# Studies on the Air Duct Ventilation with Leakage in Mines 

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## Introduction

The air duct ventilation is recognized to be one of the most important operations in mines from the standpoint of safety because it is used as a local ventilation at such places where the inflammable gas, generated in abundance, cannot be diluted within the limit of safety due to insufficient effect of the main fan. But it is difficult to prevent leakage of air through small openings on the wall or at the joints of air ducts, which has a bad influence upon the effectiveness of the ventilation.

The air duct ventilation without leakage is, no doubt, governed by a simple law; however, the circumstances becomes complicated beyond expectation when leakage occurs. There have been, however, few researches made on the air duct ventilation accompaning leakage, and the rational designing of a local ventilation is almost impossible. Considering these circumstances, the authors intend to study the two major problems; namely, air flow through minute openings and the theory of the air duct ventilation accompaning leakage.

As to the first subject, experimental or theoretical studies on flows passing through orifices or slits, in which the Reynolds' numbers are comparatively small, have been performed by several investigators such as Tsumura-Iwanami ${ }^{1)}$, Giese ${ }^{2)}$, Hansen ${ }^{3}$, Johanson ${ }^{4}$, Blasius ${ }^{5}$ and Karman ${ }^{6}$. These experiments are conducted with water and fluids of high viscosity, such as rape-seed oil, gasoline, machine oil and waterglass. But few papers are found of the instances in which the Reynolds' number are less than 200 and, further, air is used as a passing matter instead of liquids.

As to the second subject, Briggs $^{7}$ ) has carried out a theoretical study assuming that (1) the leakage air volume through any opening is directly proportional to the pressure difference between inside and outside of an air duct wall and (2) the pressure drop of air along the air duct is proportional to the distance of flow. The second assumption, however, seems not to hold true when leakage exists. M. Nakanishi ${ }^{8}$ ) has pressented a theoretical solution under the assumption that the
leakage air volume is proportional to the square root of the pressure difference, and S. Hirata ${ }^{9}$ has studied the same problem under the same assumption in a special case in which the length of air duct is infinitive. Nevertheless, there has been no strict solution concerning the theory of the air duct ventilation in which the various features of leakage phenomena are taken into account.

## Part I: A study on the air leakage through minute openings

It is a matter of fact that the Reyonlds' number of leakage air flowing through a minute opening is small, but the phenomena of leakage are not simple because of variety and irregularity of the shape and size of the openings.

The flow, in general, changes from a laminar flow to a turbulent flow as the pressure difference on both sides of the opening increases, but at what point the change takes place depends upon the conditions of the openings.

Of course the conditions of openings found on the wall of air ducts varies widely, but they may be classified into two main kinds; namely, pinholes and crevices. The typical forms of these two categories may be orifices and slits. The authors carried out a study on flows passing through orifices and slits to verify the theory of air leakage by means of experiment using full size models.

## (1) Experiments on the flow passing through orifices.

These experiments are for the purpose of finding the relation between the pressure difference $p\left(\mathrm{~kg} / \mathrm{m}^{2}\right)$ across an orifice and the leakage air volume $Q\left(\mathrm{~m}^{3} / \mathrm{s}\right)$.

## Apparatus:

The apparatus used for this experiment is shown in Fig. 1. It is so arranged that a constant flow of air is sent into vessel $J$, on one side of which is attached an orifice $O$, air flows constantly through the orifice, and the pressure difference inside and outside of the orifice can be measured. The supply of a constant flow of air into $J$ is done by flow of water into the container $A$ (also $B$ and $C$ ) at a constant speed, and latter is done by a water tank $A_{1}$, having a constant head, which let the water out by means of a device shown as $M$.


Fig. 1. The apparatus arranged for the experiments on the flow through orifices.

Changing the speed of air passing through the orifice is effected by means of varying water speed pouring into $A$ (or $B, C$ ) by controlling throttle valves $X_{1}$ or $X_{2}$. The purpose of using three vessels $A, B$ and $C$ is to be able to accurately measure the water flow quantity in a wide range by properly manipulating them. And the arrangement of vessels can be done by manipulating cocks $X_{1}, X_{2}, D$ and $F$. $M$, an overflow device, is for stabilizing water head flowing into the containers $A, B$, and $C$, and the water vessel $A_{2}$ is for the purpose of preventing water to wave at the surface. The pressure difference across the orifice is measured with an inclined manometer $K$. The thirteen kinds of orifices made on brass plates of 1.2 mm thick by 50 mm diameter are: $0.06,0.07,0.09,0.15,0.20,0.50,0.68,0.84,1.05,1.25$, 1.37, $1.80,2.50 \mathrm{~mm}$ in diameter, and the sections of which are as shown in a circle in Fig. 1.

For the purpose of noting the shape of the orifices, a few examples of photographs, magnified 120 times and taken with a universal projector (made by NihonKōgaku Co. Japan) are shown in Fig. 2. The roughness seen along the perimeter is due to fine dusts adhered to them, which are almost impossible to remove. The orifices larger than 0.15 mm in diameter, as a rule, can be considered as circular, however, those less than 0.09 mm in diameter are consider-
 ably distorted as shown in the figure.

## Results of experiment:

By using the above-described apparatus, the leakage air volume passing through each orifice is measured by changing the values of pressure difference to less than $1000 \mathrm{~kg} / \mathrm{m}^{2}$ across the orifice. The relation between the pressure difference and the measured leakage air volume is shown on a logarithmic section paper, Fig. 3. Since the line connecting these points is almost linear, the relation of $Q \propto p^{\frac{1}{n}}$ is formed and the value of $n$ is a constant existing between 1 and 2 .

Now, putting the mean velocity of air passing through the orifice as $u(\mathrm{~m} / \mathrm{s})$ and the specific weight as $\gamma\left(\mathrm{kg} / \mathrm{m}^{3}\right)$, the coefficient of discharge of the orifice is defined as:

$$
\begin{equation*}
C=u / \sqrt{2 g} p / \gamma . \tag{1}
\end{equation*}
$$

The Reynolds' number $R e$ in this case is written as

$$
\begin{equation*}
R e=2 r u / \nu \tag{2}
\end{equation*}
$$



Fig. 3. The relations between measured leakage air volume and pressure difference across the orifices.
where $\nu\left(\mathrm{m}^{2} / \mathrm{s}\right)$ is the kinematic viscosity of air, and $r(\mathrm{~m})$ is the radius of the orfice. Plotting the values of $C$ calculated from the results of experiments against Reynolds' number, we get Fig. 4. From this figure, it is found that $C$ is proportional to the square root of $R e$ for the values of $R e$ less than 50 . In consequence, an air flow passing through an orifice urder this condition can be regarded as a laminar flow and $n$ is nearly equal to 1 . And, in the case of $R e \geqq 70, C$ becomes constant regardless of $R e$, the flow in the orifice is turbulent and $n$ is nearly equal to 2 in this region. The results of experiments conducted by Giese for the flows of which Reynolds' number are larger than 1000 coincide with ours. However, Tsumura-Iwanami obtained a result shown with a dotted line in Fig. 4 and observed that a laminar flow appears when $R e$ becomes less than 10 . In the sphere of $R e \leqq 70$, the result obtained by the authors differs considerably. The reason for this difference is conceivable to be partially attributable to unsteadiness of the flow in the neighborhood of the critical Reynolds' number and partially to our inability to complete the experiment accurately

on account of smallness of the orifices employed compared with those used by Tsumura-Iwanami.

## (2) Experiments on the flow passing through slits.

Several investigations have been published on a drag exerted upon a thin flat plate placed parallel to the flow ${ }^{334(5) 66}$. In case there is no loss of energy at the entrance or when the loss at the entrance is negligible, the flow passing through a gap between the parallel flat plates can also be easily known by the above-mentioned results. But few researches are committed concerning the flow having a comparatively large loss at the entrance or when the plate is short in the direction of flow. The results of experiments for these cases generally vary widely depending upon the conditions of experiments. Hence, the authors attempt to experimentally investigate whether or not the relation


Fig. 5. The construction of the slit to be tested.
$Q \propto p^{\frac{1}{n}}$ exists in this case and, if it does, how the value of $n$ varies with the conditions of slits as well as of the flow.

## Apparatus:

The apparatus employed in this experiment is the same as that used in the preceeding experiment for orifices with the slits to be tested attached on one side of the vessel $J$ in the place of an orifice. For the parallel plates, as shown in Fig. 5, two pieces of small glass plates are set closely together. The gaps range from 0.05 to 0.43 mm and they are maintained by inserting thin metal plates at both ends.


Fig. 6. The relations between measured leakage air volumes and pressure differences across the slits.

The glass plates are of several width, which are equal to the length of the air pass. The least of them is very thin and of special construction, i. e. two razor blades closely placed opposite each other on a plane.

## Results of experiment :

The relation between the pressure difference across the slit and the air volume is obtained in the same manner as the preceeding experiment, and is shown in Fig. 6. It is noticed that the relation $Q \infty p^{\frac{1}{n}}$ exists in this case as well, and $n$ is between 1 and 2. Expressing the hydraulic mean redius of the slit by $r_{h}$, and putting $R e=4 u r_{h} / \nu$, it is found that all the Reynolds' numbers of flow in this experiment are less than 750. Since the critical Reynolds' number of a flow passing through a slit is generally considered to be about 3000 just as in the case of a circular pipe, , it is presumed that all the present conditions fall below the critical Reynolds' number and, consequently the value of $n$ to be 1 . On the contrary, the results of experiments show that $n$ varies from 1 to 2. This probably is due to the distortion of air flow at the entrance.

To investigate these


Fig. 7. The relation between the values of $n$ and $l / d$ circumstances, the value of $n$ are plotted against the length of air pass $l$ to the gap of slit $d$, as shown in Fig. 7. From this figure, it is cognizable that at $l / d>30$ the flow becomes laminar, and at $l / d<2$, turbulent.

## (3) Summary

The authors have studied the relations between the leakage air volume flowing through small crevices and the pressure difference for the purpose of investigating the phenomena of leakage seen in the air duct ventilation. For minute openings the orifices and the crevices of parallel plates are employed. As the result, it is found that the former is proportional to the $1 / n$th power of the latter. The values of $n$ were determined for several conditions of openings as well as for the varied Reyonlds' numbers.

In the actual practice of an air duct ventilation, the pressure of a fan operated underground is generally lower than $200-300 \mathrm{~kg} / \mathrm{m}^{2}$. Estimating the diameter of
pinholes on the wall of an air duct to be approximately 1 mm , the openings of crevice to be $d<1 \mathrm{~mm}$, and $l / d$ to be less than scores, the value of $n$ is regarded as 2 for pinholes and that of slits as $1 \leqq n \leqq 2$.

## Part II: A study on the theory of air duct ventilation accompaning leakage.

As has been mentioned in part I of this paper, the leakage air volume is not always proportional to the square root of pressure difference, as is generally assumed in the aforementioned researches relating to the air duct ventilation. Also, the authors find a few points to be improved in solving the former problem even if the assumption may be admitted. These circumstances have led the authors to conduct the theoretical study on the same problem.

## (1) Analysis.

The openings through which the leakage takes place are in various conditions and they are not distributed uniformly over the wall of an air duct. The analysis is conducted, however, on the assumption that all the openings are in the same condition and that their distribution is uniform all over the wall in order to simplify the analysis. Although these assumptions are somewhat imperfect, they are enevitable in making the solution possible, and we infer from these results the relations existing in practice. There are two systems of an air duct ventilation, namely suction and blowing. But, as both systems can be analyzed by the same method, the following article deals only with the blowing system.

Let the air volume flowing through an air duct of $D(\mathrm{~m})$ in diameter be $Q\left(\mathrm{~m}^{3} / \mathrm{s}\right)$, the static pressure difference between outside and inside of the pipe be $p\left(\mathrm{~kg} / \mathrm{m}^{2}\right)$, the change of air flow volume between the two sections with a distance of $d l(\mathrm{~m})$ apart be $-d Q$, then $-d Q$ is equal to the leakage air volume appearing between the two sections. The following equation is established in an air duct ventilation in mines as mentioned above:

$$
\begin{equation*}
-d Q=a \pi D p^{\frac{1}{n}} d l, \tag{3}
\end{equation*}
$$

where $a$ and $n$ are constants which are determined by the conditions of openings, the number of openings on a unit area of air duct wall and the Reynolds' number of leakage air flow. As described in part I , the value of $n$ is between 1 and 2 . Now, putting $a \pi D=K$, we get

$$
\begin{equation*}
-\frac{d Q}{d l}=K p^{\frac{1}{n^{n}}} . \tag{4}
\end{equation*}
$$

Equation (4), showing the relations among $Q, l$, and $p$, is reduced by considering only of the leakage.

On the other hand, if we consider the pressure drop in the duct, we obtain the following equation :

$$
\begin{equation*}
-\frac{d p}{d l}=\frac{8 \lambda r}{D^{5} g \pi^{2}} Q^{2} \tag{5}
\end{equation*}
$$

where $-d p\left(\mathrm{~kg} / \mathrm{m}^{2}\right)$ is the pressure drop along the distance of $d l$ and $\lambda$ is a coeffcient of resistance of the duct. Since the Reynolds' number of flow in a duct is comparatively great, it may be allowed to take $\lambda$ for a constant and to neglect the variation of $\gamma$. Putting $8 \lambda \gamma / D^{5} g \pi^{2}=R$, then, equation (5) is rewritten as:

$$
\begin{equation*}
-\frac{d p}{d l}=R Q^{2} \tag{6}
\end{equation*}
$$

Assuming that the equations (4) and (6) are established simultaneously, the following equation is obtained :

$$
\begin{equation*}
\frac{d p}{d Q}=\frac{R}{K} \frac{Q^{2}}{p^{\frac{1}{n}}} \tag{7}
\end{equation*}
$$

Integrating equation (7), and putting $p=p_{0}$ and $Q=Q_{0}$ at the entrance of the duct, and $p=0$ and $Q=Q_{x}$ at the exit of it, we have:

$$
\begin{equation*}
\frac{n}{n+1} p_{0}^{\frac{n+1}{3}}=\frac{R}{3 K} Q_{L}^{3}\left(m^{3}-1\right), \tag{8}
\end{equation*}
$$

where $m=Q_{0} / Q_{L}$. Because it is a difficult task to analyse this problem for any value of optional $n$ between 1 and 2 , the solution is sought for the two extreme values of $n$, namely 1 and 2 . By doing so the solution for any values of optional $n$ between these two limitations may be inferred to some extent.

## The case $n=1$ :

The constant $K$ in this case is notated as $K_{1}$. Differentiating equation (4) with respect to $l$, and substituting it in equation (6), and then integrating this equation from $Q=Q_{0}$ to $Q=Q_{L}$ under the condition of $d Q / d l<0$, we get

$$
\begin{equation*}
L=\frac{-1}{\sqrt{\frac{2}{3} R K_{1}}} \int_{Q_{0}}^{Q_{L}} \frac{d Q}{\sqrt{Q^{3}-Q_{L}^{3}}}, \tag{9}
\end{equation*}
$$

where $L$ is the total length of a duct, and $Q=Q_{0}, d Q / d l=-K_{1} P_{0}$ when $p=p_{0}$. Putting $Q=Q_{L} y$ to show equation (9) non-dimensionally, we get:

$$
\begin{equation*}
L=\frac{1}{\sqrt{\frac{2}{3} R K_{1} Q_{L}}} \int_{1}^{\frac{Q_{0}}{Q_{x}}} \frac{d y}{\sqrt{y^{3}-1}} . \tag{10}
\end{equation*}
$$

The integral thus obtained is the first kind of perfect elliptic integral. The necessary transformation of the variables are performed as follows;

$$
\begin{aligned}
& \cos \varphi=(\sqrt{3}+1-m) /(\sqrt{3}-1+m), \quad 0 \leqq \varphi \leqq \pi \\
& 0<k=\sqrt{2-\sqrt{3}} / 2=\sin 15^{\circ}<1
\end{aligned}
$$

and putting

$$
F\left(\varphi, 15^{\circ}\right)=\int_{0}^{\psi} \frac{d \varphi}{\sqrt{1-k^{2} \sin ^{2} \varphi}},
$$

we get :

$$
\begin{equation*}
L=\frac{F\left(\varphi, 15^{\circ}\right)}{\sqrt[4]{3} \sqrt{\frac{2}{3} R K_{1} Q_{x}}} . \tag{11}
\end{equation*}
$$

Equation (8) and (11) give the relation among $p_{0}, Q_{0}, Q_{L}$ and $L$ for the case $n=1$.

## The case $n=2$ :

The constant $K$ in this case is notated as $K_{2}$. Differentiating equation (4) with respect to $l$, and substituting it in equation (6) and putting $n=2, Q=Q_{0}, d Q / d l=$ $-K_{2} P_{0}$ when $p=p_{0}$, we get :

$$
\begin{equation*}
\frac{d Q}{d l}=A\left(Q_{L}^{3}-Q^{3}\right)^{\frac{1}{3}} \tag{12}
\end{equation*}
$$

where $A=\sqrt[3]{ } K_{2}^{2} R / 2$. Integrating equaion (12) from $l=0$ to $l=L$,

$$
\begin{equation*}
A L=\frac{1}{6} \operatorname{loge} \frac{m^{2}+m\left(m^{3}-1\right)^{\frac{1}{3}}+\left(m^{3}-1\right)^{\frac{2}{3}}}{m^{2}-2 m\left(m^{3}-1\right)^{\frac{1}{3}}+\left(m^{3}-1\right)^{\frac{2}{3}}}-\frac{1}{\sqrt{3}} \tan ^{-1} \frac{\sqrt{3}\left(m^{3}-1\right)^{\frac{1}{3}}}{2 m+\left(m^{3}-1\right)^{\frac{1}{3}}} \tag{13}
\end{equation*}
$$

Equation (8) and (13) give the relation among $p_{0}, Q_{0}, Q_{L}$ and $L$ for the case $n=2$.
Contrary to the case $n=1, m$ is determined by $R, K_{2}$ and $L$ in this case, therefore the ratio of $Q_{0}$ to $Q_{L}$ is constant for the same duct.

For reference, if no leakage occurs, the value of $Q$ becomes constant as ;

$$
\begin{equation*}
p_{0}=R Q_{0}^{2} L \tag{14}
\end{equation*}
$$

## The value of $n$ :

It is a problem to investigate what is the value of $n$. The authors have examined literatures on air duct ventilation published in the past looking for data which would help determine the value of $n$, but could not obtain satisfactory results. However, by the above discussion and also by the discussion in part I of this paper, we can assume that $n$ is either between 1 and 2 or at a point somewhat near to 2 .

## The value of $\boldsymbol{R}$ and $K$ :

Since it is difficult to assume the values $R$ and $K$ from mere observations of an air duct, the general idea of them will be shown here.


Fig. 8. The value of $R$.
The value of $R$ in this problem can be regarded, without any material error, as a function only of the diameter of an air duct and it is as shown in Fig. 8.

As to the value of $K$, a few examples will be presented below. The values of $K$ for the three kinds of air duct of 200 m long of which the air volume at the entrance is $2 \mathrm{~m}^{3} / \mathrm{s}$, and that at the exit is $80 \%, 60 \%$, and $40 \%$ respectively of the air volume at the entrance, are tabulated in Table 1 for the two cases of $n=1$ and $n=2$.

Table 1. The values of $K$ for air ducts, 200 m in length, 0.6 m in diameter.

| Class of the air duct | Air volume at the entrance | Air volume at the exit. | K |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  |  | $n=1$ | $n=2$ |
| A | $2.0 \mathrm{~m}^{3} / \mathrm{s}$ | $1.6 \mathrm{~m}^{3} / \mathrm{s}$ | $3 \times 10^{-4} \mathrm{~m}^{4} / \mathrm{kg} \cdot \mathrm{s}$ | $8 \times 10^{-4} \mathrm{~m}^{3} / \mathrm{kg}^{\frac{1}{2}} \cdot \mathrm{~s}$ |
| B | " | 1.2 | $10 \times 10^{-4}$ | $20 \times 10^{-4}$ |
| C | " | 0.8 " | $28 \times 10^{-4}$, | $40 \times 10^{-4} \quad$, |

## (2) Graphical representation of the results of analysis.

The results of analysis described above are so complicated that it is troublesome to apply them in designing or discussion of the air duct ventilation. The authors, therefore, have contrived to illustrate the results for the purpose of ready reference, and the graphs obtained are shown in Fig. 9, A and B. These graphs are constructed on the basis of equation (8), (11) and (13). The graphs are composed separately for the cases $n=1$ and $n=2$, and show the relation among $L, p_{0}$ and $m$ i. e. $Q_{0} / Q_{L}$, taking $L$ on abscissa, $p_{0}$ on ordinate, $Q_{L}$ and $m$ as parameters for the particular values of $K$ and $R$. But they can be transformed into graphs which are based upon any values of $K$ and $R$ at will by multiplying the reading of abscissa and ordinate by $\alpha$ and $\beta$ respectively. The reason for the possibility of this transformation is as follows. $\quad p_{0}$ or $L$ is simply determined by the values of $K$ and $R$ if $m$ and $Q_{L}$ remain constant because $p_{0}$ or $L$ is a function of $K, R, m$ and $Q_{L}$. Therefore, the
transformations of this graph for the value of optional $K$ and $R$ can be done by simply multiplying $p_{0}$ and $L \alpha$ times and $\beta$ times without changing the curve of parameter. The coefficients $\alpha$ and $\beta$ are given by $K$ and $R$ by the formulae shown in Table 2. The figures $2.2 \times 10^{-3}$ and so on in the table are obtainable from the special values of $K$ and $R$ adopted in constructing Fig. 9. It is recognized theoretically also that if any four of six variables, $p_{0}, Q_{0}, Q_{L}, L, K$ and $R$ are given, the remaining two are automatically determined. These circumstances seem to be quite complicated, nevertheless, the graphs are successful in showing them explicitly. And

Table 2. Formulae to evaluate values of coefficients $\alpha$ and $\beta$ for the optional values of $K$ and $R$.

| coefficient | Fig. 9. A. <br> $n=1$ | Fig. 9. B. <br> $n=2$ |
| :---: | :---: | :---: |
| $\alpha$ | $\alpha_{1}=\frac{1}{18.2} \sqrt[7]{\frac{R}{K_{1}}}$ | $\alpha_{2}=\frac{1}{7.96} \sqrt[3]{\left(\frac{R}{K_{2}}\right)^{2}}$ |
| $\beta$ | $\beta_{1}=\frac{2.20 \times 10^{-3}}{\sqrt{R K_{1}}}$ | $\beta_{2}=\frac{2.82 \times 10^{-3}}{\sqrt[3]{K_{2}^{2} R}}$ |



Fig. 9. A. $n=1, K_{1}=1.21 \times 10^{-4} \mathrm{~m}^{4} / \mathrm{kg} \cdot \mathrm{s} \quad R=4.0 \times 10^{-2} \mathrm{~kg} \cdot \mathrm{~s}^{2} / \mathrm{m}^{9}$
then the direction of use of these two graphs are explained by some calculating examples as follows.

Example 1. To find $K$ and $R$, when $Q_{0}=5.92 \mathrm{~m}^{3} / \mathrm{s}, Q_{L}=2.03 \mathrm{~m}^{3} / \mathrm{s}, L=333 \mathrm{~m}$ and $p_{0}=118 \mathrm{~kg} / \mathrm{m}^{2}$, assuming $n=1$.

On Fig. 9 A , we can read $\bar{p}_{0}=204 \mathrm{~kg} / \mathrm{m}^{2}$ and $\bar{L}=494 \mathrm{~m}$, as the coordinates of an intersecting point of two curves $m=2.92$ and $Q_{L}=2.03 \mathrm{~m}^{3} / \mathrm{s}$. It is to be remarked that $\bar{p}_{0}$ and $\bar{L}$ are notations showing the readings on the original scales. Then $\alpha_{1}$ and $\beta_{1}$ are evaluated from the ratios $p_{0} / \bar{p}_{0}$ and $L / \bar{L}$.

$$
\begin{aligned}
& \alpha_{1}=p_{0} / \bar{p}_{0}=118 \div 204=0.578 \\
& \beta_{1}=L / \bar{L}=333 \div 497=0.674
\end{aligned}
$$

From Table 1,

$$
0.578=\frac{2.2}{\sqrt{R K_{1}}} \times 10^{-3}, \quad 0.674=\frac{1}{18.2} \times \sqrt{\frac{R}{K_{1}}}
$$

Therefore


Fig. 9. B. $n=2, K_{2}=1 \times 10^{-3} \mathrm{~m}^{3} / \mathrm{kg}^{\frac{1}{2}} \cdot \mathrm{~s}, R=2.25 \times 10^{-2} \mathrm{~kg} \cdot \mathrm{~s}^{2} / \mathrm{m}^{9}$

Example 2. To find $Q_{0}, Q_{L}$ and $p_{0}$ when $K_{2}=2.06 \times 10^{-3} \mathrm{~m}^{3} / \mathrm{kg} \cdot \mathrm{s}^{\frac{1}{2}}, R=2.83 \times 10^{-2}$ $\mathrm{kg} \cdot \mathrm{s}^{2} / \mathrm{m}^{9}, L=333 \mathrm{~m}$, assuming that $n=2$ and the characteristic curve of the fan used is as shown in Fig. 10.

As $K_{2}$ and $R$ are known, $\alpha_{2}$ and $\beta_{2}$ can be calculated by referring to Table 2. The actual length of the air duct is 333 m , and it corresponds to $333 \div 0.571=583 \mathrm{~m}$ in the original scale. Now, selecting several points in a line $L=583 \mathrm{~m}$ on an original scale of Fig. 9, B, read the values of $p_{0}$ of these points, and convert them into values in true scale. On the other hand read the values of $Q_{0}=m Q_{L}$ of each point on Fig. 9, B. Then the


Fig. 10. Characteristic curve of a fan. relation of $p_{0}$ and $Q_{0}$ is illustrated by a curve in a chart which shows the characteristic curve of the fan. The intersecting point of these two curves gives the values of $p_{0}$ and $Q_{0} . Q_{L}$ is easily found by shifting the intersecting point on to Fig. 9, B. In this manner we get,

$$
p_{0}=110 \mathrm{~kg} / \mathrm{m}^{2}, \quad Q_{0}=5.55 \mathrm{~m}^{3} / \mathrm{s}, \quad Q_{L}=2.90 \mathrm{~m}^{3} / \mathrm{s}
$$

Example 3. Imagine an air duct, 250 m long, ventilating an effective air volume of $3.0 \mathrm{~m}^{3} / \mathrm{s}$ under the operation of a fan, running at $p_{0}=100 \mathrm{~kg} / \mathrm{m}^{2}$ and $Q_{0}=4.8 \mathrm{~m}^{3} / \mathrm{s}$. How much effective air volume will be obtained by keeping $p_{0}$ constant when the air duct is prolonged to 500 m ?

As the value of $n$ remains unknow in this problem, two sets of calculation on the assumption of $n=1$ and $n=2$ are carried out, so that the true air volume can be estimated from these results.

In the same manner as in example 1 and 2 , we get $Q_{L}=1.65 \mathrm{~m}^{3} / \mathrm{s}$ for $n=1$ and $Q_{L}=1.42 \mathrm{~m}^{3} / \mathrm{s}$ for $n=2$. The true value of $Q_{L}$ must be between 1.65 and $1.42 \mathrm{~m}^{3} / \mathrm{s}$. Assuming $Q_{L}$ is given by the mean of these limit values, we get $Q_{L}=1.54 \mathrm{~m}^{3} / \mathrm{s}$.

## (3) The maximum distance to be ventilated with an air duct.

The effective air volume decreases rapidly as the length of air duct increases and this tendency grows worse for the more defective air duct. This tendency is understood by the discussion that follows the similar calculation methods of example 3. Thus, it is almost impossible to give a good ventilation to a remote stope with an air duct.

For example, the relation between the necessary fan pressure to ventilate $2 \mathrm{~m}^{3} / \mathrm{s}$
of effective air volume and the total length of air duct, using $A$ class air duct,* is shown by a continuous curve in Fig. 11, and the air horse power of the fan and the initial air volume $Q_{0}$, a dotted line and a chain line respectively, against the total length of air duct are as plotted in the same figure. Thus, it is clear that, even in the case of a good air ducts as cited in the example above, the economical ventilation distance is less than 600 m .


Fig. 11. The relation among the values of $p_{0}, Q_{0}, H . P$. and $L$ to ventilate effective air volume $Q_{L}=2 \mathrm{~m}^{3} / \mathrm{s}$ using $A$ class air duct in Table 1.

## (4) Summary

The relation among the ventilating pressure, effective and leakage air volumes and the total length of an air duct is analyzed strictly for two cases; that the leakage air volume is directly proportional to the pressure difference across the wall of an air duct, and also it is proportional to the $1 / 2$ th power of the pressure difference. The results are illustrated in two graphs for ready reference. Finally, the maximum limit of distance to be efficiently ventilated with an air duct is discussed.

## Conclusion

In order to clarify the theory on the air duct ventilation accompaning leakage, the authors have preformed an experimental study on the phenemena of leakage and an analytical study on the air duct ventilation taking the loss of air volume into consideration.

The leakage air volume from an orifice or a slit is proportional to the $1 / n$th power of pressure difference across the wall of an air duct, where $n$ lies between 1 and 2, and it varies with the condition of openings and the pressure difference. For slits found usually on the wall of an air duct, $n$ seems to range from 1 to 2 , while for orifices $n$ is rather near to 2 ; therefore, for ordinary mine air ducts, it may be allowed to regard $n$ to situate in the middle of 1 and 2 or slightly nearer to 2 . Subsequently from the analytical study, the equations showing the relation among the pressure difference, effective and leakage air volume, and the total air duct

[^0]length are reduced, introducing new coefficients $K_{1}$ and $K_{2}$, which relate to leakage, for two extreme cases $n=1$ and $n=2$.

Graphs are originated that completely illustrate the results of the analysis, and a discussion on the effectiveness of an air duct ventilation is conducted by using graphs.

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[^0]:    * The letter A denotes the grade of air duct, as shown in Table 1.

