The Criterion for Instability of Steady Uniform Flows in Open Channels

By

Yoshiaki IWASA

Department of Civil Engineering

(Received August, 1954)

Synopsis In this paper, the instability criterion of steady uniform flows in open channels with any section is derived by considering the time growth or decay of an infinitesimal disturbed motion of fluid surface.

The mathematical analysis shows that the criterion by this approach, based upon the momentum equation neglecting the curvature of fluid surface, is identical with the expression of V. V. Vedernikov's criterion for instability of free surface in real fluids, using certain approximations of Saint Venant.

It is well interesting to note that this expression of the criterion for instability becomes the same condition to maintain final patterns of roll-waves in steep inclined channels, obtained by argumenting their hydraulic characteristics.

1. Introduction

The initiation of instability of free surface and the presence of progressive roll-waves on the fluid will be often observed, if Froude numbers of steady uniform flows in open channels exceed certain definite numerical values. Up to the present time, many researches and references have been made to these phenomena. Then it should be noticed that this instability is different in its essential character from the instability that leads to the transition from laminar flows to turbulent flows in turbulence theories, and it presupposes that fluid flows indicate the transition from steady uniform flows with a plane smooth surface to different flows with ripples or some transverse ridges on fluid surface.

In 1925, H. Jeffreys[1] studied this criterion for instability of open channel flows as the initiation condition of roll-waves in steep chûtes, though V. Cornish was the pioneer who observed roll-waves due to the instability of flows in open channels of the Alps. Assuming that the flow was completely turbulent and the frictional resistance was proportional to the second power of the mean velocity of flow, and considering the time growth or decay of an infinitesimal disturbance on fluid surface, he concluded that the minimum value of the Froude number necessary to initial instability was 2. In 1940, G. H. Keulegan and G. W. Patterson[2] obtained the instability criteria for the Manning and the Chézy formulas, based upon an expression for the wave celerity of volume-element due to Boussinesq, and showed that these values were 1.5 for the former and the same as in Jeffreys' for the latter.

The above described results show the special criteria for instability, based upon the definite empirical resistance formulas in two dimensional flows, so that their best range of applicability for instability will be restricted. Another approach was completed by V. V. Vedernikov[3], used certain approximations of Saint Venant, in 1945. Though it is very interesting that the result is applicable for any section and any law of resistance in open channel flows, it is unfortunate for us to know nothing but the final expression of criterion obtained by him[4]. Moreover, in recent year, it is reported[4] that A. Craya[5] has refined the above approach by considering the time growth of an infinitesimal shock with results applying also in the case of a general resistance law. And in 1950, a graphical method for investigating the instability of flows, was presented by F. F. Escoffier[6], using the method of characteristics.

The unstable disturbed waves do not grow infinitely, with the lapse of time, but they will be reduced to roll-waves with definite periodic wave patterns and become one of the causes of the initiation of air entrainment. In fact, we can easily observe these phenomena in steep chutes. And, then, the flow condition at which the instability of free surface is initiated have been considered to be the initial condition of formation of roll-waves by many investigators.

On the other hand, H. Thomas[7], R. F. Dressler[8] and the author with his colleagues[9],[10] obtained the flow condition necessary for roll-waves to maintain their final wave patterns, considering their hydraulic characteristics, and it is well interesting, however two dimensional or three dimensional the analysis may be, that it is reduced to the same one for initial instability of flows.

The method for investigating the instability of steady uniform flows in open channels will be divided into two procedures of approarch. One is the energy method in which the time growth or decay of energy of disturbed waves in steady uniform flows would be analyzed, and the other is to consider the initiation of continuous time growth of an infinitesimal disturbed wave on fluid surface, as Jeffreys first did this in 1925 for the Chézy's type of resistance. In this paper, the author's present purpose is to reveal a general criterion for initial instability of steady uniform flows, with any shape of section and any law of resistance, following the latter method.

In this study dealing with the oscillation of fluid surface, the certain approximation basing upon some reasonable assumptions and expressing with the mean velocity of flows and the depth of fluid, so-called the fundamental equation of fluid motion in open channels, is more convenient in mathematical and physical treatment of analysis than the original equation of Navier-Stokes. Usually it will be sufficient to assume that the vertical acceleration of flows in open channels will be neglected and the pressure will be given by the hydro-static law. Especially, this assumption will be satisfactory for the present infinitesimal disturbed motion.

Though Jeffreys' fundamental equation and the basic equation in shallow water wave theory adopted by Dressler and F. V. Pohle[4] are also assumed to be given by the distribution of hydro-static pressure, the hydraulic treatments with the correction coefficient due to the vertical distribution of velocity and the shearing stress on channel bed are not considered to be sufficient in this study. Keulegan and Patterson obtained the fundamental equation of turbulent flows, assuming the hydro-static distribution of pressure and neglecting all small terms, in skillful treatment. But its derivation is rather complicated. The method obtaining the momentum equation by considering the change of momentum through two sections is very simple and reasonable in treatment with every hydraulic quantities, and the results by this approach is reduced to that of Keulegan and Patterson.

Then the author deals with the momentum equation, derived by Assist, Prof. Iwagaki, as the fundamental equation of open channel flows. And the shearing stress on channel bed is assumed to be expressed by a generalized empirical formula.

Following the above described conception, the basic linear approximation of translation wave, with an infinitesimal amplitude and neglecting the curvature of fluid surface, is derived from fundamental equations of momentum and continuity of fluid. It is the basis of this study for initial instability of steady uniform flows in open channels. The analysis shows that the criterion by this approach is identical with the expression of Vedernikov's criterion for free surface instability and is the same as the flow condition necessary for roll-waves to maintain their final wave patterns. It is also interesting to note these results with respect to fluid resistance.

2. Instability of Steady Uniform Flows

(A) *Basic equation.* On the basis of the prescribed conception, the fundamental equations of fluid motion in open channels are as follows.

$$\frac{\partial u}{\partial t}+\alpha u\frac{\partial u}{\partial x}+g\cos\theta\frac{\partial h}{\partial x}+(1-\alpha)\frac{u}{A}\frac{\partial A}{\partial t}=g\sin\theta-\frac{\tau}{\rho R}, \tag{1}$$

$$\frac{\partial A}{\partial t}+u\frac{\partial A}{\partial x}+A\frac{\partial u}{\partial x}=0, \tag{2}$$

where the x-axis is taken in the downstream direction along the channel bed and the z-axis vertically upward, t: the time, u: the mean velocity, h: the depth of flow,

A: the sectional area, R: the hydraulic radius, g: the acceleration of gravity, θ: the slope angle of channel bed, τ: the shearing stress on bed, ρ: the density of fluid, and α: the correction coefficient due to the vertical distribution of velocity and assumed to be independent of x and t. Both of the sectional area and the hydraulic radius are assumed to be functions of the depth of flow. Hence it will be noticed that the sectional area becomes a simply increasing function of the depth.

Denoting values of steady uniform flows with suffix o, the shearing stress becomes

$$\tau_0 = \rho g R_0 \sin\theta. \tag{3}$$

In laminar streams, the following expression of the shearing stress applicable for the flow in wide open channels is assumed to be generally used, approximately.

$$\tau_0 = \frac{3\mu u_0}{R_0}, \tag{4}$$

where μ is the viscosity of fluid. In turbulent flows, the mean velocity is usually expressed by a suitable empirical law of resistance such as the Chézy, the Manning or the Forchheimer formula. Though each law has its own most suitable range of applicability, the author assumes the modified Vedernikov's law of resistance as the most suitable generalized formula of the mean velocity through this region. That is

$$u_0^a = \frac{1}{k} R_0^{(1+b)} \sin^m\theta, \tag{5}$$

where k is the roughness of bed and is generally assumed to be a constant having some dimensions and independent of other hydraulic quantities. From Eqs. (3) and (5), as the expression of the shearing stress in turbulent flows, the shearing stress becomes

$$\tau_0 = \rho g k^{\frac{1}{m}} u_0^{\frac{a}{m}} R_0^{1-\frac{1+b}{m}}. \tag{6}$$

Now, it is assumed that the expression of the shearing stress in the present unsteady flows is given by the same representation in steady uniform flows except for elimination of suffix o. Then the shearing stress is a function of the mean velocity and the hydraulic radius. Especially, in the representation of Chézy's type, the shearing stress is

$$\frac{\tau}{\rho R} = \frac{g u^2}{C^2 R}, \quad C = \left(\frac{1}{k}\right)^{\frac{1}{a}} R^{\frac{2(1+b)-a}{2a}} \sin^{\frac{2m-a}{2a}} \theta, \tag{7}$$

and from Eq. (7), it is clear that the Chézy's coefficient is not a constant but a function of the depth.

Considering small deviations of the velocity u' and of the depth h', in the disturbed motion, the velocity and the depth are, respectively,

$$u = u_0 + u', \quad h = h_0 + h',$$

and, then, the sectional area, the hydraulic radius, the shearing stress and the Chézy's coefficient deviate, respectively.

Substituting these quantities into the fundamental equations (1) and (2), differentiating Eq. (1) with respect to x and Eq. (2) to t, and linearizing yields the following equation for h', after eliminating u'.

$$\left[g\cos\theta - \frac{\alpha u_0^2}{A_0}\left(\frac{dA}{dh}\right)_0\right]\frac{\partial^2 h'}{\partial x^2} - \frac{2\alpha u_0}{A_0}\left(\frac{dA}{dh}\right)_0\frac{\partial^2 h'}{\partial x \partial t} - \frac{1}{A_0}\left(\frac{dA}{dh}\right)_0\frac{\partial^2 h'}{\partial t^2}$$
$$-\left[\frac{u_0}{\rho A_0 R_0}\left(\frac{dA}{dh}\right)_0\left(\frac{\partial\tau}{\partial u}\right)_0 - \frac{1}{\rho R_0}\left(\frac{\partial\tau}{\partial h}\right)_0 + \frac{\tau_0}{\rho R_0^2}\left(\frac{dR}{dh}\right)_0\right]\frac{\partial h'}{\partial x} - \frac{1}{\rho A_0 R_0}\left(\frac{dA}{dh}\right)_0\left(\frac{\partial\tau}{\partial u}\right)_0\frac{\partial h'}{\partial t} = 0, \quad (8)$$

where all terms depending upon squares and products of the deviation from the steady uniform state have been neglected. In the above approximation, parentheses with suffix o are reduced to differential derivatives of hydraulic quantities in steady uniform flows with respect to velocity or depth in the same flows. Those are

$$\left(\frac{\partial\tau}{\partial u}\right)_0 = \frac{\partial\tau_0}{\partial u_0}, \quad \left(\frac{\partial\tau}{\partial h}\right)_0 = \frac{\partial\tau_0}{\partial h_0}, \quad \left(\frac{dA}{dh}\right)_0 = \frac{dA_0}{dh_0}, \quad \left(\frac{dR}{dh}\right)_0 = \frac{dR_0}{dh_0}.$$

Eq. (8) is the second order linear equation of translation wave, with small amplitude and neglecting the curvature of fluid surface, and the basic equation to investigate the flow condition for initial instability of steady uniform flows in the present purpose.

Considering that the sectional area is a simply increasing function of the depth and the correction coefficient is equal to or greater than unity, the following inequality is obtained from Eq. (8).

$$(\alpha^2 - \alpha)\frac{u_0^2}{A_0^2}\left(\frac{dA}{dh}\right)_0^2 + \frac{g\cos\theta}{A_0}\left(\frac{dA}{dh}\right)_0 > 0 .$$

Hence, Eq. (8) is the linear partial differential equation of hyperbolic type, then the absolute velocity v_w of disturbed wave is derived, from its characteristics, as follows.

$$v_w = \left[\alpha \pm \sqrt{\alpha^2 - \alpha + \left(\frac{S_0}{F^2}\right) \Big/ \left(\frac{dA}{dh}\right)_0}\,\right] u_0 , \quad (9)$$

where F is the Froude number, expressed by $u_0/\sqrt{gR_0\cos\theta}$, and S_0 is the wetted perimeter in steady uniform flows. The upper positive sign in the above equation shows the velocity of descending wave and the lower negative sign that of the ascending wave. From Eq. (9), the ratio of the mean velocity of flows to the wave celerity on still fluid becomes

$$\frac{u_0}{v_w-u_0}=1\Big/\left[\alpha-1\pm\sqrt{\alpha^2-\alpha+\left(\frac{S_0}{F^2}\right)\Big/\left(\frac{dA}{dh}\right)_0}\,\right]. \tag{10}$$

Special expressions of Eqs. (9) and (10) in two dimensional flows, $\left(\frac{dA}{dh}\right)_0 \rightarrow S_0$ and substituting α into unity, are reduced to well-known relations in classical theories.

(B) *Flow condition for initiation of instability.* In this section, the author studies the time growth or decay of an infinitesimal disturbed wave expressed by the type

$$h'=Ae^{rt}\binom{\cos}{\sin}\left[\beta\left(x+\frac{s}{\beta}t\right)\right].$$

The above expression is rewritten into the following form.

$$h'=Ae^{\gamma t+i\beta x}, \tag{11}$$

where $\gamma=r+is$ and the real part of γ indicates the variation of amplitude of a disturbed wave, with respect to the time. Hence the criterion for stability or instability of flows depends upon the sign of the real part of γ. That is,

$$r=\Re(\gamma)\gtreqless 0 \qquad \begin{array}{l}\cdots\cdots\cdots\cdots \text{Unstable}\\ \left.\begin{array}{l}\cdots\cdots\cdots\cdots\\ \cdots\cdots\cdots\cdots\end{array}\right\}\text{Stable}\end{array}, \tag{12}$$

and the relation $\Re(\gamma)=0$ represents the limit between two parts of the flow condition. Then the insertion of Eq. (11) into the basic equation (8) yields the following relation between γ and β.

$$\frac{1}{A_0}\left(\frac{dA}{dh}\right)_0\gamma^2+2\left[\frac{1}{2\rho A_0R_0}\left(\frac{dA}{dh}\right)_0\left(\frac{\partial\tau}{\partial u}\right)_0+i\beta\frac{\alpha u_0}{A_0}\left(\frac{dA}{dh}\right)_0\right]\gamma+\beta^2\left[g\cos\theta-\frac{\alpha u_0^2}{A_0}\left(\frac{dA}{dh}\right)_0\right]$$
$$+i\beta\left[\frac{u_0}{\rho A_0R_0}\left(\frac{dA}{dh}\right)_0\left(\frac{\partial\tau}{\partial u}\right)_0-\frac{1}{\rho R_0}\left(\frac{\partial\tau}{\partial h}\right)_0+\frac{\tau_0}{\rho R_0^2}\left(\frac{dR}{dh}\right)_0\right]=0.$$

Solving for γ becomes

$$\gamma=-\left[\frac{1}{2\rho A_0R_0}\left(\frac{\partial\tau}{\partial u}\right)_0+i\beta\alpha u_0\right]\pm\sqrt{\left[\frac{1}{2\rho A_0R_0}\left(\frac{\partial\tau}{\partial u}\right)_0+i\beta\alpha u_0\right]^2-\left[A_0\Big/\left(\frac{dA}{dh}\right)_0\right]\times}$$
$$\overline{\left\{\beta^2\left[g\cos\theta-\frac{\alpha u_0^2}{A_0}\left(\frac{dA}{dh}\right)_0\right]+i\beta\left[\frac{u_0}{\rho A_0R_0}\left(\frac{dA}{dh}\right)_0\left(\frac{\partial\tau}{\partial u}\right)_0-\frac{1}{\rho R_0}\left(\frac{\partial\tau}{\partial h}\right)_0+\frac{\tau_0}{\rho R_0^2}\left(\frac{dR}{dh}\right)_0\right]\right\}}. \tag{13}$$

Applying Eq. (13) to Eq. (12), and using Eq. (10), following relations are obtained.

$$-\frac{A_0}{\left(\frac{\partial\tau}{\partial u}\right)_0}\left[\left(\frac{\partial\tau}{\partial A}\right)_0-\frac{\tau_0}{R_0}\left(\frac{dR}{dA}\right)_0\right]\lesseqgtr v_w-u_0, \tag{14}$$

or

$$-\frac{A_0}{\left(\frac{\partial\tau}{\partial u}\right)_0}\left[\left(\frac{\partial\tau}{\partial A}\right)_0-\frac{\tau_0}{R_0}\left(\frac{dR}{dA}\right)_0\right]\gtreqless\left[\alpha-1\pm\sqrt{\alpha^2-\alpha+\left(\frac{S_0}{F^2}\right)\Big/\left(\frac{dA}{dh}\right)_0}\,\right]u_0. \tag{15}$$

Eq. (14) is the expression of the criterion for initiation of instability of open channel flows in terms of the shearing stress and the wave velocity, and Eq. (15) is that for the Froude number instead of the wave velocity. But it would be hydraulically better that these expressions are represented with channel characteristics.

In the laminar region in which fluid flows in a gradually varied section, using $\alpha = 1.2$ and Eq. (4), the instability criterion becomes

$$\frac{2\,Mu_0}{v_w - u_0} \gtreqless 1 \quad , \qquad M = 1 - R_0\left(\frac{dS}{dA}\right)_0 , \tag{16}$$

or, expressing with the Froude number,

$$F^2 \gtreqless \frac{S_0}{\left(\frac{dA}{dh}\right)_0} \cdot \frac{1}{4\,M^2 - 0.8\,M - 0.2} , \tag{17}$$

and it can be reduced to the result obtained by Prof. Ishihara, Assist. Prof. Iwagaki and Y. Ishihara[11] of the author's laboratory in two dimensional flows.

In turbulent flows, using Eq. (7), Eqs. (14) and (15) become

$$\frac{M(1+b)u_0}{a(v_w - u_0)} \gtreqless 1 , \tag{18}$$

and

$$\frac{M(1+b)}{a} \gtreqless \alpha - 1 \pm \sqrt{\alpha^2 - \alpha + \left(\frac{S_0}{F^2}\right) \Big/ \left(\frac{dA}{dh}\right)_0} . \tag{19}$$

Eqs. (18) and (19) imply instability for the top sign and stability for the lower two signs. These equations are criteria for initial instability of steady uniform flows in open channels and applicable for any section and any law of resistance. Especially Eq. (18) represents the Vedernikov number derived by V. V. Vedernikov in 1945 and Eq. (19) is the expression with the Froude number and the correction coefficient.

More resently, the author with his colleagues[9),10)] obtained the necessary condition for roll-waves in steep channels to maintain their final patterns of waves, based upon Thomas' and Dressler's concept on roll-waves, by argumenting their hydraulic characteristics. The result is a generalized one applicable for any flow condition and can be reduced to the criterion for initial instability derived in this paper. It would be very interesting to be noticed that the instability is the cause on formation of roll-waves.

To show the validity of the above derived mathematical result of the initial instability criterion by considering an infinitesimal disturbed motion on free surface of fluid, there is a need for an orderly experimental research of the general character of the initiation of instability and the resulting flow. But the technique of observation on this study is very difficult and it will be impossible to determine clearly the condition of initial instability in open channel flows. An experimental study was performed by M. S. Priest and A. Baligh[12] of Cornell University in 1954 to determine the flow condition at the point from where the instability is recognized, though it is, not applicable to the problem of initial instability treating in this paper. The range of the flow conditions observed by them is in laminar and transition flows, and the experimental procedure is still primitive.

3. Factors Affecting the Instability Criterion

Relations of the criterion for initial instability in open channel flows are Eq. (16) in laminar flows and the following expression in turbulrnt flows, expressed by the Froude number,

$$F^2 \gtreqless \frac{S_0}{\left(\frac{dA}{dh}\right)_0} \cdot \frac{1}{M^2\left(\frac{1+b}{a}\right)^2 - 2(\alpha-1)M\left(\frac{1+b}{a}\right) - (\alpha-1)} . \tag{20}$$

Hence, if the determination whether a flow may be stable or unstable is expressed by the Froude number, the criterion will be affected by three parameters of the shape of section, the law of resistance and the correction coefficient due to the vertical distribution of velocity, because of Eq. (20).

In laminar flows the criterion in gradually varied channels becomes a function of one parameter of section, because the law of resistance and the correction coefficient have been derived theoretically. In this article the author may consider effects of these hydraulic factors in turbulent flows. Of course, there is the exception of the following case in this discussion. Because the Froude number reduces to zero with results being always in the stable region, from Eq. (20).

$$M\left(\frac{1+b}{a}\right) = \alpha - 1 \pm \sqrt{\alpha^2 - \alpha} .$$

(A) *Law of resistance.* Expressions of the initial instability criteria for the Manning and the Chézy formulas are respectively,

$$\text{Chézy:} \quad F^2 \gtreqless \frac{S_0}{\left(\frac{dA}{dh}\right)_0} \cdot \frac{4}{M^2 - 4(\alpha-1)M - 4(\alpha-1)} .$$

$$\text{Manning:} \quad F^2 \gtreqless \frac{S_0}{\left(\frac{dA}{dh}\right)_0} \cdot \frac{9}{4M^2 - 12(\alpha-1)M - 9(\alpha-1)} .$$

Substitution of certain definite numerical values of the shape of section and the correction coefficient into the above equations shows the results obtained Jeffreys[1], Keulegan and Patterson[2], and Dressler and Pohle[4].

Now the author studies the behaviour of the criterion for special law of resistance. In the case of $b = -1$ and $a \neq 0$, the mean velocity is independent of the hydraulic radius and the shearing stress on channel bed is proportional to the hydraulic radius. It is reported[13] that this type of resistance will be observed in the transition region for some slopes in open channel flows. In this case, it becomes easily from Eq. (20)

$$F \to \infty, \quad \text{for } \alpha = 1,$$

and for $\alpha > 1$, there is no real root of the Froude number. Hence, the author concludes that any flow governed by this type of resistance varying with any power of the mean velocity must always remain stable. The criterion for initial instability is independent of the slope of channel bed, so any value of power of the slope angle has no effect on the instability.

(B) *Shapes of section.* Although this factor has not any effect in two dimensional flows, it becomes an important one to affect the initial instability in three dimensional flows. In this study, for the sake of simplicity, steady uniform flows in trapezoidal channels may be considered, as shown in Figure 1.

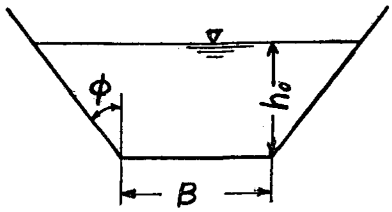

Fig. 1

There are the following relations from its geometrical figure.

$$A_0 = Bh_0 + h_0^2 \tan\phi,$$
$$S_0 = B + 2h_0 \sec\phi,$$
$$R_0 = A_0/S_0$$
$$M = \frac{B^2\cos\phi + 2h_0 B\sin\phi + 2h_0^2\tan\phi}{(B\cos\phi + 2h_0)(B + 2h_0\tan\phi)} \cdot$$

Then, the final initial instability condition becomes

$$F^2 \gtreqless \frac{\cos\phi + \frac{2h_0}{B}}{1 + \frac{2h_0}{B}\tan\phi} \cdot \frac{1}{M^2\left(\frac{1+b}{a}\right)^2 - 2(\alpha-1)M\left(\frac{1+b}{a}\right) - (\alpha-1)} \cdot \tag{23}$$

As special cases, in rectangular channels, $(\phi \to 0)$,

$$F^2 \gtreqless \frac{1}{M_r} \cdot \frac{1}{M_r^2\left(\frac{1+b}{a}\right)^2 - 2(\alpha-1)M_r\left(\frac{1+b}{a}\right) - (\alpha-1)}, \tag{24}$$

$$M_r = 1\Big/\left(1 + \frac{2h_0}{B}\right),$$

and in triangular channels, $(B \to 0)$,

$$F^2 \gtreqless \frac{1}{\sin\phi} \cdot \frac{1}{\left(\frac{1+b}{a}\right)^2 - 4(\alpha-1)\left(\frac{1+b}{a}\right) - 4(\alpha-1)} \cdot \tag{25}$$

(C) *Correction coefficient due to the vertical distribution of velocity.* It has been considered to be unity in classical and shallow water wave theories. But it is clear that its value is not unity but a definite numerical one nearly equal to unity in given flow condition. For examples, in laminar streams which flow in wide channels it is 1.2, in flows of Couette type 1.333 and in turbulent flows, if flows are governed by the logarithmic law of velocity distribution, it becomes

$$\alpha = 1 + 6.25 \frac{\tau_0}{\rho u_0^2}.$$

Usually it is considered to be nearly equal to 1.05. This is the reason in which the correction coefficient is introduced into the flow condition for initial instability by the author. Now some behaviours of the criterion for initial instability of steady uniform flows in open channels, basing upon some discussion of factors affecting the condition, may be considered.

Figure 2 shows the behaviour of formation of initial instability with the Froude number for all values of $(1+b)/a$, the coefficient due to the law of resistance, for $\alpha = 1.05$, and for some ratios of normal depth to the width in a rectangular section. And Figure 3 shows the behaviour of that in triangular channels for the angle of side slope instead of the last parameter in Figure 2.

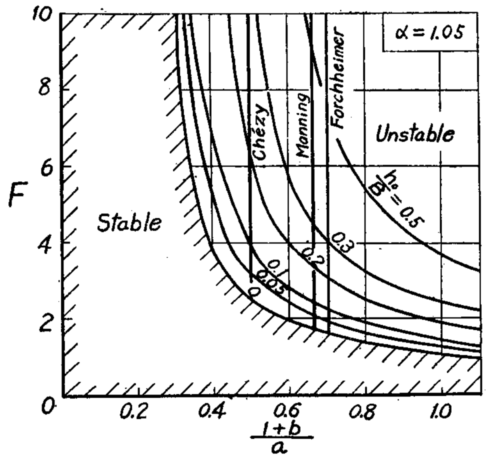

Fig. 2 Relation between F and $(1+b)/a$ in rectangular channels for the formation of initial instability

In rectangular channels, it is observed easily that in order to form the instability, the flows must be governed by the law of resistance with higher power. In triangular channels, almost all flows are considered to be stable and especially the flow governed by the Chézy's type of resistance remains always stable.

Figures 4 and 5 show relations between the Froude number and the shape of section in rectangular and triangular channels, for $\alpha = 1.05$, and for any set of law of resistance. It is observed, also, that the narrower the section is, the more stable the flow is. And Figure 6 shows the behaviour of α to the Froude number in two dimensional rectangular channels for all admissible values of $(1+b)/a$ with exception of their parts being independent of instability. α shows the restrict action for the formation of initial instability.

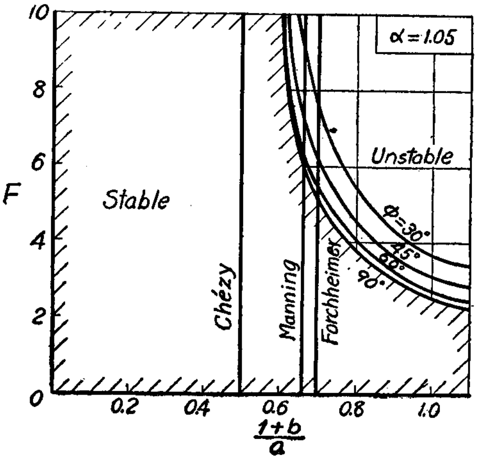

Fig. 3 Relation between F and $(1+b)/a$ in triangular channels for the formation of initial instability

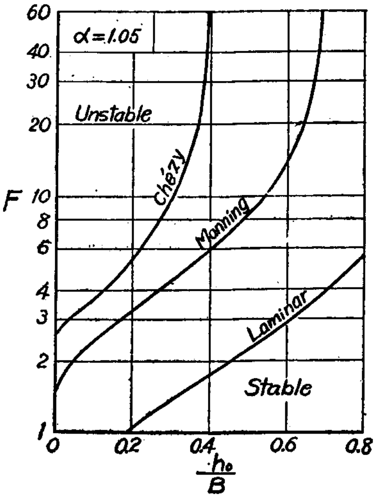

Fig. 4 Relation between F and h_0/B in rectangular channels

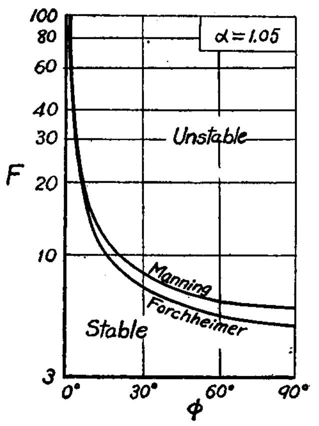

Fig. 5 Relation between F and ϕ in triangular channels

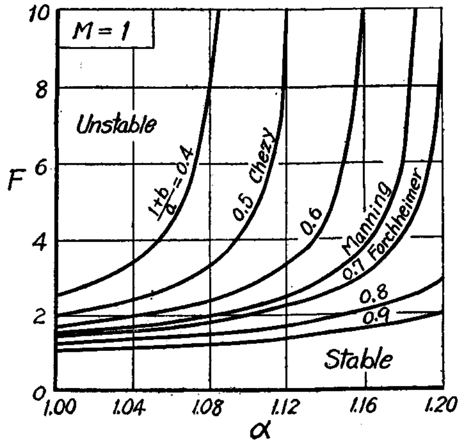

Fig. 6 Relation between F and α in two dimensional rectangular channels

In any case, there are four parameters family for the criterion of initial instability, so if three parameters of factors affecting it are assumed, the solution will be derived uniquely as a function of the last parameter.

4. Conclsion

The criterion for initial instability of steady uniform flows in open channels is derived by argumenting an infinitesimal disturbed wave superposed on original flow. The mathematical analysis shows that this criterion is expressed as the Vedernikov's criterion in any flow condition. And its hydraulic summary expressed with channel characteristics is as follows.

(1) From only hydraulic standpoint, to prevent the formation of initial instability, it is better that flows will be governed by the law of resistance with less power.

(2) Two dimensional flows have a best chance for the formation of initial instability. The narrower the rectangular section is, the more difficult the formation will become. And triangular channel flows have less chance for the formation compared with rectangular channel flows.

(3) The correction coefficient has an ability contributing to the restriction for the formation of instability.

It is the author's hope that further experimental researches on this problem will verify the above derived analytical result.

Acknowledgments

The author wishes to express his grateful appreciation to Professor T. Ishihara and Assistant Professor Y. Iwagaki of his laboratory for their considerable instruction in preparing this paper.

References

1) H. Jeffreys: The flow of water in an inclined channel of rectangular section, Phil. Mag., Series 6, Vol. 49, 1925.
2) G. H. Keulegan and G. W. Patterson: A criterion for instability of flow in steep channels, Trans. A. G. U., Part II, 1940.
3) V. V. Vedernikov: Conditions at the front of a translation waves disturbing a steady motion of a real fluid, (Comptes Rendus) Doklady Akademiya Nauk SSSR, Vol. 48, 1945.
4) R. F. Dressler and F. V. Pohle: Resistance effects on hydraulic instability, Comm. on Pure and Applied Math., Vol. 6, No. 1, 1953.
5) A. Craya: The criterion for the possibility of roll wave formation, Proc. of the Gravity Wave Symposium, National Bureau of Standards, 1951.
6) F. F. Escoffier: A graphical method for investigating the stability of flow in open channels or in closed conduits flowing partly full, Trans. A. G. U., 1950.
7) H. Thomas: The propagation of waves in steep prismatic conduits, Proc. of Hydraulic Conference, Univ. of Iowa Studies in Eng., Bull. 20, 1940.
8) R. F. Dressler: Mathematical solution of the problem of roll waves in inclined open channels, Comm. on Pure and Applied Math., Vol. 2, No. 2-3, 1949.
9) T. Ishihara, Y. Iwagaki and Y. Iwasa: Theory of the roll-wave trains in laminar water flow on a steep slope surface, Trans. J. S. C. E., Vol. 19, 1954 (in Japanese).
10) Y. Iwagaki and Y. Iwasa: Hydraulic characteristics of roll-wave trains, Proc. J. S. C. E., 1954 (in printing).
11) T. Ishihara, Y. Iwagaki and Y. Ishihara: On the roll-wave trains appearing in the water flow on s steep slope surface, Memoirs of the Faculty of Engineering, Kyoto University, Mar., 1952.
12) M. S. Priest and A. Baligh: Free surface instability of liquids in steep channels, Trans. A. G. U., Feb., 1954.
13) T. Ishihara, Y. Iwagaki and T. Goda: Studies on the thin sheet flow, 1st Report, Trans. J. S. C. E., No. 6, 1951 (in Japanese).