Studies on the Creep of Low-Carbon Steel

By

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Introduction

As a consequence of recent demand for materials of superior quality in respect to creep at elevated temperatures, it becomes necessary to know essentially about the properties of a material at elevated temperatures and to improve the material so as not to produce creep under severe stress and temperature conditions.

The authors describe in this paper about the following articles:

I) Creep testing machine and the creep test results on low-carbon steel. This article introduces the newly designed creep testing equipments, such as testing machine, automatic temperature regulator and apparatus for measuring creep strain, and the result of performed experiments, and develops discussion on creep of low-carbon steel.

II) Characteristics of creep. Consultig the results of the experiments, the authors endeavored to analyse the characteristics of creep. The general formula is introduced for the stress-strain-time relation, which is applicable even when the stress varies with the elapse of time. The characteristic values contained in the general formula are fully determined from the results of the tensile creep test.

III) Creep under bending and twisting moment. As applications, the creep under bending and twisting moment, where the distribution of stress is uneven and varies with time, are treated. It is shown that the creep test result under bending or twisting moment is determined by utilizing the characteristic values obtained numerically from the above tensile creep experiments.

I Creep Testing Machine and Creep Test Results on Low-Carbon Steel

1. Creep Testing Machine

i) Testing machine

Fig. 1 shows the tensile creep testing machine newly designed. The both ends of the specimen are screwed into adapters made of Ni-Cr steel. The other end of

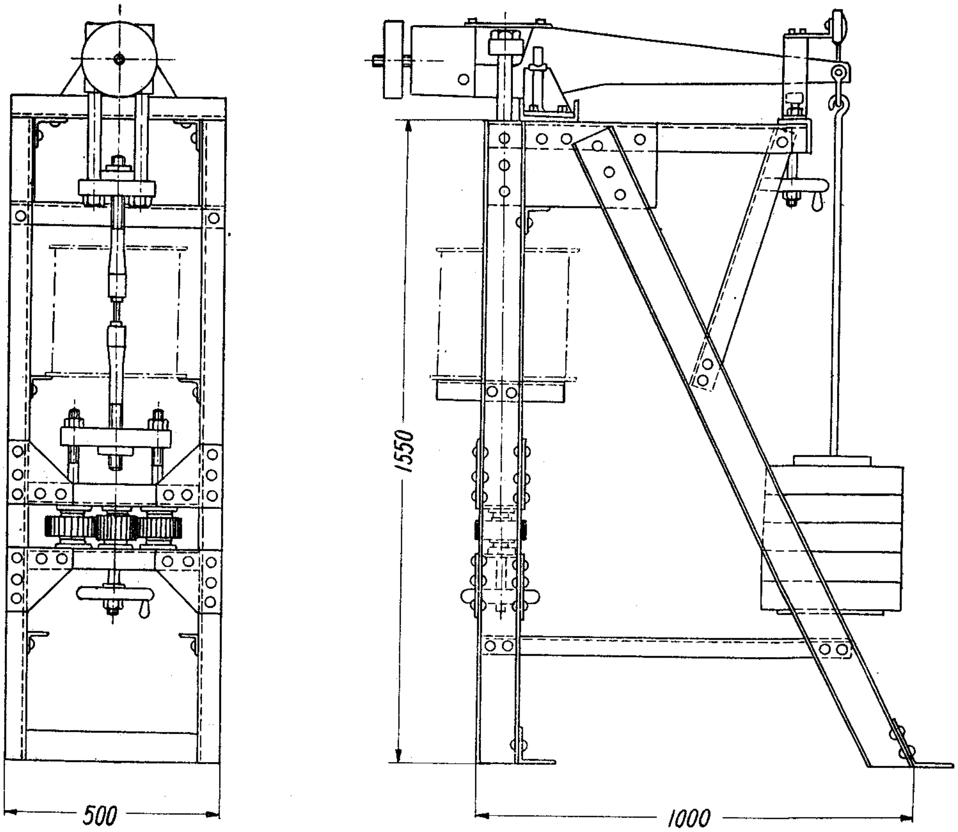

Fig. 1

the upper adapter is attached to a spherical seat hanging from a horizontal lever through knife edges. The lower adapter is connected, through a spherical seat, to the frame. The testing machine is of a dead weight type and the lever ratio is 10:1. The load up to 4.0 ton may be applied to the specimen.

For testing at elevated temperatures, a furnace is set around the specimen as shown by chain lines in the figure.

ii) The creep test specimen

Dimensions of the test specimen used are illustrated in Fig. 2. Diameter is 6 mm and gauge length 50 mm.

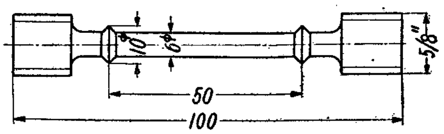

Fig. 2

iii) Measurement of extension

To measure the creep elongation, couple pairs of rods are attached parallel to the specimen, one rod of each pair at the upper collar of the specimen and another at the lower collar. Couple of dialgauges of 1/100 mm scale are mounted on each of the lower rods at the bottom, being on both side of the specimen. Mean of the readings is taken as the the value of creep elongation.

The schematic appearance of the measuring apparatus is illustrated in Fig. 3, where the specimen is denoted by *a*, the dialgauges by *b* and the heating furnace by *c*.

In order to determine the creep limit more accurately, more precise apparatus of measuring elongation, such as Martens' extensometer, may preferably be used; but for the creep rupture test, the measuring method mentioned above is sufficient.

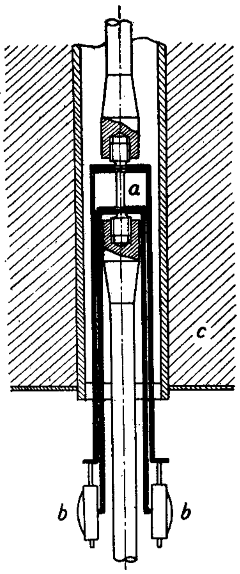

Fig. 3

iv) Heating furnace

Around a porous pat of 70 mm inner, 80 mm outer diameter and 330 mm in height, Ni–Cr wire of 1 mm diameter and 30 m length is wound. Turns of the wire are close together near the both ends and spaced apart in the middle. The space between the porous pat and the outer wall of furnace (300 mm in dia.) is stuffed with asbestos.

v) Temperature maintenance and control

The temperature control depends upon the change in electrical resistance of platinum wire wound around the porous pat. The resistance is connected to one arm of A. C. bridge network, unbalance of which is amplifiied and applied to the grid of TX 911. Charges or discharges of the valve control the current in the furnace.

On the circuit diagram, Fig. 4, R_D indicates the resistance of platinum wire of 0.15 mm diameter and 5 m length wound non-inductively around the porous pat. R_A and R_B are constant resistances and R_C is variable resistance. These four resistances and the mutual inductance M constitute A. C. bridge network, and a desired temperature can be obtained by setting R_C into suitable value. The mutual inductance M adjusts the phase of input to the amplifier. R_1 and R_2 indicate the resistances, T_1 and

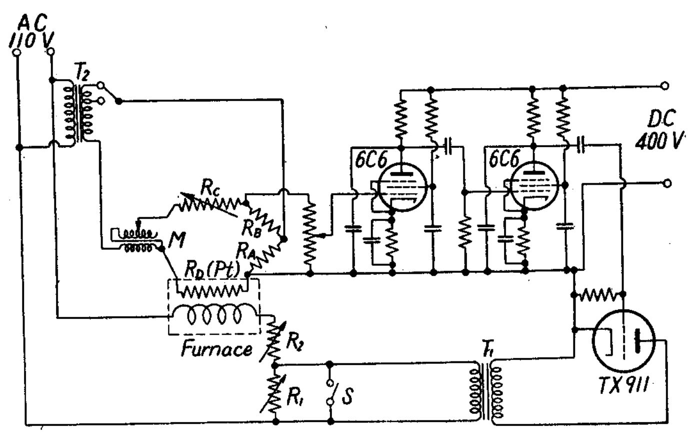

Fig. 4

T_2 the transformers. The plate voltage of TX 911 is kept at 1000 volt utilizing the resistance R_1 and the transformer T_1, while the secondary voltage of transformer T_2 is 30 volt.

Fig. 5 illustrates an experimental result of the temperature fluctuation when the heating furnace and the apparatus of temperature control mentioned above are used. There are some temperature differences existing between the middle part and the ends of the specimen. But at each point on the specimen, temperature fluctuates within the range of approximately ± 0.5°C in the lapse of time, which is a satisfactory result.

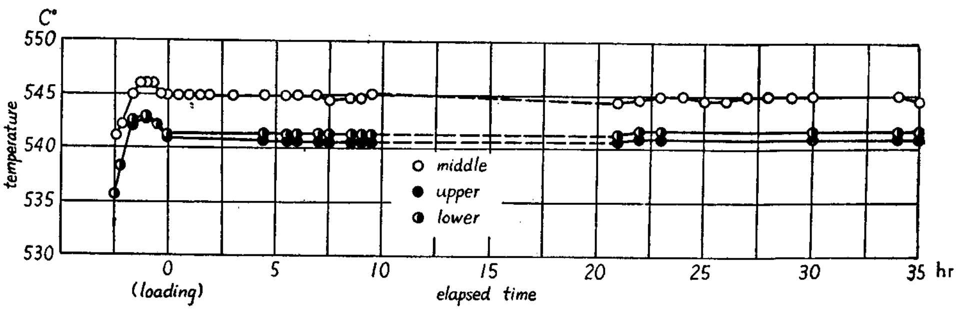

Fig. 5

From many other preparatory experiments, it is assured that the temperature fluctuation of the furnace is kept within ± 1.0°C even when the source voltage varies by ± 10 volt.

The method of temperature maintenance and control heretofore mentioned shows to be excellent to arrest vibration and shock.

vi) Measurement of temperature

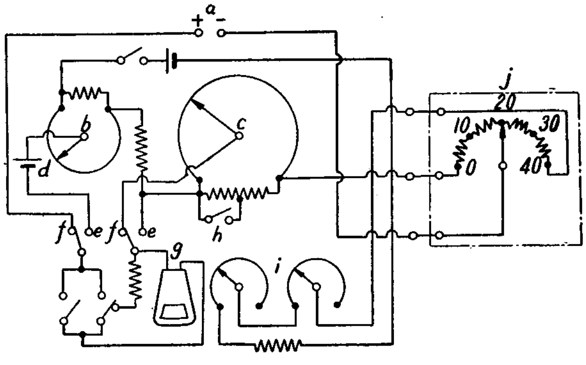

Fig. 6
a: thermocouple, *b*: calibrating resistance, *c*: potentiometer, *d*: standard cell, *ee*: measurement of current, *ff*: measurement of temperature, *g*: galvanometer, *h*: 10 mV shunt, *i*: adjusting resistance, *j*: 10 mV slide resistor.

By suitable improvement made on a ready-made potentiometer-type thermometer, the reading is precise enough to distinguish 1°C within the range of 0°～1200°C. Fig. 6 illustrates the circuit diagram of the improved thermometer. Chromel-Alumel thermocouples are led to the thermometer through an interchange switch box.

2. Creep Test on Low-Carbon Steel

i) Testing equipment

The testing machine, the heating furnace, the apparatus for controlling and measuring temperature and the creep elongation indicator mentioned above are used in the experiment.

ii) Material of specimens used

The chemical composition of the material used is shown in Table 1. The specimens were annealed for one hour at 900°C in vacuum tube and cooled in the furnace.

Table 1 (in %)

C	Si	Mn	P	S	Fe
0.10	0.16	0.60	0.040	0.030	Remainder

The properties of the specimen at room temperature are as follows;

yielding stress : $\sigma_S = 24.4\,\text{kg/mm}^2$,

tensile strength : $\sigma_B = 35.1\,\text{kg/mm}^2$.

iii) Test results

a) Test results obtained at 545°C

The experiments are carried out under initial stresses σ_0=15.2, 14.3, 13.5, 12.8, 10.0, 5.0 and 3.0 kg/mm². The first four experiments are creep rupture tests and the other three the creep tests.

Fig. 7 illustrates the time-elongation curves (t–Δl curves) obtained under various stresses, where the gauge length is 50 mm. The result obtained under the stress of 3 kg/mm² is excluded from the figure.

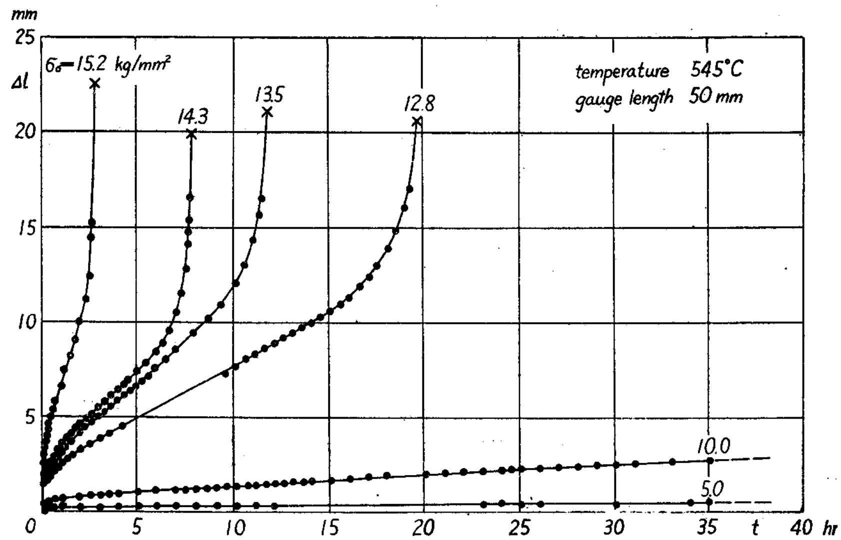

Fig. 7

From the results of the creep rupture tests, it is clarified that the relations between stress σ_0 and minimum creep rate u_{min}, rupture time t' and breaking elongation φ (gauge length 50 mm) are represented by straight lines in logarithmic scales as shown in Fig. 8. It is also seen that the elongation at rupture is almost independent of stress applied initially.

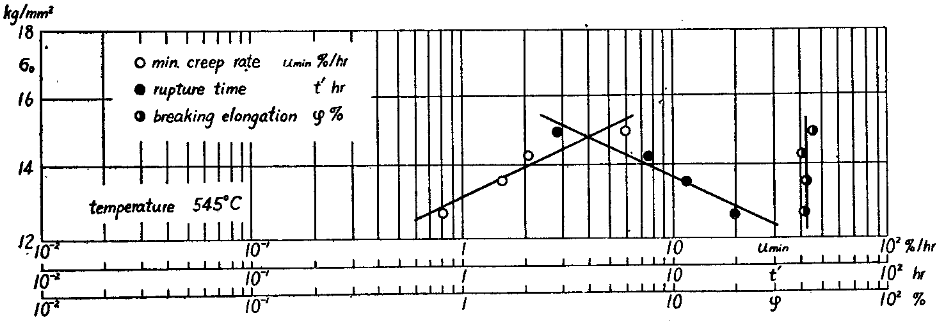

Fig. 8

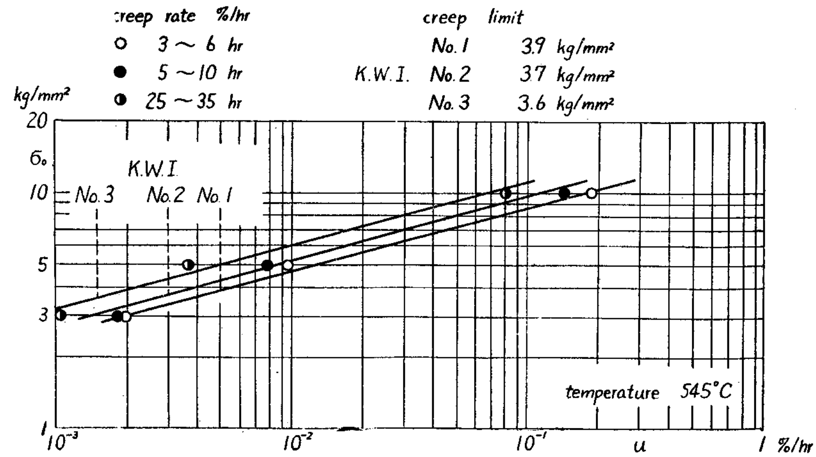

Fig. 9

From the results of the creep test, Fig. 9 is obtained by taking creep rates u of 3～6 hour, 5～10 hour and 25～35 hour as abscissa. From the diagram the creep limits based on the definitions by Kaiser Wilhelm Institut method No. 1, 2 and 3 are obtained as 3.9, 3.7 and 3.6 kg/mm² respectively. The K. W. I. methods No. 1, 2 and 3 define creep limits as the stresses that bring strain rate of $5 \cdot 10^{-3}$ %/hr within 3～6 hours, the strain rate of $3 \cdot 10^{-3}$ %/hr within 5～10 hours and the strain rate of $1.5 \cdot 10^{-3}$ %/hr within 25～35 hours respectively.

b) Test results obtained at 450°C

At the temperature of 450°C, the creep rupture tests under initial stress σ_0 of

20.0, 19.0, 18.0 and 17.0 kg/mm^2 and the creep tests under σ_0 of 10.0, 9.0, 8.0 and 7.0 kg/mm^2 are carried out.

Fig. 10 illustrates the characteristic curves of the creep rupture tests performed. By taking initial stress σ_0 as ordinate and minimum creep rate u_{min}, rupture time t' and breaking elongation φ as abscissa in logarithmic scale, the relations are linear as shown in Fig. 11.

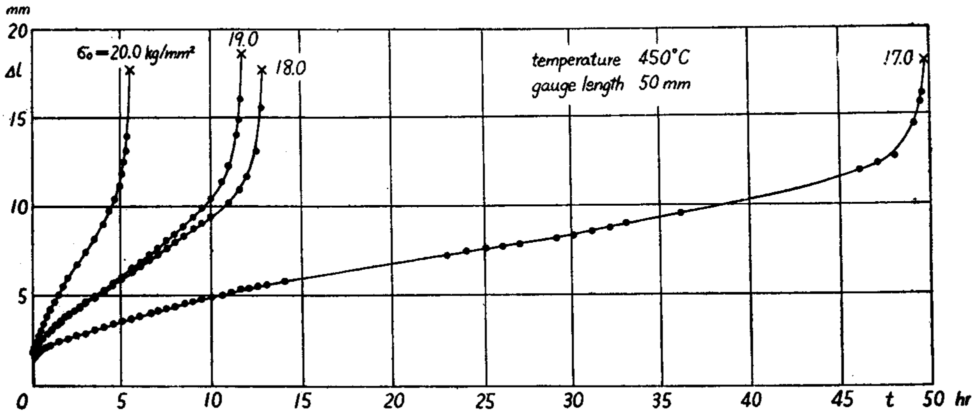

Fig. 10

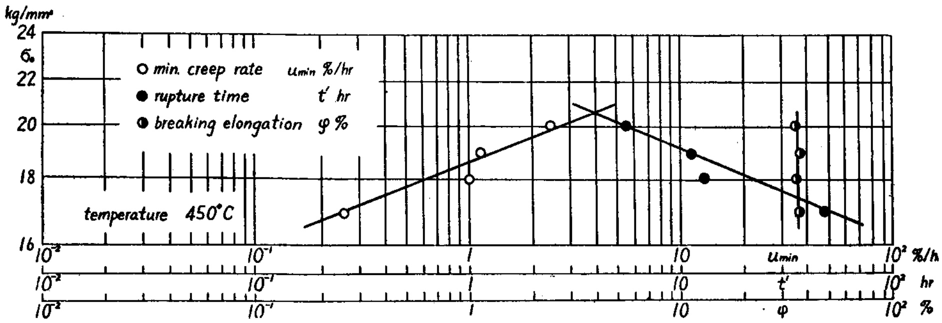

Fig. 11

From the results of the creep test, the strain rates u within 3~6 hours, 5~10 hours and 25~35 hours are illustrated in Fig. 12. The creep limits due to the K. W. I. method No. 1, 2 and 3 are determined as 7.7, 7.6 and 7.3 kg/mm^2 respectively. The points for the case of 7.0 kg/mm^2 does not lie on the lines as is seen in Fig. 12. This may be due to the fact that, taking logarithmic scales, the line representing the relation between the stress and strain rate yields near the stress of creep limit, as it is reported by other researchers.[1]

c) Test results obtained at 350°C

At the temperature of 350°C, the creep rupture tests under σ_0 of 31.0, 30.5, 29.5 and 29.0 kg/mm² are also carried out. The characteristic creep rupture curves are illustrated in Fig. 13 and the logarithmic relations between stress σ_0 and minimum creep rate u_{min}, rupture time t' and breaking elongation φ are illustrated in Fig. 14.

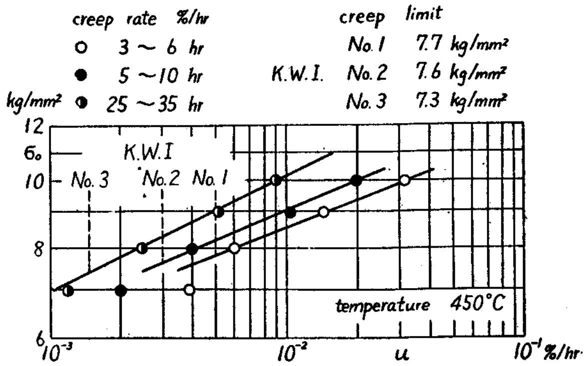

Fig. 12

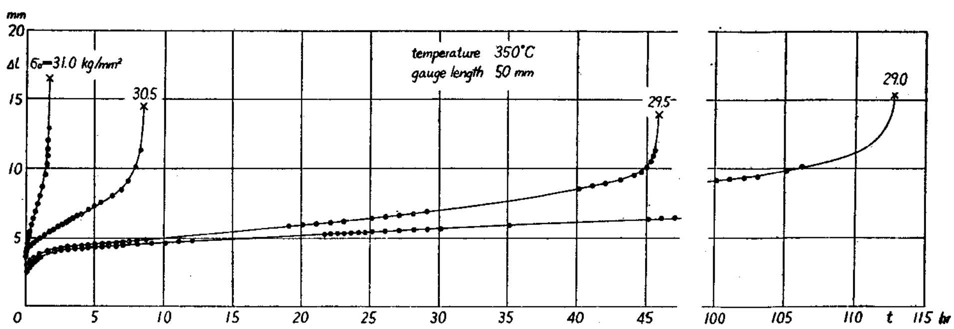

Fig. 13

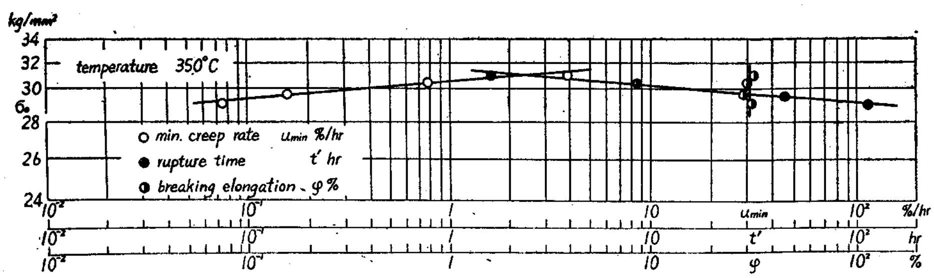

Fig. 14

3. Fundamental Consideration of Creep of Low-Carbon Steel

i) Creep curve

The general type of creep curve consists of three stages of elongation; that is, after initial extension, a stage of creep at a decelerating rate, a stage at an approximately constant rate and a stage at an accelerating rate leading to fracture. The minimum creep rate is determined from the second stage in which strain rate is nearly constant.

From the experimental results for the cases of 545°C and 450°C, the following is noticeable.

By taking the effective strain ε as ordinate and t/t' as abscissa where t is the time elapsed and t' the rupture time, we can draw the strain ε – time ratio t/t' curves for the tests under various initial stresses on the same figure. Fig. 15 is the curves for the test at 545°C and Fig. 16 at 450°C. From these figures, it may be said that the curves almost agree to each other in spite of various initial stresses.

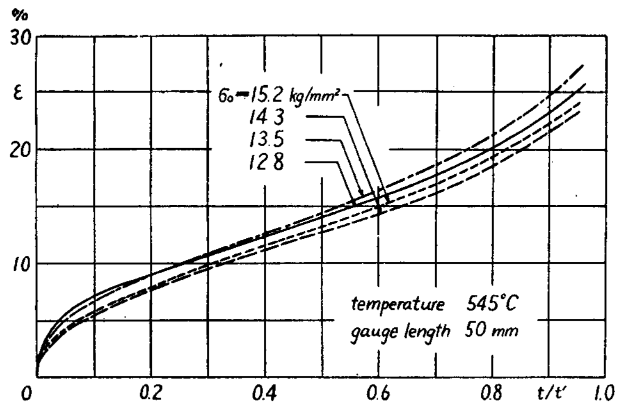

Fig. 15

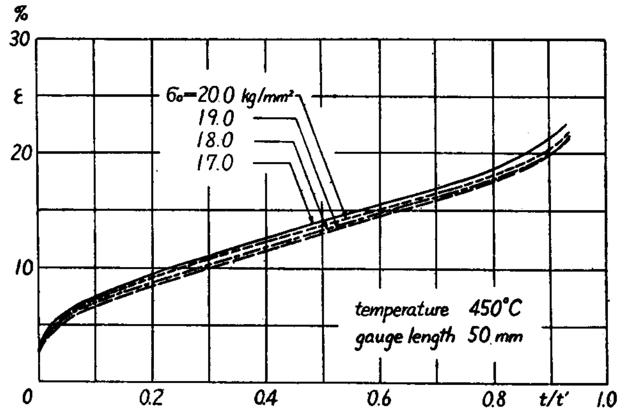

Fig. 16

This fact is the foundation of the authors' consideration on the creep of mild steel. Before proceeding into analysis, the notations are denoted in brief as follows;

σ_0	kg/mm^2 :	initial stress,
σ	kg/mm^2 :	true stress,
ε	% :	effective strain,
t'	hr :	rupture time,
t	hr :	elapsed time,
u	%/hr :	creep rate,
u_{min}	%/hr :	minimum creep rate.

ii) Creep curve at 545°C

Refering to Fig. 17, which is a typical creep curve observed in the tensile creep test, we notice that OA is the stage of creep at decelerating rate and AB the stage

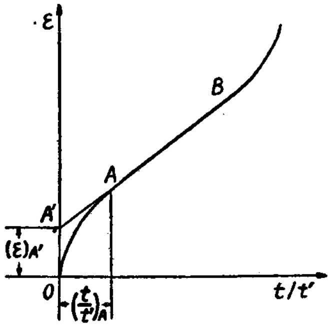

Fig. 17

Table 2 (545°C)

σ_0 kg/mm^2	$(t/t')_A$	$(\varepsilon)_{A'}\%$	C
15.2	0.18	4.6	1.24
14.3	0.16	4.4	1.21
13.5	0.16	4.8	1.28
12.8	0.17	5.6	1.19
mean	0.17	4.9	1.23

of nearly constant rate. Now, we take A' on the ordinate by extending BA. $(t/t')_A$ is the abscissa of A and $(\varepsilon)_{A'}$ the ordinate of point A'. From the experimental curves obtained, we get the values of $(t/t')_A$ and $(\varepsilon)_{A'}$ %, which are shown in Table 2. It is conceivable that those values have no bearing upon the magnitude of stresses initially applied so long as the testing temperature remains the same.

The creep rate can be represented as

$$u = \frac{d\varepsilon}{dt} = \left[d\varepsilon/d\left(\frac{t}{t'}\right)\right]\cdot\frac{1}{t'}\,. \qquad (1)$$

Taking the logarithm of each term, we get

$$\log u = \log\left[d\varepsilon/d\left(\frac{t}{t'}\right)\right] - \log t'\,,$$

that is,

$$\log u + \log t' = \log\left[d\varepsilon/d\left(\frac{t}{t'}\right)\right]. \qquad (2)$$

At the stage of minimum creep rate, we have

$$\log\left[d\varepsilon/d\left(\frac{t}{t'}\right)\right] = \text{const} = C\,,$$

and

$$\log\,(t'\cdot u_{min}) = C\,. \qquad (3)$$

The calculated values of C for the curves of various initial stresses are illustrated in the last column of Table 2. They are also nearly constant.

The $\log\sigma_0 - \log u_{min}$ and $\log\sigma_0 - \log t'$ relations are linear as are shown in Fig. 8. These linear relations, together with Eq. (3), give

$$\log u_{min} = 11.24\,\log\sigma_0 - 12.54 \qquad (4)$$

and

$$\log t' = -11.24\,\log\sigma_0 + 13.77\,. \qquad (5)$$

The two straight lines drawn in Fig. 8 represent Eqs. (4) and (5).

On the other hand, AB in Fig. 17 is represented by

$$\varepsilon = 17\,(t/t') + 4.85\,, \qquad (6)$$

and OA in the same figure is represented by exponential function

$$\varepsilon = 14.8\,(t/t')^{0.37}, \tag{7}$$

making the strain and the value of the first derivative at point A equal to the values derived from Eq. (6).

Using Eqs. (5) and (7), we obtain

$$\log u = -11.04 + 11.24\ \log \sigma_0 - 1.703 \log \varepsilon\,. \tag{8}$$

Denoting the gauge length by l_0 for the virgin specimen and l after deformation, we have

$$\frac{\varepsilon}{100} = \int_{l_0}^{l} \frac{dl}{l} = \ln \frac{l}{l_0}$$

and

$$\sigma = \sigma_0\,(l/l_0)\,.$$

Using above two equations, we get

$$\sigma = \sigma_0\, e^{\frac{\varepsilon}{100}}. \tag{9}$$

Substituting this into Eq. (8), we obtain

$$\log u = 11.24 \log \sigma - 0.049\,\varepsilon - 1.703 \log \varepsilon - 11.04\,. \tag{10}$$

This is one type of representation

$$\sigma = \sigma(\varepsilon, u)\,. \tag{11}$$

In general, stress must be written in the form

$$\sigma = \sigma(\varepsilon, u, t)\,. \tag{12}$$

Eq. (11) is the special form of Eq. (12). Here, a mention must be made of the special characteristics of the material used.

Fig. 18 and Table 3 illustrate the experimental and calculated creep strain rates at t=3～6, 5～10 and 25～35 hours under σ_0=10, 5 and 3 kg/mm².

Table 4 illustrates the creep limits due to the K. W. I. method No. 1, 2 and 3 obtained by the calculation based on Eq. (10).

iii) Creep curve at 450°C

Corresponding to Table 2, we get Table 5 from the test results at 450°C. The next Eqs. (13), (14), (15), (16), (17) and (18) are also given for the present case corresponding to Eqs. (4), (5), (6), (7), (8) and (10), i. e.

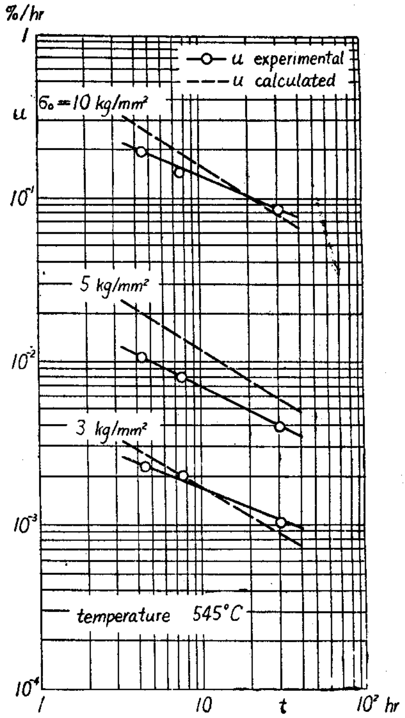

Fig. 18

$$\log u_{min} = 14.71 \log \sigma_0 - 18.70\,, \tag{13}$$

$$\log t' = -14.71 \log \sigma_0 + 19.84\,, \tag{14}$$

$$\varepsilon = 15.4\ (t/t') + 5.82\,, \tag{15}$$

$$\varepsilon = 16.1\ (t/t')^{0.368}, \tag{16}$$

$$\log u = 21.39 \log \sigma_0 - 1.72 \log \varepsilon - 25.98\,, \tag{17}$$

$$\log u = 21.39 \log \sigma - 1.72 \log \varepsilon - 0.093\,\varepsilon - 25.98\,. \tag{18}$$

Table 3 (545°C)

t hr	σ_0=10 kg/mm²		σ_0=5 kg/mm²		σ_0=3 kg/mm²	
	u %/hr	$u_{exp.}$ %/hr	u %/hr	$u_{exp.}$ %/hr	u %/hr	$u_{exp.}$ %/hr
3 ~ 6	0.257	0.195	0.0186	0.0100	0.0026	0.0021
5 ~ 10	0.191	0.146	0.0135	0.0080	0.0019	0.0019
25 ~ 35	0.079	0.082	0.0056	0.0037	0.0008	0.0010

Table 4 (545°C)

Definition		Creep limit kg/mm²	
		calculated	experimental
K.W.I. method	No. 1	3.9	3.9
	No. 2	3.8	3.7
	No. 3	3.9	3.6

Table 5 (450°C)

σ_0 kg/mm²	$(t/t')_A$	$(\varepsilon)_{A'}$ %	C
20.0	0.22	5.7	1.15
19.0	0.24	5.5	1.14
18.0	0.20	5.6	1.13
17.0	0.22	6.8	1.14
mean	0.22	5.8	1.14

Table 6 (450°C)

t hr	σ_0=10 kg/mm²		σ_0=9 kg/mm²		σ_0=8 kg/mm²	
	u %/hr	$u_{exp.}$ %/hr	u %/hr	$u_{exp.}$ %/hr	u %/hr	$u_{exp.}$ %/hr
3 ~ 6	0.0292	0.032	0.0165	0.0145	0.0086	0.0060
5 ~ 10	0.0212	0.022	0.0120	0.0105	0.0063	0.0040
25 ~ 35	0.0085	0.009	0.0049	0.0052	0.0025	0.0025

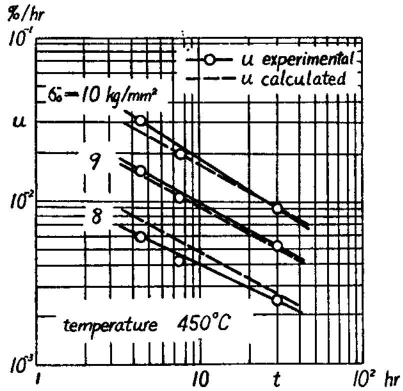

Fig. 19

Table 7 (450°C)

Definition		Creep limit kg/mm²	
		calculated	experimental
K.W.I. method	No. 1	7.2	7.7
	No. 2	7.1	7.6
	No. 3	7.2	7.3

Fig 19 and Table 6 illustrate the experimental and calculated strain rates at $t=3\sim6$, $5\sim10$ and $25\sim35$ hours under $\sigma_0=10.0$, 9.0 and 8.0 kg/mm^2.

Table 7 corresponds to Table 4 and gives the values of creep limits at 450°C.

II. Characteristics of Creep

1. Tensile Creep Test

Creep property of materials is in general investigated by using the data of the tensile creep test, in which the stress is kept as constant as possible and is independent of time. When the tensile creep data is used as the basis for analysis of other stress conditions where stress varies with time, special consideration must be taken into account.

Before proceeding to discuss the problem, we propose an idea on the behaviour of materials at elevated temperatures.

2. Mechanism of Creep

i) Creep strain

The characteristic creep curve is shown in Fig. 20, taking the elapsed time t as abscissa and the strain ε as ordinate.

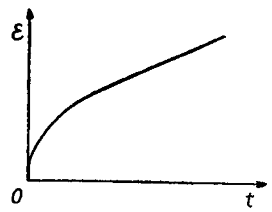

Fig. 20

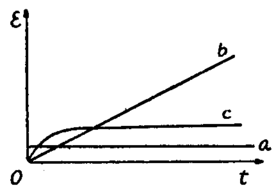

Fig. 21

We consider the creep strain is composed of three kinds of strain. That is, the one, ε_a, depends only on the stress applied and is independent of time (Fig. 21 curve a), the others, ε_b, depend on both stress and time. The latter is divided into two parts; one is the strain ε_{b1}, which increases proportionally to time (Fig. 21 curve b) and the other ε_{b2}, which increases nonlinear with time (Fig. 21 curve c).

This is formularized as

$$\varepsilon = \varepsilon_a + \varepsilon_{b1} + \varepsilon_{b2} = f_a(\sigma) + t\cdot f_{b1}(\sigma) + f_{b2}(\sigma, t)\,. \tag{19}$$

Strains ε_{b1} and ε_{b2} correspond to the strain caused by quasi-viscous flow and transient flow respectively, according to Andrade and Orowan.[2)]

ii) Strain ε_a

ε_a is the strain which is independent of time and here shows an elastic strain. It can be written as

$$\varepsilon_a = \sigma / E_0 \,, \tag{20}$$

where E_0 is the modulus of elasticity of material at a temperature elevated.

As for the tensile creep test,

$$\sigma = \sigma_0 = \text{const} \,, \tag{21}$$

therefore

$$\varepsilon_a = \sigma_0 / E_0 \,. \tag{22}$$

iii) Strain ε_{b1}

ε_{b1} is the strain due to viscous flow and the strain rate may be proportional to the stress applied, i. e.,

$$\sigma = \eta_0 \cdot \frac{d\varepsilon_{b1}}{dt} \,. \tag{23}$$

It may be considered that η_0 should be the function of applied stress or of both applied stress and the absolute value of strain. However, in the present treatise, η_0 is put constant for the sake of simplifying the mathematical treatment.

From Eq. (23), we get

$$d\varepsilon_{b1}/dt = \sigma / \eta_0 \,, \tag{24}$$

that is

$$\varepsilon_{b1} = \int_0^t \frac{\sigma(\tau)}{\eta_0} d\tau \,. \tag{25}$$

For the tensile creep test, we get

$$\varepsilon_{b1} = \sigma_0 t / \eta_0 \,. \tag{26}$$

iv) Strain ε_{b2}

ε_{b2} is the strain aroused by the applied stress accompanying time retardation and consists of n constituents, that is,

$$\varepsilon_{b2} = \varepsilon_1 + \varepsilon_2 + \cdots + \varepsilon_k + \cdots + \varepsilon_n \,. \tag{27}$$

Any constituent ε_k of strain satisfies the equation

$$\sigma - s_k = E_k \varepsilon_k + \eta_k \frac{d\varepsilon_k}{dt} \,. \qquad (\sigma \geqq s_k) \tag{28}$$

The first term of the right hand side is a force proportional to the strain, proportional constant being E_k, and the second is a force proportional to the strain rate, proportional coefficient being η_k, and s_k on the left is the resisance, being constant according to constituent number k and increasing in order of k. Only when applied stress σ overcomes the inherent resistance s_k, (28) is valid. That is, untill the stress reaches s_k, the k–th constituent is not found, ($\varepsilon_k=0$).

From Eq. (28),

$$\frac{d\varepsilon_k}{dt}=\frac{\sigma-s_k}{\eta_k}-\frac{E_k\,\varepsilon_k}{\eta_k}.$$

Integrating it, we have

$$\varepsilon_k=\exp\left(-E_k t/\eta_k\right)\left[\int_0^t \exp\left(-E_k\tau/\eta_k\right)\frac{\sigma-s_k}{\eta_k}d\tau+C\right]. \tag{29}$$

Denoting t_k as the time when the stress σ reaches s_k and considering the initial condition as

$$\tau=t_k \quad : \quad \varepsilon_k=0, \qquad (\tau\gtrsim t_k)$$

we have

$$\varepsilon_k=\frac{1}{\eta_k}\int_{t_k}^t \exp\left[-E_k(t-\tau)/\eta_k\right](\sigma-s_k)d\tau. \tag{30}$$

E_k, η_k, t_k and s_k are considered to depend upon the constituent number k. In Eq. (27) we have represented ε_{b2} as the sum of strain constituent ε_k, being defined by the discrete values of k. It may be reasonable to consider, however, that the magnitude s_k increases continuously from zero to σ in order of k from 1 to n, where n is extremely large, and that E_k, η_k and t_k are the functions of k and accordingly of s. Integrating Eq. (30) with s, we obtain

$$\begin{aligned}\varepsilon_{b2}=&\int_0^\sigma\frac{1}{\eta(s)}\int_{t(s)}^t\exp\left[-E(s)\,(t-\tau)/\eta(s)\right]\sigma(\tau)d\tau ds\\&-\int_0^\sigma\frac{s}{E(s)}\left[1-\exp\{-E(s)\,[t-t(s)]/\eta(s)\}\right]ds.\end{aligned} \tag{31}$$

On the tensile creep test, all the strain constituents of which s_k is less than σ_0 contribute from the instance the stress σ_0 is applied and consequently the following equation is obtained from Eq. (31) under the condition of $\sigma=\sigma_0$ and $t(s)=0$,

$$\varepsilon_{b2}=\int_0^{\sigma_0}\frac{(\sigma_0-s)}{E(s)}\left[1-\exp\{-E(s)t/\eta(s)\}\right]ds. \tag{32}$$

v) Tensile creep strain

From Eqs. (22), (26) and (32), the creep strain of the tensile creep test is represented as

$$\varepsilon=\frac{\sigma_0}{E_0}+\frac{\sigma_0}{\eta_0}t+\int_0^{\sigma_0}\frac{\sigma_0-s}{E(s)}\left[1-\exp\{-E(s)t/\eta(s)\}\right]ds. \tag{33}$$

For executing the integration in Eq. (33), it is necessary to know the analytical representation of $E(s)$ and $\eta(s)$. For the sake of simplicity, an approximated method is applied.

If we compare Eq. (33) with the empirical formula

$$\varepsilon = F(\sigma)\cdot G(t)\,, \tag{34}$$

in which the creep strain is assumed to be represented by the product of the function $F(\sigma)$ of stress and that $G(t)$ of time, the third term of the right hand side of Eq. (33) must have the form

$$F(\sigma)\,(1-e^{-\nu_0 t})\,,$$

where ν_0 is a constant.

Assuming that function $F(\sigma)$ is represented as usual by a power function αs^β, Eq. (34) becomes

$$\varepsilon = \alpha s^\beta (1-e^{-\nu_0 t})\,.$$

Through mathematical operation we have

$$\varepsilon = \frac{\sigma_0}{E_0} + \frac{\sigma_0}{\eta_0} t + \frac{a}{(q+1)\,(q+2)}\,(1-e^{-\nu_0 t})\sigma_0^{q+2}, \tag{35}$$

where ν_0, a and q are constants and

$$E/\eta = \nu_0\,,$$
$$1/E = as^q\,. \tag{36}$$

Using the previous experimental results obtained by 0.10%C steel at 545°C, the numerical values of η_0, a, q and ν_0 can be determined. Because the experimental results are obtained under constant load and not under constant stress, the numerical values determined are not strictly accurate but are approximate values.

As the approximate numerical values of them, we have obtained by representing ε in %, σ in kg/mm^2, t in hr,

$$\eta_0 = 69.16,$$
$$a = 2.09\times 10^{-3},$$
$$q = 2.2,$$
$$\nu_0 = 0.380\,. \tag{37}$$

And the modulus of elasticity at about 550°C may have the value

$$E_0 = 15000\,\text{kg/mm}^2. \tag{38}$$

vi) General creep strain

Using the relation (36) and Eqs. (20), (25) and (31), we obtain

$$\varepsilon=\frac{\sigma(\tau)}{E_0}+\int_0^t\frac{\sigma(\tau)}{\eta_0}d\tau+a\nu_0\int_0^\sigma s^q\int_{t(s)}^t e^{-\nu_0(t-\tau)}\,\sigma(\tau)d\tau ds$$
$$-a\int_0^\sigma s^{q+1}\left[1-e^{-\nu_0\{t-t(s)\}}\right]ds\,. \tag{39}$$

On the right hand side of the above equation, the first term is the elastic strain ε_a and the second and the third terms are the plastic strain ε_b.

As for metallic materials, especially for steel, the strain recovery may be observed when the load is reduced to zero,[3] but the magnitude of recoverable strain is rather smaller compared with the strain under consideration. So we may assume that ε_a is restored to zero when the load is removed but ε_b is hardly restored. For this reason, the integrals contained in Eq. (39) must be dealt with enough caution.

Thus we obtain Eq. (39) as an equation representing strain as a function of stress and time in general form. It has the merit in being applicable for the case when the stress varies with the lapse of time.

III Creep under Bending or Twisting Moment

1. Characteristics of Creep under Bending or Twisting Moment

In the case of creep under bending or twisting moment, the stress at each portion in the section of specimen varies with loading time even though the moment applied remains constant. That is, stress in the extreme layer and its vicinity decreases and the stress in the inner portion increases with the elapse of time, while the applied moment remains unchanged. In discussing the deformation, influence of the change of stress upon strain or strain rate must be taken into account.

The hitherto proposed mathematical methods for predicting the creep under bending or twisting moment from the tensile creep test result are done by discussing the deformation utilizing the strain or strain rate of ordinary tensile creep test without any attention paid to the fact mentioned above. These methods ignore influence of the variation of stress with time.

In the following section, the authors intend to propose a method to predict the creep under bending or twisting moment from the tensile creep test results by applying Eq. (39). The influence of stress change with the elapse of time is taken into account in the treatment.

2. Creep under Bending Moment

Under constant bending moment, let us consider a beam of rectangular cross-section, whose height is h and width b as shown in Fig. 22.

Just when the moment is applied, e. g. time $t=0$, the stress σ distributes linearly thicknesswise and may be calculated from elasticity. In the lapse of time, creep proceeds in every layer. Because the creep strain rate depends upon the magnitude of stress, the mode of creep process is different in each layer. The applied moment being kept unvaried, the stress of every layer inevitably changes. Let σ and ε denote the stress and strain in the layer distant by y from the neutral axis and $\bar{\sigma}$ and $\bar{\varepsilon}$ denote the stress and strain in the extreme layer ($y=h/2$) at any optional time t.

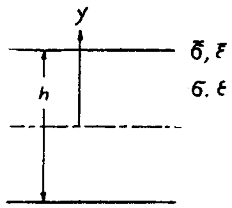

Fig. 22

Putting

$$\xi = y/\frac{h}{2}, \tag{40}$$

we have

$$\varepsilon = \xi\bar{\varepsilon}. \tag{41}$$

On the other hand, since the applied moment is kept unchanged, we have

$$M_b = 2\int_0^{\frac{h}{2}} b\sigma y\, dy = \text{const}. \tag{42}$$

In the followings, a numerical example will be presented in an approximated manner.

Consider a beam of 0.1% C steel under a bending moment at 545°C. The initial surface stress $\bar{\sigma}_e$ is 15 kg/mm². The change of surface strain may be investigated by a calculation using the necessary numerical values given from the tensile creep test. The full line in Fig. 23 shows the theoretical curve representing the change of surface strain with the time elapsed. In Fig. 24, the change of stress distribution in the cross-section in the process of creep is shown. In the figure, the line A is initial linear distribution. As the creep proceeds, it changes itself into the distribution

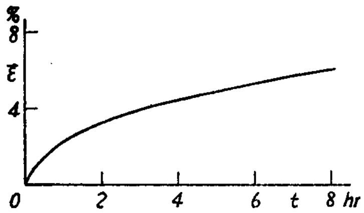

Fig. 23

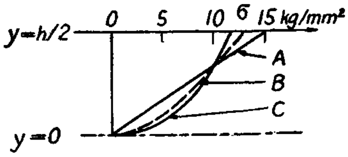

Fig. 24

shown by the curve B and, after a sufficient time, yields finally to a steady distribution as shown by the curved line C.

3. Creep under Twisting Moment

The uniaxial stress σ and the creep strain ε are inter-related through Eq. (39).

As far as the creep under twisting moment is concerned, the relation between the shear strain and the shear stress under the biaxial stress state must be known.

A. E. Johnson has suggested in this connection to adopt the octahedral shear strain and the octahedral shear stress from the result of experiments for the specimens of a thin-walled tube under combined tensile and torsional stress.[4] In the present analysis, the octahedral shear stress and the octahedral shear strain are adopted for relating the uniaxial stress state to the biaxial one.

Denoting σ_1, σ_2 and σ_3 are the principal stresses and τ_{oct} and γ_{oct} are the octahedral shear stress and strain respectively, τ_{oct} and γ_{oct} are defined as

$$\tau_{oct} = \frac{1}{3}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]^{\frac{1}{2}}, \tag{43}$$

$$\gamma_{oct} = \frac{2}{3}\left[(\varepsilon_1-\varepsilon_2)^2+(\varepsilon_2-\varepsilon_3)^2+(\varepsilon_3-\varepsilon_1)^2\right]^{\frac{1}{2}}. \tag{44}$$

At the uniaxial tension test, where the stress is σ and the strain is ε,

$$\tau_{oct} = \sqrt{2}\,\sigma/3, \tag{45}$$

$$\gamma_{oct} = \sqrt{2}\,\varepsilon. \tag{46}$$

At the torsion test, where the shear stress is τ_t and the shear strain is γ_t,

$$\tau_{oct} = \sqrt{2}\,\tau_t/\sqrt{3}, \tag{47}$$

$$\gamma_{oct} = 2\sqrt{2}\,\gamma_t/\sqrt{3}. \tag{48}$$

Consequently, the relation between the shear stress τ_t and the shear strain γ_t are otained by substituting

$$\sigma = \sqrt{3}\,\tau_t, \tag{49}$$

$$\varepsilon = 2\gamma_t/\sqrt{3}. \tag{50}$$

into the equation (39).

As a numerical example, the case of a circular rod whose radius is r_0 under twisting moment M_t is considered.

τ_t and γ_t denote the shear stress and the shear strain at an optional time t at the point of radius r, and $\bar{\tau}_t$ and $\bar{\gamma}_t$ are that of the surface ($r=r_0$), (Fig. 25). The stress with suffix e designates the initial value.

Putting

$$\zeta = r/r_0, \tag{51}$$

the following two relations must be satisfied, corresponding to Eqs. (41) and (42) of the creep under bending moment,

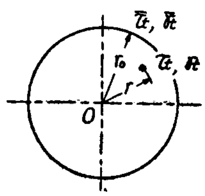

Fig. 25

$$\gamma_t = \zeta\bar{\gamma}_t, \tag{52}$$

$$M_t = \int_0^{r_0} 2\pi r^2 \tau_t \, dr. \tag{53}$$

The creep of a twist rod where the initial stress of surface is 9 kg/mm², that is $\tau_{t_0}=9$ kg/mm², is considered in the same manner as in the preceding example. The relation obtained between the shear strain at the surface and time is shown in

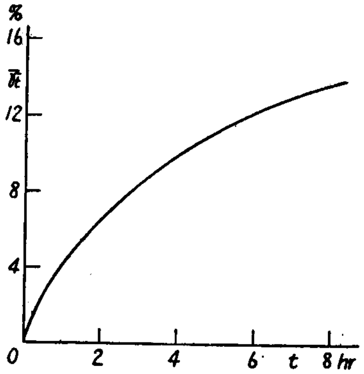

Fig. 26

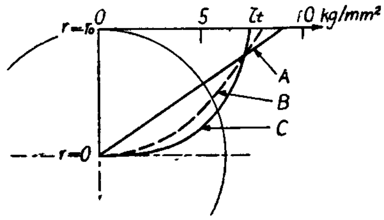

Fig. 27

Fig. 26. The initial distribution of the shear stress represented by the straight line *A* in Fig. 27 varies with time and, passing through the stress state *B*, becomes steady attaining the form of curve *C*.

IV Conclusion

In article I, the features of the creep testing machine used are described. The results of the creep rupture test performed are collected in the same figure, where the relations between the effective strain and the ratio of lapsed and rupture time are found to be approximately similar. The empirical equation which inter-relates stress, strain, and strain rate is introduced. The creep limits calculated by the equation are compared with the experimental values. The results thus obtained show no remarkable difference.

In article II, the characteristics of general creep are analysed and the authors thereby introduced a general formula for the stress-strain-time relation, which is applicable even when the stress varies with the lapse of time. At the same time, the method to determine the characteristic values contained in the formula is explained. As an example, the values are determined using the results of tensile creep test at the temperature of 545°C.

In article III, as the applications of these general formulae the creep under bending or twisting moment is treated. Because of the variation of distribution of stress with time the general formula capable to follow the stress variations, such as introduced by the authors, must be used. The result of creep test under bending or twisting moment is predicted with the formula which utilize the characteristic values obtained in article II.

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