# A Note on the Formula for the Psychrometer

By

Suekichi KAWATA and Yoshirō ŌMORI

Department of Applied Physics

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For psychrometric determination of humidity in air the generally accepted formula is

$$p = p_m' - AP(\theta - \theta_m') , \tag{1}$$

where $\theta$ and $\theta_m'$ are respectively temperatures of dry- and wet-bulbs in °C, $P$ is atmospheric pressure, $p$ and $p_m'$ are respectively water vapor pressure in question and saturation vapor pressure at $\theta_m'$ all in mm Hg, and $A$ is the so-called psychrometer constant. The formula can be derived from the condition of thermal equilibrium of a wet-bulb. We may put $Q_a = \kappa_a(\theta - \theta_m')$ for the rate of transfer of heat by conduction and convection from the surrounding air, $Q_r = \kappa_r(\theta - \theta_m')$ for that by radiation and $Q_c = \kappa_c(\theta - \theta_m')$ for that by conduction through the stem of the wet-bulb thermometer, where $\kappa_a$, $\kappa_r$ and $\kappa_c$ are constants of proportionality. Again we may take the rate of losing of latent heat by evaporation of water from the wet-bulb as $Q' = \lambda L(\delta_m - \delta)$, where $\delta_m$ and $\delta$ are respectively absolute saturation humidity at $\theta_m'$ and absolute humidity in the air, $L$ is the latent heat of vaporisation of water and $\lambda$ is a constant of proportionality. Thus, if we neglect the heat used in raising the temperature of the water vapor evaporated from the wet-bulb, we have for the condition of equilibrium

$$\lambda L(\delta_m - \delta) = (\kappa_a + \kappa_r + \kappa_c)(\theta - \theta_m') , \tag{2}$$

and by making use of the relation

$$\delta = 0.622\rho \frac{p}{P} , \tag{3}$$

where $\rho$ is the density of the air at $\theta$ and $P$, and assuming $\rho = \rho_m'$ approximately or neglecting the term $p_m'\left(1 - \frac{\rho'_m}{\rho}\right)$ (impracticable but for simplicity for the present discussion), we obtain

$$p - p_m' = -\frac{(\kappa_a + \kappa_r + \kappa_c)}{0.622\rho\lambda L} P(\theta - \theta_m') . \tag{4}$$

If heat conducted through the stem is excluded by some means (This is easily realized by adequate covering of a moistened cloth about the wet-bulb[1].) (4) will be written as

$$p = p_m' - A_a\left(1+\frac{\kappa_r}{\kappa_a}\right) P(\theta-\theta_m') , \tag{5}$$

where

$$A_a = \kappa_a/(0.622\rho\lambda L) . \tag{6}$$

The factor $1+(\kappa_r/\kappa_a)$ shows effect of radiation on the wet-bulb, and $A_a$ means, as described later, an ideal psychrometer constant influenced neither by radiation nor by conduction through the stem. If we denote coefficient of transfer of heat due to conduction and convection through air by $h_a$ and that due to radiation by $h_r$, $\kappa_r/\kappa_a$ will be equal to $h_r/h_a$, and the correction term by radiation becomes $1+(h_r/h_a)$. $h_r$ will be given approximately by

$$h_r = 4\varepsilon\sigma T_M^3 , \tag{7}$$

where $T_M$ is a mean absolute temperature of dry- and wet-bulbs, $\sigma$ is the Stefan-Boltzmann's constant and $\varepsilon$ is the total emissivity of the moistened cloth.

Now it is well-known that the temperature of a wet-bulb is largely effected by ventilation. It is depressed with increasing wind velocity but becomes nearly constant by sufficient ventilation. The constant temperature is usually taken as $\theta_m'$. With Assmann aspiration psychrometers the sufficient wind velocity is considered to be arrived at 2.5–3 m/s, while wet-junctions of thermocouple psychrometers, consisting of fine wires and covered with thin cotton thread, show such a saturated depression of temperature by wind of 10–20 cm/s[1]. We, then, intend to find the reason of the marked difference in connection with the object of obtaining the formula for the psychrometer.

To search for the formula we intended to apply the theory of analogy of transfer of mass and heat, and, for the present, to make use of the experimental formula worked out by McAdams[2] from among abundant data of various investigators on transfer of heat around circular cylinders under forced convection perpendicular to their axes. This formula can be applied fairly well for cylinders of other cross sections. The formula is given by

$$Nu = (0.35+0.47Re^{0.52})\,Pr^{0.3}, \tag{8}$$

where $Nu$ is Nusselt number, $h_aD/k$; $Re$ is Reynolds number, $vD\rho/\eta$; $Pr$ is Prandtl number, $c_p\eta/k$; and $v, k, c_p, \rho, \eta$ and $D$ denote respectively wind velocity, thermal conductivity, specific heat at constant pressure, density and viscosity of air and diameter of a cylinder. The equation (8) holds for $Re$ of 0.1–1,000. (For $Re$ from 1,000 to 50,000, $Nu = 0.26Re^{0.6}Pr^{0.3}$.) By the theory of analogy, the analogous Nusselt number corresponding to evaporation of water from a moistened cloth will be obtained by putting Schmidt number Sc, $\eta/\rho d$ ($d$ means diffusion coefficient of water vapor and air), into (8) in place of $Pr$. Then we have

$$\frac{\kappa_a}{\lambda}=\left(\frac{Pr}{Sc}\right)^{0.3}\frac{k}{d}, \tag{9}$$

and accordingly,

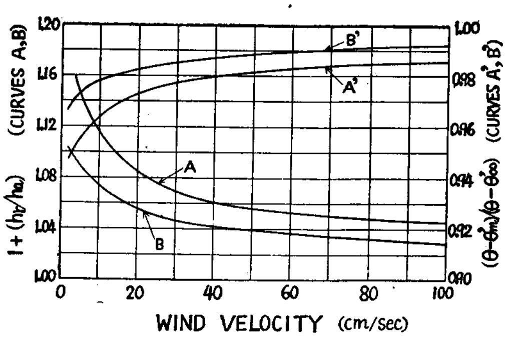
$$A_a=\frac{1}{0.622\rho L}\left(\frac{c_p\rho d}{k}\right)^{0.3}\frac{k}{d}. \tag{10}$$

Thus it is found that the ideal psychrometer constant $A_a$ is a property value represented by property values alone. Contrary to $A_a$, the correction term due to radiation, $1+(h_r/h_a)$, depends on wind velocity and also dimension of a wet-bulb because, while $h_r$ is independent of wind velocity and size of a wet-bulb, $h_a=\frac{k}{D}Nu=\frac{k}{D}\left[0.35+0.47\left(\frac{vD\rho}{\eta}\right)^{0.52}\right]\left(\frac{c_p\eta}{k}\right)^{0.3}$ increases with increase of wind velocity and with decrease of size of a wet-bulb, and it is inferred that a wet-junction of a fine wire thermocouple psychrometer will be cooled sufficiently by slight ventilation mostly due to its small size which makes $h_r/h_a$ definitely small.

In the figure, curves $A$ and $B$ show relations of $1+(h_r/h_a)$, i.e., $A/A_a$, versus wind veocity, and curves $A'$ and $B'$ show relations of "degree of depression of a wet-bulb", i.e., actual depression of wet-bulb temperature/ideal depression of wet-bulb temperature under infinite wind velocity, $\frac{\theta-\theta_m'}{\theta-\theta_\infty'}$, where $\theta_\infty'$ means wet-bulb temperature under ideal infinite wind velocity, versus wind velocity. The ratio $\theta-\theta_m'/\theta-\theta_\infty'$ is calculated by the following equation

$$\frac{\theta-\theta_m'}{\theta-\theta_\infty'}=\frac{1}{1+\frac{h_r}{h_a}\left[\frac{1}{1+\frac{1}{A_aP}\left(\frac{\Delta p_m}{\Delta\theta,}\right)}\right]} \tag{11}$$

where $\frac{\Delta p_m}{\Delta\theta}$ is the temperature coefficient of change of saturation vapor pressure at the wet-bulb temperature. The curves $A$ and $A'$ are drawn for a cylinder of 1 mm and the curve $B$ and $B'$ for that of 0.5 mm, taking $\sigma=1.36\cdot10^{-12}\mathrm{cal\cdot cm^{-2}\,sec^{-1}\,deg^{-1}}$, $\varepsilon=0.918$, $\theta=23°\mathrm{C}$ and $\theta_m'\doteqdot17°\mathrm{C}$. As it may be seen from the figure the wet-bulb temperature becomes nearly constant at wind velocity of about 20 cm/s. This seems to agree well with our observations[1].




Saturation wind velocity of 2–3 m/s for mercury-in-glass thermometer is also lead by calculation. Thus, the change of depression of wet-bulb temperature with wind velocity may be ascribed almost entirely to the effect of radiation.

In the case of natural convection instead of forced ventilation, $h_r/h_a$ can be estimated from experimental data on the heat transfer under natural convection, and a conclusion similar to the case of forced convection can be obtained.

According to (10) $A_a$ naturally depends on temperature. The effect will appear in two different ways. $L$ will depend mainly on temperature of a wet-bulb while other property values will depend on wet- and also dry-bulb temperatures (Approximately mean of the two temperatures may be assumed for the latter.).

Thus, by applying the theory of analogy of transfer of heat and mass and by adopting the formula given by McAdams, we have obtained a formula of the psychrometer including correction due to radiation, and explained the relation of size of a wet-bulb and saturation ventilation. The psychrometer constant thus obtained seems to be of magnitude of right order, and it seems to us highly probable that the procedure of deduction of the formula is in the right direction. But to ascertain the formula precisely it is necessary to know exact property values and we are confronted with difficulty especially of finding exact value of $d$, diffusion coefficient of water vapor and air.

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**References**

1) S. Kawata and Y. Ōmori, Jour. Phys. Sos. Japan, 8, 768 (1953).
2) McAdams, "Heat Transmission," Mc-Graw-Hill Co., 1942, p. 222.