Primary Power of the Hydro Electric Power Plant

By

Taturo OKUBO and Hiroshi NISHIHARA

Department of Electrical Engineering

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In this paper, a method of determining the primary power (the firm capacity) of the hydro electric power plant is developed.

I. Introduction

Upon determining the generating capacity of a hydro electric power plant, we must consider both the primary power (kW) and the total energy output (kWh) during a year or a month. Especially, on the long-term planning of electric power supply, an economic consideration for both of them based on the forecast of stream flow is essential.

The factor for determining the primary power is the annual minimum flow; while the basis used for determining the energy output (kWh) is the annual or monthly average of stream flow. The stream flow, however, not only varies seasonally, but is also influenced by the meteorological conditions, and by various complicating factors.

At present, since we have not sufficient knowledge about the conditions and mechanisms of such variations, we have to depend on the stochastic method in order to forecast the above amounts of stream flow.

Fundamentally speaking, the drought and the average flow of the stream are deduced from the stream flow duration curve. In particular, the latter is ordinarilly found by a synthetic treatment of all data covering a year or several successive years, while the former is estimated according to the stream flow in a dry season. In other words, the average stream flow which gives the energy output (kWh) of the hydro electric power plant is found from the annual or monthly duration curve of the stream flow, while the primary power is to be estimated from the data of the annual minimum flow; thus, the basic principles of these calculations are essentially different.

In this paper, the authors will at first explain the methods, which are now used in Japan, for calculating the so-called primary power of a hydro electric power plant. Then, after pointing out their defects, they will propose more reasonable methods for estimating the primary power economically.

A discussion about generated energy (kWh) will be presented on another occasion.

II. A method for calculating the primary power

The definition of the primary power of hydro electric power plants, (which is conventional and standardized in Japan), is the arithmetical mean of the smallest ten (or five) of a year's 365 daily records of stream flow discharge (m^3/sec) converted into the electric power output of the plant.* In other words, the standard value of stream flow discharge (m^3/sec) for determining the primary power is obtained by taking the average of stream flow for ten (or five) days at the lowest part of the annual stream flow duration curve. This duration curve represents the relation of duration in days versus mean value of the corresponding daily stream flow of recent N years. (N-years ordinate-mean duration curve.)**

Such a definition, however, is based upon an out-dated idea of past when the run-of-river system was exclusively applied. It is, therefore, not always sound for pondage and storage plants; neither is it always reasonable even for the run-of-river plants. For instance, the annual minimum power output of the run-of-river plants would be determined rather by the annual minimum flow. Then, also, the effects of the above mentioned smallest ten-days drought flow upon the electric power supply-demand relationship will surely be different, depending upon whether they appear successively or at intervals.

Thus, we are led to the following methods for determining the primary power of hydro electric power plants in accordance with their reserving capacity of stream flow. That is, in the annual driest season in which the power-system is suffering from the severest water shortage:

(1) For a run-of-river plant, take the minimum flow,

(2) For a pondage plant, take the minimum of the average flow during a certain interval on the hydrograph, (the length being determined in accordance with the reserving capacity of the plant),

(3) For a storage plant, take the minimum of the running average of the hydrograph during an interval chosen as in (2) but longer according to the larger reserving capacity.

Then, by taking this value of the annual drought flow of each of the N years as a sample value for calculating the primary power can be determined by the stochastic method which is described in Section V.

For a complex power-system embracing numerous groups of hydro electric

* The critical year method is prevailing in America.

** So far N is taken for 10 or 11 in Japan.

plants belonging to various categories of basins, as in Japan, the primary power of the plants must be estimated in the period in which the over-all system is in the annual driest condition; while the integrated stream flow of the system is generally estimated in proportion to the total generating capacity of the system. In this case, since the dates of the minimum flow of individual plants do not always coincide with that of the minimum flow of the system to which they are connected, then the above mentioned sample values of stream flow to be used in estimating the primary power of individual plants are the values on the day when the over-all flow of the system takes its minimum value, but not the values on the days when individual plants are suffering from the year's severest water shortage. The authors name the annual minimum-flow day of the system "the system's critical day."

Similarly, in planning a new hydro electric power plant, the flow on the critical day of the power-system to which it is to be connected will serve the purpose as the "sample value" of the drought flow in each year for the plant.

Consequently, the primary power determined independently, as at present, for individual plants seems to be of little practical significance, at least for a plant whose generating capacity is very small compared with that of the system to which it belongs.

Thus, by interconnection, the larger the number of plants located in basins of different hydrological characteristics interconnected, the greater becomes the increase of the primary power of individual plants, regardless of the annual minimum flow of the plants.

If the season, in which the load demand is at its maximum, does not coincide with the driest season, the critical day is the day on which the power-system is at its worst condition as regards the electric power supply-demand relationship.

But, since to forecast the system's critical day is considerably difficult, the running average mentioned in (2) and (3) must be taken up at random for many intervals.

Now, if the annual sample values of the drought flow are found by the above proposed method, then by arranging them in the order of magnitude, and by estimating their non-exceeding probability (the drought probability) by the stochastic method which will be shown in Section IV (an estimation of the probability distribution function of the population of drought flows), we can find the relation between the drought probability and the design value of drought flow discharge which are the bases for detemining the primary power of the plant.

The detailed discussion about the method of chosing the primary power most economically and reasonably on the basis of the non-exceeding probability will be given in Section V.

III. Stream flow duration curve and probability distribution of the minimum flow in a dry month.

Fig. 1 shows the monthly flow duration curves of a power-system in every February of the last 11 years, expressed in terms of percentage of the flow discharge corresponding to the maximum power of the system. Generally speaking, especially in dry months such as February, the monthly average flows are scattered in a range wider than that of the daily flows in each of the months, which tendency is also shown clearly in Fig. 1. For that reason, we cannot assume any population from which daily flows in every month of every year may be taken as random samples. The difficulty encountered in presuming the monthly duration curves of daily flows or of monthly minimum flows is mainly due to this fact.

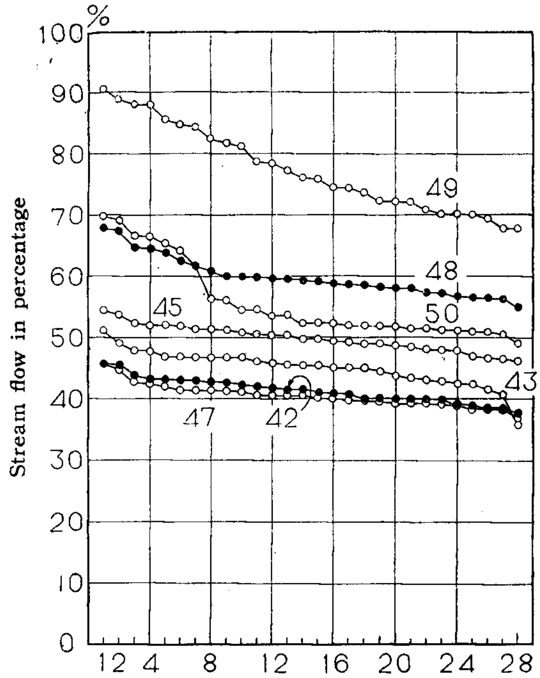

Fig. 1. Stram flow of a power-system in February.

If, however, we have data of flow discharge for N years, N being so large that the daily flow may be taken for constituting a stationary time series, then we may presume a population from which the data, such as, for example, the monthly minimum flow in every year or the drought flow on the critical day is taken as a random sample.

Our object is, in this sense, to find the probability of dearth of flow (the drought probability) on which the N-years management of the power-system may be based. This leads us to the estimation of the probability distribution function $F(x)$ of the population of the drought flow of the system's critical day in each of N years. The distribution function $F(x)$ represents the probability of drought flow Q not exceeding x, namely

$$F(x) = \text{Probability } (Q \leqq x). \tag{1}$$

Since we are now interested in the stream flow on the critical day on which the power-system is in the annual worst condition as regards the electric power supply-demand relationship, the value of N is to be taken for the longest eminent period of the time series representing the year-to-year variation of the flow discharge on that critical day of the system.

IV. The non-exceeding probability of the drought flow of the critical day (The drought probability).

Since the sample values of drought flow on the critical days defined in Section III are so far not obtainable for the authors, by way of an example, the monthly minimum flow in the annually driest month of a certain river will be dealt with by means of a stochastic method which is free from any restrictions preventing its application to the system-drought probability.

The data of the minimum flow of a certain river in Japan in every February during the last 25 years is plotted in Fig. 2. The plotting position of the data will come into question, when we plot it about the graph of the non-exceeding probability. If we arrange the sample values of size n, taken from a distribution with the distribution function $F(x)$, in ascending order of magnitude as $x_1 \leqq x_2 \leqq \cdots \leqq x_n$, the r-th sample value x_r has its own probability distribution depending upon the initial probability distribution $F(x)$, the sample size n and the order of magnitude r. Since the mode $\tilde{x}_r$ of the probability distribution of x_r for any r $(1 \leqq r \leqq n)$ is known, then the r-th observation x_r may be plotted at the point with the probability coordinate $F(\tilde{x}_r)$ (E. J. Gumbel).(3) On the other hand, since the mean value of $F(x_r)$ is $\frac{r}{n+1}$, it seems reasonable to plot x_r at the point of the non-exceeding probability:

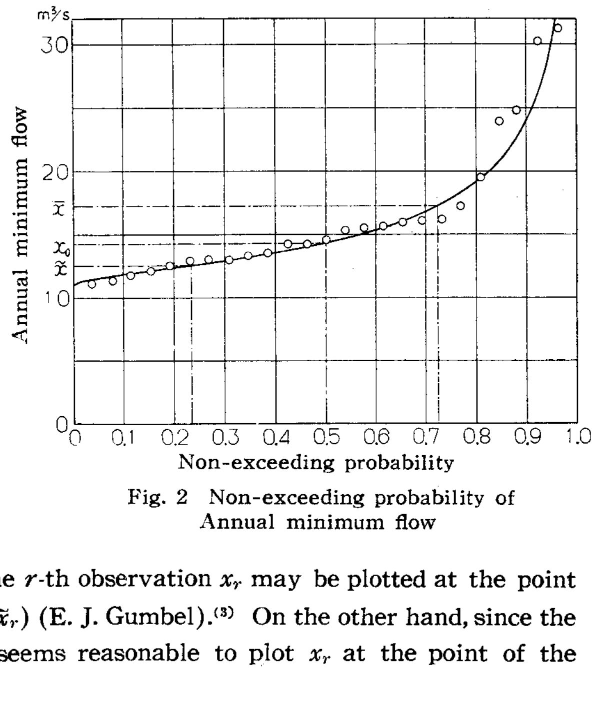

Fig. 2 Non-exceeding probability of Annual minimum flow

$$F_r = \frac{r}{n+1} \cdot \qquad \text{(H. A. Thomas)}^{(4)} \qquad (2)$$

The latter plotting seems to be more practical than the former. Observations shown in Fig. 2 are plotted by Thomas' method.

In estimating the non-exceeding probability $F(x)$ of the minimum flow from the sample values, it is known by experience that the logarithmico-normal distribution with a lower limit fits well in the observations of the minimum flow. Now an outline of methods for fitting the logarithmico-normal distribution (estimation of the population parameters) will be explained.*

Many methods for estimating the population parameters of the logarithmico-normal distribution have been published. In particular, S. Iwai, professor of civil engineering at Kyoto University in Japan, has presented detailed discussions about fitting the logarithmico-normal distribution to stream flow duration curves.(2) In the present paper, none of them are explained, but the two methods which are proposed by the authors and used in the following sections.

(1) Logarithmico-normal distribution.

Probability distribution of a population represents the probability of a sample X not exceeding x, and is denoted by $F(x)$ as mentioned in Section III. If $F(x)$ is continuous and differentiable, the probability density function or the frequency function $f(x)$ is defined by

$$F(x)=\int_{-\infty}^{x} f(x)dx \; . \tag{3}$$

Daily flow or monthly minimum flow has an asymmetric frequency curve with a lower limit b, as shown in Fig. 3. One of the most typical distributions of this type is the logarithmico-normal distribution with a lower limit, which is obtained by the logarithmic transformation:

$$z=\log(x-b) \tag{4}$$

from the normal distribution $N(m, \sigma^2)$ defined by

$$\left.\begin{aligned} \Phi(z) &= \int_{-\infty}^{z} \varphi(z)dz \; , \\ \varphi(z) &= \frac{1}{\sqrt{2\pi}\,\sigma} exp\left\{-\frac{(z-m)^2}{2\sigma^2}\right\}. \end{aligned}\right\} \tag{5}$$

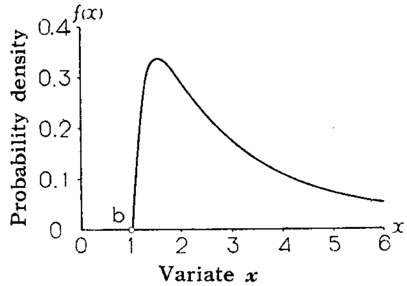

Fig. 3. Asymmetrical frequency curve.

Substituting Eq. (4) into Eq. (5), we obtain the logarithmico-normal distribution as follows:

$$\left.\begin{aligned} F(x) &= \int_{b}^{x} f(x)\, dx \; , \\ f(x) &= \frac{1}{\sqrt{2\pi}\,\sigma(x-b)} exp\left\{-\frac{[\log(x-b)-m]^2}{2\sigma^2}\right\}. \end{aligned}\right\} \tag{6}$$

* Since the observations of the minimum flow cannot be taken for the smallest of random samples from a certain distribution, the distribution of extreme values is not applicable to the drought probability, though it has been very useful in the estimation of the probability of floods.

As the common logarithm is more practical than the natural logarithm, we usually use the following transformation instead of Eq. (4).

$$z = \mathrm{Log}(x-b) \tag{7*}$$

Thus, we have

$$f(x) = \frac{M}{\sqrt{2\pi}\,\sigma(x-b)}\, exp\left\{-\frac{[\mathrm{Log}(x-b)-m]^2}{2\sigma^2}\right\}, \tag{8}$$

where $M = \mathrm{Log}\, e \doteqdot 0.4343$.

Putting

$$\xi = \frac{\mathrm{Log}(x-b)-m}{\sigma}, \tag{9}$$

we obtain

$$\left.\begin{aligned} F(x) &= \frac{1}{2} + \Phi_0(\xi)\,, \\ \Phi_0(\xi) &= \frac{1}{\sqrt{2\pi}}\int_0^{\xi} e^{-\frac{t^2}{2}}dt\,. \end{aligned}\right\} \tag{10}$$

(2) **Fitting of the logarithmico-normal distribution.**

As this distribution is characterized by three parameters m, σ^2 and b, it becomes somewhat complicated to find their statistical estimates.

(*a*) *Parameter estimation by mean, variance and median of samples.*

For the logarithmico-normal distribution, we have the following relations.

$$\left.\begin{aligned} &m_x(\text{mean of } x) = e^{m+\frac{\sigma^2}{2}} + b\,, \\ &\sigma_x^2(\text{variance of } x) = e^{2m+\sigma^2}(e^{\sigma^2}-1)\,, \\ &x_0(\text{median of } x) = e^m + b\,. \end{aligned}\right\} \tag{11}$$

Solving the equations with respect to m, σ^2 and b, we obtain

$$\left.\begin{aligned} &m = \log(x_0-b)\,, \\ &\frac{\sigma^2}{2} = \log(m_x-b) - \log(x_0-b)\,, \\ &b^3 - \left(\frac{5}{2}m_x + \frac{1}{2}x_0 - \frac{1}{2}\frac{\sigma_x^2}{m_x-x_0}\right)b^2 \\ &\quad + \left(2m_x^2 + m_x x_0 - \frac{x_0\sigma_x^2}{m_x-x_0}\right)b - \frac{1}{2}m_x^2(m_x+x_0) \\ &\quad + \frac{x_0^2\sigma_x^2}{2(m_x-x_0)} = 0\,. \end{aligned}\right\} \tag{12}$$

Let the estimates of m_x, σ_x^2 and x_0 be as follows:

* Log x is the common logarithm of x, while log x is the natural logarithm.

(i) sample median for x_0,

(ii) sample mean

$$\bar{x}=\frac{1}{n}\sum_{i=1}^{n}x_i$$

for m_x,

(iii) sample variance

$$\frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2=\frac{1}{n}\sum_{i=1}^{n}x_i^2-\bar{x}^2$$

for σ_x^2,

where $x_1, x_2, \cdots\cdots x_n$ are samples or the data given. Then substituting these estimates into Eq. (12), we obtain the estimates of the population parameters m, σ^2 and b.

For the logarithmico-normal distribution (8)～(10) expressed in terms of common logarithm, the parameters m and σ^2 in the above equations must be replaced by m/M and σ^2/M^2 respectively. Then we have

$$\left.\begin{aligned} &m=\mathrm{Log}\,(x_0-b)\ , \\ &\frac{\sigma^2}{2}=M\{\mathrm{Log}\,(m_x-b)-\mathrm{Log}\,(x_0-b)\}\ , \end{aligned}\right\} \qquad (13)$$

the equation satisfied by parameter b, being unaltered and being the same as the third equation of Eq. (12). For finding b, it is convienient to use the following equation instead of Eq. (12).

$$\left.\begin{aligned} &\xi^3+a_1\xi^2+a_2\xi+a_3=0\ , \\ &\text{where} \\ &a_1=-\frac{\alpha^2+\sigma_x^2}{2\alpha},\quad a_2=\sigma_x^2,\quad a_3=-\frac{\alpha}{2}\sigma_x^2\ ; \\ &\alpha=m_x-x_0. \qquad b=m_x-\xi. \end{aligned}\right\} \qquad (14)$$

An adjustment of sample median, by graphical interpolation, for example, will give better results in some cases.

(*b*) *The method of maximum likelihood.*

Let x_1, x_2, $\cdots\cdots\cdots$, x_n be a set of random samples. A function of the population parameters and of samples

$$L=f(x_1)f(x_2)\cdots\cdots\cdots f(x_n)$$

is called the likelihood function. The larger the likelihood L is, the more probable the set of samples may be obtained from the distribution with frequency function $f(x)$. The method of maximum likelihood gives the estimates of parameters which make the likelihood L maximum for the given samples.

The likelihood function for the logarithmico-normal distribution is

$$L=\frac{1}{(2\pi)^{n/2}}\left(\prod_{i=1}^{n}\frac{1}{x_i-b}\right)\frac{1}{\sigma^n}$$
$$\times\ exp\left\{-\frac{1}{2\sigma^2}\sum_{i=1}^{n}[\log(x_i-b)-m]^2\right\}\ . \tag{15}$$

Let $\hat{m}$, $\hat{\sigma}^2$ and $\hat{b}$ denote the maximum likelihood estimates of m, σ^2 and b respectively, which satisfy the following equations:

$$\frac{\partial\log L}{\partial m}=0,\quad \frac{\partial\log L}{\partial(\sigma^2)}=0,\quad \frac{\partial\log L}{\partial b}=0.$$

Then we have

$$\left.\begin{aligned}
&\hat{m}=\frac{1}{n}\sum_{i=1}^{n}\log(x_i-\hat{b})\ ,\\
&\hat{\sigma}^2=\frac{1}{n}\sum_{i=1}^{n}\{\log(x_i-\hat{b})\}^2-\hat{m}^2,\\
&\left(1-\frac{\hat{m}}{\hat{\sigma}^2}\right)\sum_{i=1}^{n}\frac{1}{x_i-\hat{b}}+\frac{1}{\hat{\sigma}^2}\sum_{i=1}^{n}\frac{\log(x_i-\hat{b})}{x_i-\hat{b}}=0\ .
\end{aligned}\right\} \tag{16}$$

It is not so easy to solve Eqs. (16). The method of linearization will furnish a practical way of approach to the problem by successive approximation.

Let b_0 be a first approximation of $\hat{b}$, which may be found by inspection or by some other means. The corresponding first approximations of $\hat{m}$ and $\hat{\sigma}^2$ are given by the equations

$$\left.\begin{aligned}
&m_0=\frac{1}{n}\sum_{i=1}^{n}\log(x_i-b_0)\ ,\\
&\sigma_0^2=\frac{1}{n}\sum_{i=1}^{n}\{\log(x_i-b_0)\}^2-m_0^2\ .
\end{aligned}\right\} \tag{17}$$

Putting

$$\hat{m}=m_0+\delta m,\quad \hat{\sigma}^2=\sigma_0^2+\delta\sigma^2,\quad \hat{b}=b_0+\delta b\ , \tag{18}$$

and thereby linearizing Eq. (16) with respect to the small correction terms δm, $\delta\sigma^2$ and δb, we obtain

$$\left.\begin{aligned}
&\delta m=-A_0\delta b\ ,\\
&\delta\sigma^2=2(m_0A_0-B_0)\delta b\ ,\\
&\delta b=-\frac{(\sigma_0^2-m_0)A_0+B_0}{A_0^2+\frac{2}{\sigma_0^2}(m_0A_0-B_0)^2-(m_0-\sigma_0^2+1)C_0+D_0}\ ,
\end{aligned}\right\} \tag{19}$$

where

$$\left.\begin{aligned}
&A_0=\frac{1}{n}\sum_{i=1}^{n}\frac{1}{x_i-b_0},\qquad B_0=\frac{1}{n}\sum_{i=1}^{n}\frac{\log(x_i-b_0)}{x_i-b_0},\\
&C_0=\frac{1}{n}\sum_{i=1}^{n}\left(\frac{1}{x_i-b_0}\right)^2,\quad D_0=\frac{1}{n}\sum_{i=1}^{n}\frac{\log(x_i-b_0)}{(x_i-b_0)^2}.
\end{aligned}\right\} \tag{20}$$

Adding the variations δm, $\delta\sigma^2$ and δb given by Eq. (19) to the first approximations m_0, σ_0^2 and b_0 respectively, we can find the second approximations of the parameters. If the first approximations are properly chosen, satisfactory approximations will be attained by this stage, and further processes of improving the approximation may be omitted.

When the common logarithm is used, the following equations will serve to find the estimates of parameters.

$$\left.\begin{aligned} m_0 &= \frac{1}{n}\sum_{i=1}^{n} \mathrm{Log}\,(x_i - b_0)\ , \\ \sigma_0^2 &= \frac{1}{n}\sum_{i=1}^{n} \{\mathrm{Log}\,(x_i - b_0)\}^2 - m_0^2\ ; \end{aligned}\right\} \tag{21}$$

$$\left.\begin{aligned} \delta m &= -A_0 M\,\delta b\ , \\ \delta\sigma^2 &= 2(m_0A_0 - B_0)M\,\delta b\ , \\ \delta b &= -\frac{\left(\frac{\sigma_0^2}{M} - m_0\right)A_0 + B_0}{MA_0^2 + \frac{2M}{\sigma_0^2}(m_0A_0 - B_0)^2 - \left(m_0 + M - \frac{\sigma_0^2}{M}\right)C_0 + D_0}\ ; \end{aligned}\right\} \tag{22}$$

where

$$\left.\begin{aligned} A_0 &= \frac{1}{n}\sum_{i=1}^{n}\frac{1}{x_i - b_0}, \quad B_0 = \frac{1}{n}\sum_{i=1}^{n}\frac{\mathrm{Log}\,(x_i - b_0)}{x_i - b_0}, \\ C_0 &= \frac{1}{n}\sum_{i=1}^{n}\left(\frac{1}{x_i - b_0}\right)^2, \quad D_0 = \frac{1}{n}\sum_{i=1}^{n}\frac{\mathrm{Log}\,(x_i - b_0)}{(x_i - b_0)^2}\ . \end{aligned}\right\} \tag{23}$$

A proper value of b_0 is slightly smaller than the smllest sample in most cases. The curve of non-exceeding probability of minimum flow shown in Fig. 2 was estimated by this method.

(3) Estimation of drought probability.

As the non-exceeding probability distribution of the drought flow on the critical day is thus obtained, the drought probability of the system can be estimated according to the distribution function $F(x)$. That is, if we take a flow of a_R as $F(a_R) = \frac{1}{R}$ from the graph of the drought probability, as shown in Fig. 2, for instance, then a flow less than a_R is expected to appear with probability $\frac{1}{R}$ or, in other words, the drought flow a_R is expected to appear once for R years. Thus a_R may be called the R-year drought flow with a return period of R years.

The principal values of drought flow which may serve as the standard values in planning are

(a) mean of drought flow $(\bar{x}) = exp\left(\frac{m}{M} + \frac{\sigma^2}{2M^2}\right) + b$,

(b) median of drought flow with a 50% probability $(x_0) = exp\left(\frac{m}{M}\right) + b$,

(c) mode or most probable value of drought flow $(\tilde{x})=exp\left(\frac{m}{M}-\frac{\sigma^2}{2M^2}\right)+b$.
All of them are shown in Fig. 2.

(4) Confidence limit of estimated distribution and fluctuation of data.

The R-year drought flow can be estimated as in the preceding Article (3) according to the monthly minimum flows observed on the annual critical days or of the annual minimum running average of the daily flow for N years, N being the sample size. Thereupon it may come into question to what extent we can confide in the estimated value for a rare drought (a_R for large value of R). The question will be emphasized by the fact that the records of daily stream flow are available for only five or ten years, or at most for twenty five years, in Japan. The authors intend to present a brief survey on this problem.

Let $x_1 \leqq x_2 \leqq \cdots\cdots \leqq x_n$ be a set of samples taken at random from a population with a distribution function $F(x)$. Then if we constitute a function

$$S_n(x)=\begin{cases} 0 & (x<x_1)\ , \\ \dfrac{r}{n} & (x_r \leqq x \leqq x_{r+1}\ ;\ r=1,\ 2,\cdots\cdots n-1)\ , \\ 1 & (x_n \leqq x)\ , \end{cases} \tag{24}$$

we can show that the probability α_n, whereby the following inequality holds: i. e.

$$S_n(x)-\frac{\lambda}{\sqrt{n}} \leqq F(x)\, S_n(x)+\frac{\lambda}{\sqrt{n}},\quad (\lambda>0)$$

is given as follows in the limit $n \to \infty$:

$$\alpha=\lim_{n\to\infty}\alpha_n=L(\lambda)\equiv\sum_{\nu=-\infty}^{\infty}(-1)^{\nu}e^{-2\nu^2\lambda^2}\ ,$$

$L(\lambda)$ being Dirichlet's series (Kolmogorov's theorem). Accordingly $\lambda/\sqrt{n}$ may taken for the confidence interval of $F(x)$ with a confficient $\alpha=L(\lambda)$ for sufficiently large value of n. As the estimated distribution is close to $S_n(x)$, $\lambda/\sqrt{n}$ may be considered as the confidence interval of population distribution regarding the estimated distribution.

Consequently, for a confidence coefficient $\alpha=L(\lambda)$, a drought flow corresponding to a non-exceeding probability less than $\lambda/\sqrt{n}$ seems to have only a slight significance in planning power-system economy. In Table I, $L(\lambda)$ shows the values of the confidence coefficient, while λ represents the corresponding values.

$L(\lambda)$	λ	$L(\lambda)$	λ
0.500	0.828	0.9500	1.358
0.750	1.019	0.9900	1.628
0.850	1.138	0.9950	1.731
0.900	1.224	0.9990	1.949

Table I

On the other hand, the recorded values of drought flow fluctuate about the curve representing the drought porbability $F(x)$. Again let $x_1 \leq x_2 \leq \cdots\cdots \leq x_n$ be a set of random samples from a distribution $F(x)$. It has been shown that the asymptotic distribution of an intermediate sample x_r $(0<\frac{r}{n}<1)$ for large n is a normal distribution with the population mean $\bar{x}_r(=\text{mode } \tilde{x}_r)$ and the standard variation

$$\sigma_r = \frac{1}{\sqrt{n}} \cdot \frac{\sqrt{F_r(1-F_r)}}{f_r}, \tag{25}$$

where $F_r \equiv F(\tilde{x}_r) = F(\bar{x}_r)$, $f_r \equiv f(\tilde{x}_r) = f(\bar{x}_r)$.[6] Accordingly, if the r-th sample is plotted at the point with the probability coordinate $F_r \equiv F(\tilde{x}_r) = F(\bar{x}_r)$, (i.e. Gumbel's plot), then the distribution of x_r is approximately $N(\tilde{x}_r, \sigma_r^2)$, whereas x_r is expected to fall in the interval $(\tilde{x}_r - \sigma_r, \ \tilde{x}_r + \sigma_r)$ with a 68% probability, or in $(\tilde{x}_r - 1.96\,\sigma_r, \ \tilde{x}_r + 1.96\,\sigma_r)$ with a 95% probability.[5] As σ_r depends upon only the probability F_r and the frequency f_r, the standard variation

$$\sigma = \frac{1}{\sqrt{n}} \ \frac{\sqrt{F(x)(1-F(x))}}{f(x)} \tag{26}$$

can readily be found to give the variation region of samples for any value of F (especially for $0.1<F<0.9$). The variation interval of samples must be essentially distinguished with the confidence interval of distribution. Fig. 4 shows an example of variation interval for a logarithmico-normal distribution. It will certainly be permitted to extend the idea to Thomas' plot, whereby the plotting position is $\overline{F(x_r)}$ instead of $F(x_r)$.

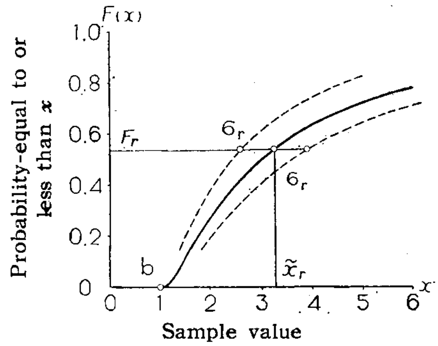

Fig. 4. Variation limit of samples.

V. A proposed definition of primary power.

In the preceding section III, we have already suggested that the probability curve of dearth of stream flow (i.e. the graph of $F(x)$) gives a theoretical basis for determining the primary power of a hydro electric power plant economically. Since the generating capacity of a plant corresponding to the flow discharge for maximum power output is designed for a stream flow several times as large as the annual minimum flow, both the power output (kW) and the energy output (kWh) of a plant decrease as the stream flow falls below the maximum usable flow discharge. Therefore, in order to secure a constant supply of electric power, a supplementary steam power plant must be installed, which can supplement deficiency of power supply in dry seasons.

In view of the essential responsibility of the electric power industry, ideal planning must be based on the drought flow with non-exceeding probability zero. However, since the present situation in Japan is as mentioned in Section II, the R-year drought flow a_R $\left(\frac{100}{R}\ \%\ \text{drought flow}\right)$ is necessarily taken as the basis of planning. Consequently, a drought flow less than a_R is expected to appear once for R years, the degree of water shortage being represented by the difference of a_R and the lower limit b. Herein, taking a_R as the planning basis, it means that the power supply corresponding to the flow discharge a_R is supplemented by steam plants. The authors, therefore, propose that the drought flow which is supplemented by steam power plants should be taken as the standard drought flow in planning a hydro electric power plant.

Now the supplementary steam power can be divided into the following two categories: one corresponds to drought flow with unity non-exceeding probability, the other to that with a probability less than unity. The former is required in all of N years and corresponds to so-called supplementary steam power, while the latter is operated in accordance with the drought probability and is the supplementary steam power for dry-year service. The supplementary steam power plants for dry-year use furnish the maximum power in a year in which the minimum stream flow corresponding to zero probability appears; while in a year in which the drought flow just takes the value corresponding to unity probability, they are out of operation.

Therefore, the choice or determination of primary power is nothing but the choice of power (kW) and energy (kWh) outputs of the supplementary steam power plants for dry-year service.

The power output of a hydro electric power plant must be compared with that of a supplementary steam power plant at the secondary bus of a primary substation situated in the suburbs of a large city, after substracting transmission losses and power consumed in the plant for house service.

The above discussions are developed exclusively for the run-of-river plants. For the pondage and storage plants, as described in the following Section VI, the auxiliary steam plants generally furnish only the deficient energy output (kWh). Their maximum capacity is determined by the duration of load division allotted to them on the daily load duration curve.

VI. Choice of primary power[1]

The particular items, which are to be taken into consideration in determining the primary power of a hydro electric power plant or power output of a supplementary steam power plant for dry-year service, vary necessarily depending upon the types of power generation.

(a) Run-of-river plant.

In order to secure perfect electric power supply even on the system's critical day, the primary power must be determined on the basis of the minimum stream flow with non-exceeding probability of zero, and the corresponding supplementary steam power must be prepared. This is the critical-year method which is adopted in some countries, wherein the power-system planning is based upon the minimum flow in the recent N years.

In Japan, the observation of daily stream flow has been made for only a few years. Hence, in establishing a long-term plan, the generating capacity of supplementary steam power plants for dry-year service must be determined allowing a margin to a certain extent over that corresponding to the smallest observation of stream flow, according to the confidence interval or the variation interval introduced in Section IV.

Moreover, a generating capacity equivalent to the capacity of one or two largest generating units of the power-system must be reserved as a stand-by for emergencies. Further, some margin is necessary for the future incease of load demand.

This is the ideal form of electric power industry. However, expansion of generating installations for the purpose of securing continuous service, and of improving reliability, will necessarily raise the original cost of electric power and become a greater burden to the consumers.

On the contrary, shortage of electric power supply in a dry year, appearing once in five to ten years, may be compensated to some extent by load-shift, (as practised now in Japan). Further, in a rarely dry year, which appears once in twenty or thirty years, we may overcome supply shortage by reducing either or both of the supply-voltage and frequency if possible. And there are some consumers who prefer a lower price for the partially interrupted power supply to a higher price for the continuous supply. Some people believe that to reserve generating facility for service in dry seasons of the driest year, and to keep a margin in addition, are excessive investments in Japan.

In such a case, determining of the primary power on the basis of drought flow of finite probability may be admissible.

Thus, the drought probability is in close relation with the power rate of electricity, while the maximum drought probability admissible for the standard drought flow in planning the primary power is to be fixed from the general view point of national economy. Here, one must take into consideration the allowable load-shift, reduction of voltage or frequency, partial interruption of supply, stand-by facility for peak-load hours, and a margin for the future load-expansion. Especially in dry years, reliability of power supply must be considered from the economical viewpoint, with partial interruption of the supply being sometimes inevitable.

If the primary power is determined on the basis of the daily-flow duration curve representing the relation of stream flow and the corresponding duration (days) during the N years, (instead of the above mentioned drought flow), then the drought flow for planning may be improperly underestimated, as the small observations in the driest year are inevitably exaggerated for the smaller values of non-exceeding probability.

Moreover, in estimating the risk of electric power supply shortage, the frequency of the dry years and the energy output (kWh) supplemented annually by steam plants are of greater importance than the total energy supplemented during N years.

Thus, the energy supplied by individual steam plant installed for supplementary use in dry years is calculated from the annual or monthly duration curves of daily flow in each of the dry years.

(b) Pondage plants.

Both power output (kW) and energy output (kWh) of a run-of-river plant decrease as stream flow falls below the planned drought flow in dry years.

The power output (kW) of a pondage plant does not necessariiy decrease even in dry years, while its energy output (kWh) is naturally reduced. Since the generating capacity of a pondage plant depends upon the daily duration (hours) of operation, its maximum power output (kW) may be kept constant irrespective of the degree of drought, if daily duration of operation can effectively be controlled by full utilization of the pondage according to the degree of stream flow shortage. In order to attain the perfect regulation as mentioned above, a pondage plant must be operated jointly with an auxiliary steam plant, changing the daily duration of operation of the latter with that of the former.

Consequently, the capacity of a pondage plant is to be determined on the basis of the minimum flow with zero probability, so that the plant carries a peak-load share of the daily duration curves. In rainy years, the generating hours of a pondage plant are to be prolonged by shifting a part of the load, (originally assigned to the auxiliary steam plants), to the pondage plant. Even in a rarely wet year, in which the annual minimum flow exceeds the drought flow with an estimated probability equal to unity the maximum power output (kW) of a pondage plant does not decrease if there is a sharing of the load.

If, however, the generating capacity of a power plant is determined in this way, the probability of auxiliary steam plants operating in peak-load hours increases with an increase of the planned drought probability. In such a situation, the generating cost becomes dear, as the pondage does not fulfil its essential function; and the steam power plant is necessarily operated under uneconomical conditions. In addition, newly planned pondage plants may have no load to carry, as a result of full utilization of the exsisting pondage plants.

On the contrary, if the primary power is determined on the basis of the drought flow with estimated non-exceeding probability equal to unity, so that the plant carries a peak-load share of the daily load duration curves, then even in dry years the pondage plant carries only the peak-load part. On the other hand, the auxiliary steam plants interconnected with the pondage plant carry the base-load part. Consequently, an economical power generation may be realized, and the pondage plant may fulfil its function satisfactorily in dry seasons.

In this case, however, the power output of a pondage plant is reduced according to the extent of dearth of stream flow. The rate of reduction depends upon the shape of the peak-load part of the daily load curves. Furthermore, the plant must be supplemented by a steam plant, whose capacity is smaller than that for a run-of-river type plant, and their joint operation is to be accomlished according to the principle described in Article (a). In the present case, however, since generating units of pondage plants suspended may be reserved as a stand-by for emergencies, extra steam plants prepared for emergencies may be omitted.

Therefore, the maximum capacity of a pondage plant is to be determined in accordance with the drought flow related to the daily load of shorter duration which is assigned to the plant. Here, one must take into account the reliability of power supply in connection with the electric power rate as well as the economical operation of the auxiliary steam power plant combined with the pondage plant.

The principles of N-year planning and the method of applying the N-year duration curve of daily flow are essentially the same as shown in Article (a).

(c) **Storage plants.**

Since all of the Japanese power-systems embrace the pondage plants carrying a peak load in dry seasons, the storage plants must carry a daily near-base load of longer duration.

In order to have an economical operation of steam power plants in dry seasons, as explained in Article (b), a careful consideration must be given in assigning a near-peak share of the daily load to the pondage plants, and for utilizing their full capacity throughout the N years. The maximum capacity of a storage plant is to be determined so that the plant carries a near-peak part of the daily load, (excluding the part assigned to the system's pondage plants in rarely dry years), and that it has a near-base load to carry which is equivalent to the maximum capacity even in rarely wet years. If such an ideal condition of joint operation is not materialized, the ratio of hydro electric power to steam power of the system as well as reservoir capacity of the storage plant must be throughly studied on the basis of drought probability.

VII. Conclusion

The primary power of the hydro electric power plant is to be determined from the general viewpoint of state economy on the basis of the non-exceeding probability of stream flows appearing in the critical days of the system in each of the N years, for the run-of-river plants. For the pondage or storage plants, the determination must be based on the running average of daily flow taken on the hydrograph over a certain interval, depending on the reserve capacity. At the same time, the reliability of electric power supply in connection with the power rate of electricity, as well as the conditions for economical operation of auxiliary steam power plants, must be considered.

The choice of primary power is nothing but the choice of power and energy output of the supplementary steam power plants for dry-year service.

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