A Study on the Mechanism of Transportation of Suspended Sediment and Its Application to Increasing the Efficiency of Sedimentation Basin

By

Takeshi GODA

Department of Civil Engineering

(Received Sept., 1953)

Synopsis

Some noteworthy facts are given concerning the meaning of factor in the law of distribution of suspended solid transported by turbulent flow and performing a flume experiment, the manner in which diffusion and deposition occur when a suspension of fine clay flows is studied. The efficiency of suspended sediment removal in sedimentation basin, which is an important application of this theory, is discussed and research results together with some opinion on increasing the efficiency are given.

1. Introduction

Sediments transported by water flow maintain the state of suspension by turbulent transfer, but the concentration, i.e., the quantity transported, varies greatly with the state of flow, therefore, it is very difficult to give an accurate estimation of the total discharge and quantity of deposit during a flood. In initial studies, the theory of distribution of suspended sediments was treated as two dimensional problem and it was discovered that the ratio of the particle settling velocity w to the material transfering coefficient η is the main factor and that in actual channels the distribution of suspended sediments in the vertical direction almost agrees with the theory quantitatively. In later studies, the turbulent transfer coefficient was sometimes handled as tensor η_{ij}, or more practically as equal to momentum transfering coefficient ϵ [1]. With the two dimensional uniform flow, however, from

$$\tau = \rho \epsilon \frac{du}{dy} \tag{1}$$

$$\tau / \tau_0 = 1 - y/h \tag{2}$$

and also from the logarithmic law of velocity distribution

$$\epsilon = u_* \kappa y(1-y/h) \tag{3}$$

where, $u_* = \sqrt{\tau_0/\rho}$, κ = universal constant, h = water depth.

Actually it is incorrect to consider that in general the scale of water fraction diffusion is equal to that of suspended sediment diffusion. When the particles are very small and the density does not differ very much from that of water, the mechanism of flow is to some extent effected by suspended matter, and the velocity gradient changes as is shown in the experimental results in 4. Namely, the result is that κ decreases below 0.4. As a result, it is considered that the transfering coefficient of suspended sediments approaches that of water fraction, and it is not unreasonable if such value of κ is adopted in eq. (3). Experimental Results of A. A. Kalinske[2] and V. A. Vanoni[3] verify this. Experimental results of the author (explained in 4) are almost the same as those of V. A. Vanoni. Therefore, in the following theory, $\eta = \epsilon$, or simplifying it further,

$$\epsilon = \text{const.} = \frac{1}{h}\int_0^h u_* \kappa y\left(1-\frac{y}{h}\right)dy = \frac{1}{6}\,\kappa u_* h \tag{4}$$

is adopted. But

$$\frac{\partial C}{\partial t} + \frac{\partial}{\partial x}(cu) = \frac{\partial}{\partial x}\left(\epsilon_x \frac{\partial C}{\partial x}\right) + \frac{\partial}{\partial y}\left(\epsilon_y \frac{\partial C}{\partial y}\right) + \frac{\partial}{\partial z}\left(\epsilon_z \frac{\partial C}{\partial z}\right) + w\frac{\partial C}{\partial z} \tag{5}$$

is difficult to solve. Therefore, in this paper, first as a general consideration, a three dimensional theory of suspended sediment distribution is derived and a detailed discussion made concerning the case when deposit mainly occur, which is an important field of application, and comparing it with experimental results, a theoretical consideration is made on the essence of purifying efficiency of sedimentation basin in water purification plant.

2. Three Dimensional Consideration

In discussing the problem of suspended sediment transportation in rivers, the analysis must be done three dimensionally, but the solution for complicated section profile is almost impossible. In this paper, for the purpose of obtaining a fundamental solution, a rectangular channel section, width $2B$, water depth h, as shown in Fig. 1 is considered and the origin taken at the bottom centre of the section of initial point and x, y and z axes taken to the downstream, crosswise

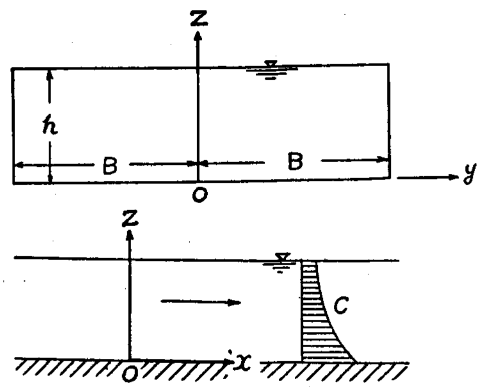


Fig. 1. Cross and Longitudinal Section of Channel.

and vertical upward directions respectively. Neglecting the first terms on both sides of eq. (5) and putting

$$u = U = \text{const.}, \quad \epsilon = \text{const.}, \text{ gives}$$

$$U\frac{\partial C}{\partial x} = \epsilon\left(\frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial z^2}\right) + w\frac{\partial C}{\partial z} \qquad (6)$$

In solving eq. (6), the fundamental equation and boundary conditions are given as below for the following 2 cases.

(i) When the distribution is uniform in x-direction: this is an ideal case,

$$\frac{\partial^2 C_1}{\partial y^2} + \frac{\partial^2 C_1}{\partial z^2} + \frac{w}{\epsilon}\frac{\partial C_1}{\partial z} = 0 \qquad (7)$$

$$\left.\begin{aligned} &\text{at } y = 0,\ B; \quad \frac{\partial C_1}{\partial y} = 0 \\ &\text{at } z = h \ ; \quad \epsilon\frac{\partial C_1}{\partial z} + wC_1 = 0 \\ &\text{at } z = 0 \ ; \quad C_1 = C_0(y) \end{aligned}\right\} \qquad (8)$$

(ii) When the distribution changes in the x=direction

$$U\frac{\partial C_2}{\partial x} = \epsilon\left(\frac{\partial^2 C_2}{\partial y^2} + \frac{\partial^2 C_2}{\partial z^2}\right) + w\frac{\partial C_2}{\partial z} \qquad (9)$$

$$\left.\begin{aligned} &\text{at } y = 0,\ B; \quad \frac{\partial C_2}{\partial y} = 0 \\ &\text{at } z = 0,\ h\ ; \quad \epsilon\frac{\partial C_2}{\partial z} + wC_2 = 0 \\ &\text{at } x = 0 \ ; \quad C_2 = C_1(y, z) \end{aligned}\right\} \qquad (10)$$

$C_0(y)$, $C_1(y, z)$ can generally be obtained in an arbitrary form, this being the case when the solution of eq. (7) is used as the boundary condition of eq. (9). The concentration is assumed symmetrical about z-axis for both (i) and (ii). For (i) the solution is given as follows.

$$C_1(y, z) = Q_0 e^{-R_z} + 2\sum_{n=1}^{\infty} Q_1 e^{-\frac{R_z}{2}} \cos k_n y \frac{2R_{wn}\cosh R_{wn}\left(1-\frac{z}{h}\right) + R_w \sinh R_{wn}\left(1-\frac{z}{h}\right)}{2R_{wn}\cosh R_{wn} + R_w \sinh R_{wn}}$$

where, $Q_0 = \frac{1}{B}\int_0^B C_0 dy, \quad Q_1 = \frac{1}{B}\int_0^B C_0 \cos k_n y dy, \quad k_n = \frac{n\pi}{B}\ (n = 1, 2, 3 \ldots\ldots)$

$$R_n = k_n h, \quad R_{wn} = \sqrt{R_n^2 + R_w^2/4}, \quad R_w = wh/\epsilon, \quad R_z = wz/\epsilon \qquad (11)$$

R_n, R_w and R_{wn} in the equation are all dimensionless terms, and R_z and R_w being typical Reynold's number. The solution for case (ii) is obtained similarly as follows:

$$C_2 = \frac{S_0}{2} e^{-R_z} + \sum_{n=1}^{\infty} S_n \exp. -\left[\frac{R_n^2}{R_u}\frac{x}{h} + R_z\right] \cos k_n y$$

where, $\frac{S_0}{2} = \frac{R_w}{Bh(1-e^{-R_w})}\int_0^h\int_0^B C_1 dydz, \quad S_n = \frac{R_w}{Bh(1-e^{-R_w})}\int_0^h\int_0^B C_1 \cos k_n y dydz$

$$R_u = Uh/\epsilon \qquad (12)$$

If (11) is substituted in C_2 and rearranged, then

$$C_2 = Q_0 e^{-R_z} + 4\sum_{n=1}^{\infty} \frac{Q_1 R_w \cos k_n y \cdot \sinh R_{wn} \cdot \exp.\left[-\left(\frac{R_n^2}{R_u}\frac{x}{h} + R_z\right)\right]}{(1-e^{-R_w})(2R_{wn}\cosh R_{wn} + R_w \sinh R_{wn})} \tag{13}$$

If suitable values are given to $C_0(y)$, the errors in the values of series (11) and (13) calculated to the 10th term would only be about 0.5 %.

In eq. (13), the most important factors are

$$R_w = wh/\epsilon, \quad R_z = wz/\epsilon\ ;$$

and if ϵ is given by eq. (4), then

$$R_w = \frac{6}{\kappa}\frac{w}{u_*} \cong 15\frac{w}{u_*} \tag{14}$$

With rivers in Japan, $Uh/\nu \cong 5\times10^7$ is attained at high water, and although the u_* also increses, U/u_* is only about 35 at most. R_w in this case is quite small. If eq. (11) is integrated, the quantity of suspended sediment of the entire section is obtained.

$$\sum C = 2\int_0^h \int_0^B C_1 dy dz = \frac{2h}{R_w}(1-e^{-R_w})\int_0^B C_0 dy \tag{15}$$

As $C_0(y)$ is given, the term effecting $\sum C$ is

$$R_* = \sum C \Big/ 2h\int_0^B C_0 dy = \frac{1-e^{-R_w}}{R_w} \tag{16}$$

$2h\int_0^B C_0 dy$ represents $\sum C$ when the concentration is uniform along the vertical from surface to bottom, and this becomes the basic unit. In eq. (16), the change in R_* due to R_w is as shown in Fig. 2, i.e., it decreases abruptly while the value of $R_w \leqq 4.0$. This means that, (a) when the flow condition is constant, a certain percentage of the finer particles ($R_w < 4.0$) maintains a perfect state of suspension, while on the

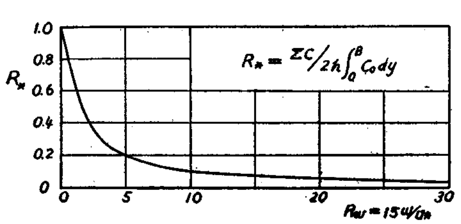


Fig. 2. Relation of R_* to R_w.

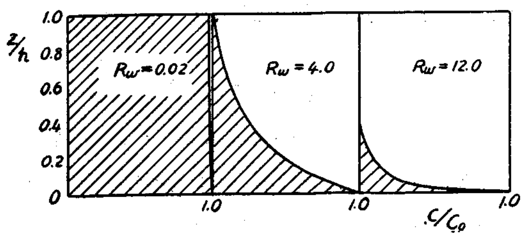


Fig. 3. Theoretical Sediment Distribution by Eq. (11).

contrary, coarser particles ($R_w > 4.0$) become distributed only near the bottom as is clear from Fig. 3. (b) With particles of almost uniform and constant settling velocity w, a slight change of the flow condition has a big effect on the amount of suspension, the boundary being the value of u_* corresponning to $R_w=4.0$. As an example of suspended sediment in high water, the result of the settling analysis experiment performed with surface layer clay collected at Yoshidayama, Kyoto, is as shown in Table 1, i.e., 51% being particles finer than $w=0.10$ cm/sec. of which about 40% was $w \cong 0.05$ cm/sec. Thus, for such small particles, $u_* \geqq 0.20$ cm/sec. gives the critical state of suspension. Applying the transfer coefficient

$$\eta = \frac{ch\sqrt{gih}\,(1-z^2)z^{\frac{1}{2}}}{\sqrt{2}\,(1+z^2)^{\frac{1}{2}}} \tag{17}$$

(c is a constant, i, slope)

Dr. Shoitiro Hayami[4] solved

$$\frac{\partial}{\partial z}\left(\eta\frac{\partial C}{\partial z}\right) + w\frac{\partial C}{\partial z} = 0$$

and acknowledged that $w/\sqrt{gih}=0.3$, which is the vicinity of $R_w=4.5$ in this paper, is the critical zone of suspension of particles. This almost agrees with the result in this paper which extending the dimension of theory. On the other hand, in discussing the stability of bottom particles, A. A. Kalinske[5] assumed v', the upward velocity occuring at the bottom as $v'=ku_*$ and giving the occurrence probability of v', considered the ratio between C_s, the concentration near the bottom, and $\Delta F(w)$, the rate of bottom particles with settling velocity corresponding to C_s, and also pointed out that this is governed by w/u_*. The relation is shown in Fig. 4 which shows that the equilibrium state of the bottom changes rather distinctly at $w/u_*=0.25$ ($R_w=3.75$). It seems that the abovementioned value of R_w, expressing the limit of the quantity of suspension, is also significant in this case and it can be understood that R_w is an important factor governing the stability of the river bed.

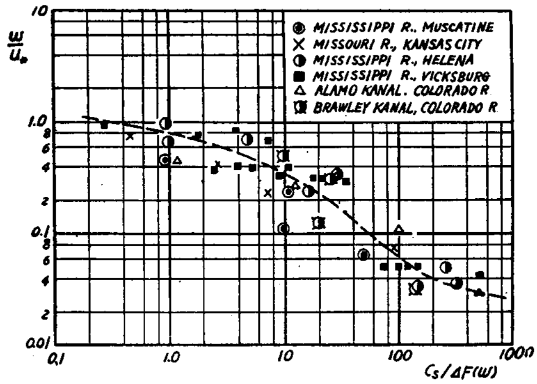


Fig. 4. Plotting of w/u_* and $Cs/\Delta F(w)$ as Determined From Field-Data (by E. W. Lane, A. A. Kalinske).

3. Two Dimensional Consideration

If analysed two dimensionally (x, z) in discussing the vertical distribution of suspended sediment, a solution identical to eq. (12) is obtained; especially in the case of a steady condition neglecting $\partial C/\partial x$, the form of the solution is simple and yet ϵ

is given as an arbitrary function of z. Using ϵ in eq. (3) and taking

$$\left.\begin{array}{ll} \text{at} \quad z=h, & \epsilon\dfrac{\partial C}{\partial z}+wC=0 \\ \text{at} \quad z=a, & C=C_a \end{array}\right\}$$

as boundary conditions, the solution is as follows.[6)]

$$\log\frac{C}{C_a}=\frac{w}{\kappa u_*}\log\left(\frac{a}{h-a}\cdot\frac{h-z}{z}\right) \tag{18}$$

This is convenient to use and worth practical application, but as only the general tendency of the distribution of suspended matter can be understood, so a different kind of calculation becomes necessary as the deposition of sediments due to flowing must actually be considered.

Particularly when w/u_* is much larger than 0.25, it can be said that once suspended matter deposits, it is hardly ever picked up again. In this case, the boundary conditions for

$$U\frac{\partial C}{\partial x}=\epsilon\frac{\partial^2 C}{\partial z^2}+w\frac{\partial C}{\partial z} \tag{19}$$

are

$$\left.\begin{array}{ll} \text{at} \quad x=0, & C=C_0 \\ \text{at} \quad z=0, & \dfrac{\partial C}{\partial z}=0 \\ \text{at} \quad z=h, & \epsilon\dfrac{\partial C}{\partial z}+wC=0 \end{array}\right\} \tag{20}$$

and the solution is obtained as follows by substituting

$$C=\xi\exp.\left[-\left(\frac{w^2}{4U\epsilon}x+\frac{w}{2\epsilon}z\right)\right]:$$

$$\frac{C}{C_0}=2\exp.\frac{1}{2}(R_w-R_z)\sum_{m=1}^{\infty}\frac{R_m\sin R_m\left(\cos\beta_m y+\dfrac{R_w}{2R_m}\sin\beta_m y\right)}{R_m^2+R_w^2/4+R_w} \tag{21}$$

where

$$R_m=\beta_m h,\quad \beta_m h=\arctan\frac{\beta_m h R_w}{(\beta_m h)^2-R_w^2/4}$$

(positive roots of β are ordered reversely to magnitude respectively, $m=1, 2, 3, \ldots\ldots$)
The solution for $C=C_0(y)$ at $x=0$ can also be obtained similarly, but will be omitted here. Series (21) converges even if only several terms are taken when $R_w<15$.

In order to examine the meaning and utility of eq. (18) and (21), the results of the flume experiment will be stated in the following.

4. Flume Experiment

The characteristic of the experiment performed lies in that a suspension of soil mixture composed of particles of various sizes is made to flow and observing the distribution at the lower flume section, the results is examined with the abovementioned theory and an investigation is made concerning the velocity law when suspended load exists.

The experiment was performed in the wooden flume (Fig. 5) within the compounds of Ujigawa Hydraulic Laboratory, Disaster Prevention Research Institute, Kyoto University. The effective length of flume is 15.0m, width 40cm, height 20 cm and slope 1/500 and a model storage reservoir with an overflow weir edge the same height as the flume bed of the end cross section adjoins.

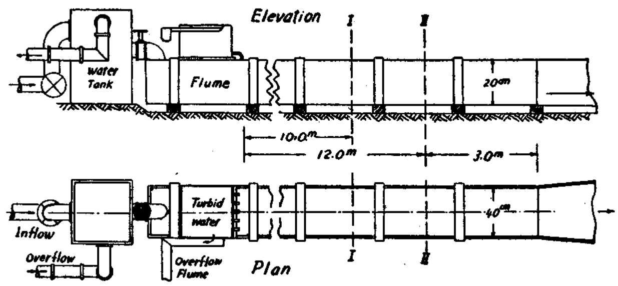
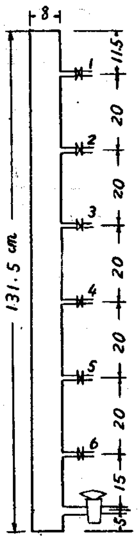


Fig. 5. Flume



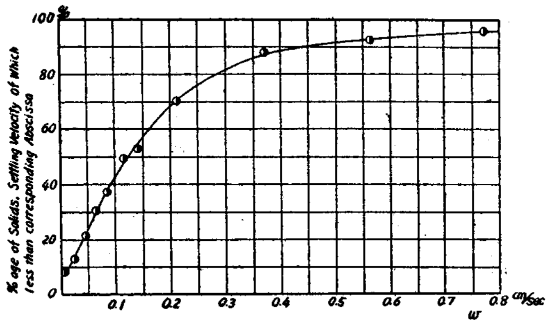


Fig. 6 (b). Settling Analysis Curve

← Fig. 6 (a). Glass Cylinder

(i) Settling analysis of soil

Soil of Yoshida-yama, Kyoto City, was used in the experiment. The majority of the soil is composed of particles diameter between 0.01 and 0.07 mm. To examine the composition, a settling analysis was performed with a glass messcylinder, length 131.5 cm and inner diameter 8 cm (Fig. 6 (a)). As the hindering effect near the bottom sampling hole is considered remarkable, samples were taken carefully at constant intervals from sampling hole No. 5 and weighed after drying; the result of which is as shown in Fig. 6 (b).

If the composition is divided according to stages of representative settling

Table 1. Composition of the Examined Soil Classified According to the Representative Settling Velocity w.

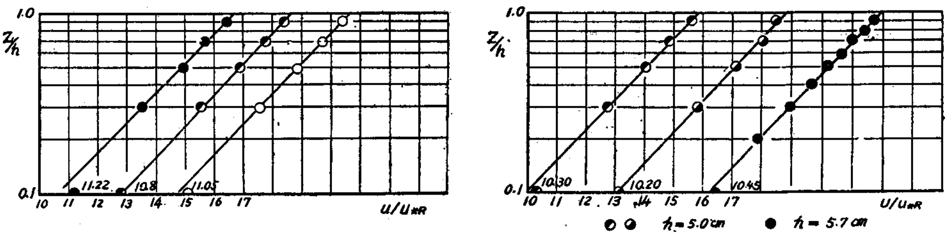
w cm/sec	0	0.05	0.10	0.15	0.20	0.25	0.35	0.60	1.50	Sum
%	13	21	17	12	9	7	10	7	4	100%

Mean settling velocity w_m=0.12 cm/sec

velocities, it becomes as shown in Table 1.

(ii) Law of velocity distribution

I and II in Fig. 5 are sections 10 m and 12 m from the point the soil is poured in. In this experiment the greatest depth of flow was about 6 cm. Using pitot tubes the velocity was measured about at 0.5 cm interval from the bottom surface



(I) (II)

Fig. 7. Velocity Distribution

along the centre vertical line of cross section. The profiles of velocity distribution with and without suspended matter are as shown in Fig. 7 and Fig. 8 respectively, and (I), (II) in Fig. 7 correspond to the measured values at section I II. In order to eliminate the three dimensional effects, the hydraulic mean depth R was adopted and put $u_{*R}=\sqrt{giR}$.

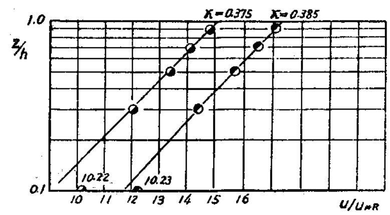


Fig. 8. Velocity Distribution in Silt Load

The profils of velocity distribution in I, II show almost the same tendency and relation

$$\frac{u-u_{\max}}{u_{*R}}=\frac{2.303}{\kappa}\log\frac{z}{h} \tag{22}$$

exists where the universal constant $\kappa=0.4$.

In the case of velocity distribution when suspended matter exist, κ=0.375~0.385 and shows a slight decrease compared to the normal case; because to avoid the effect of hindered settling the silt load was comparatively light, it being 0.15~0.30 g/litre. According to V. A. Vanoni's experiment[7], κ gradually decreases and attains a value of around 0,3 for higher silt load. Judging from experimental results given in this experiment and that of Vanoni, it can be concluded that κ decreases to about 0.38 for such low concentration as this experiment. Therefore, as stated in **1**, $\kappa=0.38$ is used hereafter in discussing the experimental results.

(iii) Distribution of Suspended Solids

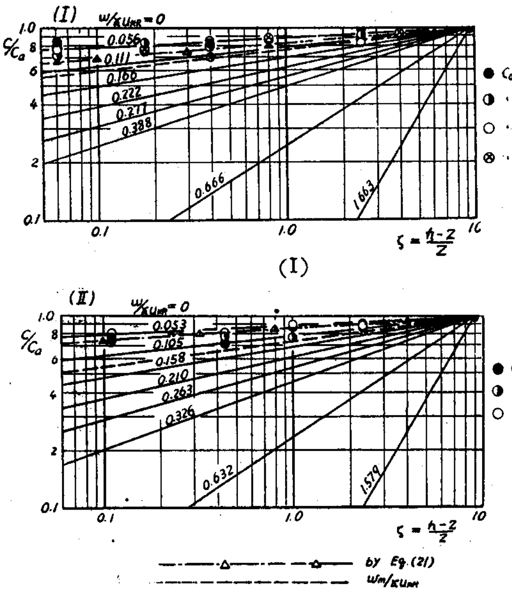
Soil, dispersed in a certain amount of water and stored in a turbid water tank, is poured into flowing water through five pipe holes attached to the tank with a constant head. There is an over flow flume at the top of the tank and the water is agitated at a constant period so that the concentration of turbid water in the tank does not decrease.

The discharge is small and the depth 5 to 6 cm, but this is rather convenient to make the turbid water pour in uniformly throughout the section at the initial point.

A siphon was used in measuring the amount of suspended solids. The siphon was made by bending a glass tube (inner diameter 3 mm) and the mouth of the tube was directed towards the flow and upon measuring the velocity, the siphon head was adjusted so that the sampling velocity was the same as the flow.

The same sections, I and II, used for velocity distribution in (ii), was used, and adjusted so that the initial concentration, immediately after the turbid water is poured in, becomes 0.10∼0.30 g/litre. A high concentration is also necessary, but the difficulty of analysing the data is anticipated because of the effect of the hinderance between the existing particles, or of the obscure estimation of the value of w/u_* must be made.

The relation between C/C_a and $h-z/z$ in eq. (18) becomes a straight line on the logarithmic paper, the slope being expressed by $w/\kappa u_{*R}$. If the obtained suspended solid distribution is plotted putting $a=0.5$ cm and taking Ca as the standard, it becomes as represented by I and II in Fig. 9. The broken lines in the figure express slope $w_m/\kappa u_{*R}=0.126$ corresponding to the average settling velocity of the entire soil, but the values are lower than the measured values for both cases I and II. Although partly due to the effect



(II)

Fig. 9. Sediment Distribution

of the composed particles not being uniform, this is result of not considering the deposition caused by flow. Actually, among the composing particles, those with larger w gradually settle and deposit. But unless w/u_{*B} is sufficiently large, deposit does not occur in the strict sense. Strictly speaking the amount of suspension must be calculated giving the equilibrium candition of the bottom for each group of particles, but this is very difficult. If the error of calculation due to a slight difference in the bottom condition is considered as remarkable only near the bottom, and a, the distance from the bottom, is taken much larger than the diameter of particles, then it is not so unreasonable to calculate C/C_a of each group for $a \leqq z \leqq h$ and compare it with the experimental data. Thus taking $a=0.5$ cm and putting $x=10$ m, 12 m for each group in table 1, each concentration distribution was calculated from eq. (21) and C/Ca was obtained by summing up the results. Plotting the results gives —·—△—·—△—·— in I and II, Fig. 9, which agrees quite well with the experimental data. Therefore, although qualitatively, this result is a suitable data to show that eq. (21) worth practically enough.

5. Settling Efficiency of Suspended Materials in Sedimentation Basin

Discussing the removal efficiency of suspended matter in sedimentation basin in water purification or sewage treatment plant, that the theory of ideal sedimentation basin does not agree with tne actual data was recognized, so that this problem is considered an important field of application of hitherto discussed theory. In the sedimentation basin, the case is different from turbulent flow in ordinary channel or river, and as the detention period is usually long, so the w/u_* value is small in comparison with Uh/ν, so that it is not unreasonable to assume that deposition only occurs at the bottom. From this point of view, the clarifying efficiency of a rectangular sedimentation basin, length l, width B, depth h, was studied three dimensionally to obtain the fundamental equation for the clarifying efficiency.[8)]

If eq. (6) is considered as the fundamental equation with the boundary condition,

$$\left.\begin{array}{lll} \text{at} & y=0,\ B/2\,; & \partial C/\partial y=0 \\ \text{at} & z=0 & ; \ \partial C/\partial z=0 \\ \text{at} & z=h & ; \ \epsilon\dfrac{\partial C}{\partial z}+wC=0 \\ \text{at} & x=0 & ; \ C=C_0(y,z) \end{array}\right\} \tag{23}$$

the solution is expressed as a double trigonometrical series (y and z). And for this the clarifying efficiency

$$E=1-\frac{\int_0^B\int_0^h Cdydz|_{x=l}}{\int_0^B\int_0^h C_0 dydz} \tag{24}$$

is obtained, and it was discovered that when C_0=const., almost the same result was obtained qualitatively as the case when treated as a problem of two dimensional.

Therefore, to simplify the calculation, if

$$E = 1 - \frac{1}{C_0 h}\int_0^h c\,dz \tag{25}$$

is calculated for C obtained by eq. (21),

$$E = 1 - 2R_w C^{\frac{R_w}{2}} \sum_{m=1}^{\infty} \frac{R_m \sin R_m \exp.\left[-\frac{1}{R_u}\left(\frac{R_w^2}{4}+R_m^2\right)\frac{l}{h}\right]}{(R_m^2+R_w^2/4+R_w)(R_m^2+R_w^2/4)} \tag{26}$$

is obtained. As the sign in the equation is the same as in eq. (21), this relation is the same as that derived by T. R. Camp[9] and W. A. Dobbins[10], only the expression being different. With this equation, Camp discusses the effect of the overflow rate w/w_0 and R_w on the clarifying efficiency. If expressed in the form of eq. (26), it becomes a universal theoretical equation, as the signs in the equation are all dimensionless terms.

However, besides the factors in the above equation which can be easily calculated from the design data, there are others which govern the depositing rate such as short circuiting and hindered settling effect which cannot be foreseen when designed. In the sedimentation basin pretreated by coagulant, the effect of flocculation occuring in the basin must be considered besides the above. Flocculation increases settling, efficiency but the other two, on the contrary, decrease it. It is considered that they have a greater effect the larger the concentration is, but there is practically no reliable theory concerning the relation to the clarifying efficiency. The effect which short circuiting or deviation of flow have on the clarifying efficiency can, however, be estimated hydraulically to some extent. In the following an opinion will be given on the effect of deviation and the relation between the clarifying efficiency and Froude's number.

Deviation and short circuiting occur due to the stability of the flow being bad. When the deviation exists the effective volume of the basin decreases, the extent being estimated generally as follows. Cl′ion is put in at the inlet eend, so from t_t, abscissa of the centre of gravity of the obtained time-concentration curve observed at the outlet end, and designed detention period t_0,

$$r = t_t/t_0 \tag{27}$$

which although is effected by diffusion to some extent, gives the approximate volumetric efficiency. With many of the representative sedimentation basins in Japan designed earlier, r=0.40～0.60. On the other hand, if the intensity of the deviation is considered as proportional to the rate of dead water zone, then r is given as follows.

That is, as $UA = U_r A_r$ where, U=designed mean flow velocity, U_r=the mean velocity of actual flowing part,

A = designed cross sectional area transverse to the direction of flow,

A_r = actually effective sectional area,

then

$$r = \frac{1}{Al}\int_0^l A_r dx = \frac{1}{l}\int_0^l \frac{U}{U_r}dx\,;$$

to make it simple, if $\frac{dU_r}{dx} = 0$,

$$r = U/U_r\,. \tag{28}$$

The mean value ϵ of the transfer coefficient changes according to u_* as mentioned before. As a result of the occurence of deviation, the value of u_* naturally changes and becomes u_{*r}. The relation between U_r and u_{*r} becomes as follows from the logarithmic mean velocity law of turbulent flow in open channel.[11]

$$\frac{U_r}{u_{*r}} = \alpha + \frac{2.303}{\kappa}\log\left(\frac{u_{*r}h}{\nu}\right), \quad \alpha \cong 3.0\,. \tag{29}$$

Therefore from

$$\frac{1}{r}\frac{U}{u_{*r}} = \alpha + \frac{2.303}{\kappa}\log\left(\frac{Uh}{\nu}\cdot\frac{u_{*r}}{U}\right) \tag{30}$$

u_{*r} correspoding to r is obtained, with which

$$\left.\begin{aligned} \epsilon_r &= \frac{1}{6}\kappa u_{*r}h \\ R_{wr} &= \frac{6}{\kappa}\frac{w}{u_{*r}} \\ R_{ur} &= \frac{6U}{\kappa r u_{*r}} \\ R_{mr} &= \arctan\frac{R_{mr}R_{wr}}{R_{mr}^2 - R_{wr}^2/4} \end{aligned}\right\} \tag{31}$$

can be calculated. Thus, clarifying efficiency E can be expressed as follows.

$$E = 1 - 2R_{wr}e^{\frac{R_{wr}}{2}}\sum_{m=1}^{\infty}\frac{R_{mr}\,\mathrm{sis}\,R_{mr}\exp.\left[-\frac{1}{R_{ur}}\left(\frac{R_{wr}^2}{4} + R_{mr}^2\right)\frac{l}{h}\right]}{(R_{mr}^2 + R_{wr}^2/4 + R_{wr})(R_{mr}^2 + R_{wr}^2/4)} \tag{32}$$

Next, a consideration will be made on Froude's number which is considered to govern the flowing condition. Although it can be understood that there exists a relation between the volumetric efficiency r and Froude's number F_u,

$$F_u = U_r^2/gh\,, \tag{33}$$

it cannot be said that the relation is always directly proportional. There are many examples of sedimentation basins in Japan with quite different values of r due to the existence of inlet flow regulating wall, or midway flow regulating wall even if the

Froude's number (by designed data) is same. By transforming eq. (32), however, Froude's number can be introduced into clarifying efficiency E as follows.

$$E=1-2R_{wr}e^{\frac{1}{2}R_{wr}}\sum_{m=1}^{\infty}\frac{R_{mr}\sin R_{mr}\exp.\left[-R_{ur}\left(\frac{F_w}{F_u}+\frac{R_{mr}^2}{R_{ur}^2}\right)\frac{l}{h}\right]}{\left(R_{mr}^2+\frac{R_{wr}^2}{4}\right)\left(R_{mr}^2+\frac{R_{wr}^2}{4}+R_{wr}\right)} \tag{34}$$

where, $F_w=w^2/gh$.

Besides the effect of flocculation and hindered settling, eq. (34) contains almost all other presumable hydraulic factors in dimensionless form, and is a very universal expression of the clarifying efficiency. Fig. 10 shows the relation of $l/R_{ur}\cdot h$ to E in eq. (32), the parameter being R_{wr}. The series in eq. (32) does not converge well for $R_{wr}>15$, and require more than 10 terms.

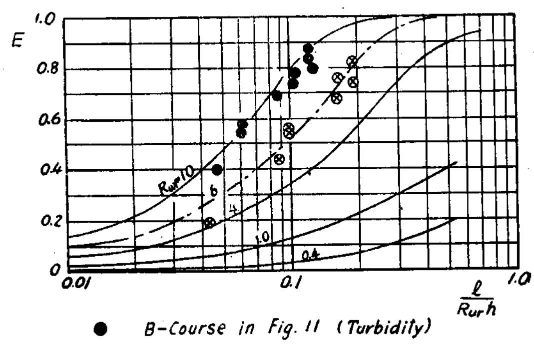


Fig. 10. Relation of E to $l/R_{ur}h$

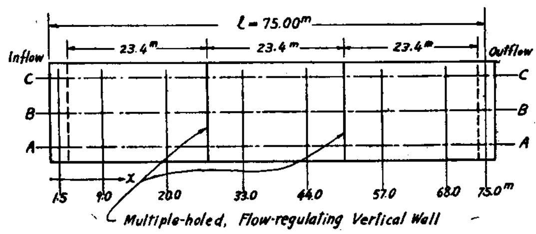


Fig. 11. Plan of No. 3 Sedimentation Basin, Amagasaki Water Purification Plant, Hanshin-Josuido-Kumiai (Rapid Filter System)

From July to Sept, 1952, the turbidity distribution and the number of general bacteria was investigated at several points along the many vertical courses shown in Fig. 11, at the pretreated sedimentation basin in Amagasaki Water Purification Plant, Hanshin-JosuidoKumiai, Japan. Turbidity removal ratio was obtained from the initial turbidity and the mean value along the each vertical line, and plotted in Fig. 10 in accordance with $x/R_{ur}h$, corresponding to $r=0.90$ (taken from observed time-concentration curve). Although there is some doubt in applying the general theoretical eq. (32) to the turbidity removal ratio, in this case it seems that this tendency agrees well with eq. (32). However, as the rate of coagulant added and the state of flocculation change with the season, the tendency cannot be said to be constant, and this will be given here as a reference data.

Next in studying the relation between F_u and E, a rectangular sedimentation basin with a constant capacity is considered as an example, and assuming the width as constant, $t=4$ hrs, $Uh=216.0$ cm²/sec, $lh=3110400$ cm², $w=0.03$ cm/sec, $Uh/\nu=18946$

(ν: at 15°C), $\kappa=0.4$ are given. Then, using eq. (30), (31), (32) or (33), the change of E due to the variation of h can be calculated. In this case r is a parameter. Fig. 12 shows that E increases with the increase of F_u, but actually as the stability of the flow increases when F_u increases, r is expected to increase and finally it can be understand that the decrease in depth h result in a considerable increase of E. The dotted line in the figure is the line obtained on the case

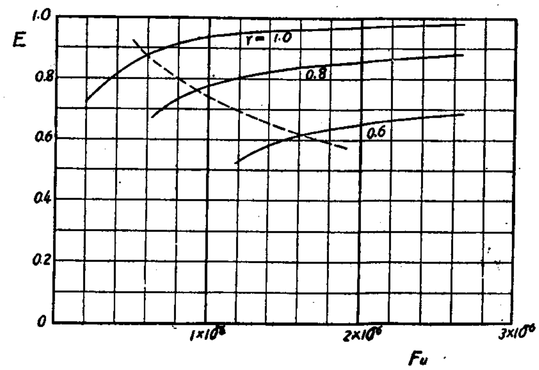


Fig. 12. Relation of E to F_u

$$t_0=\frac{l}{U}=\frac{h}{w}=4\text{ hours},$$

which is a standard line of design.

As stated by T. R. Camp[12], it was verified that when the volume of basin is constant, the effects can actually be promoted by such device as enlarging the l/h value or constructing a tray. On the contrary, when the volume of basin is variable, that to decrease the depth only is not necessarily safe in view of efficiency is shown by similar calculation as above; but the detail was omitted.

6. Summary and Conclusion

In order to comprehend the mechanism of transportation of suspended matter the meaning of w/u_* was cleared by three dimensional consideration and $w/u_*\simeq 0.25$ was given as a limit of suspension, which was also shown to be related to the stability of bottom particles. As is clear from the results of the flume experiment in 4, the two dimensional theory is practical; especially when soil particles of various sizes flow mixed and mainly deposition occurs, it was made clear that eq. (21) can be used in the calculation. In many cases, however, as the transverse distribution of concentration tends to concentrate slightly towards the centre of flow, a three dimensional consideration must be done while trying the quantitative calculation in this case. According to the measured data of the transverse concentration distribution taken after a heavy rainfall at Takano River, Kyoto, in June 1953, a higher concentration at the flow centre was recognized. This thought to be an influence of the velocity gradient, which is a theoretical problem for future study.

The effect of the various hydraulic factors on the clarifying efficiency was discussed in 5, and the followings can be given as conclusion.

(a) Concerning one of the problems confronted, i.e., what is the best design for a given certain detention period, ; it is desirable to take F_u, which is easier to understand as the standard index instead of w/u_*, larger as possible.

(b) When the capacity is not constant, merely decreasing the water depth will not necessarily increase the efficiency even if F_u increases, and a further investigation is necessary.

(c) Besides the factors in eq. (34), flocculation and hindered settling effect on efficiency E; and apart from the fact that hindered settling has a negative effect on the clarifying efficiency, there is doubt as to what extent the effect of flocculation promotes settling. Although an important problem to be studied, at the present state at least, the aid of flocculation in the settling basin should be considered secondary.

The Author is greatly indebted to Dr. T. Ishihara and Dr. S. Iwai, professors of Kyoto University, for their instruction and to Mr. I. Yagi for his cooperation in performing the experiments.

Literatures Cited

1) Vanoni, V. A.: "Transportation of suspended sediment by water", Trans. Am. Soc. Civ. Engrs., Vol. 111, p. 67, 1946.
Einstein, H. A.: "The bed-load function for sediment transportation in open channel flows", Tech. Bulletin, No. 1029, U. S. Dept. of Agr. Soil Cons. Service, Sept. 1950.
2) Kalinske, A. A. and Robertson, T. M.: "Turbulence in open channel flow", Eng. News Record, Apr. 1941.
3) Vanoni, V. A. and En-Yun Hsu: "Turbulence and diffusion as factors in sediment transportation", Proc. Am. Soc. Civ. Engrs., Preprint 67, Sept. 1952.
4) Hayami, S.: "Natural environment of North and South China as observed from the Yangtze River and Yellow River", The Sea and Land Water of the Pacific Ocean, Pacific Ocean Association, Japan, P. 755, Dec. 1943.
5) Lane, E. W. and Kalinske, A. A.: "The relation of suspended to bed material in rivers", Trans. Am. Geoph. Union, p. 637, 1939.
6) Vanoni, V. A; Einstein, H. A: 1)
7) Vanoni, V. A; 3)
8) Goda, T.: "On the excluding efficiency of settling reservoir", Proc. Japan Soc. Civ. Engrs., No. 6, p. 39, Aug. 1951.
9) Camp, T. R.: "Sedimentation and the design of settling tanks", Trans. Am. Soc. Civ. Engrs., Vol. 111, p. 895, 1946.
10) Dobbins, W. E.: "Effect of turbulence on sedimentation", Proc. Am. Soc. Civ. Engrs., Feb. 1943.
11) Ishihara, T., Iwagaki, Y. and Goda, T.: "Studies on the thin sheet flow (1st. Report)", Proc. Jap. Soc. Civ. Engrs., No. 6, p. 31, Aug. 1951.
12) Camp, T. R.: "Studies of sedimentation basin design", Sewage and Ind. Wastes, Vol. 25, No. 1, p. 1, Jan. 1953.
13) Goda, T.: "On the distribution of suspended materials in open channel", Jour. Jap. Soc. Civ. Engrs., Vol. 35, No. 10, p. 17, Oct. 1950.