A Limit of Infrared Heating for Uniform Heating

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Introduction

The two ways to heat given materials uniformly by means of electrical heating are the infrared heating and the high frequency heating.

The former is preferable in respect of the costs of installation, personnel, operation, and others. It is desirable, therefore, that we use the infrared heating for nniform heating as much as possible. Each uniform heating has its own temperature allowance permitted in its practical use. Therefore, the authors have studies and report in this paper on the conditions on which the infrared uniform heating have the same temperature allowance.

§1. Infrared heating and dielectric heating.

The most outstanding methods for heating materials by purely electrical heating are the high frequency dielectric heating and the infrared heating. When we consider of applying one of these two methods in uniform heating, it is of great importance to know which is the most suitable for given heating conditions. Then the conditions to be considered would be firstly as follows:

(1) The efficiency of the two heating methods.

In the high frequency dielectric heating an oscillator is required, whose over-all efficiency is 30%-40% in general and 50% in some cases even, if matching and other conditions of the circuit used are completely satisfied. In the infrared heating, on the other hand, the efficiency varies in a wide range depending upon its construction and is usually 25%-60%.

As the result, the comparison between the both heating methods from the comprehensive view point seems to be meaningless.

(2) Secondly, comparison should be made from the view point of the cost of installation. In the high-frequency dielectric heating, a high frequency oscillator is required in addition to heating equipments which necessitates a huge cost of installation and,

moreover, the technicians with a good knowledge of high frequency engineering are required. On the other hand, the infrared heating needs only as oven equipped with heat radiating bodies and, since the weight of the radiant bodies is light, an oven of strong type is not necessary. Consequently, this mothod requires neither huge initial cost nor special engineers. This method, therefore, is much more advantageous than the former from the economical point of view.

(3) It is a well-known fact that the construction of an oven for the infrared heating is simpler than in the case of the other when an equipment of large capacity is required.

(4) Considering the condition of materials to be heated, the high frequency dielectric heating has the effect of an inside heating by which all parts of a material are uniformly heated. However, in the case of the infrared-heating, there is an inside penetration of infrared to certain extent so that we can treat this heating as an inside heating in the case of a thin materials, but, in the case of material which is too thick or impenetrable by this ray, the method shuld be treated as outside heating. That is, when the material to be heated with infrared is thick, heating must be done by thermal conduction and as the distance from the radiated surface become farther, the temperature rise becomes slower and the peculiarity of inside heating is lost. In the other words, in view of heating, the former is much better than the latter, and speaking from this standpoint, we can heat materials uniformly in the shortest time by applying the high frequency heating with a large power in a short time by using a suitable surface heating.

(5) Now, when a given material is to be heated uniformly, it is required in general that the temperature rise in every portion of the material in a given time is desired to be in the range of $\theta \pm \Delta\theta$. Therefore, it is sufficient if, in uniform heating with the infrared heating, the temperature rise in every portion of the material in a given time lies in the range of $\theta \pm \Delta\theta$ and in the case of an electrical heating process, it is necessary to take into consideration this allowance of $\pm \Delta\theta$. In other words, it is obvious from the preceding conditions of (1) to (3), the infrared heating in general is advantageous both for installation and maintenance costs and, in every case where circumstances permit, the use of the infrared heating is preferable. On the other hand, the characteristic of the infrared heating as an inside heating vanishes with the increase of thickness of the material and a very long heating time is required until the heat becomes uniform. As the result, it will become impossible to carry out an uniform heating that meets the above conditions and we would have to rely upon the high frequency dielectric heating to satisfy the above conditions.

Now, where should the thickness of materials be set as the boundary of the two methods? And by what factors is this boundary influenced?

To answer these questions let us consider the ratio of the minimum temperature rise to the maximum temperature rise in a material at a given heating time (hereafter called the "temperature ratio" for the simplicity's sake) in the process of uniform heating. This ratio is independent of radiation intensity and is a function of thermal conductivity, thermal capacity, thermal transfer coefficient, thickness of material, and heating time as shown later in culculating formula; and if this temperature ratio becomes above the allowed value of $\theta-\Delta\theta/\theta+\Delta\theta$, which the material requires, we can amply accomplish the purpose of uniform heating by the infrared heating. In other words, in the case of the infrared heating, the radiation intensity determines the value temperature rise but has no effect on the temperature ratio. Hence, if we use this temperature ratio for determining the range within which the infrared heating can be applied, we can obtain a very rational method of this type of heating.

§ 2. Calculating formula of temperature rise in the infrared heating.

Let us now consider a material which has the infinite extension of thickness b, thermal conductivity K and absorption coefficient for infrared-ray γ, and radiate to this material infrared-ray of intensity I_1 from one side as shown in Fig. 1. Taking β as the reflection coefficient of infrared-ray entering from out of the space into the material and β' as that of emitting from the material into the space, we have the following relation,

$$I_0=(1-\beta)I_1,$$

where I_0 is the radiation intensity at the surface of the material, and the energy intensity of infrared at the distance of x from the surface, if absorption is assumed to be proportional to the intensity of that point, decreases exponentially and the intensity I is given in the form.

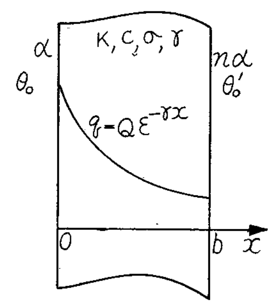

Fig. 1.

$$I=\frac{I_0\varepsilon^{-\gamma x}}{1-\beta_1^2\varepsilon^{-2\gamma b}}+\frac{\beta_1 I_0\varepsilon^{-2\gamma b+\gamma x}}{1-\beta_1^2\varepsilon^{-2\gamma b}}. \tag{1}$$

The absorption coefficient of the material being γ, the energy absorbed at the point x is given by:

$$\left.\begin{aligned} q&=\gamma I=Q_1\varepsilon^{-\gamma x}+Q_1'\varepsilon^{\gamma x}, \\ \text{where} \quad Q_1&=\frac{\gamma I_0}{1-\beta_1^2\varepsilon^{-2\gamma x}}, \qquad Q_1'=\frac{\beta_1\gamma I_0\varepsilon^{-2\gamma b}}{1-\beta_1^2\varepsilon^{-2\gamma x}}. \end{aligned}\right\} \tag{2}$$

Now, considering that the above material has been placed in the atmosphere having the temperature θ_0 and θ_0', thermal transfer coefficient α and α' respectively on both surfaces of the material, then the equations for the thermal conduction, boundary, and initial conditions will be as follows:

$$K\frac{\partial^2\theta}{\partial x^2}+q=c\sigma\frac{\partial\theta}{\partial t}, \tag{3}$$

The boundary conditions:

$$\left.\begin{aligned} &\text{at}\quad x=0,\quad \frac{\partial\theta}{\partial x}=h(\theta-\theta_0) \\ &\text{at}\quad x=b,\quad \frac{\partial\theta}{\partial x}=-nh(\theta-\theta_0').\end{aligned}\right\} \tag{4}$$

The initial conditions at $t=0$, $\theta=0$, (5)

where $h^2=\alpha/K$, $n=\dfrac{\alpha'}{\alpha}$,

and $c\sigma$ represents the thermal capacity of the material.

In Equation (2) we notice that q consists of the two terms of Q_1 and Q_1'. However, the difference between both terms has no importance since we can easily obtain the latter from the former by substituting $-\gamma$ for γ, then, it suffices to exclusively calculate the former. By the Laplace transformation the following expressions are derived:

$$\begin{aligned}\theta_1=&\frac{Q_1b^2}{K}\frac{1}{(\gamma b)^2}\frac{(\gamma b+hb)\left\{1+nhb\left(1-\frac{x}{b}\right)\right\}-(\gamma b-nhb)\left(1+hb\frac{x}{b}\right)\varepsilon^{-\gamma b}}{(n+1)hb+nhb\cdot hb} \\ &-\frac{Q_1b^2}{K}\frac{\varepsilon^{-\gamma b\frac{x}{b}}}{(\gamma b)^2}-\frac{Q_1b^2}{K}\sum_i\frac{[I]y_i}{\{(\gamma b)^2+(y_ib)^2\}[X]_{y_i}}\varepsilon^{-(y_ib)^2\frac{k}{b^2}t},\end{aligned} \tag{6}$$

$$\begin{aligned}\theta_2=&\frac{hb\theta_0\left\{1+nhb\left(1-\frac{x}{b}\right)\right\}+nhb\theta_0'\left(1+hb\frac{x}{b}\right)}{(n+1)hb+nhb\cdot hb} \\ &-hb\theta_0\sum_i\frac{[II]_{y_i}}{[X]_{y_i}}\varepsilon^{-(y_ib)^2\frac{k}{b^2}t}.\end{aligned} \tag{7}$$

θ_3 can be obtained when we substitute Q_1' and $-\gamma$ for Q_1 and γ in equation (6). Thus

$$\theta=\theta_1+\theta_2+\theta_3. \tag{8}$$

In the above equations y_i is the ith root of the equation

$$\tan yb=\frac{(n+1)hb\cdot yb}{(yb)^2-nhb\cdot hb}, \tag{9}$$

and

$$[I]_{y_i}=(\gamma b+hb)\left\{y_ib\cos y_ib\left(1-\frac{x}{b}\right)+nhb\sin y_ib\left(1-\frac{x}{b}\right)\right\}$$
$$-(\gamma b-nhb)\left(y_ib\cos y_ib\frac{x}{b}+hb\sin y_ib\frac{x}{b}\right), \tag{10}$$

$$[II]_{y_i}=y_ib\cos y_ib\left(1-\frac{x}{b}\right)+nhb\sin y_ib\left(1-\frac{x}{b}\right)$$
$$+\frac{nhb}{hn}\frac{\theta_0'}{\theta_0}\left(y_ib\cos y_ib\frac{x}{b}+hb\sin y_ib\frac{x}{b}\right), \tag{11}$$

$$[X]_{y_i}=-y_ib\left\{\frac{n+1}{2}hb+\frac{nhb\cdot hb(|y_ib)^2}{2}\right\}\cos y_ib$$
$$+\left(\frac{n+1}{2}hb+1\right)(y_ib)^2\sin y_ib. \tag{12}$$

In actual heating, setting γ as an infinite does not cause any trouble in the subsequent calculation since the absorption of infrared-ray occurs mostly near the surface.

For the simplicity's sake, we proceed in our discussion on the assumption that the thermal transfer coefficient and the air temperature are the same at each side of the material. If we put γ as an infinite and $n=1$, $\theta=\theta_0'$ in the expressions from (6) to (12), then the expressions simply becomes as follows:

$$\theta_1=\frac{I_0}{\alpha}\frac{1+hb\left(1-\frac{x}{b}\right)}{2+hb}$$
$$-\frac{I_0}{\alpha}\sum_i\frac{hb\left\{y_ib\cos y_ib\left(1-\frac{x}{b}\right)+hb\sin y_ib\left(1-\frac{x}{b}\right)\right\}\varepsilon^{-(y_ib)^2\frac{k}{b^2}t}}{[X_{n=1}]_{y_i}}, \tag{13}$$

$$\theta_2=\theta_0-\theta_0\sum_i\frac{\left\{y_ib\cos y_ib\left(1-\frac{x}{b}\right)+hb\sin y_ib\left(1-\frac{x}{b}\right)+\left(y_ib\cos y_ib\frac{x}{b}+hb\sin y_ib\frac{x}{b}\right)\right\}\varepsilon^{-(y_ib)^2\frac{k}{b^2}t}}{[X_{n=1}]_{y_i}}, \tag{14}$$

where;

$$[X_{n=1}]_{y_i}=y_ib\left\{hb+\frac{(hb)^2-(y_ib)^2}{2}\right\}\cos y_ib+(1+hb)(y_ib)^2\sin y_ib, \tag{15}$$

where y_i is the ith root of the equation

$$\tan yb=\frac{2yb\,hb}{(yb)^2-(hb)^2}. \tag{16}$$

Next, if we perform the calculation assuming the Q_1' equal to $Q_1\varepsilon^{-\gamma b}$, the value of temperature rise in the case of heating with the same radiation intensity from both sides can be obtained. This can be calculated by putting $\theta_1+\theta_3$ as θ_1.

$$\theta_1 = \frac{I_0}{\alpha} - \frac{I_0}{\alpha}\sum_i \frac{hb\left[\left\{y_i b\cos y_i b\left(1-\frac{x}{b}\right)+hb\sin y_i b\left(1-\frac{x}{b}\right)\right\}+\left(y_i b\cos y_i b\frac{x}{b}+hb\sin y_i b\frac{x}{b}\right)\right]\varepsilon^{-(y_i b)^2\frac{k}{b^2}t}}{[X_{n=1}]_{y_i}}. \tag{17}$$

θ_1 in Equation (17) obtained here takes the same form as in Equation (14).

When the temperature rise in the infrared heating is simplified in this manner, we can see that the temperature rise is maximum at the face and minimum at the back in one side heating, and it is maximum at each surface and minium at the center in the case of heating from both sides. In these cases, the temperature ratio can be classified into the following three cases;

(1) The temperature ratio caused only by the infrared heating

$$\frac{\theta_{1\ \min}}{\theta_{1\ \max}}\times 100\% \tag{18}$$

(2) The temperature ratio caused only by the convection heating

$$\frac{\theta_{2\ \min}}{\theta_{2\ \max}}\times 100\%\ , \tag{19}$$

(3) The temperature ratio in the infrared oven

$$\frac{\theta_{1\ \min}+\theta_{2\ \min}}{\theta_{1\ \max}+\theta_{2\ \max}}\times 100\%\ . \tag{20}$$

Subsequently, we discuss these cases with the fact in mind that, when γ is infinite as indicated before, the temperature rise by infrared-ray occurs in the same way as in the convection heating if we take I_0/α as the air temperature, and in case where the air temperatures of both surfaces are equal, the temperature ratio shown in the expression (2) is identical with that in the expression (1) for the both side heating. In this case the total temperature ratio given in (3) equals the temperature ratio in case (1) for the both side radiation.

§3. Temperature ratio caused by the infrared radiation only.

Now, the temperature ratio caused only by the infrared radiation is given by Equation (18) and it is a function of $hb=\alpha b/K$ and $kt/b^2=Kt/c\sigma b^2$. Then the temperature ratio is represented originally as a function of the thermal

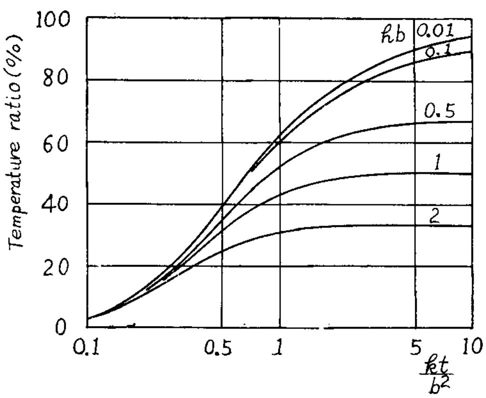

Fig. 2.

conductivity, thermal transfer coefficient, thermal capacity, thickness of material, and heating time, but it is also uniquely determined by dimensionless terms hb and kt/b^2.

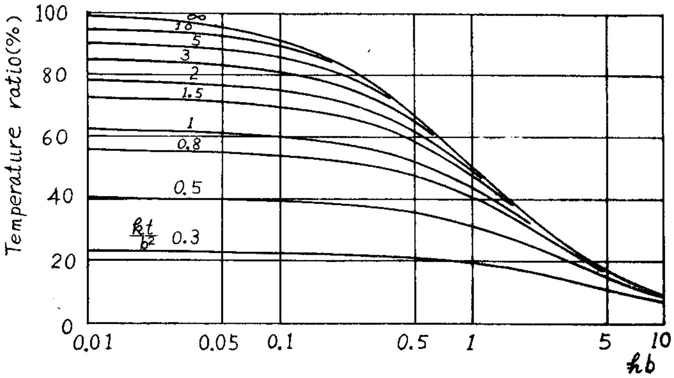

Fig. 3.

From this point of view, the relations between hb (or kt/b^2) and the temperature ratio can be represented as done in the Figs. 2 to 5 taking hb (or kt/b^2) as a parameter. In Figs. 2 and 3 the relations between hb (or kt/b^2) and temperature ratio for the one side radiation are shown, and in Figs. 4 and 5 we have the same relations for that of the both side radiation. If hb and kt/b^2 are calculated from the thermal properties and the heating conditions of a given material, the temperature ratios caused by the infrared radiation can easily be obtained by these figures. Let us consider from these figures the characteristics of the temperature ratios.

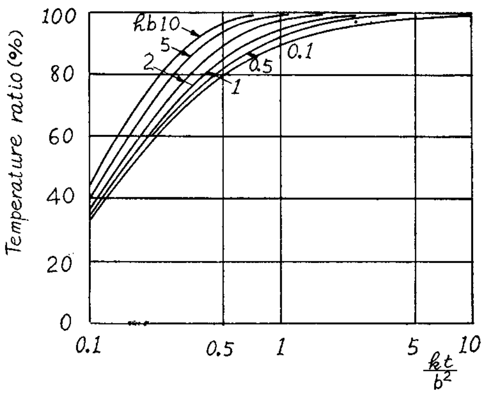

Fig. 4.

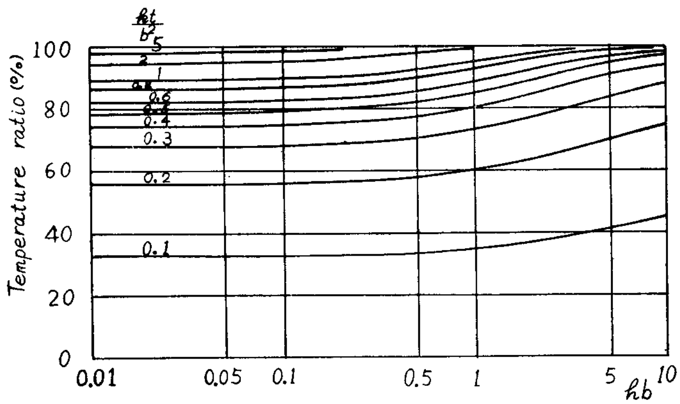

Fig. 5.

(1) In the case of the one side radiation, as hb becomes larger the temperature ratio diminished in each heating time and even the ratio at the infinite time is 50% (i.e., $hb=1$) and with the excess of hb over 1 the ratio becomes smaller than 50%.

(2) For the both side radiation the final temperature is uniformly distributed at any value of hb and the temperature ratio reaches 100%. It is evident from these figures that with the increase of hb the temperature ratio in the same heating time increases.

§4. Relations between the temperature ratio and the properties of the material.

Although we established in §3 the relations between the temperature ratio and hb or kt/b^2, we cannot explicitly illustrate with only these data how the temperature ratio varies with variations of the thickness of material, thermal conductivity, thermal capacity, and thermal transfer coefficient. Therefore, let us consider below how the temperature ratio varies with these constants.

i) Relations between the temperature ratio and thickness, b.

In the first place we consider the variation of the temperature ratio in accordance with the thickness of the material. As the material for the object of our consideration, we take a piece of wood, oak for instance, which has the thermal conductivity $K=0.15$ kcal/mhr °C, thermal capacity $c\sigma=400$ kcal/m^3 °C and thermal transfer coefficient $\alpha=7$ kcal/m^{2}hr °C. When we take the heating time as a parameter the relations between the temperature ratio and thickness become as shown in Figs. 6 and 7.

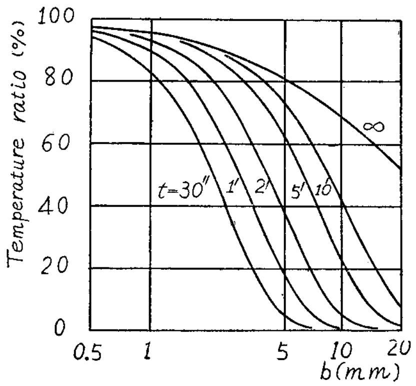

Fig. 6.
$K=0.15$ kcal/mhr °C
$c\sigma=400$ kcal/m^3 °C
$\alpha=7$ kcal/m^{2}hr °C
one side radiation

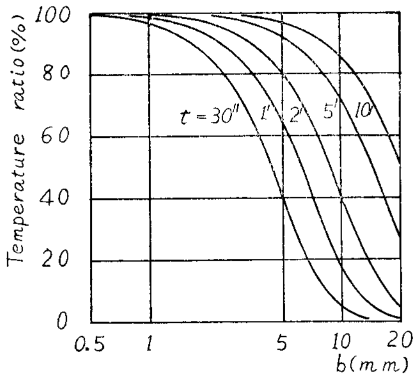

Fig. 7.
$K=0.15$ kcal/mhr °C
$c\sigma=400$ kcal/m^3 °C
$\alpha=7$ kcal/mhr °C
both side radiation

Fig. 6 corresponds to the case of the one side radiation and Fig. 7, to that of the both side radiation. In each case we take the temperature ratio for ordinate and thickness in m.m. for abscissa on logarithmic scale. In both figures heating times taken are 30″, 1′, 2′, 5′, 10′ and steady state. As it is easily understand from these two figures that the both side radiation is superior to that of the one side radiation in temperature ratio and it shows that, in case of heating a considerably thick material with infrared, the both side radiation must inevitably be applied.

ii) Relations between the temperature ratio and the thermal transfer coefficient α.

Since the thermal transfer coefficient is found in equations of temperature rise (13) and (17) in the form of $h=\alpha/K$, the relations between the temperature ratio and α of the same material in (i) having the thickness $b=10$ m.m., are shown in Figs. 8 and 9. The case of the one side radiation is shown in Fig. 8 and the case of the both side radiation in Fig. 9, the ordinates and the abscissas are the temperature

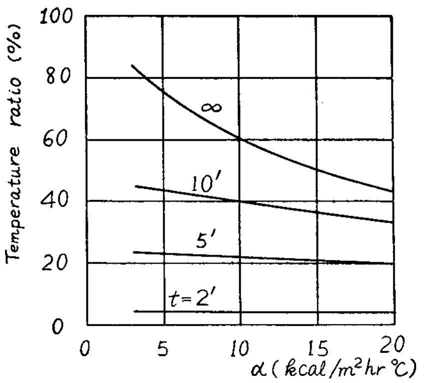

Fig. 8.
$K=0.15$ kcal/mhr °C
$c\sigma=400$ kcal/m^3 °C
$b=10$ m.m.
one side radiation

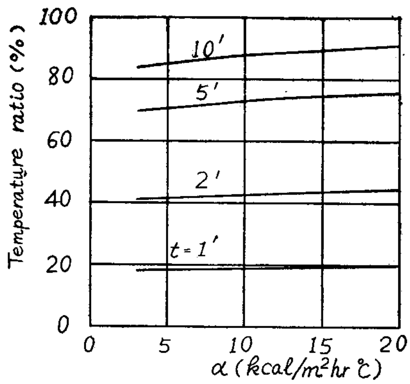

Fig. 9.
$K=0.15$ kcal/mhr °C
$c\sigma=400$ kcal/m^3 °C
$b=10$ m.m.
both side radiation

ratio in percentage and the thermal transfer coefficient in the unit of kcal/m^{2}hr °C respectivery. By these figures it is shown that α has more influence in the one side radiation than that of the both side radiation. In the one side radiation, as α become larger the temperature ratio diminishes and the longer the heating time, the greater the influence of α becomes. Such facts have been given already by Equation (13) which has expressed of a temperature rise within a material. These are caused by the thermal conduction of the material in the long run hence, with the increase of α, the thermal loss on the back increases and the temperature gradient within the material becomes steeper, and they result in the decrease of the temperature ratio.

On the contrary, variation of α in the both side radiation is not so remarkable as that in the one side radiation and, as α becomes larger, the temperature rise itself decreases but the temperature ratio tends to increase. This is easily understood in the light of the fact that the time, which is necessary to reach the steady state, is shortened with the increase of α and in Equation (18) the temperature distribution becomes linear when γ corresponds to infinite.

In the next place, even if the thermal transfer coefficient on the surface and that on the back differs each other, we can continue the calculation utilizing the result derived from the Equation (6) by setting γ infinite. For example, we show in Fig. 10 the relation between the temperature ratio and heating time for the one side radiation, where $K=0.15$ kcal/mhr C°, $c\sigma=400$ kcal/m^3 °C, $\alpha=7$ kcal/m^{2}hr °C, $b=10$ m.m. and n is taken as a parameter. It is shown in this figure that the smaller n is, i.e. the smaller the thermal loss on the back, and the larger the temperature ratio obtained in the equal heating time. For example, when the heating time is set to be 10 minutes, a better temperature ratio is obtained as n becomes smaller as shown in Table 1.

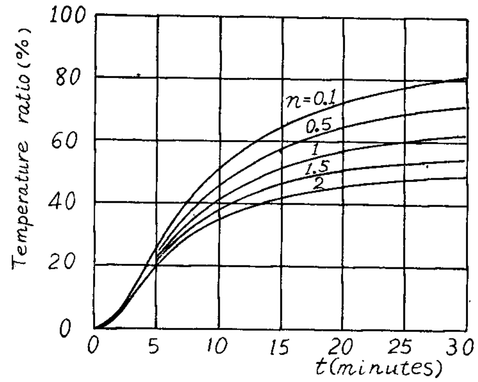

Fig. 10.
$K=0.15$ kcal/mhr °C
$c\sigma=400$ kcal/m^3 °C
$\alpha=7$ kal/m^{2}hr^2 °C
$b=10$ m.m.

Table 1.

n	0.1	0.5	1	1.5	2
Temperature ratio (%)	51.0	45.6	41.8	38.1	35.0

iii) Relations between the temperature ratio and the thermal capacity

As seen from Equations (13) and (17) relating to temperature rise, the thermal capacity $c\sigma$ appears only in the time constant of ε^{-kt/b^2} of transient terms in the form $k=K/c\sigma$, Then $c\sigma$ has no effect upon the final temperature rise but affects only in the transient state. Consequently, the temperature ratio traces the same process, and, as the result, the final temperature ratio becomes constant regardless of however the variation of $c\sigma$ may be, but the duration of time before the final ratio is attained varies. Now supposing $K=0.15$ kcal/mhr °C, $\alpha=7$ kcal/m^{2}hr °C and $b=10$ m.m., the relations between the temperature ratio and the thermal capacity are represented in Figs. 11 and 12, where heating time is taken as a parameter. Figs. 11 and 12 correspond respectively to the one side heating and the both side heating. Since $c\sigma$ is involved only in the time constant of transient term, as indicated before, the temperature ratio abruptly takes a large value in a short time and reaches the final temperature ratio when $c\sigma$ is small, but as for the large $c\sigma$ the increase of temperature ratio is small, and time necessary to reach the same temperature ratio proportionally increases for the value of $c\sigma$, i.e., for the double value of $c\sigma$, time required has to become twice as much in the light of $kt=Kt/c\sigma$. In other words, in these figures a family of curves of the same form is obtained by shifting one of them along the abscissa corresponding to the value of t. This is true also in the case of the both side radiation shown by Fig. 12 and caution must be given in case materials of large heat capacity are uniformly heated.

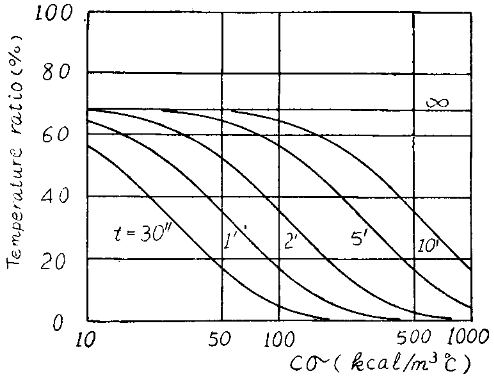

Fig. 11.
$K = 0.15$ kcal/mhr °C
$\alpha = 7$ kcal/m^{2}hr °C
$b = 10$ m.m.
one side radiation

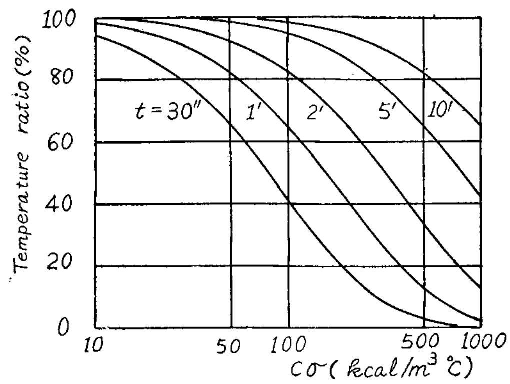

Fig. 12.
$K = 0.15$ kcal/mhr °C
$\alpha = 7$ kcal/m^{2}hr °C
$b = 10$ m.m.
both side radiation

iv) Relations between the temperature ratio and the thermal conductivity

The remaining constant that controls the temperature ratio is the thermal conductivity K. The foregoing thermal capacity is found only in the transient term which attenuates with time, but the thermal conductivity is, as indicated in Equation (13), involved not only in the transient term but also in the steady state term in the forms of $h = \alpha/K$ and $k = K/c\sigma$, and its effects upon the temperature ratio are very complicated. Assuming the other constants K, excepting, are the same as previously treated, i.e. $c\sigma = 400$ kcal/m^3 °C, $\alpha = 7$ kcal/m^{2}hr °C and $b = 10$ m.m., the relations between the temperature ratio and K are given by Fig. 13 and 14, which have the temperature ratio

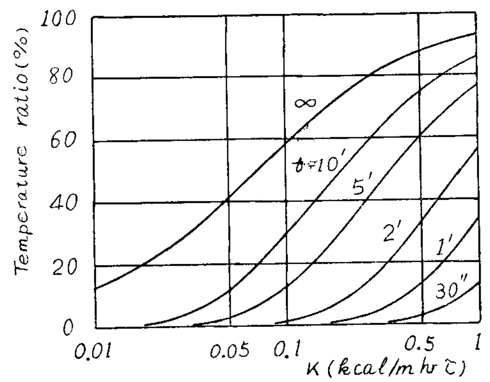

Fig. 13.
$c\sigma = 400$ kcal/m^3 °C
$\alpha = 7$ kcal/m^{2}hr °C
$b = 10$ m.m.
one side radiation

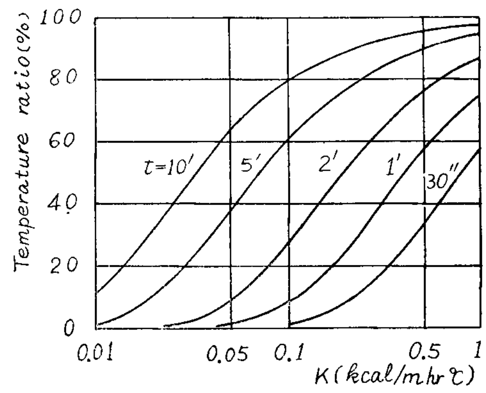

Fig. 14.
$c\sigma = 400$ kcal/m^3 °C
$\alpha = 7$ kcal/m^{2}hr °C
$b = 10$ m.m.
both side radiation

in ordinate and K in abscissa on logarithmic scale taking t as a parameter. We understand from these figures that when conductivity is small, the temperature ratio for the same heating time diminishes in both cases of the above two heating processes, and the shorter the heating time, the greater its effect. We know further that as K decreases the heating time necessary to reach the steady state is much lengthened. These results are also true for the both side radiation.

Now, in Equation (13) h and k, the constants involving K, are introduced in the form of hb or kt/b^2 respectively. Consequently we can regard the thermal conductivity as involved in this equation in the form K/b. In other words, conductivity and thickness are inversely proportional to each other in the present discussion and an increase of conductivity is equivalent to a decrease of thickness. This effect of conductivity is a natural consequence which is produced from the assumption that the material is heated on the surface under the condition that its aborption coefficient is infinite and the temperature rise within it is caused by conduction. It is shown in the above disussion that the conductivity is an important factor to determine the limit of use of the infrared heating for uniform heating.

§5. A limit of the infrared heating for uniform heating.

Previously, we indicated in §3 that the temperature ratio was given as a function of kt/b^2 and hb, and in §4 how this ratio varied with serveral constants of a material. Now, we shall discuss what kind of relation can be established between kt/b^2 and hb when the condition given in §1—when it is required to have the temperature rise at every portion of a material within the range of $\theta \pm \varDelta\theta$ in a given heating time—is to be satisfied.

If γ is infinite as in §3, we can use the same temperature ratio as in §3. Then, taking the temperature ratio as a parameter the relations between kt/b^2 and hb, necessary for obtaining a given ratio, are shown in Figs. 15 and 16. We show the case of the one side radiation in Fig. 15 and the case of the both side radiation in Fig. 16 and in each figure kt/b^2 is taken in ordinate and hb in abscissa on logarithmic scale.

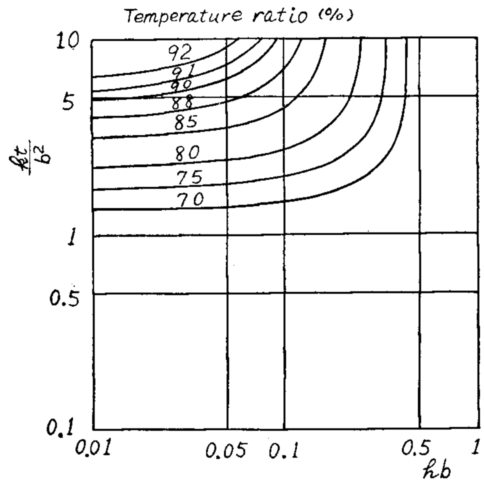

Fig. 15.

As easily seen from these figures, the relation between kt/b^2 and hb for obtaining a given temperature ratio has a complete inverse property between the both heating processes and, in the case of the one side radiation, an increase of hb results in the increase of kt/b^2 required for uniform heating and it becomes infinite at some value of it shows the fact that it hb, and is impossible to obtain this temperature ratio when hb is greater. For instance, assuming that the temperature ratio is required to be above 90%, the result will be as shown by Table 2, and the requirement will not be satisfied when hb is over 0.112.

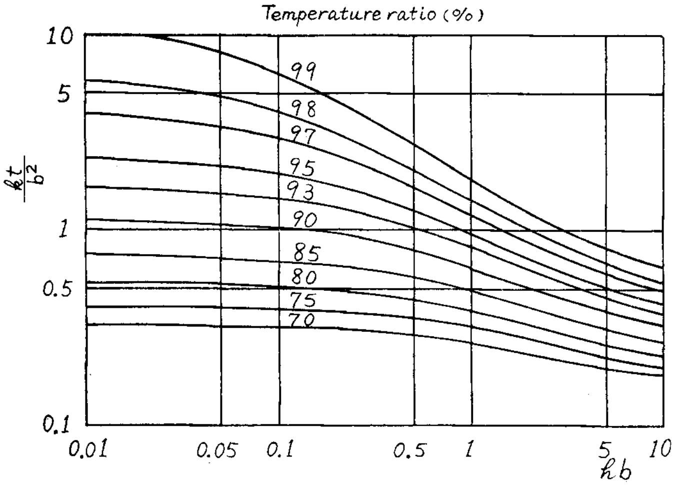

Fig 16.

Table 2.

hb	0.01	0.05	0.1	0.112	0.5	1
kt/b^2 (one side)	4.9	6.2	12.5	∞	—	—
kt/b^2 (both side)	1.15	1.10	1.05	—	0.80	0.65

In the case of the both side radiation, it is absolutely different from the one side radiation as seen from Fig. 16, and the larger the hb the less kt/b^2 for obtaining the same temperature ratio becomes, and the condition prescribed is satisfied with far less kt/b^2 than that for the one side radiation. That is to say, there is such a merit that not only the value of kt/b^2 but its variation with hb is less than that in the case of the one side radiating and, therefore, the both side radiation is favorable for uniform heating.

As has often been indicated previously, the above facts were derived on the assumption that the absorption coefficient is infinite, that is to say, energy is absorbed on the surface of a material, and this assumption corresponds to the worst condition in the infrared heating. Then in the case in which the absorption coefficient is finite, the maximum of temperature rise drops to some extent, the minimim rises and, moreover, since the term involving Q_1' is also to be taken into consideration, the temperature ratio given as the ratio of temperature rise takes far bigger value than that indicated in Figs. 15 and 16. If the conditions given by Figs. 15 and 16 are satisfied in the infrared heating, special conditions required in the practical use are generally satisfied. Hereby Figs. 15 and 16 provide us with an important index relating the limit of the infrared heating for uniform heating when some characteristics of a material and the heating conditions of it are given as described above. Now, an example is shown as to the relations between heating time and thickness for the material provided before. Setting $K=0.15$ kcal/mhr °C, $c\sigma=400$ kcal/m^3 °C, $\alpha=7$ kcal/m^{2}hr °C and $\alpha=4$ kcal/m^{2}hr °C the relations between heating time and thickness are sought to obtain the temperature ratios of 80% and 90%. They are shown by Fig. 17 in which the solid line gives the case of $\alpha=7$ kcal/m^{2}hr °C and the chain line the case of $\alpha=4$ kcal/m^{2}hr °C, ordinate and abscissa being taken as heating time in minute and thickness in m.m. respectively. In this figure it is shown that the both side radiation has better characteristics than the other; that is to say, the uniform heating of a material with more than double thickness is possible by the both side radiation with the same heating time. For instance, when $\alpha=7$ kcal/m^{2}hr °C and the temperature ratio is 90% in the heating time of one minute, the infrared heating can be applied up to $b=1.02$ m.m. for the one side radiation and up to $b=2.45$ m.m. for the both side radiation; and comparing these two figures, the latter signifies that the infrared heating can be applied in uniform heating of a material of over 2.4 times thickness as in the case of the one side heating.

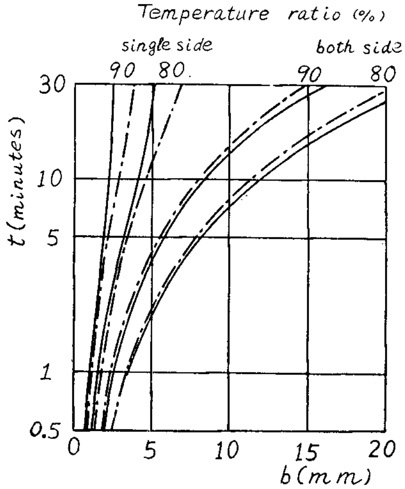

Fig. 17.
$K=0.15$ kcal/mhr °C
$c\sigma=400$ kcal/m^3 °C
—— $\alpha=7$ kcal/m^{2}hr °C
-·- $\alpha=4$ kcal/m^{2}hr °C

§ 6. Temperature rise in the infrared heating.

When a material to be heated and the conditions in heating are given, it is easy from the discussions brought forth in §3, §4 and §5 to see if the given material can be uniformly heated with the radiant energy or not, but it is not yet clear how much radiation intensity is required. In this paragraph, we shall consider about the radiation intensity required.

Although temperature rise depends upon the value of absorption coefficient γ, we shall consider of the temperature rise on the surface of a material assuming γ to be infinite for the sake of simplicity. In this case the expressions which represent temperature rise are given by Equation (13) for the one side radiation and by Equation (16) for the both side radiation respectively. From these equations, we can calculate $\alpha\theta_{x=0}/I_0$, which is a function of hb and kt/b^2, and is uniquely determined by these two constants. The results are shown in Figs. 18 to 21. $\alpha\theta_{x=0}/I_0$ for the one side radiation is represented in Figs. 18 and 19 where the abscissas are kt/b^2 and hb respectively.

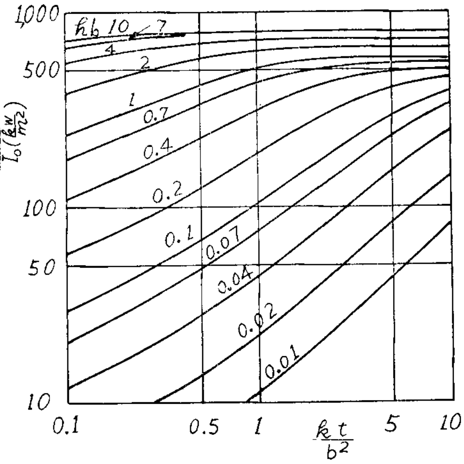

Fig. 18.

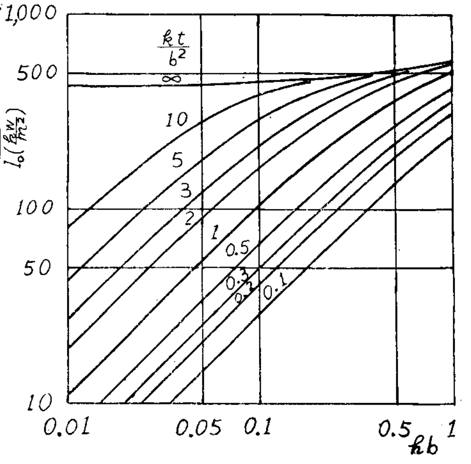

Fig. 19.

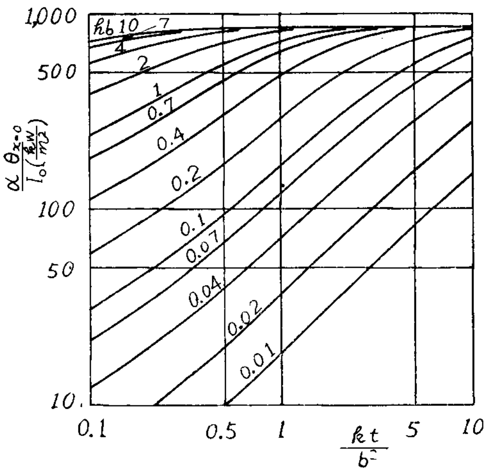

Fig. 20.

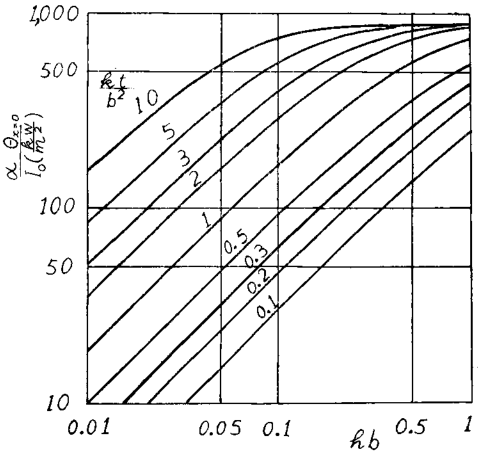

Fig. 21.

In Figs. 20 and 21 the same relations for the both side radiation are given. The radiation intensity I_0 in these figures are the energy required for the heating and is shown by kw/m^2 and the intensity I necessary in practice is given by I_0/η when the efficiency of absorption of the material is designated by η.

We see from Fig. 18 that as hb becomes larger, $\alpha\theta_{x=0}/I_0$ increases for the constant kt/b^2 and rapidly reaches the steady state.

As an example, we show in Fig. 22 how the temperature rise on the surface varies with thickness. The figure was drawn for the one side radiation where the material has constants $K=0.15$ kcal/mhr °C, $c\sigma=400$ kcal/m^3 °C, $\alpha=7$ kcal/m^{2}hr °C. The ordinate is $\theta_{x=0}/I_0$(kW/m^2) and the abscissa in thickness in m.m. on logarithmic scale, and a parameter is heating time. It is indicated that no influence from the back appears and the temperature rise draws close to the constant when the thickness becomes greater than a certain value for a fixed heating time.

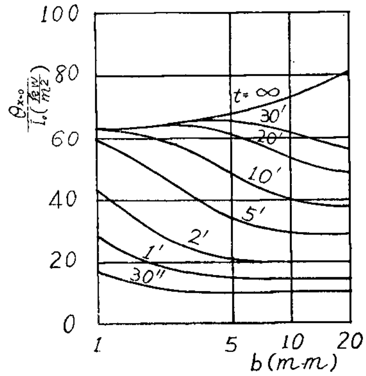

Fig. 22.
$K=0.15$ kcal/mhr °C
$c\sigma=400$ kcal/m^3 °C
$\alpha=7$ kcal/m^{2}hr °C

§7. Infrared heating with convection taken into account.

In practical case in which the infrared heating is used in the oven, neither the temperature rise produced by the air in the oven can be neglected.

Assuming that the atmospheric temperature is θ_0 and θ_0' and the thermal transfer coefficient is equal on each surface of the material to be heated, and putting $\theta_0=\theta_0'+\theta_0''$ in Equation (7), we obtain the following expressions:

$$\theta_2=\theta_0'\left[1-hb\sum_i\frac{\left\{y_ib\cos y_ib\left(1-\frac{x}{b}\right)+hb\sin y_ib\left(1-\frac{x}{b}\right)+y_ib\cos y_ib\frac{x}{b}+hb\sin y_ib\frac{x}{b}\right\}\varepsilon^{-(y_ib)^2\frac{k}{b^2}t}}{[X_{n=1}]_{y_i}}\right]$$

$$+\theta_0''\left[\frac{1+hb\left(1-\frac{x}{b}\right)}{2+hb}-hb\sum_i\frac{\left\{y_ib\cos y_ib\left(1-\frac{x}{b}\right)+hb\sin y_ib\left(1-\frac{x}{b}\right)\right\}\varepsilon^{-(y_ib)^2\frac{k}{b^2}t}}{[X_{n=1}]_{y_i}}\right] \tag{21}$$

$$[X_{n=1}]_{y_i}=-y_ib\left\{hb+\frac{(hb)^2-(y_ib)^2}{2}\right\}\cos y_ib+(1+hb)(y_ib)^2\sin y_ib$$

where y_i is the ith root of the equation

$$\tan yb=\frac{2\,hb\,yb}{(yb)^2-(hb)^2}$$

In other words, from Equation (21) the temperature rise by convection can be divided into two parts, the temperature rise from both surfaces caused by θ_0' and that from one surfaced by θ_0''. Consequently, in a case where the atmosphere have the same temperature on each surface, $\theta=\theta_0'$, then θ_0'' vanishes and the result is

$$\theta_2=\theta_0'\left[1-hb\sum_i\frac{\left\{y_ib\cos y_ib\left(1-\frac{x}{b}\right)+hb\sin y_ib\left(1-\frac{x}{b}\right)+y_ib\cos y_ib\frac{x}{b}+hb\sin y_ib\frac{x}{b}\right\}\varepsilon^{-(y_ib)^2\frac{k}{b^2}t}}{[X_{n=1}]_{y_i}}\right] \tag{22}$$

Let us compare Exquation (21) with Equations (13) and (17) obtained in the infrared heating under the assumption that the absorption coefficient is infinite. The first term of equation (21) is identical with Equation (17) in which the temperature rise in the both side radiation is expressed by substituting θ_0' for I_0/α, and the second term corresponds to Equation (13) for the one side radiation when I_0/α is replaced with θ_0'' in the same way. Accordingly, the temperature ratios concerning θ_0' and θ_0'' in the convection heating are given with the temperature ratio of the infrared radiation only, and the relations between the temperature ratio and hb or kt/b^2 are given also by Figs. 2 to 5 where as Figs. 4 and 5 stand as for θ_0', and the others as for θ_0''. The same result is given for the temperature rise except that I_0/α is replaced with θ_0' and θ_0''. This means that in Figs. 18 to 21, we have only to substitute θ/θ_0'' or θ/θ_0' for $\alpha\theta/860\,I_0$. Figs. 23 to 26 show these relations. In Figs. 23 and 24 the temperature rise corresponding to θ_0'' on the surface is represented on abscissa by kt/b^2 and hb respectively, and in Figs. 25 and 26 the same relations corresponding to θ_0' are given.

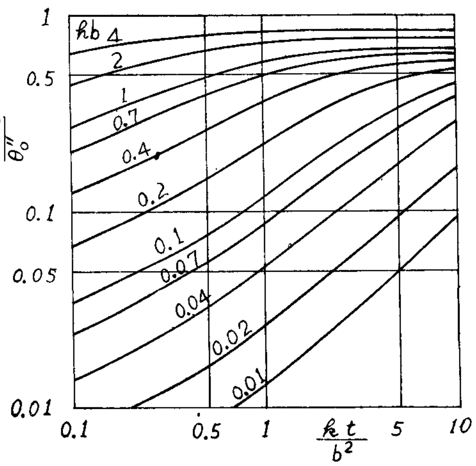

Fig. 23.

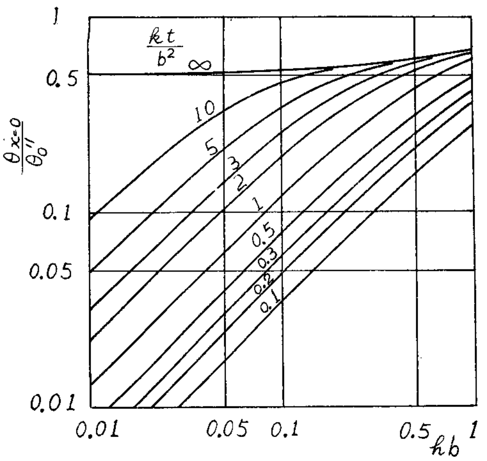

Fig. 24.

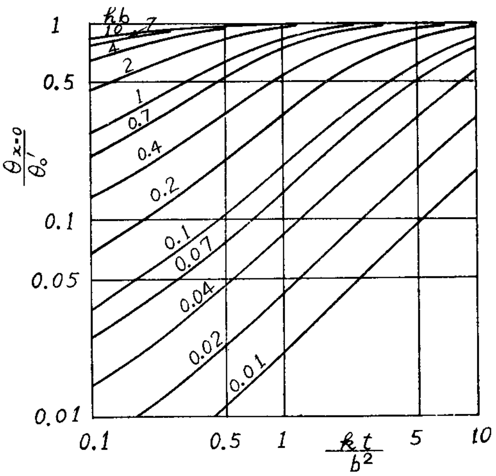

Fig. 25.

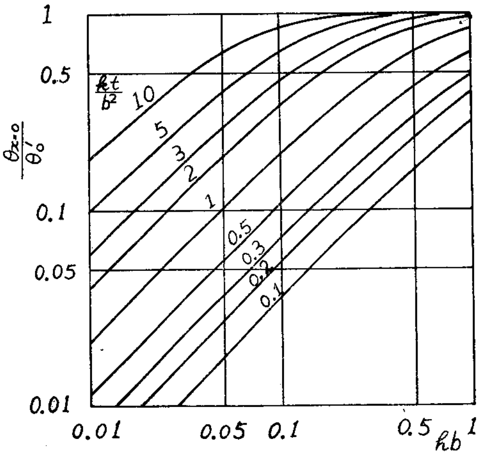

Fig. 26.

Now in practical infrared heating under the circumstance that the atmospheric temperature in the oven is so low that the effect of convection can be neglected, the limit of the infrared heating for uniform heating is naturally provided by Figs. 15 and 16 and there is no necessity of further consideration.

On the other hand, when the one side radiation is applied in practice, it often happens that there is difference in the atmosphric temperature on both surfaces of the material and atmospheric temperature of the surface not heated is left approximately as the temperature. In such case, it can be put $\theta_0'=0$ and, considering in the similar manner, the temperature ratio is furnished in Figs. 2 and 3, the limit of heating is given in Fig. 15, and the temperature rise can be figured by multiplying $[I_0/\alpha+\theta_0'']$ by the values obtained by Figs. 23 and 24.

In the case of the both side radiation, on the contrary, we are able to put $\theta_0''=0$ since we can assume that the atmospheric temperatures are nearly equal on each surface. Thus, the temperature ratio is given in Figs. 4 or 5, the limit of heating in Fig. 16, and the temperature rise can be figured by multiphying $\left[\frac{I_0}{\alpha}+\theta_0''\right]$ by the values obtained in Figs. 25 and 26.

§8. Conclusion

In this report several conditions pertaining to uniform heating with the infrared radiation has been discussed, the summary of which is as follows.

(i) Comparison was drawn between the infrared heating and the high frequency dielectric heating. The result deduced is that, although both methods are almost the same in respect to their efficiency, the former is far preferable to the latter from the point of view of the costs of installation and operation. And it has also been discussed that the infrared heating should be applied as much as possible for uniform heating.

(ii) Theoretical expressions of the temperature rise were derived from the infrared radiation by which the temperature ratio was defined.

It was also indicated that this ratio was represented by the dimensionless constants $hb=\alpha b/K$ and $kt/b^2=Kt/c\sigma b^2$. The relations between the two methods, the one side radiation and the both side radiation, were clarified. One example was cited, taking oak as a material, which showed how the temperature ratio varied with thickness, thermal transfer coefficient, thermal capacity, and thermal conductivity.

(iii) It has been discussed that, in the case of uniform heating with the infrared radiation, the allowance of temperature which is practically permissible should be utilized. From this allowance $\theta\pm\Delta\theta$, the allowable temperature ratio was defined as $\theta-\Delta\theta/\theta+\Delta\theta$ and, for the fixed ratio, the relations between hb and kt/b^2 were fiurnished whose properties were also discussed.

As the result, a boundary for the use of the infrared radiation for uniform heating has been provided.

(iv) $\alpha\theta/I_0$ representing the temperature rise on the heated surface was shown as a function of hb and kt/b^2. Relations between them were furnished in figures.

(v) Discussion has been brought forth taking into account the convection heating to be applied mostly to the infrared oven and the the treatment of temperature ratio and the state of temperature rise were considered.

The summary of this report is as listed above. However, the thermal constants of a material, by nature, undergo some changes with the variation of temperature, and the discussion on the assumption, as we did in this paper, leaves a room for question. As for the thermal transfer coefficient, the circumstances are the same. Furthermore, as to the temperature rise, the effect of radiation from a material has been neglected and this simplification also implies some approximation. Consequently, some differences between the results in pactical use and those of this treatise are inevitable. However, it is the author's please, if this paper will serve as one of the mile stones in application of infrared radiation in uniform heating.

Reference

N. Kato and Y. Takeya: Denki Hyoron, vol. 39, 6~7 (1951).