Studies on the Power Requirement of Mixing Impellers (II)

—Method of Measuring Power Consumption—

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1. Introduction.

The power requirement of mixing impellers has been studied by many investigators, the results having been reported from time to time, but the results do not always agree well.[1] To clarify the reason why the disagreements do occur, it is necessary to check the equipments used for the experiments.

However, because of the lack of any detailed description about the dynamometers used, presumption may be made use of in some cases. In the experiments of power consumption, comparatively minute values in torque must be measured compared with the experiments in other mechanics. Therefore almost all the investigators have adopted their self-assembled devices, and there are some troubles of friction in bearing parts.

2. Classification of dynamometers used by the previous investigators.

2. 1. Method of measuring the power input for commutator motor.[2),4)]

By the method of subtracting copper-loss, iron-loss and input power at no load from the observed power, net power input for motor is calculated. But it seems to be difficult to get precise values.

2. 2. Method of measuring torque produced.

2. 2. 1 Torque measurement by a turn table.

This is a method to bring out a balance on the torque produced on the side of the vessel by the rotation of the impeller in liquid. In this method vessels are to be placed on a frictionless turn-table.

2. 2. 2 Method of measuring reaction torque on a motor.

This is a method of measuring reaction torque worked upon the stator of driving motor. The equipment shown by Fig. 2. is an example made by the authors and was used at an early step of their experiments. The direct connection of an agitator shaft with a motor axis makes it difficult to get low rotation in case of ordinal motor, and the range of rotation that can change is comparatively small. These are the defects accompanying this method.

2. 3. Method of torsion dynamometer.

In this method, torque is measured by the elongation of a spring.

2. 3. 1 A dynamometer with a coiled spring[3) 11)] (See Fig. 3).

2. 3. 2 A dynamometer with a spiral spring[12) 13)] (See Fig. 5).

The method of reading the degree of elongation of a spring during the rotation differs more or less according to investigators. They have exerted themselves to assemble the dynamometers of lower static friction.

2. 3. 3 Strain gauge dynamometer[14)].

This is a method of measuring axial torsion of the rotating shaft by a strain gauge and is suitable for relatively large power measurements.

2. 3. 4 Differential gear method[15)].

The results of Rushton and his co-workers on power requirement of mixing impellers are important. The arrangement of measuring power requirement is shown by photographs, but the clear-cut details are not obtained. By the speculation of the present authors, the mechanism of the dynamometer used by Rushton may be shown graphically as in Fig. 1.

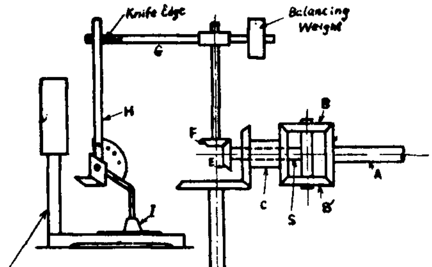

Fig. 1. Schematic diagram showing the construction of differential gear dynamometer by Rushton.

A shaft A drives C through bevel gears B and B′ and then the force is transmitted to the impeller shaft D.

A shaft S, perpendicular to the axis of B and B′, penetrate through the axis of gear C and is provided with the bevel gear E, which is coupled with the another bevel F. The axis of the gear F is transmitted vertically to a lever G. A knife edge which is found on one end of lever G, being combined with H, pushes a panel of an automatic balance at I.

The shaft S, being hampered by a pressure of I upon the balance panel, can not rotate freely. In other words, when S is fixed, the rotation of A is transmitted to C through B and B′ and is able to drive D. However, when the shaft S can rotate freely, B and B′ will rotate around C, and the shaft C and D will be kept stand still. Some resistance on D produces a force to rotate S and exerts a force on the balance through E, F, G, H and I.

Rushton and his co-workers measured the power of 8 H.P. as a highest value, in a tank of 2 m in diameter.

3. Dynamometers designed by the authors.

According to the progress of research of the present authors, three kinds of dynamometer were used. It can be concluded that the static friction of dynamometer become a source of large errors for power data. Especially, it is required to use a dynamometer having negligible static friction in order to discuss the effect of Froude number upon power consumption of impeller as mentioned later.

3. 1. Dynamometer No. 1.

This dynamometer is a type of measuring reaction torque, mentioned previously (**2. 2. 2**) and is shown by Fig. 2. B_1, B_2 are ball bearings with slight friction and suspend the weight of commutator motor of 1/20 H.P.. The lead wires to introduce electric current to the motor are dipped and slide freely in two ditches filled with mercury to minimize the friction.

The reaction torque working upon the stator of the motor can be balanced by weights or a spring balance through a fine fishing catgut. Torque required to rotate the motor as a whole, i.e., torque necessary to overcome the friction at bearing parts B_1 and B_2, is called static friction torque and denoted as *Ts*. *Ts* value of this dynamometer is about 20～30 g. cm.

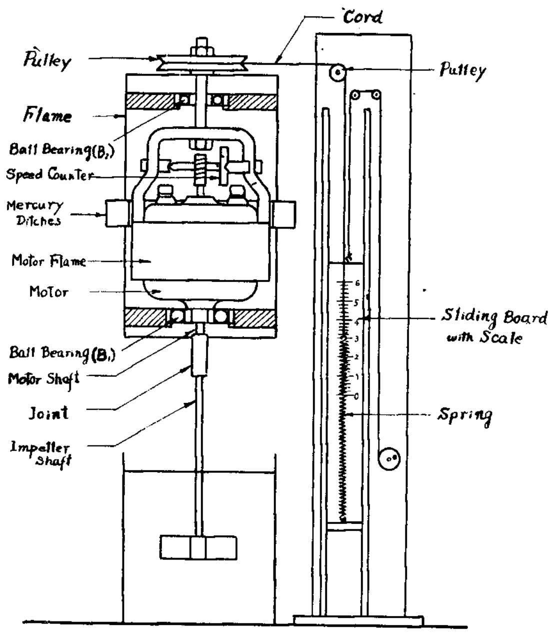

Fig. 2. Dynamometer No. 1.
(Reaction torque dynamometer)

3. 2. Dynamometer No. 2 A (Inductance gauge type).

This dynamometer is a sort of torsion dynamometer explained previously (**2. 3. 1**) and shown by Fig. 3. Two discs are set face to face, and connected by four coiled springs at 90° intervals as shown by the figure.

The upper disc is connected directly to a driving shaft.
The shaft of the lower disc passes through the hollow axis of the upper one, and can rotate concentrically on two ball bearings selected precisely.

To measure the angle of displacement between two rotating discs, the method of inductance gauge is adopted as shown by Fig. 4. A coil is fixed on the upper disc and through this coil, an iron core which is fixed on the lower disc can shift in and out freely as shown by Fig. 3 and 4. Corresponding to the elongation of the springs, the inductance of the coil may vary. This angle of displacement can be read by a microammeter (μA). The static friction (Ts) for this dynamometer can be determined by taking out the spring and measuring the minimum torque required to rotate the agitator shaft.

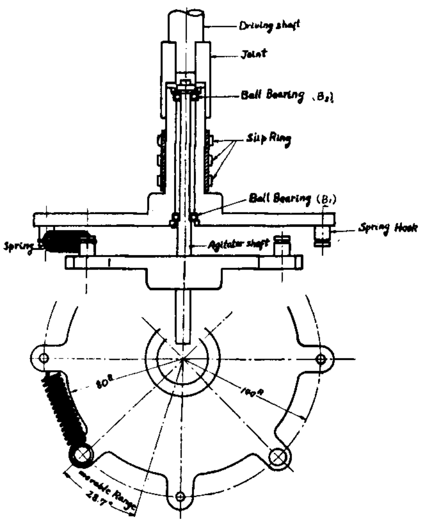

Fig. 3. Dynamometer No. 2A. (Inductance gauge type)

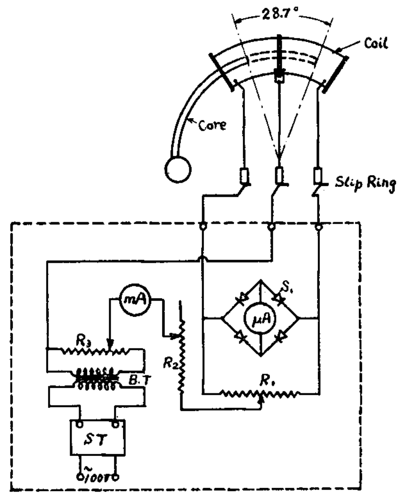

Fig. 4. Schematic diagram showing the method of measuring the displacement angles of dynamometer No. 2A.

The static friction torque Ts of this dynamometer No. 2A is 10~20 g.cm, (15 g.cm in average). This value is minute enough compared with torque to be measured.

3. 3. Dynamometer No. 2B (Sliding rheostat type).

This is a dynamometer similar to No. 2A. The mechanism of measuring angle of displacement between two rotating discs is different. Namely, an electric rheostat is fastened to the upper disc and a brush fastened on to the lower disc slides over the surface of this rheostat. By this means, Ts value becomes larger than that of type A. But the authors dared to use this type B to compare the results with those of type A so as to be able to clear up the error of observed power values caused by static friction torque as is to be shown later. Static friction torque Ts of the dynamometer No. 2B is 150∼200 g. cm (175 g. cm in average).

3. 4. Dynamometer No. 3.

This dynamometer is designed to be negligible in static friction, as shown by Fig. 5 and used to measure relatively minute power especially for a propeller type impeller. As shown by the figure, this dynamometer is provided with one easily replaceable spiral spring (s) and is an example of the type mentioned above (**2. 3. 2**). The angle of displacement can be read by a stroboscopic method.

For this apparatus, the authors exerted themselves to minimize static friction. Namely, the driving parts, as well as the agitator, must be of very light weight, therefore the authors prefered 6 mm ball bearing for the lower (B_1) and a pivot adjusted by a jewel for the upper pedestal (B_2). Thus the static friction Ts of this dynamometer can be lowered to one tenth of that of the type No. 2A, i.e., to 1 g. cm.

But the upper limit of the power that can be measured, is found to be lower compared with No. 2A.

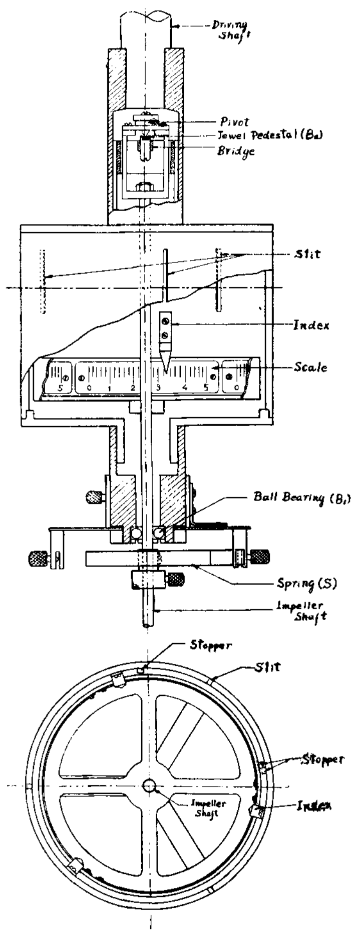

Fig. 5. Dynamometer No. 3.

4. Causes of error in power measurement.

There are many causes that may bring about errors in power measurement. Among them, discussion will be confined to the following two causes which are looked on as being very important.

(1) Errors caused by no load dynamic friction.

(2) Errors caused by static friction in torque measuring device, (errors by static friction).

4. 1. Errors caused by no load dynamic friction.

By mechanical contact of an impeller shaft with any fixed part before being put into liquid to be agitated, loss of power may be brought about. To avoid this error, it is necessary to deduct the value gained by no load measurement from the observed value.

Data obtained by an apparatus used by Olney[8], Stoops[11] and Mack[12] will contain such an error as this.

Frictions on the three slip rings of rotor in a self-synchronous motor used by Ōyama[13] may bring out this error. In Rushton's dynamometer[15], the error caused by the friction of a train of gearing A, B, C and D belongs to this category.

4. 2. Errors caused by static friction.

Static friction is that caused by moving part contained in the torque measuring system.

For example, friction on the ball bearing of turn table in (**2. 2. 1**), that on the bearing to suspend the weight of driving motor in Fig. 2, and that on the ball bearing B_1 or B_2, in Fig. 3 etc., all these belong to this category.

As a special case, in the differential gear method of Rushton shown by Fig. 1, friction in the train linkage S, E, F, G, H and I for the torque measurement belongs to the static friction.

5. Effect of static friction on power measurement.

The results of the power data measured by the dynamometer No. 2A and No. 2B are compared graphically in Fig. 6. As shown by this figure, the power data measured by the type B (drawn by dotted lines) agree well with the data measured by the type A (drawn by solid lines) at the higher ranges in power, but these two lines deviate greatly at the lower ranges of power and the power data of the type B show higher values than those of the type A.

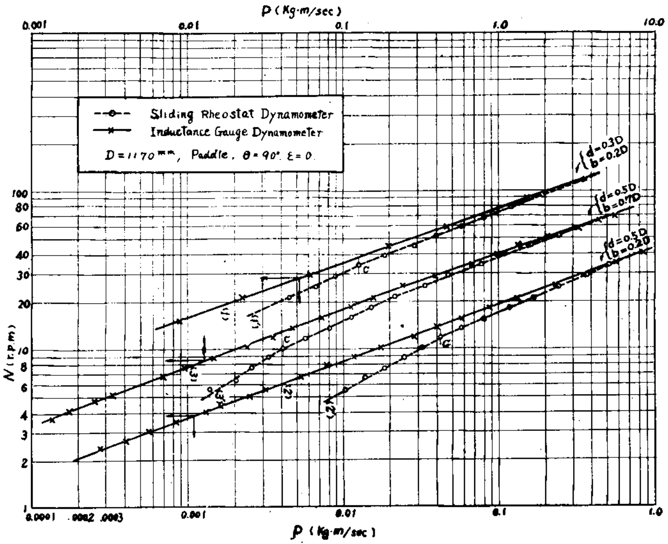

Fig. 6. Comparison of power data measured by sliding rheostat- and inductance gauge-dynamometer.

There is a bend shown by point C on the dotted line in Fig. 6. Determining such points under several conditions, the results of Table 1 are obtained. N_f, T_f and P_f in the table are the number of rotation, the torque and the power at the bend point C respectively.

Table 1. Values of N_f, T_f and P_f at the bend point C on the plots of N vs P measured with a sliding rheostat type dynamometer (No. 2 B).
Average static friction torque T_s=175 g. cm
Range of measuring torque=1,000~40,000 g. cm
D=Vessel diameter, d=Impeller diameter, b=Impeller width

Liquid used	D (cm)	d/D	b/D	N_f (r.p.m.)	T_f (g. cm.)	P_f (Kg. m./sec)
Water	117	0.5	0.2	12	3340	0.042
"	117	0.3	0.2	34	3580	0.128
"	117	0.3	0.7	34.6	3530	0.128
"	117	0.5	0.7	10.25	3770	0.0406
"	117	0.5	0.05	21.6	4000	0.0907
Machine oil 1.30 poise	58.5	0.3	0.7	79	2530	0.209
Machine oil 10 poise	58.5	0.5	0.2	34.2	4700	0.168
"	58.5	0.5	0.7	15	2670	0.042

In this table, no special relations can be found in the values of N_f and P_f. But, T_f shows nearly constant value ranging from 2,500 to 4,700 g. cm and corresponds to the value about 20 times as large in comparison with the static friction Torque Ts. From this fact, it may be possible to express the extent of error by the scale of static friction torque.

Now, let the reason be discussed why power shows larger value than the correct one by static friction.

Torque presented at the beginning of agitation is extraordinarily large in general but it begins to decrease as the swirling motion is increased until finally it reaches a constant value at steady state.

Catching up with the torque, the pointer of the meter goes down to an equilibrium position which is so much higher as corresponding to static friction, and the pointer does not get back to its correct position. This higher reading is denoted by T_h. On the contrary, at the state of an approximately stationary state, the reading of meter (i.e., the elongation of spring) can be lowered by a temporary increase in rotation of the shaft of agitator side by grasping and accelerating the impeller speed. After the operation, let the shaft be free; then the meter reading begins to increase up again. However, it can never get back to the correct position, but stops at a lower point than correct according to static friction. This lower reading is denoted by T_l.

On the measurement of power consumption of agitators, the difference between T_h and T_l is large in type B dynamometer but is very little in type A.

Therefore, by the method of measuring power with step up rotation, a larger value corresponding to static friction will be observed, for the initial torque is larger than that of stationary state.

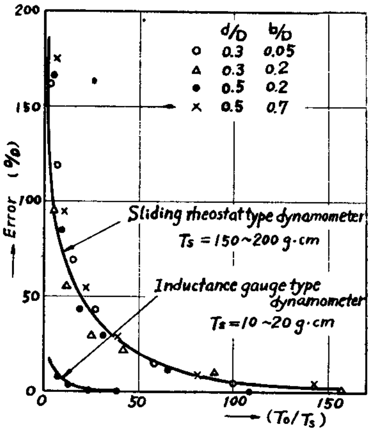

Fig. 7. Errors caused by static friction.

Denote the mean in T_h and T_l measured by the No. 2A dynamometer as T_0. This value T_0 may be taken as correct. In the type A dynamometer, the torque T_A greater than $T_s \times 20 = 15 \times 20 = 300$ g. cm, is equal to T_0, in other words T_h is equal to T_l, therefore the observed data can be fixed.

By drawing the curve $\{(T_h - T_0)/T_0\} \cdot (100)$ as ordinate versus (T_0/T_s) as abscissa, Fig. 7 is obtained.

Table 2 shows an example of calculation on the error of data obtained by the dynamometer No. 2B for the paddle of $d=0.5$ D, $b=0.2$ D. In this case T_0 can be taken as being equal to T_A.

Table 2. Sample calculation on the error of data obtained by the dynamometer No. 2B.
Vessel diameter $D=117$ cm.
Liquid used=water
Static friction torque for No. 2B dynamometer $(T_s)_B=175$ g. cm
Static friction torque for No. 2A dynamometer $(T_s)_A=15$ g. cm

Agitator speed N(r.p.m.)	Sliding rheostat type (No. 2B)		Inductance gauge type (No. 2A)		$T_B - T_0$ $(\doteqdot T_B - T_A)$	Error % $\{(T_B - T_0)/T_0\} \cdot 100$	$T_0/(T_s)_B$
	Power P_B (Kg.m/sec)	Torque T_B (g.cm)	Power P_A (Kg.m/sec)	Torque T_A (g.cm)			
5	0.0084	1605	0.0024	460	1145	249	2.63
7	0.0160	2185	0.0060	820	1365	166.5	4.69
10	0.0305	2900	0.0165	1570	1330	84.7	8.96
15	0.072	4590	0.050	3190	1400	43.9	18.2
20	0.145	6950	0.112	5360	1590	29.7	30.7
30	0.400	12720	0.360	11450	1270	11.1	65.4
40	0.800	19100	0.800	19100	0	0	109.0

Just as clearly shown by Fig. 7, the exact value of torque can only be obtained when the torque to be measured is more than twenty times larger, that is, more than 300 g. cm in the case of type A dynamometer whose static friction T_s is equal to 15 g. cm.

In the case of type B dynamometer whose static friction torque T_s is equal to

175 g. cm, a reliable value can never be obtained unless the observed values are greater than 100~150 times as large compared with T_s, i.e., above 1.75×10^4~2.63×10^4 g. cm.

6. Effect of Froude number on power data.

Rushton and his co-workers revealed the effect of Froude number upon power number as a term $(dn^2/g)^{(a-\log Re)/b}$ and presented the numerical values of the factor a and b for propeller-and turbine-agitators[15].

By the experiments carried out by the present authors, the plotted curves of N_p versus R_e for a paddle agitator are shown in Fig. 8.

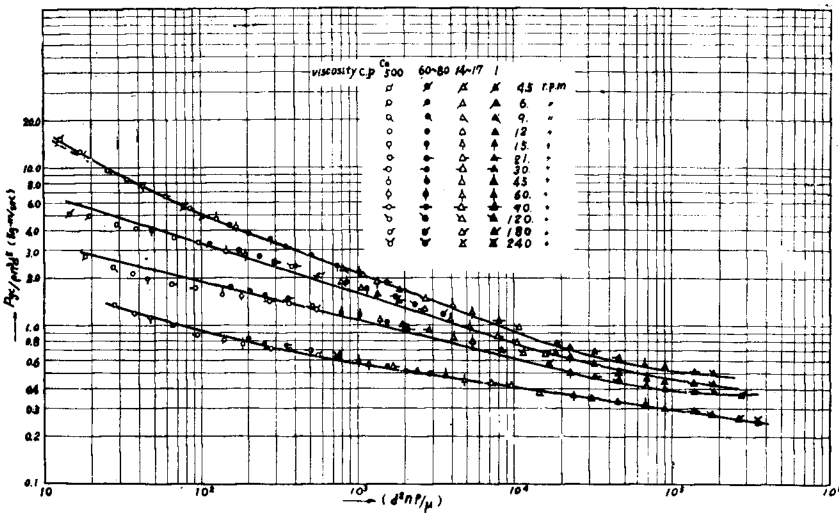

Fig. 8. Diagrams showing *Np* vs *Re*. (Effect of Froude number)

Plotted points for liquids of several viscosities lie almost on one curve, and the correction for Froude number becomes negligible. Of course, in such a system as agitated vessel that has a free liquid surface, theoretically speaking, it may be correct to consider the Froude number effect. However, to take up the effect of Froude number term, the accuracy of dynamometer should be very high.

At the present state, the curves of plotted points of N_p versus R_e for liquids of several viscosities overlap one another so that the correction for Froude number becomes useless so far as the technical accuracy is concerned.

Fig. 9 shows the relation of N_p versus R_e where curves (1), (2), (3) are the experimental results of Rushton's marine propeller. If these results be correct, it is necessary to consider the correction for Froude number. But the results of the present authors for the similar type of impeller obtained by the dynamometer with least static friction (No. 3) are presented by curves (4), and the effect of Froude number seems to be negligible.

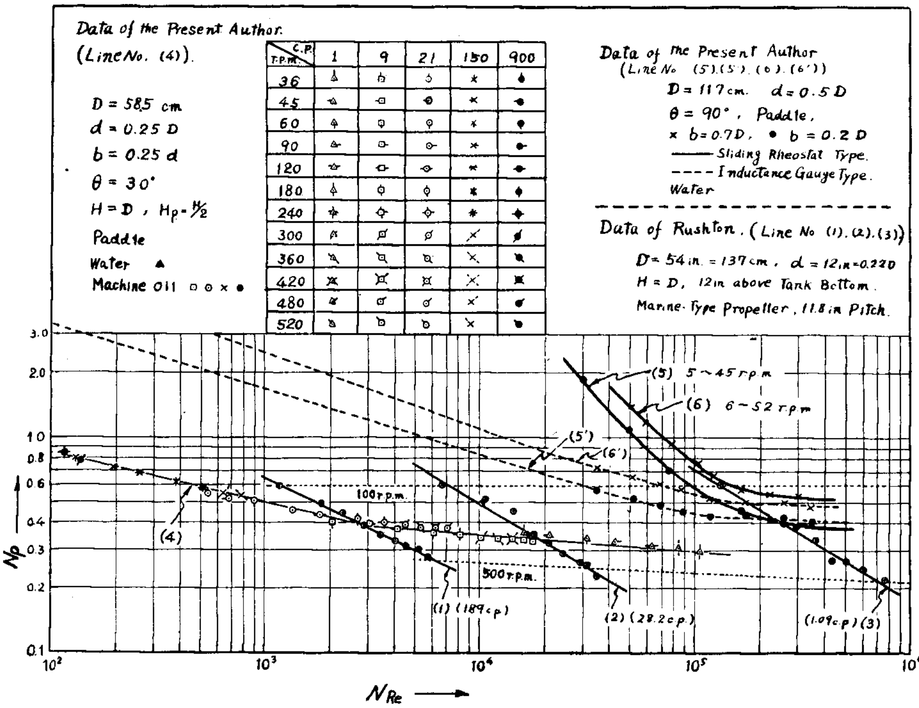

Fig. 9. Comparison of the data by Rushton with those by the present authors and the effects of error caused by static friction.

If the power data shown by curve (2′) and (3′) in Fig. 6 which are measured by the dynamometer of sliding rheostat type (No. 2B) are drawn on Fig. 9, the plotted points are shown by curves (5) and (6) which are similar to those of Rushton's. While the power data shown by curve (2) and (3) in Fig. 6 which are measured by the dynamometer of inductance gauge type (No. 2A) are replotted on Fig. 9, curves (5′) and (6′) are obtained.

Judging from the results gained in this way, it can be said that the observed values by Rushton et. al. may include errors caused by static friction so that the correction term for Froude number by them may be erroneous.

7. Conclusion.

Classification of the equipments to measure power requirement used by previous investigators was done.

The causes of the errors produced by the measuring equipments were discussed.

As the result, it was concluded that static friction should be a serious cause of error and the correction term for Froude number proposed by Rushton et. al. would contain the errors that can be traced back to this source.

Notations used

D : Vessel diameter cm
d : Impeller diameter cm
b : Impeller width cm
$N_p = Pgc/\rho n^3 d^5$: Power number —
$R_e = d^2 n\rho/\mu$: Reynolds number —
$F_r = dn^2/g$: Froude number —
N : Impeller speed in r.p.m. 1/min
n : Impeller speed in r.p.s. 1/sec.
P : Power consumption of impeller Kg.m/sec.
T : Torque of impeller g.cm
T_s : Static friction torque g.cm
T_0 : Correct torque g.cm
suffix f : Values at the bend point C in Fig. 6

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