Research of the Resolving Power of the Electron Microscope

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Main notations

General rules

(1) Letters dotted (.) above them represent differentiation against time.

(2) Letters which have a dash (′) on their right shoulders represent differentiation against variable x.

(3) Letters written in Gothic represent vector quantities.

Notations

$\boldsymbol{A}$ vector potential,

a position of object,

b position of exit pupil or stop,

c position of image plane,

d value of resolving power,

e, m charge and mass of electron, (m is also used as direction cosine, but in such case it is notified.),

E Eikonal function,

$\boldsymbol{E}$ electric field,

f focal length of lens,

H strength of magnetic field on symmetrical axis,

$\boldsymbol{H}$ general magnetic field,

i unit of imaginery,

$\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$ unit vector for rectangular coordinate,

$k = \frac{2\pi}{\lambda}$

λ wave length,

l, m, n direction cosine,

$\boldsymbol{n}$ normal vector,

r, ψ, x cylindrical coordinate,

$\boldsymbol{r} = \boldsymbol{i}x + \boldsymbol{j}y + \boldsymbol{k}z$, position vector,

r_b radius of aperture of a stop,

r_α aberration free electron path of a electron emitted from a point of a object, parallel to the symmetrical axis,

r_β aberration free electron path of a electron emitted from a point of a object, with a inclination of 45° to the symmetrical axis,

S Seidel's Eikonal,

S_B area on the surface of a stop,

$\boldsymbol{s} = \boldsymbol{i}l + \boldsymbol{j}m + \boldsymbol{k}n$ unit vector,

s electron path length,

t time

$t = kr_b \frac{r_0}{f}$

(this notation is distinguished according to the place that is used.)

r_0 distance from the symmetrical axis on a object plane,

$v = y + iz$,

x, y, z rectangular coordinate,

y_0, z_0 a point on a object plane,

$w_0 = y_0 + iz_0$ is used,

η_0, ζ_0 a point on a entrance Pupil Plane,

η_1, ζ_1 a point on a exit Pupil Plane,

$w_1 = \eta_1 + i\zeta_1$, is used,

y_1, z_1 a point on a image plane,

φ space potential,

Φ Potential on the symmetrical axis,

μ Refraction Index.

Outlines of Contents

From the point of view that, at present, the theory of resolving power of the optical microcope has been applied to the resolving power of electron microscope and has not been given any difinite meaning of its own yet, I accepted the request of the Electron Microscope General Research Committee for this problem to try to describe the meaning and definition of the resolving power of the electrom microscope and to give theoretical method to decide it. This report is the result, which consists of 5 chapters.

In Chapter 1, through the consideration of difference between the optical microscope and the electron microscope observed from the standpoint of the resolving power and factors which affect on the resolving power of the electron microscope, and by explaining the classification of the resolving power introduced by the Electron Microscope Society of America's Committee on Resolution, we give a conclusive definition to Resolving Power of Electron Microscope to propose that half width of intensity distribution curve should be used as the limit of Resolving Power. At the same time, we suggest to use a value, called Resolution to indicate the quality of the electron micrograph, while a limit of Resolving Power is used to represent the quality of the Electron Microscope. This suggestion is equally followed by the description of the meaning, definition, and the method of measuring Resolution, and then examined by experiment including preparation of suitable material for the measurement.

In Chapter 2, on the assumption of the symmetrical distribution of electromagnetic field against an axis, we show the calculation of chromatic aberration at the simultaneous fluctuation of accelerating voltage and exciting current. And also we calculated spherical aberration under such influence to find Eikonal function in such case.

Next, we calculated aberration of axis asymmetrical electron lens from Eikonal function. In order to do this calculation, we made a formula of Eikonal function for general electro-magnetic field and find general formula for electro-magnetic field. Then calculated the aberration with these formulas. The result of this calculation coincides with the one of Mr. Suzuki and Mr. Inoue's calculation using differential equation of electron path.

In Chapter 3, we show a calculation of intensity distribution curve from Eikonal function in the case that diffraction aberration, chromatic aberration and axis asymmetrical astigmatism are assumed to exist. Then assuring this calculation to be right when there asssumed only diffraction aberration and axis asymmetrical astigmatism, we find a theoretical formula of Resolving Power under the existence of the above every aberrations. Then into this theoretical formula, we applied the figures actually measured with the electron microscope which now exists in Japan and presumed the limit of Resolving Power of the electron microscope in Japan.

Chapter 4 is the conclusion of the research.

CHAPTER 1

Definition of Resolving Power and Resolution of the Electron Microscope

Section 1, Definition of Resolving Power

To state the quality of the electron microscope exactly, the following factors should be taken in account,

(1) In the test of the optical microscope, a light source which can be considered as nearly a point may be prepared, but in the electron microscope it is very difficult to make a electron source which can be considered as a point-source of electron. Therefore it is actually impossible to obtain experimentally the shortest distance that can be distinguished as two point-sources of electron.

(2) Though in Optics, it is possible to assume a black body or to make an object considerable to be approximately a black, body, but in the electron microscope, an object such as to absorb all the incident electrons cannot be considered. Owing to the charge of electron, every object which absorb electron shows charging phenomena which makes it unabsorbable of any extra charge over a definite limit. i.e. in the electron microscope it is impossible to make a point-source of electron and a black body available to use as a standard sample, so the way of stating the quality of the electron microscope should be different from the one of the opitical microscope. In another words, the definition of Resolving Power of the optical microscope cannot be applied to that of the electron microscope directly. Electron microscope must have its own definition which suit to its activity. Thereupon we are to compare the activities of the above two microscopes.

Owing to the easy portability of the optical microscope, we can compare two optical microscopes by putting them together in a certain place and directly seeing the standard samples with the naked eyes. But on the other hand, electron microscope is much complicate and bulky that it is quite difficult to move. Accordingly, in order to compare two electron microscopes, it is a practical way to compare the two on the micrographs by taking them of an image of any suitable samples. As a matter of fact, in electron microscope, as it takes considerable procedures before having an image of a sample on a fluorescent screen, micrographing is usually performed at the same time when appeared an image of it. So, the dry plate or the film is generally considered as a part of the electron microscope and not as a fixture of it. Therefore in the disscussion of the quality of the elcctron microscope, it may be a practical way to distinguish the quality of the microscope body itself and of the synthetic quality of electron microscope including microscope body and dry plate. It is because, inspite of the completeness of micrscope body and electrical power system, to transfer the responsibility for the dry plate to the maker of the electron microscope by judging this microscope system unsatisfactory when defective micrograph have been obtained owing to the inferiority of the plate. There are scarecely any firms which produce both of electron microscope and dry plate so that the sources of responsibility can not be identified when the judgement of the quality of electron microscope and dry plate has not been given separately. In the other hand, in many cases, clearness of the micrograph is required. In such cases, the above mentioned synthetic quality may become a problem.

The followings are considered as the factors which influence over the quality of the electrom microscope,

(1) Due to the construction of the electron microscope itself.

One dues to the construction of microscope body and the other, to the construction of the electrical power system,

(2) Due to the sample to be applied to the electron microscope.

The sort of sample holding film, difference in the sample with big electron scattering and that with little scattering,

(3) Due to the quality of the dry plete.

Exposure of dry plate, difference in developing procedures, and characteristics of sensitivity.

(4) Due to the operation of the microscope.

Discrepancy of centering axis, misadjustment of focus, dirtiness of stop, difference of magnification, etc.

Thereupon, the Electron Microscope Society America's Committee[1] of Resolution enumerates the following five items as the kinds of Resolving Power,

(a) Quality......quality of micrograph

(b) Resolution of a micrograph......Resolving Power measured by the micrograph affected by the above (1), (2), (3), (4).

(c) Instrumental Resolving Power......Resolving Power in the best condition of above items (1), (2), (3), (4).

(d) Calculated limit of Resolving Power......Resolving Power calculated after deciding respective aberrations.

(e) Theoretical limit of Resolving Power......value of $\frac{0.61\lambda}{\sin\alpha}$

As a matter of fact, Inspite of the above detailed discrimination, each of them can not supposed to be used ditinctively and respectively. Therefore, it is necessary and sufficient to distinguish as following two items.

(1) Resolving Power necessary to judge the quality of electron microscope body.

(2) Synthetic Resolving Power necessary to judge the quality of electron micrograph.

The item (1) will be called *Resolving Power*, and item (2), *Resolution*. In other words, the former represents the quality of microscope body and the latter represents that of a electron micrograph. Accordingly, resolving power is the peculiar value for the system and resolution varies so much by the kind of samples. And an inferiority of the limit value of resolution does not always means inferior Resolving Power. However, inferior resolving power inevitably means poor resolution.

Next, we go on to the definition of Resolving Power of the electron microscope. In the Optical microscope, it is said that,

"Optical Resolving Power"......The Power of an optical system using light of one colour to form images of two point sources which are close together, so that the images may be recognized as distinct." It may be considered sufficient to put the word electron in place of light in this definition. That is,

" *The power of an Electron Microscope available to discriminate the image of two adjacent point-sources of electrons which produce electrons of the same velocity, is called Resolving Power of the Electron Microscope.*"

and we consider the above definition is sufficient. However, it is so difficult to make a point-source of electron that we can hardly decided the value of resolving power from measuring by the way stated in the above definition. Another method to decide resolving power should be find out.

Then we will next to thik over the limit of Resolving Power. As long as the forms of a minute source of electron are recognized by using electron microscope, it may be concidered to remain in the limit of Resolving Power, but while it can not be recognized, it may be out of the limit. In another words, when we apply the magnification of an electron microscope to the maximum and then vary sizes of an minute source of electron to smaller in order, its form can be recognized for the first time, but when they become smaller than a certain size, there must be a size of a source of electron with image of definite largeness however smaller the size of a source of electron may varies. This boundary is the true limit within where electron microscope is available. But in this case, as frenel's fringe may be appeared around the image, what should be recognized as an image is uncertain. So that according to the above definition of Resolving Power,

" The shortest distance with which two adjacent point-sources of electrons can be distinguished."

will be the limit of Resolving Power. The word distinguished in the sentence means that as long as the two point-sources make contrast, they may said distinguishable and when they are uniformaly bright or their intensity distribution curve become flat, they may said undistinguishable and the boundary may be considered as the limit of Resolving Power· Abbe has little different ideas from this. He said the shortest distance is " A distance from center axis to the first dark ring of diffraction pattern. Culculating this value,

$$d = \frac{0.61\lambda}{\sin\alpha} \qquad (1)$$

λ: wave length of electron wave

α: aperture angle

will be obtained.

This formula has been used hitherto in optical microscope and has also been used in electron microscope considering as available. In optical microscope it has been a formula of resolving power for the self-luminous body. When defining the limit of resolving power to the position where the whole parts of sight become uniformly, bright or unformly dark, the value of this formula should become much smaller. For this reason, Abbe's formula represented by formula (1) has been criticized that it does not correctly agree with real case.

The above formula is available when a lens without any aberration is used and it can not be applied to an electron microscope that always have aberrations. To microscope which can not avoid aberration, it is suitable to adopt half width of intensity distribution curve as the limit of Resolving Power. As shown in Fig. 1, when distance between two images become equal to half width, intersection of these two curves are respectively in the point of half value and a resultant of these two curves become nearly flat. In other words, condition of "uniform brightness" or "unform darkness" will be almost satisfied though not perfectly, when half width is adopted. To obtain a formula of resolving power, in case of no aberration, when half width is adopted, the value of intensity distrbution I_P of electron beam on the surface of image plane which pass through a stop will be given by the following equation,

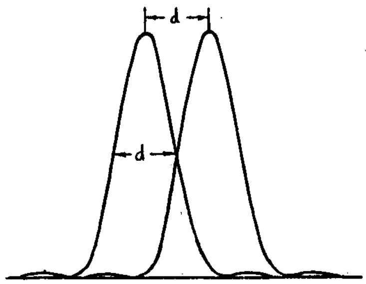


Fig. 1.

$$I_p = \left[\frac{2J_1(x)}{x}\right]^2 \tag{2}$$

so

$$\left[\frac{2J_1(x)}{x}\right]^2 = \frac{1}{2} \tag{3}$$

i. e., to solve the equation

$$J_1(x) = \frac{x}{2\sqrt{2}} \tag{3'}$$

is necessary, $J_1(x)$ represents Bessel function. Solving this equation

$$x = 0.512\,\pi \tag{4}$$

will be obtained. By the quite same procedure to obtain resolving power of formula (1)

$$d = \frac{0.512\,\lambda}{\sin\alpha}$$

will be obtained as a formula of resolving power in the above case, Fig. 2 shows the intensity distribution curves (thick lines) given by equation (2), according to the definition of above half width, at the limit position where two points can be distinguished as two points and resultant intensity distribution curves (dotted line) of the two curves. To decide the value of resolving Power experi-

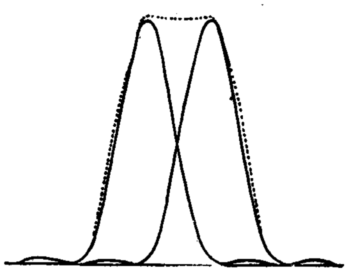


Fig. 2.

mentally, measurement of half width of intensity distribution curve will be enough, so only one point-source is necessary to decide the value of resolving Power and result in much conveniency.

Now, we try brief explanation about a way to decide resolving Power. The value of resolving Power of electron microscope is decided chiefly by four aberrations of spherical, axis asymmetrical astigmatism, chromatic, and diffraction aberration and for this reason these are measured[2]. The value of these aberrations are measurable but only diffraction aberration is better to decided by calculation. Then deciding an incident angle of electron ray, and composing aberrations obtained, a compound aberration will be obtained. It means that the magnitude of aberration of an image of a point-source of electron is obtained. Now that the value of aberration is decided, the value of resolving Power can be obtained. Detailed explanation of this will be discussed in the following chapters.

Section 2, Definition of Resolution

Definition of resolution in optical microscope is as follows,

" Resolution : — The act or property of rendering visible the separate parts of an object."

Basing on this definition,

" The resolution is the power of electron micrograph rendering visible the separate parts of an object." may be said in electron micrograph and this may be considered as sufficient. There may be seen a little difference from optical microscope in considering the ability of electron micrograph instead of electron microscope. As the limit of resolution,

" The limit of Resolution is the shortest distance which two points of objects are distinguishable on a micrograph."

may be adopted. In this case, two points, of course, means the sample with its size about the limit of Resolution and its size must be about several m. μ.. And the shortest distance means the same meaning of shortest distance in the case of resolving Power.

Defining as above, the difference between resolving Power and resolution may be clearified in the following points,

(1) In resolving Power, two point sources are taken in account, and therefore incoherent beam is considered, but in resolution, the case of illuminating the sample with one source of electron is taken in accout and for this reason incoherent beam plus coherent beam must be considered. Therefore, equation (1) or (2) can not be applied to resolution without any change.

(2) To the value of Resolution, what sort of sample for the resolution measurement

is used must be said. For, no black body is considered for electron beam, the sort of sample will vary the value of resolution. In addition, it will also be influenced by the sort of the photographic dry plate, the sort of plate also should be designated. For resolving Power such consideration is unnecessary.

(2) According to the roport of Hillier's experiment[3], when he reported the resolution of $1\,\text{m.}\mu.$, it said that

accelerating voltage 50,000V ($\lambda = 0.005\,\text{m.}\mu.$)
sample illuminating angle $\alpha = 10^{-4}$
magnificatıon $m = 14{,}600$.

While frenel's fringe can not be seen in the slightest degree in his micrograph. This may be considered that by the effect of the sample holding film, frenel's fringe can not be so effective enough to be taken micrograph even if it appears. If frenel's fringe does not apper, resolution will become much better by making α as small as possible. However, as small α results a dark image, cathode should be as bright as possible. Hillier used a cathode which is brighter twenty times than that of usual electron microscope. In such a case resolution may be improved but there can not be any improvement in resolving Power. There are no way but to remove each aberrtation to improve resolving Power.

Now we consider over a method of measuring. In order to measure resolution, there are many methods which have been considered, for instance to measure the interval between frenel's fringe lines[1], to decide it from the degree of dullness of an sharp edge[1], and so on. But as resolution is defined as the shortest distance visible of two adjacent point objects, it is the best way to measure as the definition says. The only problem is whether we can make samples such as to be considered as point, or to make them to be at the shortest distance that is visible even if they may be made. When it becomes clear that the measuring method by the definition becomes impossible, the above mentıoned way of measuring will become problem. Our experiment proved it possible to measure resolution in accordance with the definition.

In preparing samples for measuring resolution, the simplest way which does not require any particular equipment and can be made by anybody is to be desired. For this purpose, it is the most favorable way to make metalic particle by vacuum evaporation. The reason is every laboratory where electron microscope is installed has without exception a vacuum evaporating installation from the need of shadow casting.

We made the following experiment. For the sample to be evaporated, silver was used and its arrangement is shown in Fig. 3. The whole arrangement of Fig. 3 will be put into a vacum jar, which then will be evacuated to evaporate

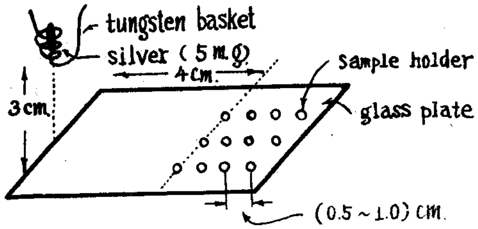


Fig. 3.

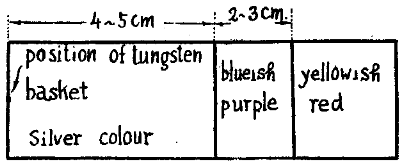


Fig. 4.

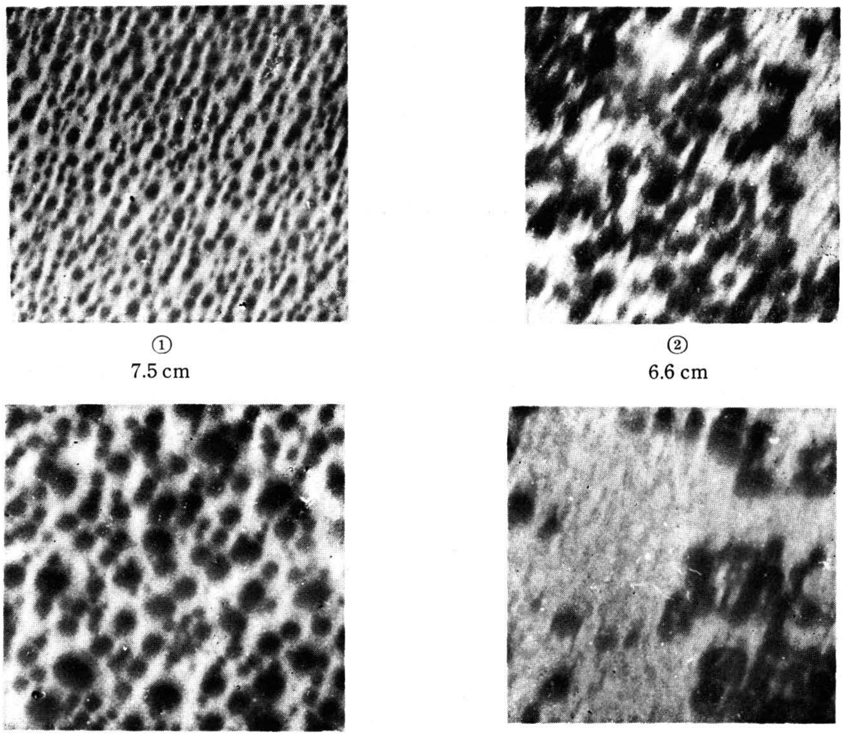
the silver. At first, sample holder on the glass plate is removed and only the latter will be put into the vacuum jar to evaporate and when the evaporation is made, then silver particle will be deposited on the glass plate. In this case, colours of the glass plate may vary as shown on Fig. 4. The part of silver has a rather thicker layer with colour itself and the remoter from the basket has a thinner layer and its colour changes to blueish purple. At the remotest, it changes to yellowish red. since this thin layerwill be granulated respectively, thick part of the layer has rough particles and the part with thin layer has fine grains. The suitable part for electron microscope is that of blueish purple.

The better result will be obtained when using evaporation filament with the temperature of red heated state and by evaporating gradually but instantaneous evaporation with white heated filament will also give fine grains. After assuring the position of blueish purple on the glass plate, evaporated again putting sample holder on this position. Then, various samples with voluntary roughness from rough grain to that of fine all at once will be obtained. Among these samples selected the one suitable for measurement of Resolution. And this will be understood as simple procedure for making samples. As the vacuum evaporting installation is Provided wherever the electron microscope is installed, the experiment can be performed at every where.

When we made an image of electron microscope of 20,000 direct magnification as large as of 200,000 total magnification by enlarging it 10 times with eyepiece and when adjusted condenser lens current gradually to make smaller the electron heam angle, we could see with the naked oye the granules which were faintly recognized. Tho size of the granules in this case were presumed about 2~3 m. μ.. The yellowish-red part on Fig. 4 may be considered as granules of smaller than 1 m. μ. but they could not recognize. Fig. 5 shows electron micrographs of silver grains produced by vacuum evaporation and their distance from tungsten basket are taken as follows

① 7.4 cm ② 6.6 cm ③ 5.8 cm ④ 5.0 cm.

Examining by electron microscope the samples, prepared by the above mentioned procedures, from the rougher to the finer in turn, those that stand near the boundary



① 7.5 cm

② 6.6 cm

③ 5.8 cm

④ 5.0 cm

direct magnification 20,000
total magnification 10^5
accelerating voltage 50,000 V

enlargement 5

aperture angle 8×10^{-3}

Fig. 5.

between visiqle and invisible will be found easily. Generally this may be considered as sample of the grain size about the limit of Resolution. We are to find out a couple of points in the shortest distance which is visible as two points after taking micrograph of the sample that stands at near the limit of resolution. And according to the definition, this shortest distance will give resolution, though it is uncertain that whether such a couple of points will be obtainable or not. If they can not be found, the average radius of the finest grain on micrograph can approximately be regarded as the limit value of resolution. The reason for this is as follows.

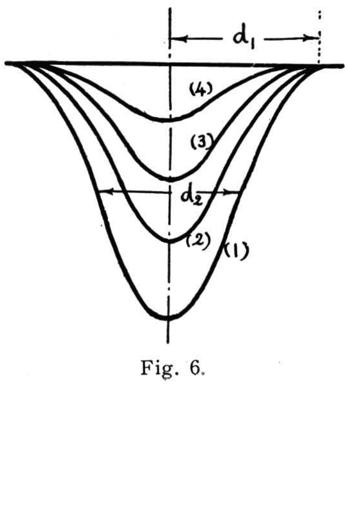


Fig. 6.

When the size of any object is made smaller, its image will disappear through following three stages.

(1) Shape of an object can be recognized clearly.

(2) Existence can be assured but not clear.

(3) Nothing can be seen.

Boundary between (1) and (2) is resolution in real sense. In stage (2) intensity distribution curve changes as (1)…(2)…(3)…(4) shown in Fig. 6, when roughness of the grain being varied to smaller. In this case, the state of scattering of electron is considered, of course, to be the same. The value d_1 is the limit of resolution but it differs from half width. Therefore correctly speaking it is not right. By adopting these method success of measurement through only one preparation of samples will be assured. It is quite enough to have a mean value by two or three measurements and need not to repeat much more. When it is wished to take micrographs of some bacteria of any other samples and at the same time to measure resolution of electron microscope, it is sufficient to put fine grain in a part of the visual field.

Measurement of resolution using colloidal gold is sometimes tried as it offers extremely fine grains. But in this case, there is a defect that it is impossible to vary the size of samples from rough one to fine grain in turn, there may be some anxiety whether the grain are about the size of the limit of resolution and yet there is no way to assure it.

Next, we had examined whether silver grain deforms its shape when it is illuminated with intense electron beams. Using accelerated voltage of about 50,000 V, illuminated it with intense electron beams but no change was acknowleged. However, according to Mr. Tsunesaburo Watanabe's[4)] report, it is said,

" When the silver layer on a glas produced by vacuum evaporation is left in the open air, spots often grow on it. Causes of this considered to be as follows;

(a) If there exist acid vapour in the air, spots are apt to grow and when density of this acid is greater, a thin silver layer reacts sooner.

(b) The thickness of silver film to where spots grow, is about a part of dark purple to pale purple."

Therefore it is thinkable that when we left samples in the open air for a long time there may ocurr deformation in its shape. But this will give no harm to the actual measurement. Using gold instead of silver, we can get fine grains of gold by vacuum evaporation and with this golden grain we can measure resolution. But silver grain can be produced more easily. Since resolution will be influenced considerably by the degree of magnification of electron microscope, a sort of dry plate, and the way of developing and printing process, etc., it is meaningless to use the value of resolution to compare the quality of various maker's electron microscope. Resolving Power should be used for comparison. Qualities of various electron microscope is comparable by the magnitude of spherical aberration and axis asymmetrical astigmatism for a given aperture angle. From this reason, there is no need of settling standard for the measurement of resolution.

Conclusion

(1) Definition of Resolving Power.

The Power of an electron microscope available to discriminate the image of two adjacent point-sources of electrons which produce electrons of the same velocity, is called resolving Power of the electron microscope.

(2) Method of determining resolving power.

By determining every aberrations such as spherical, axis asymmetrical, diffraction and chromatic etc., we can determine the value of resolving power of electron microscope by calculation for some illuminating electron beam angle. Result of the calculation will be described as follow,

R. P. 1 m. μ. (calcu), M = 10,000, V = 50,000 V

(3) Definition of Resolution.

The power of electron micrograph rendering visible the separate parts of an object is called resolution of a electron micrograph and the limit of resolution is the shortest distance which two points of objects are visible on a micrograph.

(4) Method of determining Resolution.

Making granules of which roughness can be optionally changed (for example golden or silver granules obtained by vacuum evaporation) and applying them to an electron microscope in turn from rougher to finer to find the limit of resolution, taking an electron micrograph of those of the roughness of the granules near the limit of resolution, finding two adjacent points which can be recognized as distinct, we can determine the resolution by measuring the distance between these two points. If there are no such points, average radius of the smallest granules on the micrograph will be taken as the approximate limit of resolution. Result of the measurement will be described as follow,

Reso., 3 m. μ. (silver granules), M = 10,000, V = 50,000 V

Process dry plate was used.

If measurement of the resolution of an electron micrograph of bacteria or any other material is desired, it will be enough to deposit adequate granules on a part of visual field when taking the micrograph.

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CHAPTER 2

Calculation of Chromatic Aberration, Spherical Aberration And Axis Asymmetrical Astigmatism.

As described in chapter 1, the chief aberrations which affect on resolving Power are

1. Diffraction Aberration
2. Chromatic Aberration and Spherical Aberration
3. Axis Asymmetrical Astigmatism

so that every aberrations stated above should be calculated. Diffraction aberration is the same in Optics and in election Optics. Therefore diffraction aberration enumerated as item 1 above is alreadly settled. So we here calculate chromatic aberration and spherical aberration and axis asymmetrical astigmatism.

However precisely finished electron lens is used for the electron microscope, axis asymmetry will remain to some extent and perfect axis symmetrical distribution on electromagnetic field can not be acquired. Accordingly, for the calculation of aberration, all aberrations should be considered refering to this axis asymmetrical electromagnetic field. While axis asymmetrical chromatic aberration or spherical aberration is formed by axis symmetrical aberration with influence of axis asymmetry but this is considered to give very little effect to axis symmetrical aberration. Therefore, we here neglect the influence of axis asymmetry for the sake of convenience to consider axis symmetrical chromatic or spherical aberration.

Now, potential φ in the space is to be measured from the origin where velocity of electron is O, and $\boldsymbol{A}$ represents vector potential and $\boldsymbol{s}$ shows unit vector of the direction of electron beam path. Then the refractive index μ for the general electromagnetic field is known to be given with the formula of

$$\mu=\sqrt{\varphi}-\eta \boldsymbol{A}\boldsymbol{s}, \quad \eta=\sqrt{\frac{e}{2m}} \qquad \cdots\cdots(1)$$

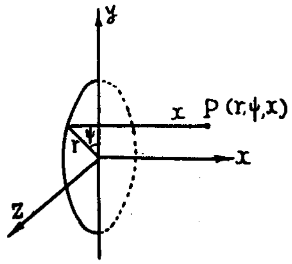

Fig. 1.

Now assuming that electromagnetic field is distributed symmetrically against the axis as shown in Fig. 1, adopt cylindrical coordinate (r, ψ, x). For this coordinate system

$$A_r=A_x=0.$$

Representing Eikonal function by E, this will be given as

$$E=\int_{P_0}^{P_1}(\sqrt{\varphi}-\eta \boldsymbol{A}\boldsymbol{s})ds, \qquad \cdots\cdots(2)$$

s represents the length of electron path. Now, taking coordinate axis, object plane, exit pupil plane and image plane as shown in Fig. 2, we assume that existence of electromagnetic field is limited only between the object plane and the exit pupil plane.

In electron microscope voltage fluctuation may occur at the following four places,

1. Anode accelerating voltage
2. Condenser lens
3. Objective lens
4. Projection lens

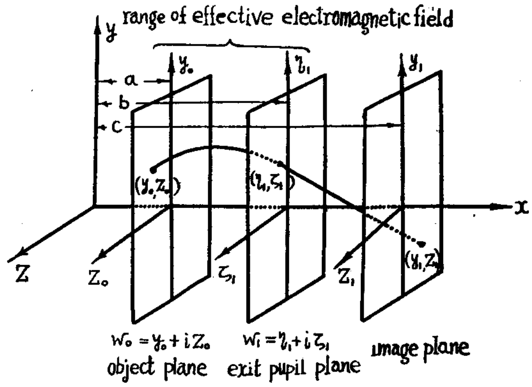

Fig. 2.

But among the above four items, fluctuation of anode voltage and current fluctuation at objective lens always come into question, so our consideration will be limited to these two hereafter. And to this, we consider only when it is axis symmetrical, then the case of axis asymmetrical will be treated on the bases of the above consideration. Since certain limit of the case of axis asymmetry is the case of axis symmetry, these two can not bear entirely different result. The degree of difference between the two will be decided by the degree of axis asymmetry, can be said.

Representing formula (2) in cyrindrical coordinate,

$$\left.\begin{aligned} E &= \int_{P_0}^{P_1} \left(\sqrt{\varphi} - \eta A_\psi \cdot r \frac{d\psi}{ds}\right) ds = \int_a^b F dx \\ F &= \sqrt{\varphi(1+y'^2+z'^2)} - \eta A_\psi \cdot r\psi' \end{aligned}\right\} \quad \cdots\cdots(3)$$

is obtained. Now suppose electric potential φ and vector potential A_ψ changed their amount φ_0, $A_{\psi 0}$ respectively, by fluctuation of Power source voltage, then

$$F = \sqrt{(\varphi+\varphi_0)(1+y'^2+z'^2)} - \eta(A_\psi + A_{\psi 0}) r\psi' . \quad \cdots\cdots(4)$$

The value of φ and A_ψ in the case of axis symmetrical magnetic field distribution are easily calculated. Providing that r_α and r_β are the solutions of the equation of electron path without aberration, we can put as

$$w(x) = c_\alpha r_\alpha + c_\beta r_\beta \quad \cdots\cdots(5)$$

and assume as this equation satisfy the conditions generally used as the initial condition of

$$\begin{cases} r_\alpha(a) = 1 \\ r'_\alpha(a) = 0 \end{cases} \qquad \begin{cases} r_\beta(a) = 0 \\ r'_\beta(a) = 1 \end{cases} \quad \cdots\cdots(6)$$

putting

$$w(a) = w_0 = y_0 + iz_0$$
$$w(b) = w_1 = \eta_1 + i\zeta_1$$
$$R = y_0^2 + z_0^2, \quad \rho = \eta_1^2 + \zeta_1^2, \quad \kappa = y_0\eta_1 + z_0\zeta_1, \quad \sigma = y_0\zeta_1 - z_0\eta_1$$

we have

$$\left. \begin{aligned} w(x) &= h(\eta_1 + i\zeta_1) - g(y_0 + iz_0) \\ h &= \frac{r_\beta(x)}{r_\beta(b)}, \quad g = \frac{r_\alpha(b)}{r_\beta(b)} r_\beta(x) - r_\alpha(x) \end{aligned} \right\} . \qquad \cdots\cdots\cdots\cdots\cdots\cdots (7)$$

Picking up what is necessary among each terms of Eikonal function, we have

$$E = a_{2c}\rho + a_{3c}\kappa + a_{4c}\sigma + b_{2c}\rho^2. \qquad \cdots\cdots (8)$$

From this equation and Fig. 3, we have the terms of the first order aberration, that is

$$y_1 + iz_1 = A_2 w_1 + (M + A_3) w_0 + iA_4 w_0 \qquad \cdots\cdots (9)$$

and the value of A_2, A_3, A_4 will be

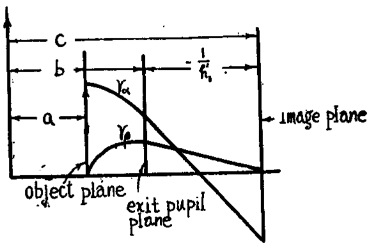

Fig. 3.

$$\left. \begin{aligned} A_2 &= 1 - \frac{2a_{2c}}{\mu_1 h'_1} = -\frac{1}{\mu_1 h'_1} \int_a^c \left[\frac{\varepsilon_1}{8\sqrt{\Phi}} (\Phi'' + \eta^2 H^2) h^2 + \varepsilon_1 \frac{\sqrt{\Phi}}{2} h'^2 - \frac{\varepsilon_2 \eta^2 H^2}{\sqrt{\Phi}} h^2 \right] dx, \\ A_3 &= -M - \frac{a_{3c}}{\mu_1 h_1'} = \frac{1}{\mu_1 h_1'} \int_a^c \left[\frac{\varepsilon_1}{8\sqrt{\Phi}} (\Phi'' + \eta^2 H^2) gh - \frac{\varepsilon_2 \eta^2 H^2}{\sqrt{\Phi}} gh + \varepsilon_1 \frac{\sqrt{\Phi}}{2} g'h' \right] dx, \\ A_4 &= -\frac{a_{4a}}{\mu_1 h_1'} = -\frac{\mu_0}{\mu_1 h_1' r_\beta(b)} \left(\frac{\varepsilon_1}{4} - \varepsilon_2 \right) \int_a^c \frac{\eta H}{\mu} dx. \end{aligned} \right\} (10)$$

These are the coefficient of the chromatic aberration. Providing that in equation (9) $A_2 \neq 0$, $A_3 = A_4 = 0$, it will become clear that the position of image is decided only by the position w_1 on the exit pupil plane. This is so called chromatic aberration. A_3 indicates the occurrence of incorrect magnification. A_4 also represents the incorrect magnification, but it somewhat differs from A_3 in the point that the direction of the distorsion is at right angle to that of A_3.

From equation (8), (10), we have

$$E = \frac{\mu_1 h_1'}{2}(1 - A_2)\rho - \mu_1 h_1'(M + A_3)\kappa - \mu_1 h_1' A_4 \sigma - \frac{\mu_1 h_1'}{4} B_c \rho^2 \qquad \cdots\cdots\cdots\cdots (11)$$

B_c means the coefficient of spherical aberration.

No we go on to calculate the aberration of axis asymmetrical electron lens. A similar calculation has beeen reported by Mr. Suzuki and Mr. Inoue,[1),2)] but they decided the aberration from the solution of differential equation that their method was quite inconvenient for the calculation of the Resolving Power. For, the intensity distribution curve of electron image should be decided beforehand to calculate the resolving Power of the electron microscope. To obtain an intensity distribution curv, phases of waves must be compounded at the first on the basis of Huygens and Frenel's view of point. For this, the Eikonal function should be decided. As there is no reference literature that gives Eikonal function to the axis asymmetrical electro magnetic field, we shall start from the calculation of it. The value of a refractive index μ for an general electromagnetic field is

$$\mu = \sqrt{\varphi} - \eta As, \quad \eta = \sqrt{\frac{e}{2m}}. \qquad \cdots\cdots(12)$$

From this we have the coordinate of the image on the image plane by ordinary calculation, the result is

$$y_1 + iz_1 = M(y_0 + iz_0) - \frac{2\bar{b}_2}{\mu_1 h_1'} - \frac{i\eta H_0}{\mu_1 h_1'} \cdot \frac{e^{i\psi_1}}{2} h(\eta_1 + i\zeta_1)$$
$$- \frac{i\eta H_0}{\mu_1 h_1'} \cdot \frac{e^{i\psi_1}}{2} g(y_0 + iz_0) - \frac{4\bar{c}_2}{\mu_1 h_1'}(\eta_1 - i\zeta_1) - \frac{2\bar{c}_3}{\mu_1 h_1'}(y_0 - iz_0). \qquad \cdots\cdots(13)$$

Each terms on the right side of the equation (13) represent the aberrations.

The first term on the right represents the size of image without the aberration and their ratio means magnification. As the second term is the constant term, this shows that the position of image moves in parallel in the whole and there appears no image distortion, so this term will not be aberration. The third is to be given as function of the position (η_1, ζ_1) of the exit pupil that this represents the aberration of which the position of focus moves just as the chromatic aberration. Their difference is that in the chromatic aberration the position of focus always changes against time and in that of the third therm its position of focus does not move aginst time. Therefore this aberration can easily be eliminated by moderately regulating Gauss's image plane. Accordingly this aberration is quite out of question. The aberration of the fourth term is determined only by the position of object and has relation to magnification. But it does not give serious influence over the resolving power as it comes to 0 at the center of object. Only the value of magnification varies and there does not appear the distorsion of image. The fifth term evidently represents the aberration with is particular to the axis asymetrical electron lens.

$$-\frac{4\bar{c}_2}{\mu_1 h'_1}(\eta_1 - i\zeta_1) \cdots\cdots \text{axis asymmetrical astigmatism,} \qquad \cdots\cdots(14)$$

$$-\frac{2\bar{c}_3}{\mu_1 h_1'}(y_0 - iz_0) \cdots\cdots \text{axis asymmetrical distorsion.} \qquad \cdots\cdots(15)$$

Among these aberrations, axis asymmetrical distorsion will be 0 at the center i. e. $y_0 = z_0 = 0$, and gives comparatively small influence over the resolving power.

On the contrary, astigmatism appears equally at any part of the visual field and can not be neglected. Therefore, as a problem for the resolving power, aberration given by $\bar{c}_2$ should be considered most seriously. The reason why $\bar{c}_2$ is called by the name of axis asymmetrical astigmatism will be explained as follows. To consider $\bar{c}_2$ in equation (13), its relation between object and image will be

$$y_1+iz_1=M(y_0+iz_0)-\frac{4\bar{c}_2}{\mu_1 h_1'}(\eta_1-i\zeta_1)\,. \qquad \cdots\cdots(16)$$

In this equation we concider what sort of a image will the electron beam which have come out of the center of the visual field $y_0=z_0=0$ reproduce. For convenience's sake we put

$$\frac{4\bar{c}_2}{\mu_1 h_1'}=\bar{C}_2, \qquad \bar{c}_2=\frac{\mu_1 h_1'}{4}\bar{C}_2\,.$$

Since the electron beam, which has come out from the point $y_0=z_0=0$ on the object plane, comes to a position of

$$\frac{4\bar{c}_2}{\mu_1 h_1'}r_b e^{-i\theta_B}=\bar{C}_2 r_b e^{-i\theta_B},$$

on the image plane by passing through $r_b e^{i\theta_B}$ on the stop plane as shown on Fig. 4, the electron beam which comes out from 4 on the stop plane in Fig. 4 will pass through 4 on the image plane. And those that pass through

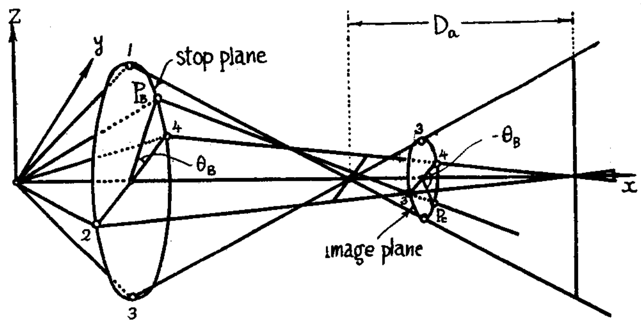

Fig. 4.

P_B on the stop plane will also pass through P_C on the image plane. If the electron beams rotate as 1–2–3–4 on the stop plane, they will rotate in the same manner 1–2–3–4 on the image plane, and its direction of rotation is reversed of on the image plane. Accordingly its form will be similar to the symmetrical aberration which is called astigmatism in optics. This figure is drawn under the assumption that c_2 is a real number. The Eikonal function when the influence by this aberration is taken into account will be

$$E=\frac{\mu_1 h_1'}{2}(1-A_2)\rho-\mu_1 h_1'(M+A_3)k-\mu_1 h_1' A_4\sigma$$
$$+\frac{\mu_1 h_1'}{4}\bar{C}_2(\eta_1-i\zeta_1)^2-\frac{\mu_1 h_1'}{4}B_c\rho^2\,. \qquad \cdots\cdots(17)$$

Originally Eikonal function should be a positive real number. The reason is that Eikonal function represents the length of an "Optical path" and as the length of an "Optical path" means the time necessary for the light to proceed from a point to another one, that should naturally be a positive real number. This is evident from the Eikonal function which have a term $c_2(\eta_1+i\zeta_1)^2$ then the conjugate term $\bar{c}_2(\eta_1-i\zeta_1)^2$ against that have, and their sum should always be a real number. While it is $\bar{c}_2$ that we are now considering and c_2 is omitted. Owing to this, it become Eikonal function of imaginery number but we can solve the problem without the influence of this. In order to decide the value of the astigmatic distance D_a in Fig. 4, we give the astigmatic distance as shown on Fig. 5. Taking the position of image plane at the position of the least circle of confusion as Fig. 5 shows, we find the relation between D_a and $\bar{C}_2$, If $\bar{C}_2$ is a real number, we have from this circle,

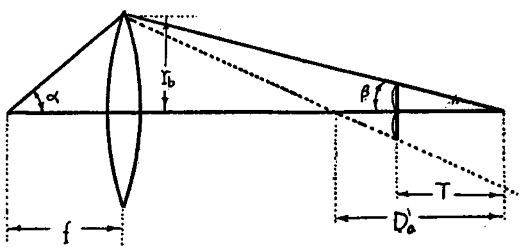

Fig. 5.

$$\left.\begin{aligned}\beta T &= \bar{C}_2\cdot r_b\\ \beta(D_a-T) &= \bar{C}_2\cdot r_b\end{aligned}\right\} \qquad \cdots\cdots(18)$$

and thus

$$\bar{C}_2=\frac{\beta D_a}{2r_b}=\frac{\alpha D_a}{2Mr_b}=\frac{D_a}{2Mf}. \qquad \cdots\cdots(19)$$

In Fig. 5, the distance between object and the lens is given as f, this approximately correct in the case of the electron microscope.

From equation (19) the value of the radius of the aberration circle βT will be

$$\beta T=\frac{1}{2}\frac{D_a}{fM}r_b=\frac{1}{2}\frac{D_a}{M}\alpha=\frac{1}{2}D_a'\cdot\alpha \qquad \cdots\cdots(20)$$

where

$$D_a'=\frac{D_a}{M}. \qquad \cdots\cdots(21)$$

The case of imaginary number should additionary be thought of, as $\bar{C}_2$ generally takes imaginary number. But in experiment, it is seldom appear that it becomes maginary number. Therefore this case is much unnecessary.

Conclusion

(1) The first order of the aberration of axis asymmetrical electron lens causes the parallel transition, variation of magnification and dislocation of the focus, etc., but

they will not give serious influence over the resolving power too much.

(2) Besides them, there exist axis asymmetrical astigmatism and axis asymmetrical distorsion, in the aberration of the first order, but it is astigmatism that gives serious influence over the resolving power. These are all peculiar to axis asymmetrical electron lens.

(3) The value of astigmatic difference can be measured experimentally without difficulty. From this

$$D'_a = \frac{D_a}{M} \qquad (M;\ \text{magnification})$$

thus from the value of above D_a', we can decide the value of aberration by the following formula.

$$\frac{1}{2} D_a' \cdot \alpha \qquad (\alpha;\ \text{aperture angle})$$

Reference Literatures

1) S. Suzuki, T. Inoue and N. Kato; Relationship of Electron Microscope Lens making and its Aberration. Shimazu Hyoron, Jan. 1948, p. 23.

2) T. Inoue; Axis Asymmetrical Aberration of the Electron Microscope. Jour. of Denki Shikenjo. Vol. 13, No. 6.

3(N. Kato and T. Inoue; The Theory of Aberration in Geometrical-Electron Optics. Kikai and Denki, Vol. 4, No. 12.

CHAPTER 3

Calculation of the Resolving Power of the Electron Microscope

By the calculations shows in the preceding chapters all the necessary aberrations have been decided. So now the rational calculation to obtain the value of the resolving power should be practised by compounding them all.

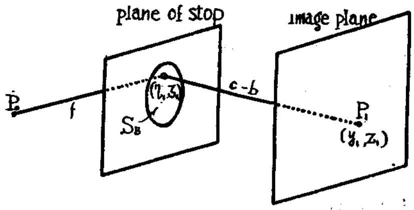

Fig. 1.

We will take the relative position of the plane of stop and the image plane as shown in Fig. 1. where the wave function U_{P_1} will be given as

$$U_{P_1} = -\frac{i}{\lambda f(c-b)} \iint e^{i\frac{2\pi}{E}} dS_B \qquad \cdots\cdots\cdots\cdots(1)$$

at the point P_1. As the E in this equation, equation (17) in Chapter 2 may be available. In equation (17)

$$A_2 = A_3 = A_4 = \bar{C}_2 = B_c = 0$$

i. e. when there exist no chromatic aberration, spherical aberration and axis asymmetrical astigmatism, equation (17) will be

$$E = \frac{\mu_1 h_1'}{2}\rho - \mu_1 h_1' M\kappa . \qquad \cdots\cdots\cdots\cdots(2)$$

Since the first term of the right side is a function of $\rho = \eta_1^2 + \zeta_1^2$, it means that incident wave to the stop is not the plane wave but curved one. In the electron microscope, it is usual that an object is set near the focus of the electron lens to make magnification bigger, and the opening of the stop is made very smaller to prevent aberration from getting greater. In such state, waves may be regarded almost as plane when passing through the stop. Therefore in equation (2) we take as

$$\frac{\mu_1 h_1'}{2}\rho = 0$$

to consider plain wave. Thus, equation (2) will be

$$E = -\mu_1 h_1' M\kappa . \qquad \cdots\cdots\cdots\cdots(3)$$

While from Fig. 3 of chapter 2 and Fig. 1 of this chapter, we have

$$-\mu_1 h_1' M = \frac{\sqrt{\Phi_c}}{c-b} M . \qquad \cdots\cdots\cdots\cdots(4)$$

Usually refractive index is to be represented by the form

$$\mu = \sqrt{\frac{\Phi_b}{\Phi_a}} \qquad \cdots\cdots\cdots\cdots(5)$$

while as it is obvious from equation (1) in chapter 2, it is taken as $\mu = \sqrt{\varphi} - \eta As$. Accordingly this will be sufficient for the calculation of the value of the aberration, but it does not bring exact dimension in the case of calculating the absolute value of the "optical path." Therefore, the form of equation (5) must be adopted in such case.

Thus, equation (4) should be taken as

$$-\mu_1 h_1' M = \frac{M}{c-b} \doteqdot \frac{1}{f}. \qquad \cdots\cdots\cdots\cdots(6)$$

By putting this in equation (3), we have

$$E = \frac{1}{f}(y_0\eta_1 + z_0\zeta_1) \qquad \cdots\cdots\cdots\cdots(7)$$

and also by putting this into equation (1), we have

$$U_{P_1} = -\frac{i}{\lambda f(c-b)} \iint e^{i\frac{k}{f}(y_0\eta_1 + z_0\varsigma_1)}\, d\eta_1 d\zeta_1 \qquad \cdots\cdots\cdots\cdots(8)$$

$$(k=\pi/\lambda).$$

Since this equation is symmetrical for y_0 and z_0, providing that

$$\eta_1 = r_b \cos\psi, \quad \zeta_1 = r_b \sin\psi,$$

taking as

$$y_0 = r_0 \cos\psi_0, \quad z_0 = r_0 \sin\psi_0,$$

we have

$$U_{P_1} = -\frac{i}{\lambda f(c-b)} \iint e^{i\frac{k}{f} r_0 r_b \cos(\psi-\psi_0)}\, r_b dr_b d\psi.$$

So integrating this, we have

$$U_{P_1} = -\frac{i}{\lambda f(c-b)} \cdot \pi r_b^2 \cdot \frac{J_1\left(k r_b \frac{r_0}{f}\right)}{k r_b \frac{r_0}{f}}. \qquad \cdots\cdots\cdots\cdots(9)$$

In discussing the resolving power of the electron microscope it will be sufficient to consider about the neighbourhood of the center of the visual field i. e. $y_0 = z_0 = 0$. Therefore in equation (17) in chapter 2, it may be taken as

$$kA_3 \doteqdot \sigma A_4 \doteqdot 0. \qquad \cdots\cdots\cdots\cdots(10)$$

By putting the conditions of plane wave, refractive index of equation (5) and the condition of the center of the visual field in equation (10) into equation (17) in chapter 2, we have,

$$E = -\frac{1}{2} h_1' A_2 \rho - h_1 M\kappa + \frac{h'_1}{4} \bar{C}_2(\eta_1 - i\zeta_2)^2 - \frac{h_1'}{4} B_e \rho^2 . \quad \cdots\cdots(11)$$

While from equation (10), (21) in chapter 2,

$$\left.\begin{aligned} A_2 &= -\frac{M}{r_\beta(b)} A_2' , \\ B_e &= \frac{M}{r_\beta^3(b)} B_e' , \\ \bar{C}_2 &= \frac{D_a'}{2f} , \end{aligned}\right\} \quad \cdots\cdots(12)$$

and from Fig. 2 and Fig. 3 in chapter 2 we have

$$h_1' = -\frac{1}{c-b} \fallingdotseq -\frac{1}{M r_\beta(b)} \fallingdotseq -\frac{1}{Mf}, \quad \cdots\cdots(13)$$

$$(\text{as } r_\beta'(a) = 1, \; r_\beta(b) \fallingdotseq f)$$

then equation (12) will be

$$\left.\begin{aligned} A_2 &= -\frac{MA_2'}{f} \\ B_e &= +\frac{MB'_e}{f^3} \\ \bar{C}_2 &= +\frac{D_a'}{2f} \end{aligned}\right\} \quad \cdots\cdots(14)$$

by putting equation (13) and (14) into equation (11), we have

$$E = \frac{\eta_1 y_0 + \zeta_1 z_0}{f} + \frac{B_e'}{4f^4}(\eta_1^2 + \zeta_1^2)^2 - \frac{A_2'}{2f^2}(\eta_1^2 + \zeta_1^2) - \frac{D_a'}{8f^2 M}(\eta_1 - i\zeta_1)^2 \cdots\cdots(15)$$

Though this equation should be that of real number, it will come to contain an imaginary number by the reason stated in chapter 2. Putting this equation (15) into equation (1), we will obtain the value of U_{p1}.

$$U_{p1} = -\frac{i}{\lambda f(c-b)} \iint e^{i\frac{2\pi}{\lambda}E} d\eta_1 d\zeta_1 \quad \cdots\cdots(16)$$

Providing equation (15) as

$$E = \frac{1}{f}(y_0\eta_1 + z_0\zeta_1) + \delta E \quad \cdots\cdots(17)$$

equation (16) will approximately become

$$U_{p1} = -\frac{i}{\lambda f(c-b)} \iint e^{i\frac{k}{f}(y_0\eta_1 + z_0\zeta_1)} (1 + ik\delta E) d\eta_1 d\zeta_1 \quad \cdots\cdots(18)$$

$$(k = 2\pi/\lambda)$$

Now providing as

$$\left.\begin{aligned} U_{pq} &= \iint \eta_1^p \zeta_1^q e^{i\frac{k}{f}(y_0\eta_1+z_0\zeta_1)} d\eta_1 d\zeta_1 \\ U_{00} &= \iint e^{i\frac{k}{f}(y_0\eta_1+z_0\zeta_1)} d\eta_1 d\zeta_1 \end{aligned}\right\} \qquad \cdots\cdots(19)$$

there exist the relation of

$$U_{pq} = \frac{1}{\left(i\frac{k}{f}\right)^{p+q}} \cdot \frac{\partial^{p+q} U_{00}}{\partial y_0^p \partial z_0^q} \qquad \cdots\cdots(20)$$

In other word we can obtain all the value of U_{pq} when the value of U_{00} were to be decided. But the integral of equation (8) represents the same value of the integral of this U_{00}, and we at once obtain

$$U_{00} = 2\pi r_b^2 \cdot \frac{J_1\left(kr_b\frac{r_0}{f}\right)}{kr_b\frac{r_0}{f}} \qquad \cdots\cdots(21)$$

Putting equation (19) into equation (18), we have

$$\begin{aligned} U_{p1} = -\frac{i}{\lambda f(c-b)}\Big[& U_{00} + ik\Big\{\frac{B_e'}{4f^4}(U_{40}+2U_{22}+U_{04}) - \frac{A_2'}{2f^2}(U_{20}+U_{02}) \\ & + \frac{D_a'}{3f^2M}(_{02}-U_{20}) + 2i\frac{D_a'}{8f^2M}U_{11}\Big] \end{aligned} \qquad \cdots\cdots(22)$$

Providing as follows to obtain the value of the respective terms in this equation

$$t = kr_b\frac{r_0}{f} \qquad \cdots\cdots(23)$$

thus we have

$$U_{00} = 2\pi r_b^2 \frac{J_1(t)}{t} \qquad \cdots\cdots(24)$$

Differentiating equation (24) by using the formula,

$$\frac{d}{dt}\left(\frac{J_p(t)}{t^p}\right) = -\frac{J_{p+1}(t)}{t^p} \qquad \cdots\cdots(25)$$

We have

$$\left.\begin{aligned} U_{11} &= -2\pi r_b^6 \cdot \frac{J_3(t)}{t^3} y_0 z_0 \left(\frac{k}{f}\right)^2 \\ U_{20}+U_{02} &= +2\pi r_b^4 \left(\frac{2J_2(t)}{t^2} - \frac{J_3(t)}{t}\right) \\ U_{20}-U_{02} &= -2\pi r_b^6 \cdot \frac{J_3}{t^3}\left(\frac{k}{f}\right)^2 (y_0^2 - z_0^2) \\ U_{40}+2U_{22}+U_{04} &= 2\pi r_b^6 \left(\frac{J_s}{t} - \frac{8J_4}{t^2} + \frac{8J_3}{t^3}\right) \end{aligned}\right\} \qquad \cdots\cdots(26)$$

And also using the relation

$$J_{\nu+1} = \frac{2\nu}{t} J_\nu - J_{\nu-1} \qquad \cdots\cdots(27)$$

We change and simplify the right side of equation (26), and putting this into equation (22), we have

$$U_{p1} = -\frac{i2\pi r_b^2}{\lambda f(c-b)}\Big\{F_1 + 2k\Big(\frac{D_a'}{8M}\alpha^2\Big)(k\alpha)^2 y_0 z_0 F_4 + ik\frac{B_c'\alpha^4}{4}F_2 + ik\frac{A_2'}{2}\alpha^2 F_3 + ik\Big(\frac{D_a'}{8M}\alpha^2\Big)(k\alpha)^2(y_0^2 - z_0^2)F_4\Big\} \quad \cdots\cdots(28)$$

where

$$\left.\begin{aligned} \alpha &= r_b/f \\ F_1 &= \frac{J_1(t)}{t} \\ F_2 &= 8\frac{J_3(t)}{t^3} - 4\frac{J_2(t)}{t^2} + \frac{J_1(t)}{t} \\ F_3 &= 2\frac{J_2(t)}{t^2} - \frac{J_1(t)}{t} \\ F_4 &= \frac{J_3(t)}{t_3} \end{aligned}\right\} \quad \cdots\cdots(29)$$

To decide the value of intensity distribution I_p from the above equation, we must at first decide the coefficient of aberration A_2', B_c', ... by experimental measurement, then by assuming aperture angle α and calculating the value of F_1, F_2, F_3, and F_4 by table 1, we will obtain the real part and imaginary part of U_{p1}. Then we can dicide the intensity distribution by the equation $I_p = |U_{p1}|^2$.

Now to make out sure that equation (28) is true, we assume $B_c' = A_2' = 0$ and exist only diffraction aberration and astigmatism, and examine the intensity distribution curve. In this case, equation (28) will become

$$U_{p1} = -i\frac{2\pi r_b^2}{\lambda f(c-b)}\Big\{\frac{J_1(t)}{t} + 2k\Big(\frac{D_a'}{8M}\alpha^2\Big)\frac{J_3(t)}{t^3}y_0 z_0 (k\alpha)^2 + ik\Big(\frac{D_a'}{8M}\alpha^2\Big)\frac{J_3(t)}{t^3}(y_0^2 - z_0^2)(k\alpha)^2\Big\} \quad \cdots\cdots(30)$$

From this, we have

$$I_{p1} = \Big\{\frac{2\pi r_b^2}{\lambda f(c-b)}\Big\}^2\Big[\Big(\frac{J_1}{t}\Big)^2 + \Big(\frac{D_a'}{8M}\alpha^2\Big)^2(k\alpha)^4 r_0^4\frac{J_3^2}{t^6} + 4\Big(\frac{D_2'}{8M}\alpha\Big)(k\alpha)^3\frac{J_1 J_3}{t^4}y_0 z_0\Big] \cdots\cdots(31)$$

(a) If $D_a' = 0$, only the first term of the right side of the equation will remain, it will become the equation only with diffraction aberration.

(b) When the first and the second terms are always positive, and r_0 is constant, its value becomes constant and distribution will be symmetrical against origin.

(c) As in the third term, $J_1 J_3 > 0$ when r_0 is small, therefore it will be positive when $y_0 z_0 > 0$, and negative when $y_0 z_0 < 0$. Accordingly it will be added at the first and third quadrant and subtracted at the second and fourth quadrant. Small r_0 means that the neighbourhood of the origin is considered, namely that is a

Table 1

t	F_1	F_2	F_3	F_4	F_1^2	F_2^2	F_3^2	F_4^2
0	0.5	0.1250	−0.25	0.021	0.25	0.01561	0.0625	0.000442
0.2	0.4975	0.12385	−0.24835	0.0208	0.248	0.01535	0.0620	0.000433
0.4	0.49005	0.1205	−0.2434	0.0206	0.2401	0.01455	0.0592	0.000425
0.6	0.4779	0.1149	−0.2350	0.0204	0.2295	0.01320	0.0552	0.000417
0.8	0.46105	0.1073	−0.2241	0.0200	0.2130	0.01153	0.0507	0.000400
1.0	0.44005	0.09705	−0.21025	0.0194	0.1940	0.00945	0.0442	0.000377
1.2	0.41525	0.0867	−0.1939	0.0190	0.1722	0.00753	0.0377	0.000362
1.4	0.3871	0.0747	−0.1755	0.0185	0.1500	0.00560	0.0308	0.000343
1.6	0.3502	0.0609	−0.1555	0.0177	0.1272	0.00372	0.02425	0.000314
1.8	0.32305	0.0467	−0.1340	0.0169	0.1045	0.00218	0.0180	0.000286
2.0	0.28835	0.0322	−0.1119	0.0161	0.0832	0.00104	0.01251	0.000260
2.2	0.2527	0.0176	−0.0894	0.0152	0.0641	0.000310	0.00800	0.000232
2.4	0.21675	0.00345	−0.06715	0.0143	0.0472	0.00001195	0.00452	0.000205
2.6	0.1811	−0.0102	−0.0453	0.0134	0.0329	0.000104	0.00206	0.000180
2.8	0.1463	−0.0229	−0.0244	0.0124	0.0214	0.000525	0.000597	0.000154
3.0	0.1130	−0.0343	−0.0050	0.0114	0.01279	0.00118	0.000025	0.000130
3.2	0.08165	−0.0444	+0.0128	0.0105	0.00659	0.00198	0.0001640	0.0001102
3.4	0.0527	−0.0528	+0.0285	0.0095	0.00278	0.00280	0.000812	0.0000905
3.6	0.0265	−0.0595	+0.0422	0.0085	0.000705	0.00355	0.001785	0.0000725
3.8	0.00337	−0.0673	+0.0563	0.0076	0.0000114	0.00455	0.003175	0.0000580
4.0	−0.01651	−0.06725	+0.06205	0.0067	0.000273	0.00454	0.00385	0.0000450
4.2	−0.03301	−0.0682	+0.0682	0.0059	0.00109	0.00466	0.00466	0.0000349

position of intense part of beam is considered spatially. Therefore it corresponds to consider the neighbourhood of lens.

(d) When $J_1 = 0$, only the second term will remain. This means symmetrical distribution.

(e) When r_0 is large, $J_1 J_3 < 0$ and the sign of the third term will vary. From this, spatial change of intensity distribution is as shown in Fig. 2 and this represents astigmatism.

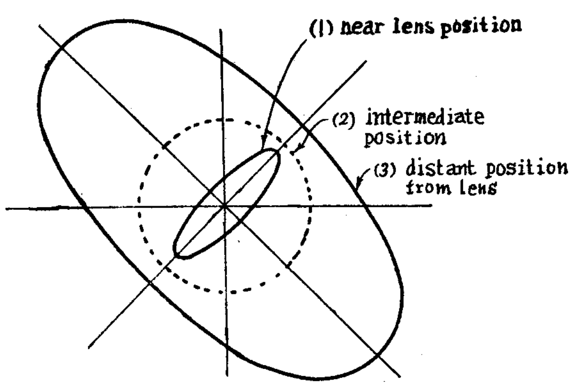

Fig. 2.

Next, when we consider only about the diffraction and spherical aberrations, we have

$$I_{p1} = \left(\frac{2\pi r_b^2}{\lambda f(c-b)}\right)^2 \left\{\left(\frac{J_1}{t}\right)^2 + \left(k\frac{B_e'\alpha^4}{4}\right)^2 F_2^2\right\} \qquad \cdots\cdots(32)$$

and a result of the calculation $I_p/I_{p\,\max}$ on the provision of

$$\lambda = 5.35\times 10^{-8}\ \text{m.m.}\ (50{,}000\text{V})$$
$$\alpha = 5\times 10^{-3}\ \text{and}\ \alpha = 7\times 10^{-3}$$
$$B_e' = 10\ \text{m.m.}$$

is shown in Fig. 3. As it is evident on the figure, the greater the influence of spherical aberration is, the shorter becomes the distance from the origin to the first dark ring. According to Abbe's difinition of the revolving power, the distance up to the first dark ring is the limit of the revolving power when spherical aberration is neglected. But when aberration exist, this way of thinking causes absurdity which shows that the greater spherical aberration is, the better is the resolving power.

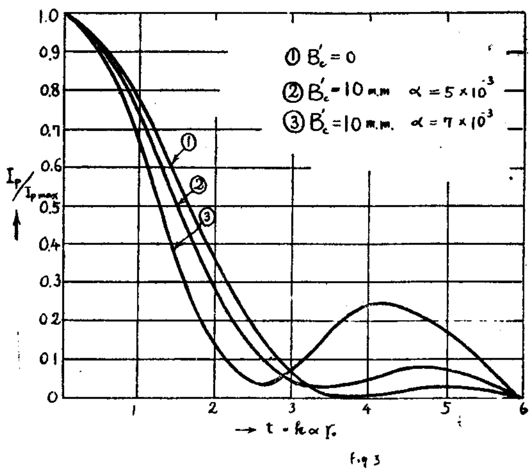

Fig. 3.

As diffraction aberration is one caused by the wave property of light, it is impossible to compose with other aberrations in geometrical optics such as spherical aberration or astigmatism and else. To do this composition, a new definition should be required. As Seidel's Eikonal $S^{4)}$ and its aberrations are given by the equation as follows,

$$\left.\begin{aligned} S^{(4)} &= \frac{A}{4}(y_0^2+z_0^2)^2 + \frac{B}{4}(\eta_1^2+\zeta_1^2)^2 + C(y_0\eta_1+z_0\zeta_1)^2 + \cdots\cdots \\ -\frac{\partial S^{(4)}}{\partial \eta_1} &= y_1 - y_0 = \Delta y \\ -\frac{\partial S^{(4)}}{\partial \zeta_1} &= z_1 - z_0 = \Delta z \end{aligned}\right\} \qquad \cdots\cdots(33)$$

In order to compose Seidel's five aberations it is obvious that, a algebraic summation of these aberrations is sufficient. But diffraction aberration can not be added as it belongs to a different kind of aberration. The way which Ardenne[1] had adopted is as follows. He assumed that every aberration can be represented by error curve. Accordingly when error curves are provided that

$$\left.\begin{aligned}\varphi_1(x) &= \frac{h_1}{\sqrt{\pi}} e^{-h_1^2 x^2} \\ \varphi_2(x) &= \frac{h_2}{\sqrt{\pi}} e^{-h_2^2 x^2}\end{aligned}\right\} \quad \cdots\cdots\cdots(34)$$

then compound of these becomes

$$\left.\begin{aligned}\varphi_{ges}(x) &= \int_{-\infty}^{\infty} \varphi_1(x+y)\varphi_2(y)dy = \frac{h}{\sqrt{\pi}} e^{-h^2x^2} \\ h &= \frac{h_1 h_2}{\sqrt{h_1^2+h_2^2}}.\end{aligned}\right\} \quad \cdots\cdots\cdots(35)$$

Now representing deviation as ε, we have

$$\varepsilon_1 = \frac{1}{\sqrt{2}\,h_1}, \quad \varepsilon_2 = \frac{1}{\sqrt{2}\,h_2}$$

and their compound will be

$$\varepsilon_{ges} = \sqrt{\varepsilon_1^2+\varepsilon_2^2}$$

And on the analogy of this equation, we compound the aberrations by calculating as

$$\text{Compounded aberration} = \sqrt{d_{diff}^2 + d_{space}^2 + d_{sph}^2 + d_{chrom}^2} \quad \cdots\cdots\cdots(36)$$

Where d_{diff}, d_{space}, indicate the value of aberrations. Now what becomes problem here is that error curve and intensity distribution curve of spherical aberration differ considerably as shown in Fig. 4, and the same is said to the other aberrations too.

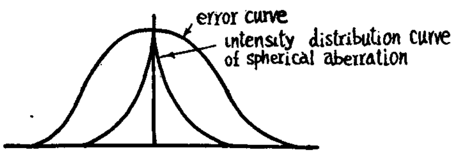

Fig. 4.

Therefore the result which has been com posed in such manner is very doubtful to what extent it is reliable. Secondary, this calculation merely composes aberration and does not represents the limit of the resolving power. Since the limit of the resolving power is the shortest distance which the two point sources are distinguishable and compound of aberrations will not be the limit of the resolving power.

The first term in the parenthesis of the right side of equation (32) represents the distribution curve of diffraction aberration and the second term represents that of spherical aberration. Though this equation is a approximate one, it shows that the addition of the distribution curve of diffraction aberration and that of spherical aberration makes the synthetic distribution curve. The extent of the distribution curve extends the greater the value of resolving power becomes.
And it approximately may said as

Compounded resolving power = Resolving power of diffraction aberration
+Resolving power of spherical aberration

Though the magnitude of spherical aberration may given by $B_e'\alpha^3$, what percent of the value of this aberration will be taken as magnitude of the resolving power will become a Problem. Rebsh[2] decided the limit of the resolving power by x_{sph} which satisfy the following equation

$$\frac{\int_0^{x_{sph}} I_p dx}{\int_0^{\infty} I_p dx} \gtreqless 50\%$$

where the intensity distribution curve of spherical aberration is provided as I_p. After calculation, this value becomes

$$x_{sph} = 0.4\, d_{sph} = 0.4 \times B_e'\alpha^3$$

Therefore the resolving power d_{tot} will be

$$d_{tot} = d_{diff} + 0.4\, d_{sph}$$

This is the formula of Rebsh. For the value of diffraction aberration it is reasonable to adopt the half width. When chromatic aberration, spherical aberration and axis asymmetrical astigmatism respectively take the half width, the result of the calculation using table 1 will be

$$\begin{aligned} \text{half width} &= 0.48 \times d_{sph} = 0.48 \times B_e'\alpha^3 \\ \text{,,} \quad &= 0.46 \times d_{chrom} = 0.46 \times A_2'\alpha \\ \text{,,} \quad &= 0.44 \times d_{astig} = 0.44 \times \frac{D_a'\alpha}{2} \end{aligned}$$

Diffraction aberration taking half width will be given as stated in Chapter 1,

$$d_{diff} = \frac{0.51\,\lambda}{\alpha}$$

Accordingly, we have

$$\begin{aligned} d_{tot} &= \frac{0.51\,\lambda}{\alpha} + 0.48\, B_e'\alpha^3 + 0.46\, A_2'\alpha + 0.44 \times \frac{D_a'\alpha}{2} \\ &\fallingdotseq \frac{1}{2}\left(\frac{\lambda}{\alpha} + B_e'\alpha^3 + A_2'\alpha + 0.5\, D_a'\alpha\right) \end{aligned}$$

i. e., approximately

$$\begin{aligned} d_{tot} &= 0.5\left(\frac{\lambda}{\alpha} + B_e'\alpha^3 + A_2'\alpha + 0.5\, D_a'\alpha\right) \\ &= \frac{0.5}{\alpha} + 0.5(d_{sph} + d_{chrom} + d_{astig}) \end{aligned}$$

and we decide this as the formula of the resolving power.
Now, adopting following figures

$\lambda \ = 0.535\times10^{-8}$ m.m. ($50{,}000^{V}$), $\alpha = 5\times10^{-3}$

$B_c' = 12$ m.m. (experimentally measured on Hidachi electron microscope[3])

$A_2' = 2.73\times10^{-4}$ m.m. ($\varepsilon_1 = 10^{-4}$, $\varepsilon_2 = 0.5\times10^{-4}$ figures adopted for Hidachi electron microscope)

$D_a' = 2\mu$ ($D_a' = 100\sim0.5\,\mu$, ordinary $D_a' = 2\mu$)[3]

we have

$$d_{diff} = 0.5\times\frac{\lambda}{\alpha} = 0.535\times10^{-6} \text{ m.m.}$$

$$d_{sph} = B_c'\cdot\alpha^3 = 1.5\times10^{-6} \text{ m.m.}$$

$$d_{chrom} = A_2'\cdot\alpha = 1.365\times10^{-6} \text{ m.m.}$$

$$d_{astig} = 0.5\,D_a'\cdot\alpha = 5.0\times10^{-6} \text{ m.m.}$$

$$d_{tot} = 4.47\times10^{-6} \text{ m.m.}$$

And also providing $\alpha = 3\times10^{-3}$ and other constants are the same to the above, we have

$$d_{diff} = 0.892\times10^{-6} \text{ m.m.}$$

$$d_{sph} = 0.324\times10^{-6} \text{ m.m.}$$

$$d_{chrom} = 0.819\times10^{-6} \text{ m.m.}$$

$$d_{astig} = 3\times10^{-6} \text{ m.m.}$$

$$d_{tot} = 2.96\times10^{-6} \text{ m.m.}$$

Conclusion

(1) Axis asymmetrical astigmatism is remarkably greater than the other aberrations.

(2) The resolving power of the electron microscope made in Japan have the value of about several m.μ. or more.

(3) The resolving power will be remarkably improved when axis asymmetrical astigmatism is made smaller.

(4) It is suitable to use grains of several m.μ. for measuring resolution.

(5) The resolving power of the electron microscope is given by

$$d_{tot} = 0.5\left(\frac{\lambda}{\alpha} + B_c'\alpha^3 + A_2'\alpha + 0.5\,D_a'\alpha\right)$$

or

$$d_{tot} = \frac{0.5\,\lambda}{\alpha} + 0.5(d_{sph} + d_{chrom} + d_{astig})$$

where

λ ; wave length

α ; aperture angle of electron beam

B_e' ; coefficient of spherical aberration
D_a' ; astigmatic difference
A_2' ; coefficient of chromatic aberration

Reference Literatures

1) Ardenne; Ultra Electron Microscope. Translated by Proffessional Education Bureaue of Ministry of Education. p. 62.
2) R. Rebsh; Das theoretische Aufloesungsvermoegen des Elektronenmikroskops. Annalen der phys. 5 Folge. B. 31, 1938.
3) K. Koizumi, N. Morito; Measuring Method of Spherical Aberration of the Electron Microscope, Report oJ the Electron Microscope General Investigation Committee, Dec. 10, 1951.

CHAPTER 4

Conclusion

Completing the results obtained in the foregoing chapters, it becomes as follows. In considering the resolving power of an electron microscope, it is necessary to distinguish the resolving power which represents the ability of an electron microscope itself from one which represents the ability of an electron micrograph. The former will be called the resolving power and the latter the resolution. Difinition of the resolving power will be decided as follows.

" Power to distinguish the images of two adjacent point sources of electron producing electrons of the same velocity is called the resolving power of an electron microscope. The shortest distance which is visible enough the adjacent point sources of electron is called the Limit of the Resolving Power."

In order to decide the resolving power of an electron microscope, we must actually measure the aberrations such as spherical aberration, axis asymmetrical astigmatism, diffraction aberration, and chromatic aberration etc. which give serious influence over the resolving power and we will obtain the value of the resolving power by calculation assuming an aperture angle of incident electron beam.

For the calculation of the resolving power, we should used the following formula.

$$d_{tot} = \frac{0.5\,\lambda}{\alpha} + 0.5(d_{sph} + d_{chron} + d_{astig})$$

or

$$d_{tot} = 0.5\left(\frac{\lambda}{\alpha} + B_c'\alpha^3 + A_2'\alpha + 0.5\,D_a'\alpha\right)$$

where

	$\frac{0.51\,\lambda}{\alpha}$	diffraction aberration
d_{sph}	$= B_c'\alpha^3$	spherical aberration
d_{chrom}	$= A_2'\alpha$	chromatic aberration
d_{astig}	$= 0.5\,D_a'\alpha$	axis asymmetrical astigmatism

It is quite reasonable to take half width and we prosecute calculation by all half width to decide the value of the resolving power from spherical aberration, diffraction aberration and so on.

We obtained several m.μ. for the value of the resolving power of the electron microscopes made in Japan by using the above formula and measuring respective aberrations of the electron microscopes.

Next, the resolution which represents the quality of an electron micrograph will be defined as follows.

" Power to distinguish every part of inspecting object is called the Resolution and the shortest distance that can distinguish two points of objects is called the Limit of the Resolution."

To measure this resolution, the following procedure should be taken. We first prepare the samples of which roughness of the grains can be optionally varied (for instance, the grains obtained by vacuum evaporation of silver or gold), and apply them to an electron microscope from rougher to the finer to decide the limit of the resolution. Then, we take micrographs of the grains near the limit and decide the resolution by the distance between two points finding out the shortest distance which can distinguish as two points on the micrographs. In the case when such two points can not be found, the mean radius of the minimum grains in the micrographs will be taken as a approximate value of the resolution.

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