Hold-up in a Wetted Wall Tower

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1. Introduction

The hold-up in a wetted wall tower is closely related to the mechanical operation of the tower and to the heat and mass transfer rate in the tower as well as the hold-up in a plate tower and a packed tower.

Many inquiries have been made, theoretically and experimentally, into the hold-up without gas flow, but, as will be mentioned later, certain problems remain unsolved. Moreover the hold-up with gas flow has not been investigated, because suitable methods for measurements have not been known.

The authors made their experiments with an accurate new method, a so-called balancing tower method, and investigated the relationship between hold-up and various factors about the tower and the liquid used. This method was applicable both with gas flow as well as without.

2. Previous Investigations

Nusselt[11)] engaged in theoretical studies of the flow of a falling liquid film. He found that the thickness of the laminar liquid film correlated with the flow rate and the physical constants of the liquid by the following equations,

$$B=(3L'\mu_L/g\rho_L^2)^{1/3}=(3\mu_L^2/4g\rho_L^2)^{1/3}\quad Re_L^{1/3} \tag{1}$$

on a vertical plane wall, and

$$B(1-2B/d)^{1/3}=(3\mu_L^2/4g\rho_L^2)^{1/3}\quad Re_L^{1/3} \tag{2}$$

on an inner wall of a vertical pipe,
where

$$Re_L=4L'/\mu_L=4L/\pi d\mu_L\,.$$

The experimental studies which have been done previously may be classified into two groups according to the method used. One method is by direct measurement of film thickness with a micrometer, which was adopted by Hopf⁽7⁾, Chwang⁽1⁾, Schoklitsch⁽12⁾, Kirkbride⁽10⁾ et al. Kirkbride said that the theory of Nusselt held good in the region of $Re_L<8$, but the observed values of the thickness at $Re_L>8$ were greater than the theoretical value because of the appearance of ripples on the surface of the liquid film. But this method seems unreliable, because it is not the average film thickness which is measured but the height of the crests of the ripples, and it is difficult to see whether the pointer of the micrometer exactly touches the surface of the film. Dukler & Bergelin⁽4⁾, by the electrical method, obtained results which indicated that the observed values of the thickness at $Re_L<1000$ satisfied the theory and the values at $Re_L>1000$ agreed with the following semi-theoretical equation derived by them.

$$(Re_L/4)+64=2.5Y\ln Y+3Y, \tag{3}$$

where

$$Y=(g\rho_L{}^2 B^3/\mu_L{}^2)^{1/2}.$$

The other experimental method is by direct measurement of the weight or the volume of the liquid remaining in the tower when the feed and the exhaust of the liquid are stopped simultaneously. This method was used by Claassen⁽2⁾, Warden⁽13⁾, Fallah, Hunter & Nash⁽5⁾, Cooper & Willey⁽3⁾, Friedman & Miller⁽6⁾ et al., and all of them reported that the theory was correct at $Re_L<1000\sim1500$. But this method is also inaccurate because it is difficult to stop simultaneously both the feed and the exhaust. Friedman & Miller, moreover, measured the surface velocity of the liquid film, and obtained values greater than the theoretical at $Re_L>20\sim30$. They attributed the discrepancy of the observed and theoretical values to the ripples generated on the surface of the film, as Kirkbride had done, and called the flow of the film at $Re_L=25\sim1000$ a pseudo-streamline flow.

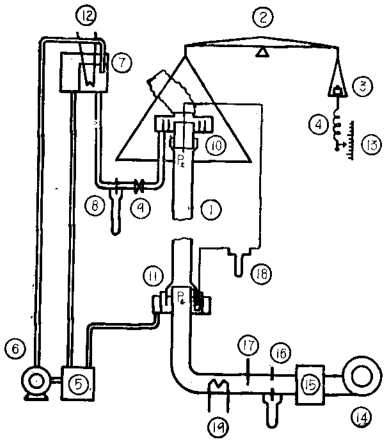
Nothing has been reported about the hold-up with gas flow, although hold-up without gas flow has been thoroughly investigated. That is because the apparatus and the method of measurements mentioned above are not applicable in this case.

3. Apparptus and Procedure

In the authors' investigations, the hold-up was weighed while the liquid was falling vertically along the wall both in the case without gas flow and with counter-current gas flow.

Fig. 1 is a schematic diagram of the apparatus used. Seven different sized towers made of glass tube were used, and their principal dimensions are shown in Table 1. The tower was suspended from one end of the beam of a balance, and at the other end a pan a and spring were attached.

In the case where there is no gas flow, the dry wall tower was first balanced and



① Tower
② Balance
③ Pan
④ Spring
⑤ Lower liquid tank
⑥ Pump
⑦ Upper liquid tank
⑧ Liquid flowmeter
⑨ Liquid flow regulating valve
⑩ Liquid inlet plate
⑪ Liquid outlet plate
⑫ Liquid heater
⑬ Scale
⑭ Blower
⑮ Gas tank
⑯ Gas flowmeter
⑰ Damper
⑱ Micromanometer
⑲ Gas heater

Fig. 1. Apparatus.

Table 1. Dimensions of Towers.

Tower	Diameter (cm)	Length (cm)	Tower	Diameter (cm)	Length (cm)
T_1	4.91	ca. 250	t_1	5.09	ca. 100
T_2	4.18	"	t_2	4.22	"
T_3	3.16	"	t_3	2.97	"
T_4	1.90	"			

then fed with liquid. The increased weight of the tower due to the weight of the liquid film flowing on the wall caused the tower to become unbalanced. The sum of the additional weights and the tension of the spring added necessary to balance the tower gave the weight of the hold-up. The sensitivity of this balance was 0.5 gr.

In the case where gas flows countercurrently, the same method was used and in addition the pressure drop of the gas flow between the two points p_b and p_t was measured. The upward force which was given to the tower by the gas flow can be represented as follows.

$$F=(p_b-P)(S'-S)+(p_b-p_t)S \tag{4}$$

The weight of the hold-up per unit wetted area, therefore, was

$$H=(W+F)/\pi dl\,. \tag{5}$$

However with countercurrent gas flow the weight necessary to balance the tower was unstable and so the measurements were rougher than those without gas flow.

In both cases the relation between H and B is

$$H=\rho_L B(1-B/d)\,. \tag{6}$$

The liquids used for this experiment were water, soapless soap solution and millet-jelly solution. The density, viscosity and surface tension of the liquids varied, respectively from 1.0 to 1.26 gr/cm^3, from 0.011 to 0.562 poise and from 33.3 to 77.9 dyne/cm. The liquid flow rate per unit wetted periphery of the tower was 0.06 to 4.4 gr/cm sec, that is Re_L was 4 to 16,700. The gas was air only.

4. Results and Discussion (I)

—Hold-up without air flow—

(1) Conditions of the surface of the falling liquid film.

The ripples appeared on the surface of the liquid film irrespective of the liquid flow rate. It is due to the disturbance of the liquid at the time of its feed and also to the friction with air at the surface. The lower the surface tension of the liquid, the fewer the ripples to the point where they could hardly be seen, and this was more marked at the low Re_L. The influence of the viscosity was not so distinct as the surface tension, but the distance between the crests of the ripples grew larger with increase in viscosity.

The relation between B and Re_L is, as mentioned later, divided into that in the laminar flow region and in the turbulent flow region. Perfect laminar flow cannot be expected to exist because of the turbulence caused by the ripples except in the case of a very low flow rate, and thus the theory of Nusselt does not seem to hold good. The effect of the turbulence will become more marked in the turbulent flow region. The factors which have an influence on B may be d, L, μ_L, ρ_L, ρ_L, σ_L and g in both regions.

(2) Variation of the film thickness along the current of the liquid.

The falling liquid film that has been discussed is assumed to be of constant thickness. Eqs. (1), (2) and (3) represent the thickness of such a film. But near the top and the bottom of the tower the liquid flow is accerelated or decelerated due to the influence of the feed and the exhaust of the liquid, and so the film thickness may well vary along the tower.

In order to know this variation the thickness must be measured directly. With the method used here, only its average value over both parts of constant film thickness and of varying film thickness can be obtained. But the difference of the weights of the hold-up in two towers which have equal diameters and different lengths gives the weight of the hold-up in the middle part of the tower where the film thickness is constant. Thus, by comparing the thickness calculated from the hold-up of this part with that of the whole part of the tower, the general character of the variation of the thickness can be known. Fig. 2 shows the comparison of the film thickness in the towers T_1 and t_1. The film thickness in the tower T_1 is slightly greater than that in the tower t_1 and is almost equal to that in the middle part. This means that

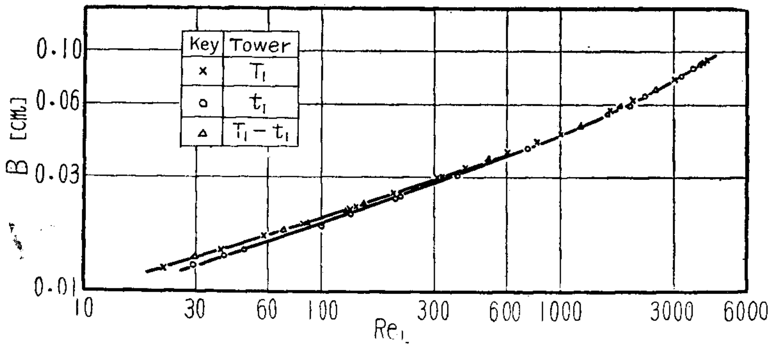


Fig. 2. Variation of film thickness along current of liquid.

the variation in film thickness has some influence on the average value of B in the tower of about 100 cm long, but need not be taken into consideration in the tower of about 250 cm long. All discussions from now, therefore, concern only the long towers T_1, T_2, T_3 and T_4.

(3) Influence of the diameter of the tower

In the theory of Nusselt, the effect of the diameter in the same Re_L is represented by the term $(1-2B/d)^{1/3}$ as eq. (2) shows. This was compared with the observed values.

Fig. 3 shows the plots of $B(1-2B/d)^{1/3}$ against L/d in the towers of various diameters on a log-log paper. The plotted points lie on a line and so the influence of d may be represented by the same term $(1-2B/d)^{1/3}$ as in the theory, regardless of the condition of the surface of the liquid film.

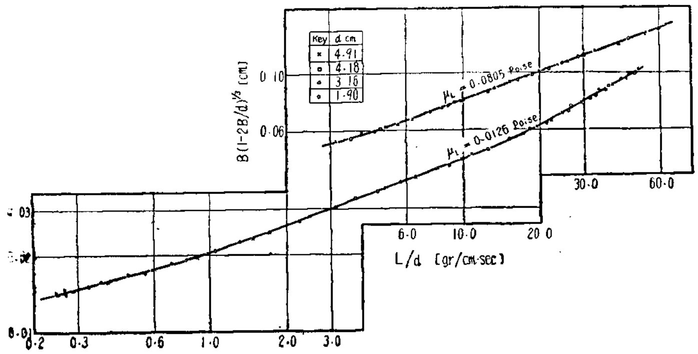


Fig. 3. Influence of tower diameter on film thickness.

The variation of the thickness of the film with the diameter of the tower should exist also in the case of turbulent flow. The plots at $L/d > 16.0$ i.e. $Re_L > 1650$ with the liquid having $\mu_L = 0.0126$ poise in Fig. 3 shows the data for the turbulent liquid film. As the plotted points lie on a line, it may be said the correction term $(1-2B/d)^{1/3}$ is also applicable for the turbulent liquid film.

(4) Relation between the film thickness, the flow rate and the physical properties of the liquid.

Fig. 4 shows the plots of $B(1-2B/d)^{1/3}$ against Re_L in the laminar flow region of the various liquids. The straight lines N corresponding to each μ_L/ρ_L represent eq. (2) and have a slope of 1/3. The plots of the points of the same μ_L/ρ_L can be distinguished by σ_L, which is enlarged in Fig. 5. The following matters are with reference to this figure.

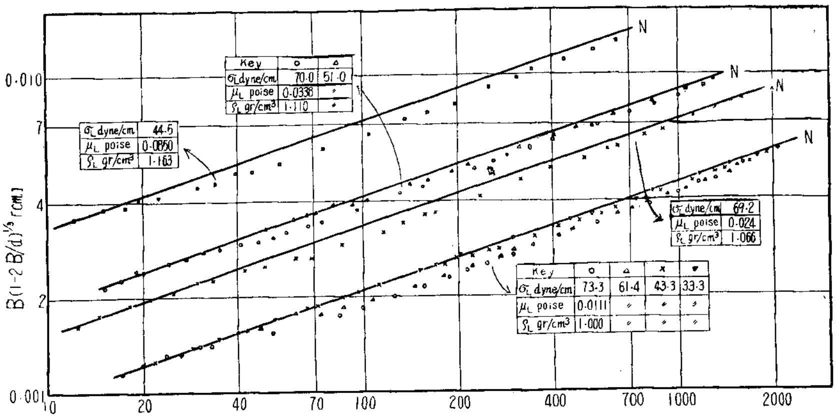


Fig. 4. Correlation between $B(1-2B/d)^{1/3}$ and Re_L in perfect laminar and pseudo-laminar regions.

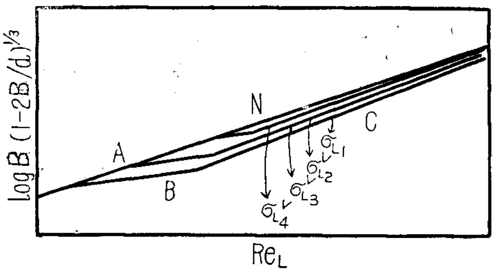


Fig. 5. Schematic diagram of Fig. 4.

(i) In the part A where Re_L is very small, the plotted points lie on a straight line N corresponding to μ_L/ρ_L of the liquid and are independent of σ_L. This part is considered to be a perfect laminar region.

(ii) As Re_L increases, the points deviate downwards from the line N and lie on a curve concave upwards. This curve can be represented by two straight lines B and C. The slope of the line B is less than 1/3 and that of the line C is greater than 1/3. This part is influenced by the ripples and is considered to be pseudo-laminar as was suggested by Friedman & Miller.

(iii) Re_L at the point of intersection of the parts A and B increases rapidly with the decrease of σ_L. In the case of water this critical Re_L is 30 to 40.

(iv) The degree of deviation from the line N in the pseudo-laminar region varies according to σ_L and becomes smaller as σ_L decreases. Generally the surface tension of the liquid is closely related to the generation of the ripples. As the liquid film flows vertically in a wetted wall tower, it is only the surface tension that generates the horizontal waves. As described in (1), the ripples are practically negligible when σ_L is very small, and therefore the flow of the liquid is nearly perfect laminar. The observed values about such a liquid may agree with the theoretical line.

(v) The same thing is observed with the liquid of large μ_L/ρ_L. The plotted points approach the line N as μ_L/ρ_L increases, and the critical Re_L between the perfect laminar and pseudo-laminar regions decreases with the increase of μ_L/ρ_L. The authors are assured that the kinematic viscosity has a function of pacifying the ripples and the generation of the ripples has a direct relation with the velocity of the liquid film.

From this observation it is considered to be convenient to represent the film thickness in the pseudo-laminar region, especially in the part C, by the following type of equation in order to compare it with the theoretical equation.

$$B(1-2B/d)^{1/3}=B_{Nu}(1-2B_{Nu}/d)^{1/3}(1-\varphi)Re_L^{\psi} \tag{7}$$

Both φ and ψ are functions of σ_L, μ_L and ρ_L and should satisfy the following conditions according to the statement in (iv).

$$\lim_{\sigma_L\to 0}\varphi=0 \qquad \lim_{\sigma_L\to 0}\psi=0 \tag{8}$$

φ and ψ are expected to be dimensionless, but calculated from the data, φ depends on σ_L, μ_L and ρ_L, ψ is a function of σ_L only, and they can not be represented as dimensionless forms. Substituting these φ and ψ in eq. (7) gives

$$B(1-2B/d)^{1/3}=B_{Nu}(1-2B_{Nu}/d)^{1/3}(1-5.8\times10^{-6}\sigma_L{}^{2.38}\,\rho_L{}^{0.1}\,\mu_L{}^{-0.1})\times Re_L^{8\times10^{-8}\sigma_L^3} \tag{9}$$

The critical Re_L between the perfect laminar and pseudo-laminar regions is given by the equation,

$$Re_{LC_1}=1.63\times10^7\,\sigma_L{}^{-3.43}(\rho_L/\mu_L)^{0.398} \tag*{(10)$_1$}$$

The surface velocity of the liquid film is

$$u_{LS}=(9g/128)^{1/3}\,(\mu_L/\rho_L)^{1/3}\quad Re_L{}^{2/3}, \tag{11}$$

according to the theory of Nusselt. Eliminating Re_L from eqs. $(10)_1$ and (11), the critical surface velocity is represented as follows.

$$u_{LSC_1}=2.64\times10^5\,\sigma_L{}^{-2.29}\,(\mu_L/\rho_L)^{0.068} \tag*{(10)$_2$}$$

As shown by this equation, u_{LSC_1} depends mainly on σ_L and varies very little with μ_L/ρ_L. It is generally the Froude number that evaluates the conditions of the surface of the liquid film such as waves, and the Froude number is proportional to the second power of the velocity and independent of the viscosity. Therefore, eq. $(10)_2$ is preferable to eq. $(10)_1$ for the analysis of the critical condition. Some of the problems about the waves of viscous liquids are still unsolved. The above empirical equation does not help to explain the phenomenon reasonably.

By the use of this new method of measurements it became obvious that the observed thickness of the liquid film was smaller than the theoretical. Thus the actual velocity of the film must be necessarily greater than the theoretical values. If the effect of the ripples is limited to the vicinity of the surface, the velocity at the surface will increase extraordinarily. This is a qualitative interpretation given to the results of Friedman & Miller.

The relation between $B(1-2B/d)^{1/3}$ and Re_L in the turbulent region is shown in Fig. 6. The straight line N represents eq. (2) and the curve D represents eq. (3). The observed values are plotted between the line N and the curve D and are not distinguished by σ_L. About the liquid whose viscosity is less than 0.03 poise the following equation is obtained.

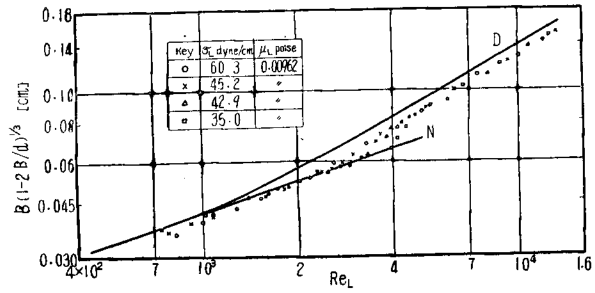


Fig. 6. Correlation between $B(1\text{-}2\,B/d)^{1/3}$ and Re_L in turbulent flow region.

$$B(1-2B/d)^{1/3}=0.0140\ (\mu_L/\rho_L)^{0.68}\ \ Re_L^{0.578} \tag{12}$$

B on a vertical plane wall is given theoretically by the equation

$$B=0.0136\,(\mu_L/\rho_L)^{2/3}\ \ Re_L^{7/12}, \tag{13}$$

if the friction factor of the flow on the wall is calculated from the Blasius formula. This equation is almost the same as the empirical equation (12). The change of the flow from the pseudo-laminar to the turbulent is enlarged in Fig. 7. Re_{LC_2} varies considerably according to σ_L but is not so definite as Re_{LC_1}. The flow of the liquid of high σ_L enters the transitional region at $Re\simeq1000$ and shows the turbulent condition at $Re_L\simeq2000$, and Re_{LC_2} is indefinite. On the contrary the flow of the liquid of $\sigma_L=35$ dyne/cm stays in the pseudo-laminar condition until $Re_L\simeq4000$ and becomes fully turbulent at $Re_L\simeq4500$. The liquid with the middle value of σ_L changes to turbulent flow at the value of Re_L intermediate between 2000 and 4500. Therefore, the higher the value of σ_L, the greater is the range of Re_L in the transitional region, and as σ_L decreases, the change from pseudo-laminar to turbulent flow is rather sudden.

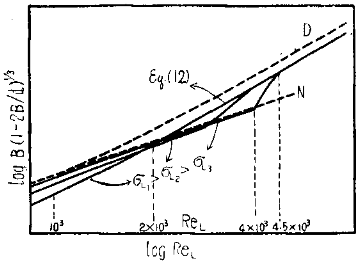


Fig. 7. Transitional region between pseudo-laminar and turbulent regions.

5. Results and Discussion (II)

—Hold-up with countercurrent air flow—

(1) The theoretical value of the thickness of the liquid film with countercurrent air flow.

The generation of the ripples on the surface of the liquid film increases due to the air flow, and becomes more active especially near the flooding point as reported previously⁸⁾. The theoretical analysis is extremely difficult under this condition. Now, assuming that there are no ripples and that the flow of the liquin film is laminar in spite of the countercurrent air flow, the relation between L' and B is,

$$L' = g\rho_L^2 B^3/3\mu_L - \tau_s \rho_L B^2/2\mu_L \tag{14}$$

on a vertical plane wall, and

$$L' = \{g\rho_L^2 B^3/3\mu_L - (\tau_s \rho_L B^2/2\mu_L)(1-2B/3d)\}(1-2B/d) \tag{15}$$

on an inner wall of a vertical pipe.

Denoting B without air flow by B_0,

$$(B/B_0)^3 = 1 + (3\tau_s/2\rho_L g B_0)(B/B_0)^2 \tag{16}$$

on a vertical plane wall, from eqs. (1) and (14), and

$$(B/B_0)^3 = 1 + 2(B-B_0)/d + 3(\tau_s/2\rho_L g B_0)(B/B_0)^2(1-2B/3d) \tag{17}$$

on an inner wall of a vertical pipe, from eqs. (2) and (15). B/B_0 increases with the increase of τ_s on both these walls. As the effect of d in eq. (10) is very small, eq. (16) may be used as long as B is not very large.

In the turbulent liquid film, the action of τ_s cannot be analysed theoretically.

(2) Observed values

Eq. (16) is derived on the assumption of perfect laminar flow and B_0 should be the value calculated from eq. (1). As the observed B_0 is a little smaller than the theoretical as shown in the foregoing paragraph, it may be questionable to substitute this value in eq. (16). But since B with air flow will also be smaller than that given by eq. (14) or (15), the observed B/B_0 may be compared with the theoretical values by using eq. (16).

With the air flowing, the tower swings violently and irregularly and the accuracy of the balance decreases. The observed p_b and p_t show a large fluctuation with the swinging of the tower. It is inevitable that the observed B is more or less in error under such conditions.

B/B_0 of the water film is plotted against τ_s in Fig. 8. The lines show the values calculated from eq. (16) corresponding to each B_0. The points scatter and most of them fall far down from the theoretical lines, especially at the large liquid flow rate. This may be due not only to the loss of accuracy in the weight measurements but also to the fact that some liquid remains stagnant on the wall due to the friction of air flow and the hold-up may vary irregulaly.

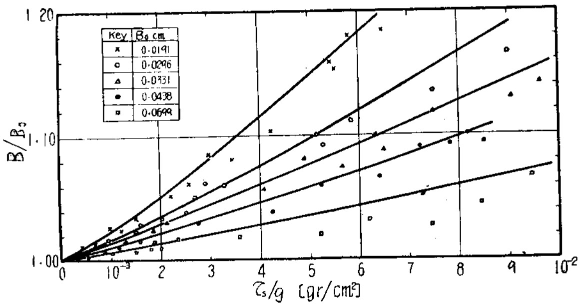


Fig. 8. B/B_0 of water.

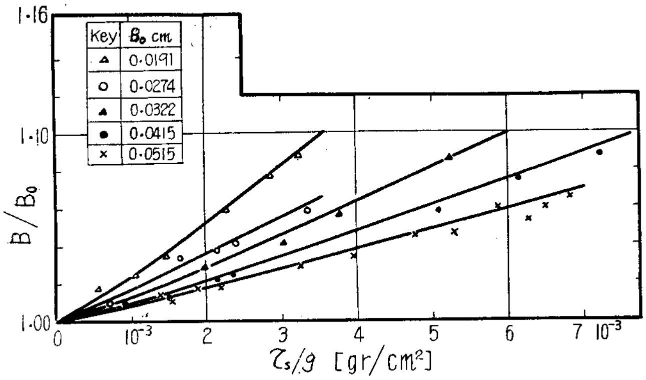


Fig. 9. B/B_0 of low surface tension liquid.

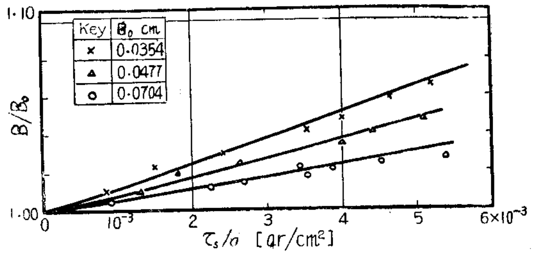


Fig. 10. B/B_0 of viscous liquid.

Fig. 9 and Fig. 10 represent the data about the liquid of the low σ_L and of the high μ_L respectively. The plotted points in these figures do not scatter so much as in Fig. 8 and they lie almost on the theoretical lines. It seems that the decrease of σ_L and the increase of μ_L inhibit the generation of the ripples and the localized hold-up of liquid as mentioned in the case of water does not appear. The fact that σ_L and μ_L are important factors influencing the flooding in a wetted wall tower[9] has been known for some time now.

The measurement of the thickness of the turbulent liquid film is almost impossible. Judging from the fact that with the increase of the liquid flow rate the points deviate farther from the theoretical lines in the above figures, B/B_0 seems not to become so large as in the case of laminar liquid films.

τ_s that is necessary for $u_{LS}=0$ is correlated with B as follows.

$$\tau_s = B\,\rho_L\,g/2 \tag{18}$$

When τ_s becomes greater than the value calculated from this equation, the surface of the film begins to flow upwards and the flooding occurs. Substituting τ_s from eq. (18) in eq. (16) and solving as to B/B_0 gives

$$B/B_0=(4)^{1/3}=1.59 \tag{19}$$

According to the results of observations of the flooding conditions in a wetted wall tower[9] the flooding occurs before B/B_0 takes this value. It shows that the flooding conditions can not be so simply predicted as by $u_{LS}=0$.

Notations

B : Thickness of falling liquid film (cm)
d : Diameter of tower (cm)
F : Force acting on surface of falling liquid film (dyne)
g : Accelerating of gravity (cm/sec^2)
H : Hold-up per unit wetted area (gr/cm^2)
L : Liquid flow rate (gr/sec)
L' : Liquid flow rate per unit wetted periphery of tower (gr/cm sec)
l : Axial length of wetted part of tower (cm)
P : Barometric pressure (dyne/cm^2)
p : Pressure of gas in tower (dyne/cm^2)
Re : Reynolds number (—)
S : Sectional area of tower (cm^2)
S' : Sectional area of enlarged part at lower end of tower (cm^2)
u : Linear velocity (cm/se)
W : Weight of wetted tower weighed by balance (gr)
Y : Parameter$=(g\rho_L^2 B^3/\mu_L^2)^{1/2}$
μ : Viscosity (poise)
π : Ratio of circumference of circle to its diameter (—)
ρ : Density (gr/cm^3)
σ : Surface tension (dyne/cm)
τ : Shearing stress (dyne/cm^2)
φ : Function
ϕ : Function

Subscripts

b : Bottom of tower
c_1 : Critical point between perfect laminar and preudo-laminar
c_2 : Critical point between pseudo-laminar and turbulent
L : Liquid
Nu : Theory of Nusselt
S : Surface of falling liquid film
t : Top of tower
0 : Without gas flow

Literature Cited

1) Chwang., C. T., S. M. thesis, M. I. T. (1928).
2) Classen, H., Zentr. Zuckerind., **26**, 497 (1918).
3) Cooper, C. M. and Willey, G. S., unpublished memorundum to W. H. McAdams (1930).
4) Dukler, A. E. and Bergelin, O. P., Chem. Eng. Prog., **48**, 557 (1952).
5) Fallah, R., Hunter, J. G. and Nash, A. W., J. Soc. Chem. Ind. London, **50**, 369 T (1934).
6) Friedman, S. J. and Miller, C. O., Ind. Eng. Chem., **33**, 885 (1941).
7) Hopf, L., Ann. Physik., **32**, 777 (1910).
8) Kamei, S. and Oishi, J., Chem. Eng. (Japan), **18**, 421 (1954).
9) Kamei, S., Oishi, J. and Okane, T., ibid., **18**, 364 (1954).
10) Kirkbride, C. G., Trans. Am. Inst. Chem. Engrs., **30**, 170 (1933-1934).
11) Nusselt, W., Z. V. D. I., **60**, 541 (1916).
12) Schoklitsch, A., Akad. Wiss. Wien, Math-Naturw. **129 II A**, 895 (1920).
13) Warden, C. P., S. M. thesis, M. I. T. (1930).