# Transient Voltages across Series Capacitors on Three-Phase Transmission Line due to Non-Simultaneous Switching 

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## Synopsis

Series capacitors have been utilized recently in order to increase the capacity of transmission systems by compensating the detrimental effects of inductance. In such cases, these capacitors have to be switched in or off according to the variation in load.

This is necessarilly accompanied by power disturbances, abnormal voltage rises etc.. In order to insert the capacitors, the circuit breakers that are installed in parallel with them are opened, in which case it is scarcely possible to open all three poles of the breakers simultaneously.

The present paper deals analytically the abnormal transient voltages across the capacitors arising due to non-simultaneous tripping of the poles, and the validity of the analytical results are affirmed by comparing with many experiments recently executed.

## 1. Non-grounded System

Let us first consider a simple case as shown in Fig. 1, where the impedance functions of the source, the transmission line and the load, viz. $z_{0}(p), z_{1}(p)$ and $z_{2}(p)$ are all balanced, and the series capacitors $C$ 's are connected at the receiving end.

Let us choose the origin for the time scale the instant at which the pole of the breaker $s_{a}$ is opened.

Let the operational functions of the steady phase-currents that


Fig. 1. Non-grounded system. would flow if the poles $s_{a}, s_{b}$ and $s_{c}$ were closed, be denoted by the symbols $I_{a 0}^{0}(p)$, $I_{b 0}^{0}(p)$ and $I_{g 0}^{0}(p)$ respectively; i.e.
and

$$
\left.\begin{array}{c}
I_{a 0}^{0}(p)=\mathscr{S}^{-1} J_{a 0}^{0} \sin \left(\omega t+\theta_{a 0}^{0}\right) H(t) \\
I_{b 0}^{0}(p)=\mathscr{5}^{-1} J_{b 0}^{0} \sin \left(\omega t+\theta_{b 0}^{0}\right) H(t)  \tag{1.2}\\
I_{c 0}^{0}(p)=\mathfrak{S}^{-1} J_{c 0}^{0} \sin \left(\omega t+\theta_{c 0}^{0}\right) H(t)
\end{array}\right\}
$$

and the symbols $\mathscr{S}^{-1}$ and $\mathscr{F}$ denote
or

$$
\left.\begin{array}{l}
\oiint^{-1} f(t)=p \int_{0}^{\infty} f(t) \varepsilon^{-p t} d t  \tag{1.3}\\
\wp F(p)=\frac{1}{2 \pi j} \int_{c-j \infty}^{c+j \infty} \frac{F(p)}{p} \varepsilon^{p t} d p, c>0
\end{array}\right\}
$$

As already described, let the instant of the tripping of $s_{a}$ be $t=0$, that of $s_{b}, t_{1}$, and that of $s_{c}, t_{1}+t_{2}$ respectively. At first, when the pole $s_{a}$ is broken, the resulting effects can be calculated by superposing the phenomena that would prevail were the pole not to be opened, on those that result by forcing the current $i_{a_{0}}^{0}(t)=\mathfrak{g} I_{a 0}^{0}(p)$ $=J_{a 0}^{0} \sin \left(\omega t+\theta_{a 0}^{0}\right) H(t)$ through $s_{a}$ in the opposite direction.

The operational function for the voltage across the capacitor of phase- $a$ that will be generated by suddenly forcing $i_{a 0}^{0}(t)$ in the opposite direction through $s_{a}$ will be
where

$$
\begin{equation*}
V_{a_{0}}(p)=\frac{3 z(p)}{3 z(p) C p+2} I_{a 0}^{0}(p), \tag{1.4}
\end{equation*}
$$

and the operational functions for the disturbance currents in the phases will be
and

$$
\left.\begin{array}{l}
I_{a 0}^{\prime}(p)=\frac{2}{3 z(p) C p+2} I_{a 0}^{0}(p)  \tag{1.6}\\
I_{b 0}^{\prime}(p)=I_{c 0}^{\prime}(p)=\frac{1}{3 z(p) C p+2} I_{a 0}^{0}(p)
\end{array}\right\}
$$

Hence the operational values for the transient currents on the three phases after breaking the pole $s_{a}$ are
and

$$
\left.\begin{array}{l}
I_{a 0}(p)=I_{00}^{0}(p)-I_{a 0}^{\prime}(p),  \tag{1.7}\\
I_{b 0}(p)=I_{b 0}^{0}(p)+I_{b 0}^{\prime}(p) \\
I_{c 0}(p)=I_{c 0}^{0}(p)+I_{c 0}^{\prime}(p) .
\end{array}\right\}
$$

Next, assume that the pole $s_{b}$ is broken at the time $t_{1}$, and that $I_{b t_{1}}^{0}(p)$ denotes the operational function such as

$$
\begin{equation*}
I_{b t_{1}}^{0}(p)=\mathscr{S}^{-1}\left\{H\left(t-t_{1}\right) \mathscr{g} I_{b_{0}}(p)\right\} \tag{1.8}
\end{equation*}
$$

where it must be noticed that the time function $\delta_{8} I_{b 0}(p)$ starts at $t=0$.

In this case also, the resulting phenomena are obtained by similar procedure as in the previous case, where $s_{a}$ is opened, i.e. by forcing the current $i_{b t_{1}}^{0}(t)=\oint_{b} I_{b t_{1}}^{n}(p)$ through $s_{b}$ in the opposite direction.

The operational functions for the voltages across the capacitors in $b$ - and $a$-phases that will be generated by forcing in $i_{b t}{ }_{1}(t)$ as above described will be
and

$$
\left.\begin{array}{rl}
V_{b t_{1}}(p) & =\frac{z(p)\{3 z(p) C p+2\}}{\{z(p) C p+1\}\{3 z(p) C p+1\}} I_{b t_{1}}^{0}(p)  \tag{1.9}\\
V_{a t_{1}}(p) & =\frac{z(p)}{\{z(p) C p+1\}\{3 z(p) C p+1\}} I_{b t_{1}}^{0}(p)
\end{array}\right\}
$$

and the disturbance currents (operational functions) in the phases will be
and

$$
\begin{align*}
I_{a t_{1}}^{\prime}(p) & =\frac{z(p) C p}{\{z(p) C p+1\}\{3 z(p) C p+1\}} I_{b t_{1}}^{0}(p) \\
I_{b t_{1}}^{\prime}(p) & =\frac{2 z(p) C p+1}{\{z(p) C p+1\}\{3 z(p) C p+1\}} I_{b t_{1}}^{o}(p)  \tag{1.10}\\
I_{c t_{1}}^{\prime}(p) & =\frac{1}{3 z(p) C p+1} I_{b t_{1}}^{0}(p)
\end{align*}
$$

Hence the transient currents (operational functions) on the three phases after breaking the poles $s_{a}$ and $s_{b}$ are
and

$$
\left.\begin{array}{l}
I_{a t_{1}}(p)=I_{a 0}(p)+I_{a t_{1}}^{\prime}(p)  \tag{1.11}\\
I_{b t_{1}}(p)=I_{b 0}(p)-I_{b t_{1}}^{\prime}(p) \\
I_{c t_{1}}(p)=I_{c 0}(p)+I_{c t_{1}}^{\prime}(p) .
\end{array}\right\}
$$

Finally, the pole $s_{c}$ is assumed to be opened $t_{2}$ seconds after, i.e. at the instant $t=t_{1}+t_{2}$.
And furthermore assume $I_{c t_{2}}^{0}(p)$ to denote the operational function such as

$$
\begin{equation*}
\left.I_{c t_{2}}^{0}(p)=\mathscr{夕}^{-1}\left\{H\left(t-\overline{t_{1}+t_{2}}\right)\right)_{c} I_{c t_{1}}(p)\right\}, \tag{1.12}
\end{equation*}
$$

where it must be noticed that the time function $\mathscr{S}_{5} I_{c t_{2}}^{0}(p)$ starts actually at $t=t_{1}+t_{2}$.
As discussed previously, the voltages across the capacitors that will be generated by forcing $i_{c t_{2}}^{0}(t)=\mathfrak{F} I_{c t_{2}}^{0}(\boldsymbol{p})$ through $s_{c}$ in the opposite direction will be
and

$$
\left.\begin{array}{l}
V_{c t_{2}}(p)=\left\{\frac{1}{3 C p}+\frac{2}{3} \frac{z(p)}{z(p) C p+1}\right\} I_{c t_{2}}^{0}(p)  \tag{1.13}\\
V_{a t_{2}}(p)=V_{b t_{2}}(p)=\left\{\frac{1}{3 C p}-\frac{1}{3} \frac{z(p)}{z(p) C p+1}\right\} I_{c t_{2}}^{0}(p)
\end{array}\right\}
$$

and the disturbance current-operators will be
and

$$
\left.\begin{array}{rl}
I_{a t_{2}}^{\prime}(p)=I_{b t_{2}}^{\prime}(p) & =\frac{1}{3} \frac{1}{z(p) C p+1} I_{c t_{2}}^{0}(p)  \tag{1.14}\\
I_{c t_{2}}^{\prime}(p) & =\frac{2}{3} \frac{1}{z(p) C p+1} I_{c t_{2}}^{0}(p)
\end{array}\right\}
$$

Thus the transient currents (operational functions) in the phases after breaking all the three poles are
and

$$
\left.\begin{array}{l}
I_{a t_{2}}(p)=I_{a t_{1}}(p)+I_{a t t_{2}}^{\prime}(p)  \tag{1.15}\\
I_{b t_{2}}(p)=I_{b t_{1}}(p)+I_{b t_{2}}^{\prime}(p) \\
I_{c t_{2}}(p)=I_{c t_{1}}(p)-I_{c t_{2}}^{\prime}(p)
\end{array}\right\}
$$

The operational functions for the terminal voltages across the capacitors after all the three poles of the circuit breakers are opened will be obtained by superposition of equations (1.4), (1.9) and (1.13), viz.

$$
\left.\begin{array}{lr}
V_{a}(p)=V_{a 0}(p)+V_{a t_{1}}(p)+V_{a t_{2}}(p)  \tag{1.16}\\
V_{b}(p)= & V_{b t_{1}}(p)+V_{b t_{2}}(p) \\
V_{c}(p)= & V_{c t_{2}}(p)
\end{array}\right\}
$$

## 2. System with the Neutral of the Source grounded

Next, let us consider a system with the neutral of the source grounded through a generalized impedance $z_{n}(p)$, which may consist of an inductance and resistance, or directly grounded. The equivalent circuit of such a network is illustrated in Fig. 2,


Fig. 2. System with the neutral of the source grounded. assuming the transmission line to be represented as a $T$-type network having an impedance $z_{1}(p)$ and capacity to ground $C_{1}$. Let the impedances of the source and the load be $z_{0}(p)$ and $\boldsymbol{z}_{2}(p)$ respectively, and the series capacitors $C$ 's. are assumed to be connected at the receiving end.

By applying a similar consideration as for the previous case, viz., for the system with isolated neutral, the following relations are obtained.

$$
\begin{align*}
& V_{a 0}(p)=\frac{3 \beta(p)}{\alpha(p)} I_{c 0}^{0}(p),  \tag{2.1}\\
& I_{b 0}^{\prime}(p)=I_{c 0}^{\prime}(p)=\frac{\gamma(p)}{\alpha(p)} I_{c 0}^{0}(p),  \tag{2.2}\\
& I_{b 0}(p)=I_{b 0}^{0}(p)+I_{b 0}^{\prime}(p),  \tag{2.3}\\
& I_{c 0}(p)=I_{c 0}^{0}(p)+I_{c 0}^{\prime}(p),  \tag{2.4}\\
& I_{b t_{1}}^{0}(p)=\mathfrak{F}^{-1}\left\{H\left(t-t_{1}\right)\left\{I_{b 0}(p)\right\},\right.
\end{align*}
$$

$$
\left.\left.\begin{array}{c}
V_{b t_{1}}(p)=\frac{\alpha(p) \beta(p)}{\delta(p) \varepsilon(p)} I_{b t_{1}}^{0}(p), \\
V_{a t_{1}}(p)=\frac{\beta(p) \gamma(p)}{\partial(p) \varepsilon(p)} I_{b t_{1}}^{0}(p),
\end{array}\right\}, \begin{array}{c}
I_{c t_{1}}^{\prime}(p)=\frac{\gamma(p)}{\varepsilon(p)} I_{b t_{1}}^{0}(p), \\
I_{c t_{1}}(p)=I_{c 0}(p)+I_{c t_{1}}^{\prime}(p), \\
I_{c t_{2}}^{0}(p)=\delta_{\delta}^{-1}\left\{H \left(t-\overline{\left.t_{1}+t_{2}\right)}\left\{I_{c t_{1}}(p)\right\},\right.\right. \\
V_{c t_{2}}(p)=\left\{\frac{1}{3 C p}+\frac{2}{3} \frac{\beta(p)}{\delta(p)}\right\} I_{c t_{2}}^{0}(p)  \tag{2.9}\\
\text { and } \quad V_{a t_{2}}(p)=V_{b t_{2}}(p)=\left\{\frac{1}{3 C p}-\frac{1}{3} \frac{\beta(p)}{\delta(p)}\right\} I_{c t_{2}}^{0}(p),
\end{array}\right\}
$$

where

$$
\begin{align*}
\alpha(p)= & 3\left\{z_{1}(p)+2 z_{2}(p)\right\}\left\{z_{1}(p)+2 z_{0}(p)\right\} C C_{1} p^{2}+12\left\{z_{1}(p)+z_{2}(p)+z_{0}(p)\right\} C p \\
& +4\left\{z_{1}(p)+2 z_{0}(p)\right\} C_{1} p+8, \\
\beta(p)= & \left\{z_{1}(p)+2 z_{2}(p)\right\}\left\{z_{1}(p)+2 z_{0}(p)\right\} C_{1} p+4\left\{z_{1}(p)+z_{2}(p)+z_{0}(p)\right\}, \\
\gamma(p)= & 2\left[\left\{z_{1}(p)+2 z_{0}(p)\right\} C_{1} p+2\right], \\
\delta(p)= & \frac{1}{3}\{\alpha(p)+\gamma(p)\} \\
\text { and } \varepsilon(p)= & \alpha(p)-\gamma(p) . \tag{2.10}
\end{align*}
$$

Thus, the operational functions for the terminal voltages across the capacitors after all the three poles are opened will be obtained from equation (1.16).

The above operational functions can be transformed to their time functions by applying the Laplace transformation (1.3).

## 3. D.C. and Zero-Phase-Sequence Voltages

Let us consider the first term $\frac{1}{3 C p} I_{c t_{2}}^{0}(p)$ in equations (1.13) and (2.9). Assumming $i_{c t_{2}}^{0}(t)=\$ I_{c_{2}}^{0}(p)$ to be of the form

$$
\begin{equation*}
i_{c t_{2}}^{0}(t)=\sum_{n=0}^{n} J_{n} \varepsilon^{-\boldsymbol{a}_{n}\left(t-\overline{t_{1}-\bar{t}_{2}}\right)} \sin \left\{\omega_{n}\left(t-\overline{t_{1}+t_{2}}\right)+\varphi_{n}\right\} H\left(t-\overline{t_{1}+t_{2}}\right), \tag{3.1}
\end{equation*}
$$

where $\alpha_{0}=0, \omega_{0}=$ commercial angular frequency, and applying the Laplace transformation (1.3) to this term, there results

$$
\begin{align*}
\mathscr{\delta} \frac{1}{3 C p} I_{c t_{2}}^{0}(p)= & \frac{1}{3 C} \sum_{n=0}^{n} J_{n} \frac{1}{\sqrt{\alpha_{n}^{2}+\omega_{n}^{2}}}\left[\sin \left(\varphi_{n}+\psi_{n}\right)\right. \\
& \left.-\varepsilon^{-\alpha_{n}\left(t-\overline{t_{1}+t_{2}}\right)} \sin \left\{\omega_{n}\left(t-\overline{t_{1}+t_{2}}\right)+\varphi_{n}+\psi_{n}\right\}\right] H\left(t-\overline{t_{1}+t_{2}}\right), \tag{3.2}
\end{align*}
$$

where

$$
\tan \psi_{n}=\frac{\omega_{n}}{\alpha_{n}}
$$

The first term on the r.h.s. of equation (3.2) i.e. $\frac{1}{3 C} \sum_{n=0}^{n} \frac{J_{n}}{\sqrt{\alpha_{n}^{2}+\omega_{n}^{2}}} \sin \left(\varphi_{n}+\psi_{n}\right)$ $\times H\left(t-\overline{t_{1}+t_{2}}\right)$ gives a d.c. voltage at $t>t_{1}+t_{2}$. This d.c. component originates only after the last capacitor has been switched in.

Let us consider, next, the zero-phase-sequence voltage in order to study still more this d.c. component.

The zero-phase-sequence component (operational function) will be

$$
\begin{equation*}
V_{0}(p)=V_{00}(p)+V_{0 t_{1}}(p)+V_{0 t_{2}}(p) \tag{3.3}
\end{equation*}
$$

where
and

$$
\begin{align*}
V_{00}(p) & =\frac{\beta(p)}{\alpha(p)} I_{00}^{0}(p), \\
V_{0 t_{1}}(p) & =\frac{\beta(p)}{\varepsilon(p)} I_{b t_{1}}^{0}(p)  \tag{3.4}\\
V_{0 t_{2}}(p) & =\frac{1}{3 C p} I_{c t_{2}}^{0}(p)
\end{align*}
$$

and in which we put
and

$$
\left.\begin{array}{rl}
\alpha(p) & =3 z(p) C p+2  \tag{3.5}\\
\beta(p) & =z(p) \\
\varepsilon(p) & =3 z(p) C p+1, \quad[\gamma(p)=1]
\end{array}\right\}
$$

for the isolated system.
It remains to calculate the time functions $v_{00}(t), v_{0 t_{1}}(t)$ and $v_{0 t_{2}}(t)$. From (2.10) and (3.5), there are
and

$$
\left.\begin{array}{l}
\frac{\beta(p)}{\alpha(p)}=\frac{1}{3 C p}\left\{1-2 \frac{\gamma(p)}{\alpha(p)}\right\}  \tag{3.6}\\
\frac{\beta(p)}{\varepsilon(p)}=\frac{1}{3 C p}\left\{1-\frac{\gamma(p)}{\varepsilon(p)}\right\}
\end{array}\right\}
$$

Let
and

$$
\begin{align*}
& 5 \frac{\eta(p)}{\alpha(p)}=A(t)  \tag{3.7}\\
& 5 \frac{\eta(p)}{\varepsilon(p)}=B(t)
\end{align*}
$$

and $A(t)$ and $B(t)$ be of the forms
and

$$
\left.\begin{array}{l}
A(t)=A_{0}+\sum^{n} A_{n} \varepsilon^{p_{n} t}  \tag{3.8}\\
B(t)=B_{0}+\sum^{m} B_{m} \varepsilon^{q_{m} t} .
\end{array}\right\}
$$

Evidently putting $p=0$ in (3.7) we have $A_{0}=\frac{1}{2}$ and $B_{0}=1$, and for $A(0)=0$ and $B(0)=0$ there are
and $\left.\quad \begin{array}{l}\frac{1}{2}+\sum^{n} A_{n}=0 \\ 1+\sum^{m} B_{m}=0 .\end{array}\right\}$
Again, from (3.6) and (3.7), the following relations are obtained.
and

$$
\left.\begin{array}{l}
\delta \frac{\beta(p)}{\alpha(p)}=\frac{2}{3 C} \sum^{n} \frac{A_{n}}{p_{n}}\left(1-\varepsilon^{p_{n} t}\right)  \tag{3.10}\\
\mathfrak{f} \frac{\beta(p)}{\varepsilon(p)}=\frac{1}{3 C} \sum^{m} \frac{B_{m}}{q_{m}}\left(1-\varepsilon^{q_{m} t}\right)
\end{array}\right\}
$$

Let the initial currents $I_{a 0}^{0}(p), I_{80}^{0}(p)$ and $I_{c 0}^{0}(p)$ be of the forms
and

$$
\left.\begin{array}{l}
I_{00}^{0}(p)=夕^{-1} \sin (\omega t+\theta) H(t) \\
I_{00}^{0}(p)=\xi^{-1} \sin \left(\omega t+\theta \pm \frac{2 \pi}{3}\right) H(t)  \tag{3.11}\\
I_{c 0}^{0}(p)=5^{-1} \sin \left(\omega t+\theta \mp \frac{2 \pi}{3}\right) H(t)
\end{array}\right\}
$$

Substituting the first of equation (3.11) in (1.6) or (2.2) and using (3.8), $\oiint I_{b 0}^{\prime}(p)$ and $\xi^{\prime} I_{c 0}^{\prime}(p)$ are obtained. Again, from (3.11) and (1.7) or (2.3), $\mathscr{S}_{b_{0}}(p)$ and $I_{c_{0}}(p)$ can be calculated, and hence $I_{b t_{1}}^{0}(p)$ from (1.8) or (2.4).
Substituting this value of $I_{b t_{1}}^{0}(p)$ in (1.10) or (2.6), and using (3.8), $\xi_{8} I_{c t_{1}}^{\prime}(p)$ is obtained and hence $I_{\text {ct }_{2}}^{0}(p)$ from (1.11) and (1.12) or (2.7) and (2.8). Thus all the quantities of the r.h.s. of (3.4) are known and the time functions of the three voltages can be calculated by using (3.10)
Let $\tilde{v}_{0 t_{2}}(t)$ be equal to $5 V_{0 t_{2}}(p)$ minus the d.c. comporent and using (3.9), there is

$$
\begin{equation*}
v_{00}(t)+v_{0 t_{1}}(t)+\tilde{v}_{0 t_{2}}(t)=0 \quad \text { for } \quad t>t_{1}+t_{2}, \tag{3.12}
\end{equation*}
$$

where $v_{00}(t)=\mathfrak{g} V_{00}(p), v_{0 t_{1}}(t)=\mathfrak{g} V_{0 t_{1}}(p)$ and $v_{0 t_{2}}(t)=\mathfrak{g} V_{0 t_{2}}(p)$.
This shows that after the last capacitor has been switched in, the zero-phasesequence voltage has only a d.c. component and does not have any oscillatory part. So, hence $v_{0 t_{2}}\left(t_{1}+t_{2}+0\right)=0$ from (3.4), the magnitude of the d.c. component is equal to that of the zero-phase-sequence voltage just before the last capacitor is switched in.

Therefore, the d.c. component originates with its magnitude equal to one third the sum of the $a$ - and $b$-phase terminal voltages just before the last capacitor is switched in
(From experiments, it seems that the charge developed by this component discharges through the zero-phase-sequence circuit after some time.)

In the system with the neutral of the source grounded, it is evident from its equivalent circuit that the neutral potential of the source does not float, but that of the load floats with the zero-phase-sequence voltage.

## 4. Simultaneous Switching

When the initial currents are balanced and the three poles of the circuit breakers are opened simultaneously, it is enough for us to consider only one phase.

Let the operational function of the steady phase current that would flow if the pole were closed, be denoted by the symbol $I^{0}(p)$; i.e.

$$
\begin{equation*}
I^{0}(p)=\mathfrak{g}^{-1} J^{0} \sin \left(\omega t+\theta^{0}\right) H(t) \tag{4.1}
\end{equation*}
$$

The operational function for the terminal voltage across the capacitor that will be generated by suddenly forcing $\mathscr{S}^{-1} I^{0}(p)$ in the opposite direction through the circuit breaker will be

$$
\begin{equation*}
V(p)=\frac{z(p)}{z(p) C p+1} I^{0}(p) \tag{4.2}
\end{equation*}
$$

for the non-grounded system, and

$$
\begin{equation*}
V(p)=\frac{\beta(p)}{\delta(p)} I^{0}(p) \tag{4.3}
\end{equation*}
$$

for the system with the grounded source.
In this case, it is generally difficult to find the value of $\theta^{\circ}$, the phase-angle of switching, which gives the highest voltage.

But, when the generated natural oscillations are only of low frequency and even if high frequency oscillations of small amplitude are present, the following method of solution will be useful.
Let

$$
\left.\begin{array}{rl}
G(t) & =\mathscr{\delta} \frac{z(p)}{z(p) C p+1},  \tag{4.4}\\
& =\mathscr{\delta} \frac{\beta(p)}{\delta(p)} \quad, \quad \text { for the the grounded system. }
\end{array}\right\}
$$

Applying Duhamel's theorem for (4.2) or (4.3), there results

$$
\begin{align*}
v(t) & =\mathfrak{F} V(p) \\
& =J^{0}\left\{G(0) \sin \left(\omega t+\theta^{0}\right)+\int_{0}^{t} G^{\prime}(\tau) \sin \left(\omega \overline{t-\tau}+\theta^{0}\right) d \tau\right\} \tag{4.5}
\end{align*}
$$

At the maximum and minimum values of $v, \frac{\partial v}{\partial t}=0$ and $\frac{\partial v}{\partial \theta}=0$. Then, from (4.5), one of the necessary conditions satisfying the above equations will be

$$
\begin{equation*}
G^{\prime}(t) \sin \theta=0 \tag{4.6}
\end{equation*}
$$

When $G^{\prime}(t) \neq 0 ; t=t, \theta=n \pi(n=0, \pm 1, \cdots \cdots)$ are the solutions and hence the maximum and minimum values of $v(t)$ will be calculated. Also when $G^{\prime}(t)=0 ; t=t$, the following consideration can be applied. In order to calculate the transients, it is enough for us to consider only the first few points given by $G^{\prime}(t)=0$ for the low frequency oscillation, the high frequency oscillation being neglected.

Substituting the resulting value $t_{n}$ in (4.5) yields a relation between $v$ and $\theta^{\circ}$ for every value of $t_{n}$. The ultimate value of $\theta^{\circ}$ for which $v$ takes its maximum crest value can then be obtained analytically or graphically and this maximum value is compared with that obtained when $\theta=n \pi$, and whichever is highest can be retained.

## 5. Numerical Calculation

a) Non-grounded System

Consider a system with

$$
\begin{aligned}
& L_{0}=0.101 \mathrm{H}, \quad R_{0}=0, \\
& L_{1}=0.477 \mathrm{H}, \quad R_{1}=28 \Omega \\
& L_{2}=R_{2}=0, \\
& C=35.6 \mu \mathrm{~F}, \\
& i_{a 0}^{0}=\sin (\omega t+\theta), \\
& i_{b 0}^{0}=\sin \left(\omega t+\theta \pm \frac{2 \pi}{3}\right), \\
& i_{c 0}^{0}=\sin \left(\omega t+\theta \mp \frac{2 \pi}{3}\right), \quad \text { where } \omega=377 .
\end{aligned}
$$

b) System with the Neutral of the Source grounded Assume that

$$
\begin{array}{ll}
L_{0}=R_{0}=0, & \\
L_{1}=0.209 \mathrm{H}, & R_{1}=20.8 \Omega, \\
L_{2}=0.0297 \mathrm{H}, & R_{2}=3.4 \Omega, \\
C_{1}=1.35 \mu \mathrm{~F}, & \\
C=73.7 \mu \mathrm{~F}, &
\end{array}
$$

and also that the initial currents are the same as in the above example.

(i)

(iii)

(ii)
(a) $\theta=0, t_{1}^{+}=2.78 \times 10^{-3} \mathrm{sec} ., t_{2}=2.5 \times 10^{-3} \mathrm{sec}$.

Fig. 3. Non-grounded system.
Ordinate=actual voltage/the amplitude of steady terminal voltage due to simultaneous switching abscissa $=t$ in cps The superfix + or - of $t_{1}$ is to be the same as the sign of the phase angle of $i_{b 0}^{0}$, i.e. $+\frac{2 \pi}{3}$ or $-\frac{2 \pi}{3}$.

The highest terminal voltage due to simultaneous switching is represented by the dotted lines.


(d) $\theta=0, t_{1}=4.17 \times 10^{-3} \mathrm{sec}$.,


Fig. 4. Magnitude of d.c. component (non-grounded system).
The ordinates are graduated the same as in Fig. 3. abscissa $=t_{2}$ in cps


Fig. 5. System with the neutral of the source grounded.
The ordinates and the abscissae are graduated the same as in Fig. 3.
The superfix and the dotted lines have the same meaning as in Fig. 3.




Fig. 6. Magnitude of d.c. component (system with the neutral of the source grounded). The ordinates and the abscissae are graduated the same as in Fig. 4.

## Conclusion

Transient terminal voltages across series capacitors resulting from non-simultaneous switching have been calculated by assuming a simplified equivalent circuit for a power network. A numerical examples illustrating the generation of higher terminal voltages than those in the case of simultaneous switching have been worked out and it seems that these excessive voltages are partly due to a d.c. voltage which originates after the three capacitors are switched in non-simultaneously. This d.c. voltage is equal to the zero-phase-sequence voltage just before the last capacitor is switched in. Also, the zero-phase-sequence voltage consists of only d.c. component after all the three capacitors are switched in.

The method of the calculation of highest voltage due to simultaneous switching has been given in detail. But the procedure for calculating the highest voltage resulting from non-simultaneous switching is rather involved. However, it is believed that the analysis presented here will be guiding for analogue computations since the mechanism of the generation of abnormal voltages is evident in some measure. This analysis will prove useful in the design of protective devices for series capacitors.

