, 1950.

5) S. Ban: Journal of the Architectural Institute of Japan, Vol. 49, No. 603, 1953.