On the Analysis of Mechanism of River-Bed Variation by Characteristics

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Synopsis

In this paper, a new method using the characteristic curve for analysing the mechanism of river-bed variation in a river in which the sediment is mainly transported as bed-load is presented.

This method is introduced on the basis of the assumption that both of stream flow and river-bed profile vary gradually and the computation of river-bed variation is proceeded step by step. Various interesting phenomena of river-bed variation resulting from the sediment transport can be explained clearly by this characteristics method. As some examples, the deformations of mound and depression of river-bed in uniform flow of water and the deposition of sediment in reservoir are treated, and then the mechanism of their behaviours are ascertained.

1. Introduction

Sometimes it is said that the river is alive. It means that the feature of river varies with the lapse of time, and the variation of river-bed will be one of main factors from which the variation of river feature results.

The sediment transported by flowing water from mountain-side and others deposits at the place where the stream flow of water has not the capability to transport its sediment, and contrarily if the flow is capable of transporting more sediment, the bed material at this part in the river is picked up and transported downstream, so that the river-bed erosion occurs. This phenomenon may be in the concrete explained by comparing the quantities of sediment transported through two cross-sections of upper- and lowerstream in a river; that is, when the quantity of sediment through the upperstream cross-section is larger than that through the lowerstream cross-section, the deposition of sediment occurs on an average between these two cross-sections, and contrarily when the former is smaller than the latter, the river-bed erosion occurs on an average between these two cross-sections. This fact is easily understood, if the deposition of debris and sediment in soil-saving dam basins or reservoirs is taken as examples; that is, since the sediment hardly flows out at the downstream end of reservoir and, on the other hand, much quantity of sediment flows into the reservoir at the upstream end during flood stages, almost of the sediment transported into a reservoir deposits there and consequently the problem of filling of reservoir, which concerns with the life of reservoir, results. The detailed process of reservoir sedimentation, however, can not be ascertained with such a rough treatment as described above. To study the mechanism of river-bed variation including reservoir sedimentation, it is necessary to analyse this mechanism after expressing it mathematically.

In this paper, as a method for analysing the mechanism of profile variation of river or reservoir bottom on which the sediment is mainly transported as bed-load, the step by step computing method using characteristics is proposed, and moreover some interesting variation phenomena of bed profiles of river and reservoir are explained by this method.

2. Fundamental Equations on the River-Bed Variation

Taking the x-axis in the downstream direction along the river-bed, the equation of motion neglecting the vertical acceleration and the equation of continuity are, respectively[1],

$$\cos\theta_s\frac{\partial h}{\partial x}+\frac{1}{g}\frac{\partial u}{\partial t}+\alpha\frac{\partial}{\partial x}\left(\frac{u^2}{2g}\right)+(1-\alpha)\frac{u}{gA}\frac{\partial A}{\partial t}=\sin\theta_s-\frac{u^{*2}}{gR}, \tag{1}$$

$$\frac{\partial A}{\partial t}+\frac{\partial}{\partial x}(Au)=0, \tag{2}$$

where h is the water depth, u the mean velocity, A the cross-sectional area of flowing water, u^* the friction velocity, θ_s the river bed declination, α the momentum correction factor, R the hydraulic radius, g the gravity acceleration and t the time.

Since the rate of change of water flow motion in a river is generally small and also the variation of river-bed is slow, the idea that flow of water approximates steady at every instant may be permitted. Based on this idea, the terms of $\partial/\partial t$ in Eqs. (1) and (2) are omitted and moreover, for the sake of simplicity, the value of α in Eq. (1) is put as unity. In addition, as the case of small river-bed declination θ_s is treated, taking $\cos\theta_s$ unity and putting $\sin\theta_s=J_s$, Eqs. (1) and (2) are simplified as follows respectively:

$$\frac{dh}{dx}=J_s-\frac{d}{dx}\left(\frac{u^2}{2g}\right)-\frac{u^{*2}}{gR}, \tag{1$'$}$$

$$Au=Q=\text{const}, \tag{2$'$}$$

where Q is the river discharge.

On the other hand, according to the expression by A. A. Kalinske, the rate of bed-load transport along the stream bed per unit width (volume of sediment) q_B is expressed[2),3)]

$$\frac{q_B}{u^*d}=f\left(\frac{u^{*2}}{\{(\sigma/\rho)-1\}gd}\right), \tag{3}$$

where d is the diameter of sand grain, and σ and ρ are the density of grain and water, respectively. This expression, however, is also applicable, even though the sediment transport becomes to include the transportation in suspension with bed-load movement[3)]. In such a case, introducing the critical friction velocity u_c^* in Eq. (3) and moreover rewriting q_B as q_T denoting the rate of total sediment transport per unit width, Eq. (3) may generally be expressed

$$\frac{q_T}{u^*d}=K\left[\frac{u^{*2}-u_c^{*2}}{\{(\sigma/\rho)-1\}gd}\right]^m, \tag{4}$$

or

$$q_T=\alpha' u^*(u^{*2}-u_c^{*2})^m, \tag{4$'$}$$

where K and m are constants and $\alpha'=Kd/[\{(\sigma/\rho)-1\}gd]^m$. C. B. Brown has shown that when the friction velocity u^* is high as compared with the critical friction velocity for movement u_c^* and consequently the sediment transport includes some suspended-load movement, $K=10$ and $m=2$. But, the value of K obtained by the author's experiment was 30. Discussion on this difference will be made at another opportunity and in this paper Eq. (4) or Eq. (4)$'$ is applied to an approach as the relation connecting the rate of sediment transport with the friction velocity. Although this equation is the one presented for uniform flow, it is assumed to be applicable even for nonuniform flow. However, when the sediment is silt and transported almost as suspended load, q_T is determined by not only the hydraulic factors at the section of channel, but the boundary and initial conditions for the distribution of concentration of suspended material. Consequently, in order to analyse strictly the phenomena of sediment transportation in such a case, the general differential equation for the transport of material in suspension must be solved. This performance is very difficult except under special conditions.

Let Z be the river-bed elevation measured from any datum surface, then for the rectangular cross-section of width B the equation of continuity with respect to sediment transported is expressed

$$\frac{\partial Z}{\partial t}+\frac{1}{B(1-\lambda)}\frac{\partial(q_T B)}{\partial x}=0, \tag{5}$$

where λ is the porosity of deposits.

Denoting the slope of datum surface by J_0, the river-bed slope J_s is written as

$$J_s=J_0-\frac{\partial Z}{\partial x}, \tag{6}$$

and when the resistance law of Manning type $u^{*2}/gR=n^2u^2/R^{4/3}$ is used and the cross-section is regarded as a wide and rectangular shape, the friction velocity is given by

$$u^{*}=g^{1/2}nQ/h^{7/6}B. \tag{7}$$

Now, computing the term of $\partial(q_T B)/\partial x$ in Eq. (5) by using Eqs. (4)′ and (7) and assuming that α' and m are constant, Eq. (5) becomes

$$\frac{\partial Z}{\partial t}=A'\left(B'\frac{\partial h}{\partial x}+C'\frac{dB}{dx}\right), \tag{8}$$

where

$$A'=\frac{\alpha' n^{2m+1}g^{m+(1/2)}Q^{2m+1}}{(1-\lambda)B^{2m+1}h^{13/6}}\left(\frac{1}{h^{7/3}}-\frac{1}{h_k^{7/3}}\right)^{m-1},$$

$$B'=\frac{7}{6}\left\{\left(\frac{1}{h^{7/3}}-\frac{1}{h_k^{7/3}}\right)+\frac{2m}{h^{7/3}}\right\},\quad C'=\frac{2m}{Bh^{4/3}},$$

and h_k is the critical water depth for bed-load movement. The relation between this critical depth and the critical friction velocity is expressed by

$$h_k=(ng^{1/2}Q/Bu_c^{*})^{6/7}. \tag{9}$$

Using Eq. (1)′ for $\partial h/\partial x$ in Eq. (8), based on the assumption that the stream flow approximates to be steady at every instant, and also Eq. (6) for J_s, Eq. (8) is rewritten as follows:

$$\frac{\partial Z}{\partial t}+A'B'\frac{\partial Z}{\partial x}=A'\left[B'\left\{J_0-\frac{d}{dx}\left(\frac{u^2}{2g}\right)-\frac{n^2u^2}{R^{4/3}}\right\}+C'\frac{dB}{dx}\right]. \tag{10}$$

Expressing this relation by the characteristic equation, it becomes

$$\frac{dx}{dt}=A'B', \tag{11}$$

$$\frac{dZ}{dt}=A'\left[B'\left\{J_0-\frac{d}{dx}\left(\frac{u^2}{2g}\right)-\frac{n^2u^2}{R^{4/3}}\right\}+C'\frac{dB}{dx}\right]. \tag{12}$$

That is, it shows that Eq. (12) holds on the characteristic curves expressed by Eq. (11). If the datum surface and its slope J_0 are replaced with the river-bed surface and its slope J_s respectively, Eq. (12) is rewritten as follows:

$$\frac{dZ}{dt}=A'\left(B'\frac{dh}{dx}+C'\frac{dB}{dx}\right). \tag{12$'$}$$

Then, denoting dZ, dt and dx by ΔZ, Δt and Δx respectively and assuming that the hydraulic factors such as the discharge Q, the water depth h, the roughness coefficient n and others are constant during a time period Δt, and using the average values between the distance Δx as B, dB/dx, h, dh/dx and n, the depth of deposition or erosion on the river-bed ΔZ after a time period Δt can, in principle, be computed by replacing the actual cross-sectional shape of river with rectangular shape.

3. Mechanism of River-Bed Variation and Some Examples of its Applications

In considering the mechanism of river-bed variation based on the characteristic equation Eqs. (11) and (12)′, the case of constant width of river, i.e. $dB/dx=0$, for the sake of simplicity, will be treated in this section. Accordingly, in this case, Eq. (12)′ becomes

$$\frac{dZ}{dt}=A'B'\frac{dh}{dx}. \tag{12''}$$

Since, at the part where the bed material load is transported, u^* is larger than u_c^*, it is found from Eq. (9) that if Q and n are constant, the water depth h must always be less than the critical water depth for movement h_k. Consequently, the values of A' and B' are always positive, and therefore dh/dx and dZ/dt have the same sign as seen from Eq. (12)″, and also when $dh/dx=0$, $dZ/dt=0$. Since the datum surface of Z in Eq. (12)″ is always the river-bed surface, it means that the bed-material load deposits, when dZ/dt is positive, and the river-bed is eroded, when dZ/dt is negative.

Therefore, it is seen that when the bed-material load is briefly transported as bed-load and the characters of bed-material, such as the diameter of grain and the distribution of grain size, don't change along the stream, the bed-material load deposits in the case of $dh/dx>0$ and contrarily the river-bed is eroded in the case of $dh/dx<0$, and especially the variation of river-bed doesn't take place in the case of uniform flow. Such an idea will also be applied to treating the problem of equilibrium slope of river.

Since the value of $A'B'$ is positive except the section of river where the water depth h is equal to the critical water depth h_k, dx/dt is always positive as seen from Eq. (11) and consequently it is seen that the characteristic curve starting any section proceeds towards the positive direction of x (downstream). In this case, the smaller h, the greater the value of dx/dt. And at the section of river where $h=h_k$, both values of dx/dt and dZ/dt are zero, because of $A'=0$.

Now, the behaviour of river-bed variation and bed-material load deposit in reservoir will be explained below by some examples of application.

The first description deals with the following condition as shown in Fig. 1: there exists a mound on river-bed of which the average slope is gentler than the critical slope, i.e. the flow is subcritical, and also the stream is uniform flow at both of upper and lower reaches of that mound. In such a case, the water surface profile has minimum water depth at the top of the mound as shown by (a) in Fig. 1. But, if the bottom friction is considered, the minimum water depth may appear at the point a little downstream from the top. The figure denoted by (b) in Fig. 1 shows the characteristic curves on $x-t$ plane, which are expressed by straight lines during a time period Δt in this case. For instance, the characteristic curve starting the point a at $t=0$ arrives at the point a′ after $t=\Delta t$. From the fact described above that the deeper the water depth is, the smaller the value of dx/dt is, that is, the gradients of the characteristic curves in Fig. 1 become steep with the increase of water depth, the gentlest characteristic curve is that starting the top point c of mound, (a little upstream the point c, if the average between Δx is considered), and the steepest is that in the part of uniform flow. Therefore, the characteristic curves become gentle gradually as they approach the top of mound from the upper reach and contrarily, become steep as they leave the mound downstream till they have a definite slope in the region of uniform flow. The figure (c) in Fig. 1 shows $Z-x$ plane and x-axis is taken along the river-bed surface, adopted as the datum surface of Z. And, in this case, it represents the river-bed surface i.e. the shape of mound at $t=0$.

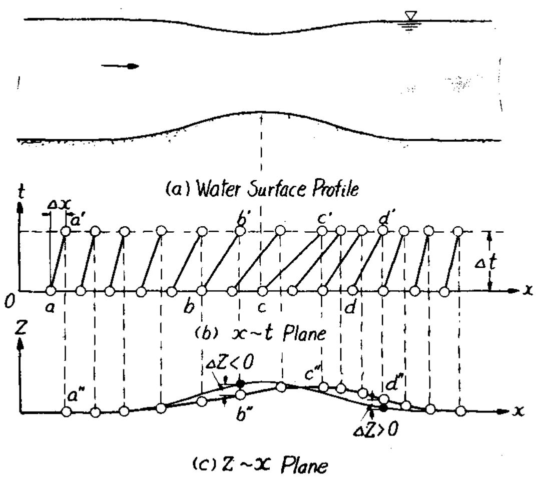

Fig. 1. Illustrative diagram for analysing the mechanism of river-bed variation in subcritical flow by characteristics.

Since at the point a the stream is uniform flow, $dh/dx=0$, and hence $\Delta Z=0$ from Eq. (12)″. Accordingly, at the point a″ the variation of river-bed doesn't occur. At the point b, since $dh/dx<0$, then $\Delta Z<0$, and hence the river-bed descends at the point b″ as shown in the figure. On the other hand, at the point d, since $dh/dx>0$, then $\Delta Z>0$, so that the river-bed at the point d″ ascends. Although $\Delta Z=0$ since $dh/dx=0$ at the point c, if an average between Δx is taken, the average value of dh/dx becomes greater than 0 and the river-bed at the point c″ ascends in some degree as shown in the figure. Consequently, it is found that while the mound moves downstream gradually, the lower side slope of mound surface becomes steeper more and more as seen in the figure and the mound tends to be deformed in the shape like the profile[4)] of sand ripple in the case of low Froude Number. This phenomenon is just alike to the behaviour of breaker near the beach.

Fig. 2 shows the mechanism of river-bed variation in the case of supercritical flow where the river-bed slope is steeper than the critical slope. If the stream at both of upper and lower reaches of the mound is uniform flow, the water surface profile becomes as shown in the figure (a), so that the water depth is maximum at the top of mound. Accordingly, the characteristic curves on $x-t$ plane in the figure (b) have the gentlest gradient in the reach of uniform flow and the steepest at the top of mound. Moreover, the deformation of mound is contrary to the case of subcritical flow described above, as shown on $Z-x$ plane in the figure (c). That is, it is seen that while the mound moves upstream gradually, the upper surface of the mound becomes to form steeper profile more and more. The behaviour of the mound makes us expect that the sand ripple moves upstream in the case of high Froude Number.

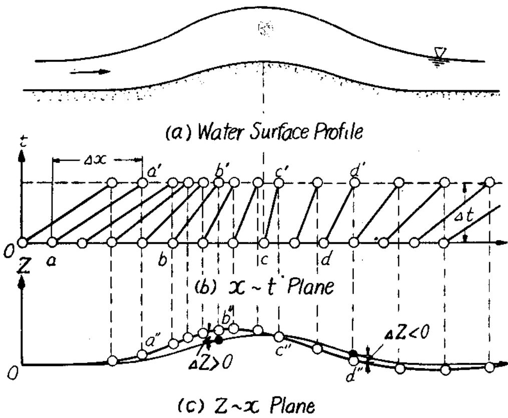

Fig. 2. Illustrative diagram for analysing the mechanism of river-bed variation in super-critical flow by characteristics.

Furthermore, the deformation of depression of river-bed can be explained by the same method as described above for the deformation of mound. It is also concluded that in this case, the depression proceeds downstream when the flow is sub-critical and contrarily it moves upstream when the flow is super-critical.

Fig. 3 illustrates the mechanism of deposition of the bed-material load in reservoir by the method of characteristics. In this case, the deposition is treated for the water surface profile of backwater produced by dam as shown in the figure (a). If there exists the water depth equal to the critical water depth h_k for bed-material movement at a certain cross-section in the reservoir, since the value of dx/dt is 0 at this cross-section, the characteristic curve at this point k on $x-t$ plane in the figure (b) is vertical. And the slopes of the characteristic

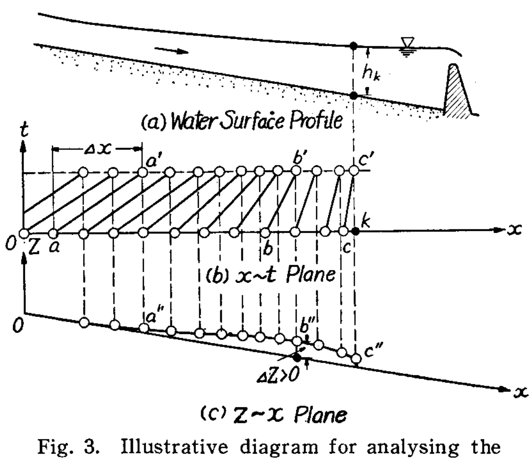

Fig. 3. Illustrative diagram for analysing the process of sediment deposition in reservoir by characteristics.

curves become gentle as they are going toward upstream until they have a definite gradient at the reach of uniform flow. Now, if the characteristic curve starting the point c comes to the point c′ after a time period Δt, all characteristic curves starting every point from c to k cross the vertical characteristic curve starting the point k. This is similar to the expression of the phenomenon of breaker near the beach by the characteristic curves. Although there are many values of ΔZ at the point k, since the greatest value of ΔZ at this point is that for the characteristic curve cc′ in this case, this value should be adopted as ΔZ at the point c″ on $Z-x$ plane in the figure (c). As $dh/dx>0$ in the reach of backwater, $\Delta Z>0$ and then the continual deposition occurs along the shallower reach than the critical water depth in the reservoir. But when the river-bed is uneven and there exists any reach with the condition $dh/dx<0$ near the upper part of reservoir, the scouring of bottom will be able to occur there.

After the lapse of time Δt, the bed-material load deposits with the profile as shown in the figure (c) on the river-bed being a straight line profile at $t=0$. Moreover, when the water depth at the point c″ becomes smaller than h_k resulting from the deposition, the bed-material load begins to be transported downstream through this cross-section and then deposits also at the lower part of the point c″ after the lapse of time Δt. It is very difficult to analyse how the sediment transported deposits at the lower reach of the point c″. It is, however, expected that the bed-material load transported through the cross-section of the point c″ deposits so that the water depth at the foot of delta deposits may always be equal to h_k and the foot of delta deposits thus approaches progressively the dam. Since such a mechanism of bed-material load deposition is all alike to that in a soil-saving dam pool, the process of filling of this pool by the sediment deposition can also be explained by this method.

This step by step computing method by the characteristics approach was applied to the analysis of the experimental results on the sedimentation above a weir by Mr. S. Sugio under the author's instruction[5]. And also, the annual change of the bottom profile of Omine Reservoir of the Uji River in Japan was computed practically by the author. Both of the results by Mr. S. Sugio and the author were successful, and the process of sedimentation in the pool of reservoir could be well explained though some difficult problems were introduced in determining variables in the practical applications.

4. Conclusion

Since there are various kinds of river-bed variations from the small sand ripple having a wave length of several cm to the sediment deposition in a great reservoir, it may be impossible to analyse their mechanism by the same theory. In this paper, the river-bed variations of large scale compared with the water depth were treated and their mechanisms in the most simplified conditions were considered. The difficult problem on the mechanism of river-bed variation, which has hardly been treated up to the present day, could be solved to some extent by the method of characteristics proposed here by the author.

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