On the Theoretical Solution of Highway Traffic Capacity Under Mixed Traffic

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1. Introduction

Highway traffic in our country is the so-called mixed traffic, and is very complicated because besides the various kinds of vehicles with different speeds, there are high and low speed vehicles even among the same kind (automobiles). In considering the method of computing the traffic capacity of highways, highway traffic should be treated as mixed traffic, because it is considered that no great change will appear in the traffic condition for the time being. Considering the case where high and low speed vehicles travel under a mixed condition on a two-way two-lane or a three-lane highway, the former must pass over the latter. In this passing over, the high speed vehicle occupies the on-coming lane when a two-lane road, and the center lane (an exclusive lane for passing over) when a three-lane road, so the passing over is possible only when no vehicle approaches from the opposite direction. Therefore, in order to maintain safe travel, it is necessary to decide a critical amount of traffic that shall not be congested, by keeping the probability of this passing over above a certain value.

This paper, based on the assumption of Poisson's distribution applied to the traffic stream, points out that the traffic capacity can be calculated by obtaining the probability of passing over and determining this probability as a certain value. This assumption of Poisson's distribution applied to the traffic stream is to consider that when b is the traffic volume per unit time passing through a certain point, the number of vehicles passing through this point in τ, time interval decided arbitrary, becomes the Poisson's distribution of which $b\tau$ is the expected value. Namely, if ν is the number of vehicles passing through in τ, $P(\nu)$, the probability of ν vehicles passing through in τ is obtained by the next formula.

$$P(\nu)=\frac{(b\tau)^{\nu}e^{-b\tau}}{\nu\,!} \tag{1}$$

From this formula (1) the probability of passing over is obtained in the following.

2. The Passing Probability Applied to Two-lane Roads

2. 1. The possible probability of passing over* (the case of passing over a low speed vehicle)

The passing over of the right-bound traffic will be considered with the case of a two-lane highway with a traffic volume of a v. p. h. and b v. p. h. in the respective directions as shown in Fig. 1. Now, in considering the case when a

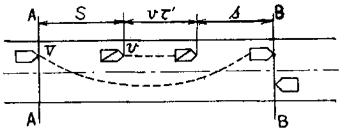

Fig. 1. Passing a slower vehicle on the two-lane road.

high speed vehicle (speed V) passes over a slower-moving one (speed v), let

S: the least head clearance in which the high speed vehicle can follow the preceding slower-moving one without deceleration,

s: the least head clearance in which the low speed vehicle can follow the preceding high speed one.

τ', the time necessary for the high speed vehicle to travel from A to B is

$$\tau' = \frac{S+s}{V-v} \tag{2}$$

It is not allowed for any vehicles to appear in the on-coming lane before the passing over motion is completed. The traveling time of the on-coming vehicle from B to A is determined as follows.

1) If it is a high speed vehicle appearing at B in the opposing lane, the time is τ'.
2) If it is a low speed vehicle appearing at B in the opposing lane, the time is $\mu\tau'$, $\left(\mu = \frac{V}{v}\right)$.

Now, if b is the vehicles per hour in the opposing lane, b_2 the high speed vehicles and b_1 the low speed vehicles, then the average time necessary for a vehicle traveling in the opposing lane from B to A is

$$\frac{b_2\tau' + b_1\mu\tau'}{b} = \tau'\{1+(\mu-1)\psi\}$$

* The passing probability when the condition necessary to pass over occurs.

where

$$\mu = \frac{V}{v}, \qquad \psi = \frac{b_1}{b}$$

τ, the time of one cycle, namely, the time required from the instant a vehicle starts to pass over at A till an opposing vehicle reaches A arriving at B at the same instant as the former after completing the passing over, is obtained as the sum of these traveling times. The relation is as follows.

$$\tau = \frac{S+s}{V-v}\{2+(\mu-1)\psi\} \tag{3}$$

In making observations at A, passing over is impossible when there is any vehicle in the on-coming lane, but possible when there is no on-coming traffic for at least τ. From formula (1), the probability of no vehicle appearing in the on-coming lane during τ is obtained as follows.

$$P(0) = e^{-b\tau}$$

This is the probability of a high speed vehicle immediately starting to pass over upon overtaking a low speed vehicle. Observation point A may be chosen at any point on the highway.

The probability that a high speed vehicle overtaking a slow one immediately starts to pass over is

$$q_0 = e^{-b\tau}$$

The probability of a high speed vehicle starting to pass over after following a slow one for τ' is

$$q_{\tau'} = q_0 + (1-e^{-b\tau})e^{-b\tau}$$

The probability of a high speed vehicle starting to pass over after following a slow one for $2\tau'$ is

$$q_{2\tau'} = q_{\tau'} + (1-e^{-b\tau})^2 e^{-b\tau}$$

The probability of a high speed vehicle starting to pass over after following a slow one for $n\tau'$ is

$$q_{n\tau'} = q_{(n-1)\tau'} + (1-e^{-b\tau})^n e^{-b\tau}$$

Likewise, the probability in general is obtained by summing up all the relations above.

$$q_{n\tau'} = 1-(1-e^{-b\tau})^{n+1} \tag{4}$$

This is the possible probability of a vehicle starting to pass over after following a slow one for $n\tau'$.

2. 2. The potential probability of passing over

In article 2. 1., considerations were made on the mixed traffic consisting of only two kinds of vehicles, that is, high speed vehicles and low speed vehicles. In this article, the mixed traffic will be considered in the general case, in which there are many different kinds of vehicles with different speeds of $v_1, v_2, \cdots\cdots, v_r$.

Let a v.p.h. represent the traffic volume in one direction and $a_1, a_2, \cdots\cdots, a_r$ the number of vehicles with speeds of $v_1, v_2, \cdots\cdots, v_r$ respectively. And likewise with the opposing direction, let b represent the traffic volume and $b_1, b_2, \cdots\cdots, b_p$ the number of vehicles with speeds of $v_1, v_2, \cdots\cdots, v_p$ respectively. Then the following relations are obvious.

$$\left.\begin{aligned} a_1+a_2+\cdots\cdots\cdots\cdots\cdots\cdots+a_r=a \\ b_1+b_2+\cdots\cdots\cdots\cdots\cdots\cdots+b_p=b \end{aligned}\right\} \tag{5}$$

If any one of $a_1, a_2, \cdots\cdots, a_r$ is represented as a_t, and any one of $b_1, b_2, \cdots\cdots, b_p$ as b_t, ψ_t should be defined as follows.

$$\psi_{ta}=\frac{a_t}{a}, \qquad \psi_{tb}=\frac{b_t}{b} \tag{6}$$

Generally, if V denotes the speed of the overtaking vehicle and v_{ta} the speed of the overtaken vehicles, then clearly

$$V>v_{ta}$$

The time necessary for passing a slower-moving vehicle is given by Eq. (2). Expressing that time by τ_1',

$$\tau_1'=\frac{S+s}{V-v_{ta}}$$

Generally speaking, time τ_1' necessary to pass over ν slower-moving vehicles in queue may be shown by the following relation (referring to Fig. 2)

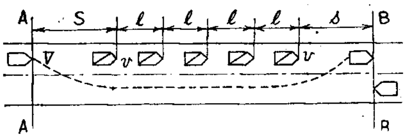

Fig. 2. Passing ν continuous slower-moving vehicles on the two-lane road.

$$\tau_\nu'=\frac{S+s+(\nu-1)l}{V-v_{ta}}$$

where $(\nu-1)l$ is the distance between the head vehicle and the rear vehicle of the slower-moving vehicles in queue, and l is the headway of the slower-moving vehicles. In order to pass over ν vehicles, the following condition is necessary.

$$l<S+s$$

v_{ia} in the above formula is to be chosen as $V > v_{ia}$. If the number of v_i satisfying $V > v_{ia}$ is equal to r', the number of combinations of V and v_{ia} is r'. Assuming that the probability of the speed of the vehicle to be passed over being v_i is proportional to the ratio of the number of vehicles with a speed of v_i to the total number of vehicles, the most probable value of τ_ν' is given by the next expression.

$$\tau_\nu' = \frac{1}{\sum_{i=1}^{r'} \psi_{ia}} \sum_{i=1}^{r'} \frac{S+s+(\nu-1)l}{V-v_{ia}} \psi_{ia} \tag{7}$$

In the case of passing ν slower-moving vehicles in queue in Fig. 2, the formula giving the most probable value of the time for one cycle of section A-B is obtained as follows.

$$\tau_\nu = \frac{2-\sum_{i=1}^{p} \psi_{ib} + \sum_{i=1}^{p} \mu_{ib} \psi_{ib}}{\sum_{i=1}^{r'} \psi_{ia}} \sum_{i=1}^{r'} \frac{S+s+(\nu-1)l}{V-v_{ia}} \psi_{ia} \tag{8}$$

where $\mu_{ib} = \frac{V}{v_{ib}}$ and $\sum_{i=1}^{p}$ is the total sum excluding the value of i corresponding to $v_i = V$.

It is governed by the probability $q_0 = e^{-b\tau_\nu}$, whether the high speed vehicle of a speed of V can pass over the ν slower-moving vehicles in queue or not. Corresponding to Eq. (4), the possible probability of passing over after following them for $n\tau_\nu'$ is given by

$$q_{n\tau'_\nu} = 1-(1-e^{-b\tau_\nu})^{n+1} \tag{9}$$

This formula (9) means the passing probability after overtaking ν continuous slower-moving vehicles. Considering this sort of passing over, the value obtained by multiplying the probability necessitating the pass over with the possible probability of passing over is defined as the potential probability of the passing over phenomenon. As the moment it became necessary to pass over has been considered so far, this corresponds to the case of the probability necessitating the pass over being equal to 1.

Now consider the case when one vehicle passes over a slower-moving vehicle on the free traveling road. Time element t is put

$$t = \frac{S+s}{v_{ia}} \tag{10}$$

As the passing phenomenon occurs when only one vehicle belonging to a arrives during time t and no vehicle belonging to b appears during time τ_1, the potential probability is shown by the next relation for $\nu=1$ according to the assumption of Poisson's distribution.

$$P(V > v_{ia}) ate^{-at}\{1-(1-e^{-b\tau_1})^{n+1}\}$$

Next when two vehicles are passed over; as the passing phenomenon occurs when two vehicles belonging to a during time t and no vehicle belonging to b during time τ_2 appear, the potential probability is shown by the next expression for $\nu=2$.

$$P(V>v_{ia})\frac{(at)^2e^{-at}}{2!}\{1-(1-e^{-b\tau_2})^{n+1}\}$$

Generally, the potential probability of obtaining the phenomenon of passing over ν vehicles is as follows.

$$P(V>v_{ia})\frac{(at)^\nu e^{-at}}{\nu!}\{1-(1-e^{-b\tau_\nu})^{n+1}\}$$

Hence, the potential probability of obtaining the phenomenon of passing over is given by the following formula.

$$T_{n\tau'_\nu}\equiv P(V>v_{ia})\sum_{\nu=1}^{\infty}\frac{(at)^\nu e^{-at}}{\nu!}\{1-(1-e^{-b\tau_\nu})^{n+1}\} \tag{11}$$

where $P(V>v_{ia})$ indicates the probability of the speed of the vehicle belonging to a appearing during the chosen time element t being smaller than the speed of the given overtaking vehicle, in other words, the probability for the problem of passing over to occur. Assuming $P(V>v_{ia})$ is expressed by the ratio of the number of vehicles which satisfy the condition $V>v_{ia}$ to the total number of vehicles, the following relation is obtained.

$$P(V>v_{ia})=\sum_{i=1}^{r'}\psi_{ia}$$

Thus formula (11) may be written as follows,

$$T_{n\tau'_\nu}=\sum_{i=1}^{r'}\psi_{ia}\sum_{\nu=1}^{\infty}\frac{(at)^\nu e^{-at}}{\nu!}\{1-(1-e^{-b\tau_\nu})^{n+1}\} \tag{12}$$

where t and τ_ν can be obtained from Eq. (10) and (8).

In observing the number of vehicles passing during a time interval of t hours at a certain place, an attempt is made to select a vehicle with a speed of V, for the purpose of checking the passing over phenomenon. In this case formula (12) represents the ratio of the number of time intervals which the passing over phenomenon occured to total number of time intervals. When deriving Eq. (11), (12), it is assumed that when ν vehicles come during interval t, the headway of the vehicles is constantly l independent of the value of ν. It is obvious from formula (12) and Fig. 3 that value T is a function which increases with the increase of a, and decreases when a exceeds a certain value, finally converging to zero as $a\to\infty$, but constantly decreases with the increase of b. Furthermore, the value of T takes several values according to which value of v_1, v_2, ……, v_r is taken as the value of V.

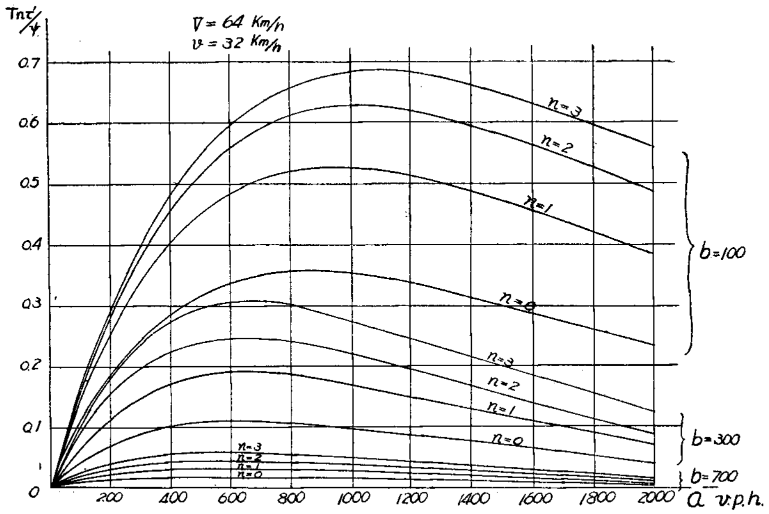

Fig. 3. Potential probability curves, $a \neq b$

In the above formulae, traffic flows containing various vehicles with r classes of different speeds are treated. In the case of mixed traffic flows containing two classes of vehicles, the high speed vehicles and the low speed vehicles, the following formulae can be obtained.

$$\tau_\nu' = \frac{S+s+(\nu-1)l}{V-v} \qquad (7')$$

$$\tau_\nu = \frac{S+s+(\nu-1)l}{V-v}\{2+(\mu-1)\psi_b\} \qquad (8')$$

$$T_{nT'_\nu} = \psi_a \sum_{\nu=1}^{\infty} \frac{(at)^\nu e^{-at}}{\nu!}\{1-(1-e^{-b\tau_\nu})^{n+1}\} \qquad (12')$$

Assuming that the speed of the passing vehicle is equal to that of the high speed vehicle V, then ψ_a represents the ratio of the number of low speed vehicles to the total number of vehicles. Accordingly the potential probability of passing over also increases as the number of low speed vehicles increases, and when $\psi_a=0$, $T=0$.

Fig. 3 is a graphical representation showing the value of $T_{nT'}$ calculated for the various values of a and b, where $T_{nT'}$ is obtained for the case of $n=0$, 1, 2, 3, for $V=64$ km/h, $v=32$ km/h. Graph of $T_{nT'}$ for various combinations of V and v may be useful, if prepared.

2. 3. The possible probability of passing over (for the case considering the passing of ν slower-moving vehicles in queue)

Now consider an instant when it becomes necessary for a certain high speed vehicle to pass another vehicle. It is not known how many vehicles are preceding ahead the high speed vehicle when it wanted to pass over. In the previous article

2. 1., it was assumed that only one vehicle is passed over. But in this article, the following assumption is made to solve this problem. Namely, when the driver of the high speed vehicle decided to pass over, the probability $P'(\nu)$ is assumed to be proportional to probability $P(\nu)$, where $P'(\nu)$ denotes the probability of the number of vehicles to be passed over being equal to ν and $P(\nu)$ the probability of ν vehicles arriving in time interval t.

Then

$$\frac{P'(1)}{P(1)}=\frac{P'(2)}{P(2)}=\frac{P'(3)}{P(3)}=\cdots\cdots=\frac{P'(\nu)}{P(\nu)}=\cdots\cdots$$

Therefore,

$$\frac{P'(\nu)}{P(\nu)}=\frac{P'(1)+P'(2)+\cdots\cdots}{P(1)+P(2)+\cdots\cdots}=\frac{1}{1-P(0)}=\frac{1}{1-e^{-at}}$$

$$\therefore\ P'(\nu)=\frac{1}{1-e^{-at}}\cdot\frac{(at)^{\nu}e^{-at}}{\nu\,!} \tag{13}$$

Hence, when the driver decided to pass over, the possible probability of passing over after following for $n\tau_{\nu}'$ is obtained from Eq. (9) and (13) as follows.

$$q_{n\tau'_{\nu}}=\frac{1}{1-e^{-at}}\sum_{\nu=1}^{\infty}\frac{(at)^{\nu}e^{-at}}{\nu\,!}\{1-(1-e^{-b\tau_{\nu}})^{n+1}\} \tag{14}$$

Eq. (14) is the basic equation of the possible probability of passing over.

As is known from the above equation, q becomes an indeterminate form $\frac{0}{0}$ if $a=0$, and becomes 1 if $b=0$. If $a=0$, it is insignificant to consider a passing over, whereas, when $b=0$, there is no vehicle on the on-coming lane, so that passing over is always possible.

Expressing q by potential probability T of the pass over phenomenon, gives

$$q_{n\tau'_{\nu}}=\frac{T_{n\tau'_{\nu}}}{\psi_{a}(1-e^{-at})} \tag{15}$$

Computing $q_{n\tau'}$ from Eq. (15) by changing the value of b corresponding to several fixed values of a, a monotonous decreasing curve as shown in Fig. 4 is obtained. This curve has no extreme value which was observed in the calculation of T.

2. 4. Computation of traffic capacity

If one value of q is given in Eqs. (14) and (15), the values of a and b corresponding to it are obtained. When one value of the passing over probability is determined, various sets of the combination of a and b are found, and it should be decided from the existing situation which of them is to be chosen. As it is difficult to solve Eqs. (14) and (15) for a and b, it is preferable to compute q and

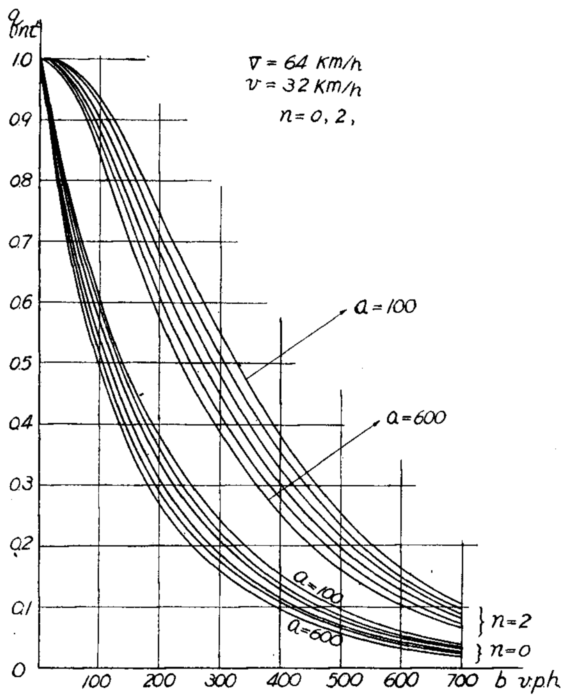

Fig. 4. 1) Possible probability curves, $a \neq b$, $n=0$, 2

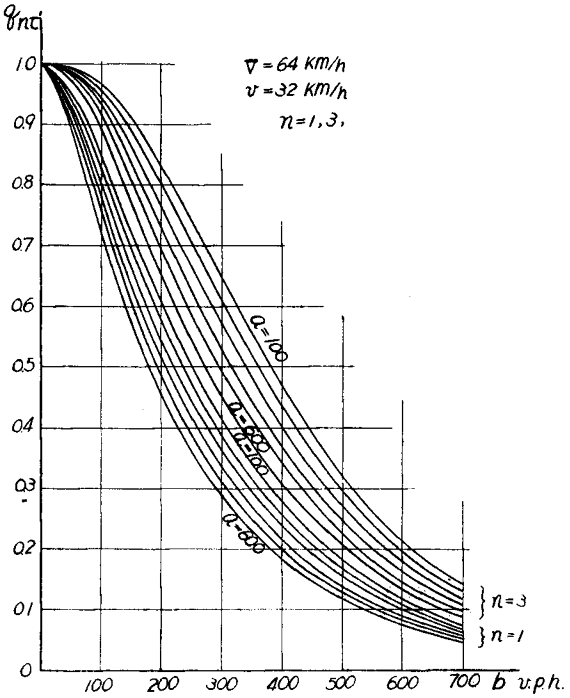

Fig. 4. 2) Possible probability curves, $a \neq b$, $n=1$, 3.

illustrate them in diagrams previously for various values of a and b. For this purpose, Fig. 4 was made taking the speed of high speed vehicles $V=64$ km/h and that of low speed vehicles $v=32$ km/h. But it is necessary to make these diagrams for various combinations of a and b in order to make this method of computing traffic capacities practical.

With ordinary roads, it may be said that the observed traffic volumes in "a" direction and in "b" direction are nearly equal. For such a road, assuming $a=b\equiv x$, the following equation for the possible probability of passing over in both directions is obtained.

$$q_{nT'_\nu}=\frac{1}{1-e^{-tx}}\sum_{\nu=1}^{\infty}\frac{(tx)^\nu e^{-tx}}{\nu!}\{1-(1-e^{-\tau_\nu x})^{n+1}\} \tag{16}$$

Applying an adequate value of $q_{nT'}$ in this Eq. (16), the corresponding traffic volume x can be found. Fig. 5 shows the values of q computed for the various

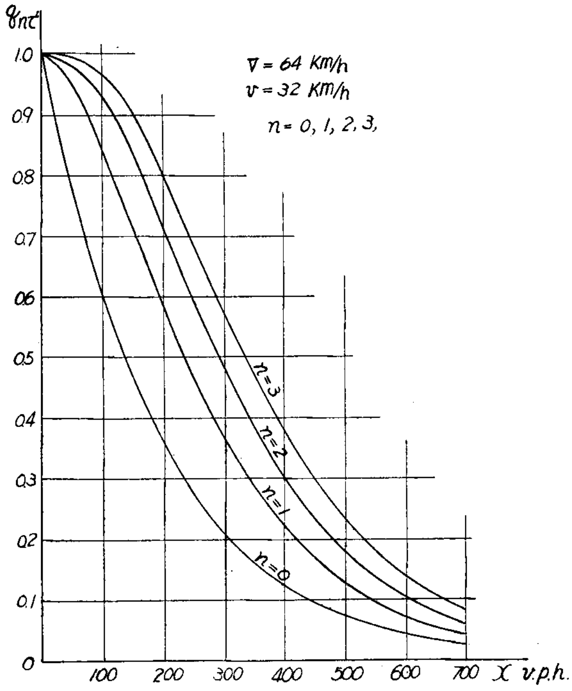

Fig. 5. Possible probability curves, $a=b\equiv x$

values of x. If "hour" is chosen as the unit of t and τ_ν, the unit of the traffic volume x will be in vehicles per hour.

According to routine observations, b becomes larger as the traffic volume a increases. Also, as is known from Fig. 4, the passing probability depends greatly upon b, and effected little by a. Considering from these facts, it can be safely said that Eq. (16) can be used to compute $q_{n\tau'}$ with a high degree of accuracy even if a is not equal to b.

The potential probability of passing over phenomenon T which is convenient to compute Eq. (16) is

$$T_{n\tau'_{\nu}}=\psi_a\sum_{\nu=1}^{\infty}\frac{(tx)^{\nu}e^{-tx}}{\nu!}\{1-(1-e^{-\tau_{\nu}x})^{n+1}\} \tag{17}$$

and Fig. 6 illustrates the values of $\frac{T_{n\tau'_{\nu}}}{\psi_a}$ for several values of x.

Above numerical calculations were performed under the following assumptions, i. e. $\psi_b=0.5$ and $l=S+s$. These values were introduced in Eq. (8′) to compute τ_{ν}. Again, $S+s$ is regarded as the sum of S' and s', where S' is the least headway between both high speed vehicles and s' is that of both low speed vehicles.

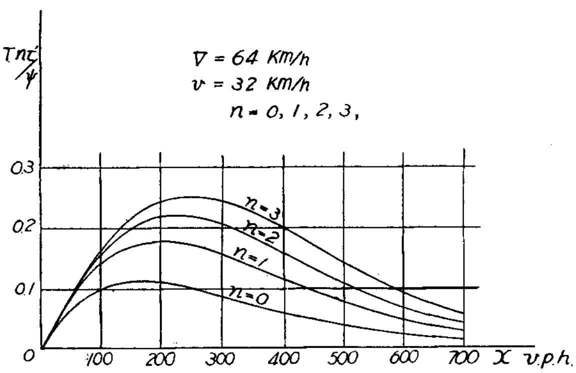

Fig. 6. Potential probability curves, $a=b\equiv x$

$$\therefore \quad S+s \doteqdot S'+s'=0.0373(V^2+v^2)+1.47(V+v)+30 \quad (ft)$$

V and v in this equation are expressed in mile per hour.

2. 5. Comparison of the values of the passing over probability observed and computed

In calculating the traffic capacity of a two-lane road by the method described in 2. 4., it is required, at first, to determine the value of the passing probability. Therefore a field observation was made and the actual value of the passing probability under routine mixed traffic was measured.

(1) Method of measurement

The section from Kusatu to Seta (6.6km long, total width 7.5m, width of pavement 6.0m) on the National Highway Route No. 1 was taken as an experiment section, which was favourable because of less access connections. In this experiment section, a test vehicle was run with the stream of traffic at a known speed (50km/h) and several data were taken. The ratio of the number of passings without delay to the number of total passing maneuvers gives q_0, the passing over probability without any delay. Likewise, $q_{\tau'_{\nu}}$, the passing over probability after following slower-moving vehicles for or within τ_{ν}', was found. τ_{ν} is computed from τ_{ν}', the time required to pass over ν slower-moving vehicles in queue,

$$\tau_\nu=\tau_\nu'\{2+(\mu-1)\psi\}$$

where $\psi=\dfrac{\text{number of slower-moving vehicles in the stream}}{\text{number of total traveling vehicles}}$,

$$\mu=\frac{V}{v}.$$

(2) Measured values

Measured results are as shown in Table 1.

Table 1

Date of observations	Mean traffic volume (x)	Total number of pass overs	$q_{n\tau'}$ $n=0$	$q_{n\tau'}$ $n=1$
Apr. 28, 1954 10.30-11.30	82	19	0.580	0.895
〃 11.30-12.30	65	15	0.734	0.867
〃 13.30-14.30	87	13	0.538	0.922
May 11, 1954 14.30-15.30	71	11	0.546	0.820
〃 15.30-16.30	73	18	0.667	0.890
〃 16.30-17.30	75	19	0.580	0.947
〃 17.30-18.30	78	22	0.500	0.864
		Total 117		

Total average of the measured values in Table 1 are as follows:

$$q_0=\frac{69}{117}=0.590,\qquad q_{\tau'}=\frac{104}{117}=0.890,\qquad x_{mean}=79.5 \text{ v. p. h.}$$

(3) Computed values

The passing probability is computed by Eq. (16).

$$q_{n\tau'}=\frac{1}{1-e^{-tx}}\sum_{\nu=1}^{\infty}\frac{(tx)^\nu e^{-tx}}{\nu!}\{1-(1-e^{-\tau_\nu x})^{n+1}\} \tag{16}$$

As the speed of the test vehicle was 50 km/h in this experiment, the slower-moving vehicles mean vehicles traveling at speeds less than 50 km/h. At a census point in the experiment section, an observation of the speed distribution of all traffic was made and Table 2 obtained. In the table, speeds being classified every 5 km/h, all speeds included in each interval are represented by the class mark. As the number of slower-moving vehicles, the sum of the number of vehicles corresponding to the class mark less than 45 km/h was taken. Then $\psi=0.86$ is given. 49 km/h being the average calculated from the distance travelled and the required time, this value was taken as the actual speed of the test vehicle V. Hence, the value $V=49$ km/h was used, and the value $v=37$ km/h of slower-moving vehicles was computed as the weighted mean of vehicles belonging to and below

Table 2

Speed (km/h)	Number of vehicles
17.5-22.5	2
22.5-27.5	6
27.5-32.5	18
32.5-37.5	28
37.5-42.5	37
42.5-47.5	22
47.5-52.5	6
52.5-57.5	5
57.5-62.5	4
62.5-67.5	0
67.5-72.5	1
72.5-77.5	0
77.5-82.5	2

the class mark 45 km/h in Table 2. Strictly speaking, although it can not be expected that all vehicles travel the entire length of the section at a constant speed observed at the census point, the value of μ was computed tentatively assuming that this value represents the average speed of the slower vehicles.

Next, τ_ν' the time required to pass over ν slower-moving vehicles in queue is considered as the function of V and v, but the form of the function is not yet known. Therefore, the relation of τ_ν' to relative speed $V-v$ is shown graphically in Fig. 7. As is known from Fig. 7, hardly no correlation exists between the

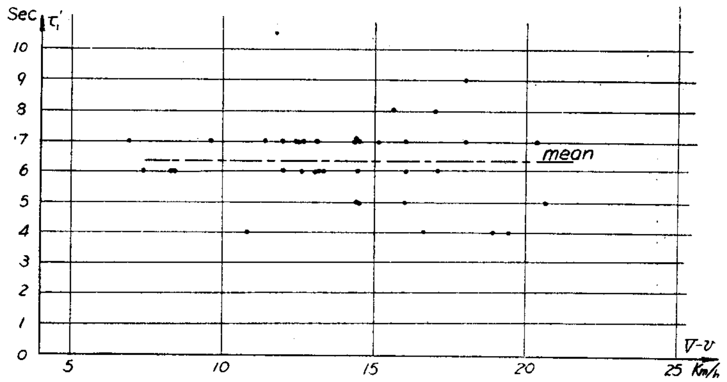

Fig. 7. Correlation between the relative speed and the time required to pass over.

relative speed and the passing time within the range of the relative speed in this experiment. Hence, $\tau\nu'$ is regarded almost constant regardless of any $V-v$. In practice, τ_ν' being regarded as constant in the range of the relative speed in this experiment, the follow ng values were obtained.

$$\tau_1'=6.4 \text{ sec}, \qquad \tau_2'=8.3 \text{ sec}$$

Although these are the average values of the time required in passing, it is almost impossible practically for the test vehicle to meet an on-coming vehicle at the instant it has finished passing. Thus, 3 seconds were given as an allowance time to τ_ν' when it is used to calculate τ_ν. Then,

$$\tau_1'=9.4 \text{ sec}, \qquad \tau_2'=11.3 \text{ sec}$$

This allowance time 3 sec. is the adequate value confirmed by the observer in the test vehicle. Now, as for the values of τ_ν' $\nu=1$ and $\nu=2$ alone were computed, because $\nu\geqq3$ didn't occur in this experiment.

Next, the method of computing the relative speed is as follows. When the test car passes any vehicle, one observer in the vehicle takes records of the kind and type of the overtaken vehicle, and the second observer measures time Δt to pass over the wheel base L of the overtaken vehicle. Then

$$V-v=\frac{L}{\Delta t}$$

If τ_1' is constant as above mentioned, $S+s$ is given from the relation

$$\tau_1'=\frac{S+s}{V-v}$$

and then $t=(S+s)/v$ is computed. In this case, allowance time for τ_1' is never added to calculate $S+s$.

Table 3

x	$q_{n\tau'}$ $n=0$	$q_{n\tau'}$ $n=1$
82	0.620	0.855
65	0.680	0.895
87	0.595	0.837
71	0.659	0.887
73	0.650	0.877
75	0.638	0.869
78	0.629	0.861

Table 3 shows the computed results of $q_{n\tau'}$ by Eq. (16) using the figures above obtained, and also the computed values of the passing probability corresponding to the traffic volume, $x=0\sim200$ v.p.h. are given in Table 4.

Fig. 8 reveals that the measured values of the passing probability in Table 1 and the computed values above obtained nearly coincide. In view of the fact that only a few data were used to plot these points in Fig. 8, it seems possible to calculate the passing probability with sufficient accuracy in the case of a two-lane road with a sufficient width.

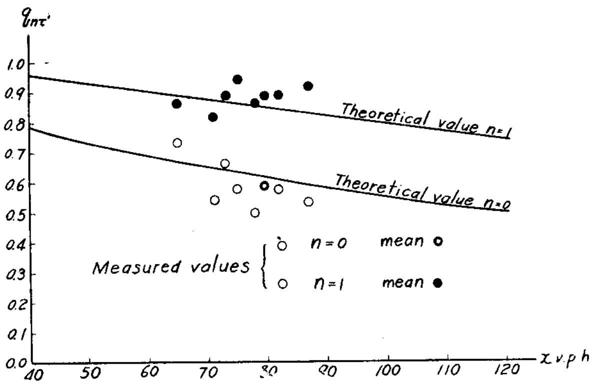

Fig. 8. Passing probability on a two-lane road.

Table 4

x		0	20	40	60	80	100	120	140	160	180	200
$q_{n\tau'}$	$n=0$	1	0.896	0.787	0.700	0.620	0.550	0.495	0.433	0.382	0.340	0.301
	$n=1$	1	0.980	0.960	0.910	0.855	0.798	0.750	0.680	0.616	0.561	0.513

3. The Passing Probability on the Three-lane Roads

3. 1. Mixed traffic consisting of two classes of traffic

In Fig. 9, a right-bound high speed vehicle (speed V) which is about to begin to pass over a slower-moving vehicle (speed v) at A is considered. The high speed vehicle is prohibited to pass over when a high speed vehicle on the on-coming lane is passing over beyond the site D. If, therefore, a left-bound vehicle does not pass over while the right-bound high speed vehicle travels between A and D, it is evidently possible for the right-bound vehicle to pass over. It is unknown how many vehicles in queue are being overtaken by the left-bound vehicle while the right-bound high speed vehicle is passing ν vehicles in queue in section A-B. Now, for simplicity, it is assumed that ν vehicles in queue are to be passed over in the left-bound lane, also.

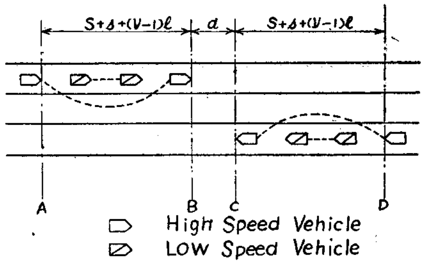

Fig. 9.
Passing on the three-lane road.

As was mentioned in the case of two-lane roads, the travel time of the high speed vehicle from A to B is given as follows,

$$\tau_\nu' = \frac{S+s+(\nu-1)l}{V-v}$$

Then, the travel time τ_ν of the right-bound high speed vehicle traveling from A to D is

$$\tau_\nu = 2(\tau_\nu' + \zeta)$$

where

$$2\zeta = \frac{d}{V}$$

ζ means allowance time, and d is the allowance distance to complete the passing over safely which is closely connected with the road width.

Assuming that the right-bound traffic is a v.p.h., the left-bound traffic is b v. p. h. (low speed vehicles b_1, high speed vehicles b_2), and the number of pass overs per unit time is p, which is necessary for the left-bound traffic, then choosing τ_ν arbitrary, m, the average number of pass overs during time element τ_ν is given from the following relation.

$$V : b_2 p \tau_\nu = V\tau_\nu : m$$

$$\therefore \quad m = b_2 p \tau_\nu^2 \tag{18}$$

Now the following assumption is introduced. "The number of pass overs which occurs in an arbitrary time element will conform to Poisson's distribution around the expected number of pass overs in that period of time." Namely, the probability of generating k pass overs during τ_ν is expressed as

$$\frac{m^k e^{-m}}{k!}$$

Therefore, the probability of generating no pass over in the on-coming lane during this time element τ_ν becomes

$$e^{-b_2 p \tau_\nu^2}$$

Thus, the probability for the right-bound vehicle to pass over without any delay is obtained as follows.

$$q_0 = e^{-b_2 p \tau_\nu^2} \tag{19}$$

In general, the probability of enabling to pass over ν vehicles in queue after following for a time of $n\tau_\nu$ is obtained

$$q_{n\tau_\nu} = 1-(1-e^{-b_2 p \tau_\nu^2})^{n+1}$$

Thus, as in the case of two-lane roads, the possible probability of passing over is given by the following relation.

$$q_{n\tau_\nu} = \frac{1}{1-e^{-at}} \sum_{\nu=1}^{\infty} \frac{(at)^\nu e^{-at}}{\nu!} \{1-(1-e^{-b_2 p \tau_\nu^2})^{n+1}\} \tag{20}$$

where

$$t = \frac{S+s}{v}$$

In this relation, p is found as follows. Consider a stream of vehicles moving along a section of the road in one direction consisting of b_1 slow vehicles and b_2 fast vehicles. If the observer travels with the stream on a high speed vehicle and records the number of overtaken vehicles, the test vehicle travels V km per unit time, whereas slower-moving vehicles travel v km only, and thus the number of vehicles overtaken by the test car per unit time will be greater than that when the slower-moving vehicles are counted at the census point per unit time. If b_1 vehicles were observed per unit time at the census point on the road, these slower vehicles may be observed on the test vehicle as $(\mu-1)\ b_1$ vehicles, which is equal to p. Then, putting $b_1 = \psi b$

$$p = (\mu-1)b_1 = \psi(\mu-1)b \tag{21}$$

Strictly speaking, when a driver wants to pass over, he will experience a certain delay in waiting for a chance to pass over, the value of p diminishing to some extent. But, if the traffic volume is not too high, it is recognized that Eq. (21) can be used in practice.

3. 2. Mixed traffic consisting of R classes of traffic

Actual traffic flow on the three-lane roads is not the mixed traffic of only two classes, but is consisted of various vehicles of different speeds, and the passing over phenomena occur even among the slower-moving vehicles. Since Eq. (20) does not give the actual passing probability, a further research will be made considering the mixed traffic consisting of R classes of traffic.

The number of pass overs per unit time which is necessary for a high speed vehicle of a speed of v_r will be expressed by p_r. Assuming

$$v_1 < v_2 < \cdots\cdots < v_r < \cdots\cdots < v_R$$

subscript r is used to indicate class r, and subscripts a, b to distinguish right-bound and left-bound respectively.

Now the passing probability of the vehicle of a speed of v_r will be obtained. Referring to Fig. 9, $\tau_{\nu}'_{ra}$, the time required for the high speed vehicle to travel between A and B, will be given from the next expression.

$$\tau'_{\nu ra} = \frac{1}{\sum_{i=1}^{r-1} \psi_{ia}} \sum_{i=1}^{r-1} \frac{S+s+(\nu-1)l}{v_r - v_i} \psi_{ia} \doteqdot \frac{S+s+(\nu-1)l}{v_r - \overline{v_{r-1}}} \tag{22}$$

where $\overline{v_{r-1}}$ means the average speed of all vehicles belonging to classes below the $(r-1)$th class. The travel time $\tau_{\nu}'_{b}$ during which the high speed vehicle travels between C and D will be

$$\tau'_{\nu b} = \sum_{r=2}^{R} \left(\sum_{i=1}^{r-1} \frac{S+s+(\nu-1)l}{v_r - v_i} \frac{\psi_{ib}}{\sum_{i=1}^{r-1} \psi_{ib}} \right) \frac{\psi_{rb}}{1-\psi_{1b}} \doteqdot \sum_{r=2}^{R} \frac{S+s+(\nu-1)l}{v_r - \overline{v_{r-1}}} \frac{\psi_{rb}}{1-\psi_{1b}} \,. \tag{23}$$

Hence, the travel time $\tau_{\nu r}$ for section A-D is given as

$$\tau_{\nu r} = \tau'_{\nu ra} + \tau'_{\nu b} + 2\zeta \tag{24}$$

The average number of pass overs for the r th class which will happen during this time element $\tau_{\nu r}$ will be given from Eq. (18) as

$$b_r p_r \tau^2_{\nu r}$$

and for all the classes as

$$\sum_{r=2}^{R} b_r p_r \tau^2_{\nu r}$$

The probability of generating no pass over in the on-coming lane during this time element $\tau_{\nu r}$ becomes

$$q_0 = e^{-\sum_{r=2}^{R} b_r p_r \tau^2_{\nu r}} \tag{25}$$

Eq. (25) is the probability of the right-bound vehicle being able to pass over without any delay at the instant it desires to pass over. Accordingly, the possible probability of passing over after following for a time of $n\tau_{\nu r}$ is obtained similar to Eq. (20).

$$q_{n\tau}=\frac{1}{1-e^{-at}}\sum_{\nu=1}^{\infty}\frac{(at)^{\nu}e^{-at}}{\nu!}[1-\{1-exp(-\sum_{r=2}^{R}b_r p_r\tau^2{}_{\nu r})\}^{n+1}] \tag{26}$$

In this equation p_r denotes the number of pass overs per unit time which the r th class vehicle will perform. It will be given by the following

$$p_r=\sum_{i=1}^{r-1}\psi_i(\mu_i-1)b\doteqdot(\mu_r-1)b\sum_{i=1}^{r-1}\psi_{ib} \tag{27}$$

where

$$\mu_i=\frac{v_r}{v_i}\ ,\qquad \mu_r=\frac{v_r}{v_{r-1}}$$

As has been explained with the case of two-lane roads, putting $a=b\equiv x$, gives

$$q_{n\tau}=\frac{1}{1-e^{-tx}}\sum_{\nu=1}^{\infty}\frac{(tx)^{\nu}e^{-tx}}{\nu!}[1-\{1-exp(-\sum_{i=2}^{R}x_r p_r\tau^2{}_{\nu r})\}^{n+1}] \tag{28}$$

$$p_r=\sum_{i=1}^{r-1}\psi_i(\mu_i-1)x\doteqdot(\mu_r-1)x\sum_{i=1}^{r-1}\psi_i \tag{29}$$

Eq. (22) and (23) from which $\tau_{\nu r}$ is obtained will be written as follows.

$$\tau'_{\nu r}=\frac{1}{\sum_{i=1}^{r-1}\psi_i}\sum_{i=1}^{r-1}\frac{S+s+(\nu-1)l}{v_r-v_i}\psi_i \tag{30}$$

$$\tau'_{\nu}=\sum_{r=2}^{R}\left(\sum_{i=1}^{r-1}\frac{S+s+(\nu-1)l}{v_r-v_i}\frac{\psi_i}{\sum_{i=1}^{r-1}\psi_i}\right)\frac{\psi_r}{1-\psi_1}\doteqdot\sum_{r=2}^{R}\frac{S+s+(\nu-1)l}{v_r-v_{r-1}}\frac{\psi_r}{1-\psi_1} \tag{31}$$

3. 3. Experimental observation

In our country, although there are no three-lane roads in the strict meaning with lane markings, several routes of a width of 10 m or more operating like the three-lane roads were selected and making field measurement on these roads, the appropriateness of Eq. (28) was examined.

One example of the test drives was performed at a 10.1 km section between Sendai and Siogama. The road surface of this section consists of a pavement 9.0 m wide and a shoulder 1.4 to 2.0 m wide on each side. Since the surface condition is in a good state of concrete pavement, all vehicles can travel at high speed. The average traffic volume is less than 100 v.p.h., a volume which can be carried by a two-lane road. Therefore, in spite of the walking traffics such as hand-carts and wagons which were considerable obstacles for the drivers, all the pass over maneuvers were performed by $n=0$. In this case, the measured values of passing probability q_0 coincide with the computed values from Eq. (28) as are shown in Table 5 and Fig. 10. On the contrary, these measured values differ greatly from the computed values by Eq. (16) for two-lane roads. These verify the appropriateness of Eq. (28), and signify that such a road must be treated as a three-lane road.

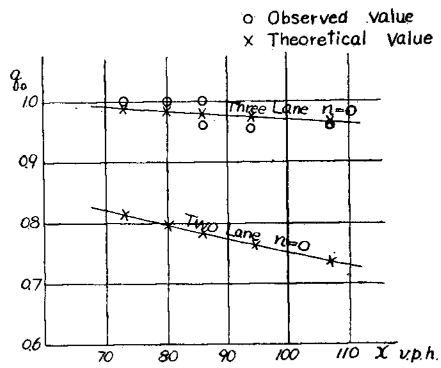

Fig. 10.
Measured example on the three-lane road.

Table 5

Date of observations	Mean traffic volume (x)	q_0		
		Measured value	Three-lane formula	Two-lane formula
Aug. 26, 1954 14.00-15.00	86	1.000	0.980	0.783
〃 15.00-16.00	73	1.000	0.989	0.813
〃 16.00-17.00	94	0.954	0.975	0.764
Aug. 27, 1954 10.00-11.00	107	0.959	0.965	0.736
〃 11.00-12.00	86	0.960	0.980	0.783
〃 12.00-13.00	80	1.000	0.985	0.795

4. Conclusion

The passing probability of the two-way two-lane and three-lane roads under mixed traffic of r classes was investigated, and some analytical formulae derived thereof. In this paper, a report was made on the results of experiments which were performed in order to verify the appropriateness of the formulae above obtained. The following is the summary of the conclusions reached as the result of this investigation.

1) Passing probability is affected remarkably by the traffic volume in the on-coming lane, but not greatly affected by the volume in the same direction.

2) The computed and measured values of the passing probability for the two-lane roads and the three-lane roads do not differ greatly.

3) Hardly no correlation exists between the relative speed and the time required to pass over.

4) Necessary number of pass overs per unit time can be determined.

5) The values of τ'_ν, τ_ν, $S+s$ and the actual value of the passing probability are found.

6) When a good road has a width equivalent to that of three-lanes with sufficient shoulders on each side and the road surface is very good, it displays the function of a three-lane road even if there is no lane marking.
7) By observing the traffic flow of broad roads with widths wider than two-lanes, it can easily be judged whether the overall roadway width is being utilized fully or not. It is believed that a road whose roadway width is not utilized fully can be converted to a high traffic capacity road by means of controlling how to use the lanes by drawing the lane marking.
8) It was ascertained that defects and obstacles on the roads such as insufficient shoulders or roadside parking vehicles would check traffic flows strikingly.

In calculating the traffic capacity of roads, the problem of determining the value of the passing probability is the main subject which must be studied hereafter. The author intends to develop the method of computing traffic capacity, the outline being as follows:

(1) introduce congestion index as a function of the passing probability,
(2) determine the value of the passing probability to restrain the congestion index from not exceeding a certain limit,
(3) calculate the traffic capacity corresponding to this passing probability.

The description in this paper are all applicable to a perfect two-lane or perfect three-lane road. Therefore, the author will make further investigations on the passing probability of the roads narrower than two-lane or three-lane roads.

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