On the Theoretical Capacity of an Off-Street Parking Space

By

Eizi KOMETANI and Akira KATO

Department of Civil Engineering

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1. Introduction

In the planning of an off-street parking space in the big cities in our country, it will be necessary only to deal with the short time parking, because almost all vehicles parked will be used for the purpose of business and not by those shopping or going to office. As the rate of turn-over in our parking space is relatively high, a moderately small area will be sufficient. On the other hand, differing from the parking spaces used by those attending office, the site of the parking space must be as close as possible to the destination. Namely, it must be located adjacent to the central business district. As it is difficult to acquire sufficient areas in such places where the lands are very expensive a close scientific investigation must be made in determining the site and the size.

At a parking space we find some cars entering, leaving and parked. The most efficient capacity is such that the parking space is always utilized fully but is able to accept a new vehicle intending to park. Namely, the ideal condition is that when some vehicles depart, the same number of vehicles enter. In this case the problem arises when some vehicles can not park there because all the parking lots might be occupied when many vehicles rush in a short period. But it is a matter of course from the point of economy to minimize the number of the parking lots against the total number of vehicles parked. This is regarded as of great importance especially in the central business district of the cities where the land cost is very expensive. Therefore the problem is how to determine the size of a parking space to avoid the impossibility of parking and to gain the maximum efficiency with the minimum parking space area.

Regardless of the observation results, parking demand shows a continuous increase year after year, and hourly or daily fluctuation of the cars parked in a district which generates the parking demand can be clearly observed. Thus the frequency of the parking demand shows a certain tendency which fluctuates hourly, but if we consider a short time period as a unit, then,

(1) parking occurs individually at random,
(2) some vehicles are parked during the short limited time.

If the above conditions (1) and (2) exist, it is obvious that the probability for the occurrence of the parking has a certain type of distribution and it is also known that the parking duration of each vehicle conforms with a certain type of distribution. Investigating these two types of distribution, we have derived the probability to end a parking from the probability distribution for the occurrence of parking and the distribution of parking duration, and thus have computed the theoretical capacity of a parking space corresponding to a known parking demand by applying the condition of the statistical equilibrium to the above obtained probability to end a parking.

2. The Probability and its Distribution for the Occurrence of Parking Demand

When we consider an area within which parking demands occur, the following theoretical solution is derived by considering that all vehicles intending to park in the district over a certain time do park in the off-street parking space within the area. This corresponds to the case when parking freely at any place other than the fixed parking lots is entirely prohibited. At first in order to indicate the quantitative expression of the parking, we define as follows. Average number of parking demand which will occur in a certain survey block within a unit time period is defined as parking demand a, and the number of vehicles capable of occupying vacant parking lots is named parking volume a_c. The ratio of the number of cars which can not park due to the parking lots being occupied when the cars arrive at the parking space to the number of parking demand is defined as the factor of parking impossibility L, which is given by the following equation.

$$L = \frac{a-a_c}{a} \qquad \cdots\cdots(1)$$

If the probability of any lot being occupied during a random time interval t is equal, then the parking demand which arises in this survey district during any short time interval Δt is equal to $a\Delta t$. If the percentage of occupied lots in the whole district at any instant is denoted by Φ, and the chance for a lot, unoccupied at the beginning of the time interval, to be occupied within that time is denoted by α, the rate of lots occupied during a time interval Δt is given by the following relation

$$\alpha\Delta t = \frac{a\Delta t}{(1-\Phi)N} \qquad \cdots\cdots(2)$$

where N is the total sum of the capacities of all parking spaces in this survey district. Now, Φ the rate of occupied lots is considered as follows.

$$\Phi = \frac{a(1-L)}{N} \qquad \cdots\cdots(3)$$

When the parking spaces for r vehicles are occupied, the probability of the additional i lots out of the remaining $(N-r)$ lots being filled within the time Δt is expressed as follows.

$$Q^{(i)} = {}_{N-r}C_i(\alpha\Delta t)^i + O(\alpha\Delta t)^{i+1} \qquad \cdots\cdots(4)$$

The second term denotes the case when a car parked at the beginning of Δt leaves and a different car parks in the lot during the period of Δt.

Denoting the probability of i vehicles entering a parking space within a certain time period t by $f(i, t)$, and expressing

$$t = t_1 + t_2$$
$$i = i_1 + i_2$$

the following relation is derived.

$$f(i, t) = \sum_{i_1+i_2=i} f(i_1, t_1)\cdot f(i_2, t_2) \qquad \cdots\cdots(5)$$

Now substituting $t+\Delta t$, t, Δt instead of t, t_1, t_2, we derive Eq. (6) from the property of Markoff process, because Eq. (5) is the relation of the discrete stochastic process.

$$\begin{aligned} f(i, t+\Delta t) &= \sum_{i_1+i_2=i} f(i_1, t)\, f(i_2, \Delta t) \\ &= f(i, t)\, f(0, \Delta t) + f(\overline{i-1}, t)\, f(1, \Delta t) + O(\Delta t)^2 \end{aligned} \qquad \cdots\cdots(6)$$

$f(i, t)$ is a function of i, t, and the parking demand a, expressed by $a(t)$, is a function of t and not a constant. As $f(0, \Delta t)$, $f(1, \Delta t)$ represent the probability for zero or one vehicle to enter a parking space during a time period Δt respectively, Eq. (7) is derived.

$$\left.\begin{aligned} f(0, \Delta t) &= 1 - a(t)\,\Delta t + O(\Delta t)^2 \\ f(1, \Delta t) &= a(t)\,\Delta t \end{aligned}\right\} \qquad \cdots\cdots(7)$$

From Eqs. (6) and (7)

$$f(i, t+\Delta t) = f(i, t)(1 - a(t)\,\Delta t) + f(\overline{i-1}, t)\, a(t)\,\Delta t + O(\Delta t)^2 \qquad \cdots\cdots(8)$$

Eq. (8) gives Eq. (9) when Δt tends to zero.

$$\frac{1}{a(t)}\frac{d}{dt} f(i, t) = f(\overline{i-1}, t) - f(i, t) \qquad \cdots\cdots(9)$$

Eq. (9) is a finite difference differential equation, because i may be any one of $0, 1, 2, \cdots$. From Eqs. (7) and (9)

$$\frac{1}{a(t)}\frac{d}{dt} f(0, t) = -f(0, t)$$

Thus considering the initial condition $f(0,\ 0)=1$, Eq. (10) exists.

$$f(0,\ t)=e^{-\int_0^t a(t)\,dt} \qquad \cdots\cdots(10)$$

To solve the finite difference differential equation (9), we introduce a generating function $G(x,\ t)$.

$$G(x,\ t)=\sum_{i=0}^{\infty} x^i f(i,\ t)$$

Multiplying x^i to both sides of Eq. (9) and summing up the equations obtained by putting $i=0, 1, 2, \cdots$, Eq. (11) is derived as the following relation exists so far as $G(x,\ t)$ converges.

$$\frac{1}{a(t)}\frac{d}{dt}\sum_{i=0}^{\infty} x^i f(i,\ t)=\sum_{i=0}^{\infty} x^{i+1} f(i,\ t)-\sum_{i=0}^{\infty} x^i f(i,\ t)$$

$$\therefore\quad \frac{1}{a(t)}\frac{d}{dt}G(x,\ t)=(x-1)\,G(x,\ t) \qquad \cdots\cdots(11)$$

Solving this equation we obtain

$$G(x,\ t)=A\exp\left\{-(1-x)\int_0^t a(t)dt\right\}$$

where A: integration constant.

$$\therefore\quad f(i,\ t)=A\cdot\frac{\left\{\int_0^t a(t)\,dt\right\}^i}{i\,!}\exp\left\{-\int_0^t a(t)\,dt\right\} \qquad \cdots\cdots(12)$$

Eq. (12) has to satisfy Eq. (10), so $A=1$

$$\therefore\quad f(i,\ t)=\frac{\left\{\int_0^t a(t)\,dt\right\}^i}{i\,!}\exp\left\{-\int_0^t a(t)dt\right\} \qquad \cdots\cdots(13)$$

Eq. (13) represents Poisson's distribution of which the average value is

$$m=\int_0^t a(t)\,dt\,.$$

Now the origin of time t may be taken arbitrary, and m is the value of expectation for parking demand to occur. That is to say, if we know the expectation of the parking demand which occurs during the time t, $f(i,\ t)$ the probability for the occurrence of parking is found independent of the length of t and of the attitude $a(t)$ at which the parking demand occurs. As the expectation of the parking demand assumed to occur during a short time interval Δt is expressed by $a\Delta t$ as just mentioned above, the probability $f(i,\ \Delta t)$ for the parking demand of i vehicles to occur during Δt is given by the Poisson's distribution as follows.

$$f(i,\ \Delta t)=Q^{(i)}=e^{-m}\frac{m^i}{i\,!}=\frac{(a\Delta t)^i\,e^{-a\Delta t}}{i\,!} \qquad \cdots\cdots(14)$$

Observed results for the probability distribution of the occurrence of parking within certain blocks along a street in the central business district of Kyôto are shown in Fig. 1 (a), Fig. 2 (a) and Table 1.

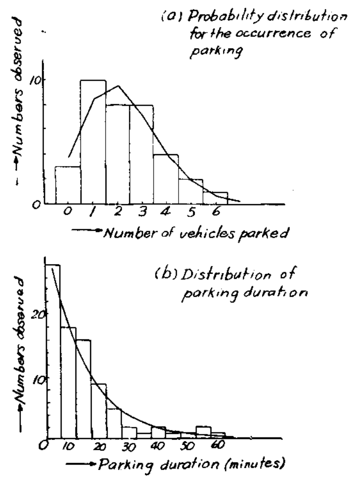

Fig. 1. Diagram to analyze the parking near Kawaramati-Bukkôzi in Kyôto City.

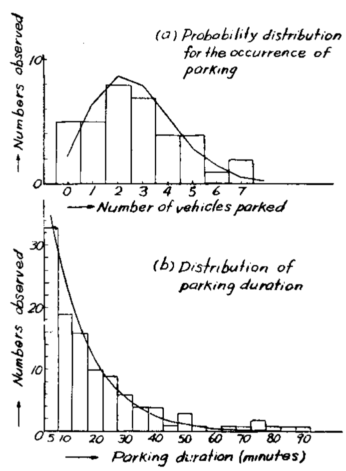

Fig. 2. Diagram to analyze the parking near the Kyôto City Hall.

Table 1. Probability distribution for the occurrence of parking. (t=5 minutes)

Vehicles parked	Number of occurrences observed	Theoretical number of occurrences
(a) Near Bukkôzi on the Kawaramati Street		
0	3	3.68
1	10	8.38
2	8	9.55
3	8	7.27
4	4	4.14
5	2	1.91
6	1	0.72
7	0	0.35
Total	36	36.00
Parking demand (Weighted mean of the parked vehicles) a=2.28		
(b) Near the Kyôto City Hall		
0	5	2.36
1	5	6.44
2	8	8.76
3	7	7.95
4	4	5.40
5	4	2.94
6	1	1.43
7	2	0.52
8	0	0.20
Total	36	36.00
Parking demand a=2.72		

3. The Distribution of Parking Duration and the Probability of Ending the Parking

Parking duration of each parked vehicle is not constant but is considered as showing a certain distribution. Taking a time interval τ as time unit, it is well known that the probability for the parking duration to be greater than t is ordinarily expressed as an exponential distribution. Therefore the probability of a vehicle to park from t_{i-1} to t_i is expressed by

$$H(t)=\int_{t_{i-1}}^{t_i}\frac{1}{\beta\tau}e^{-\frac{1}{\beta\tau}t}dt \qquad \cdots\cdots(15)$$

Now denoting the average parking duration as $\bar{t}$,

$$\bar{t}\fallingdotseq\beta\tau$$

where β is the cardinal number of the time interval which contains the average value, and τ is, as mentioned, above a time interval taken as time unit. Examples of the distribution of parking duration observed in a street in the central business district of Kyôto are given in Fig. 1 (b), Fig. 2 (b) and Table 2.

Let the parking duration of any one vehicle be t_i, $i=1, 2, 3, \cdots, g$ (g is the number corresponding to the observed maximum parking duration). Considering t_i as being equal to any one of the following,

$$0\leqq t_1<t_2<\cdots<t_{i-1}<t_i<\cdots<t_g,$$

and denoting the probability of any one vehicle to park for a period of t_i as h_i, the following relations exist.

$$\left.\begin{aligned}\sum_{i=1}^{g}h_i&=1\\ \sum_{i=1}^{g}t_ih_i&=1\end{aligned}\right\} \qquad \cdots\cdots(16)$$

$H(t)$ is defined as follows.

$$\begin{array}{ll} 0\leqq t\leqq t_1 & H(t)=1\\ t_{i-1}<t\leqq t_i \quad (i=2, 3, \cdots, g) & H(t)=\sum_{i=i}^{g}h_i\\ t_g<t & H(t)=0\end{array}$$

The probability of a parking lot occupied by a vehicle with a parking duration of t_i at an instant t_0 to become unoccupied between t and $t+\Delta t$ is as follows,

$$\begin{array}{lll}\dfrac{\Delta t}{t_i} & \text{for} & 0\leqq t\leqq t_i\\ 0 & \text{for} & t_i<t.\end{array}$$

The reason is it is certain that the duration during which this vehicle was parked in the lot is from 0 to t_i preceding t_0, but it is impossible to say at which instant the parking probably occurred. As it is sure that the parking lot becomes unoccupied

Table 2. The distribution of parking durations.
(τ=5 minutes)

Parking duration (minutes)	Number of occurrences observed	Theoretical number of occurrences
(a) Near Bukkôzi on the Kawaramati Street		
2-5	28	27.6
6-10	18	18.6
11-15	16	12.7
16-20	9	8.5
21-25	5	5.7
26-30	2	3.8
31-35	1	2.6
36-40	2	1.8
41-45	1	1.1
46-50	0	0.8
51-55	2	0.5
56-60	1	0.3
Further omitted		
(b) Near the Kyôto City Hall		
2-5	33	34.8
6-10	19	24.6
11-15	16	16.7
16-20	10	13.1
21-25	9	7.9
26-30	6	5.6
31-35	4	3.7
36-40	4	2.6
41-45	1	1.7
46-50	3	1.3
51-55	1	0.8
56-60	0	0.5
61-65	1	0.4
66-70	1	0.3
71-75	2	0.2
76-80	1	0.1
81-85	1	0.1
86-90	1	0.1
Further omitted		

N. B. The first row only of the parking duration was defined as 2-5 minutes, because the vehicles which stayed less than 2 minutes were not taken as parked vehicles but considered as merel stopping.

after t_0 and before t_i, the probability of the parking lot to become idle at the end of the time interval Δt may be considered as above.

Furthermore, as the probability of the parking duration of a vehicle occupying a parking lot at an instant t_0 to be t_i is proportional to the probability for the occurrence of the parking and to the parking duration t_i itself, the following relation is derived from Eq. (16).

$$\frac{t_i h_i}{\sum_{i=1}^{g} t_i h_i} = t_i h_i$$

Thus the probability $h(t)$ for any one vehicle in the parking lots at an instant t_0 to end the parking after t_0 and between t and $t+\Delta t$ may be given as follows, if $t_{i-1} < t \leqq t_i$

$$\begin{aligned} h(t)\ \Delta t &= \sum_{i=1}^{g} \frac{\Delta t}{t_i} t_i h_i \\ &= H(t)\ \Delta t \end{aligned} \qquad \cdots\cdots(17)$$

Therefore, the probability of the parking ending within Δt after t_0 may be expressed as follows considering the relation (15),

$$h(0)\ \Delta t = \frac{1}{\beta\tau}\ \Delta t \qquad \cdots\cdots(18)$$

As we can suppose that the r vehicles in the parking lots are individual to each other, the probability Q_j of any j vehicles out of the r vehicles ending their parking during Δt is given by Eq. (18) as follows.

$$Q_j = {}_rC_j\left(\frac{\Delta t}{\beta\tau}\right)^j + O\left(\frac{\Delta t}{\beta\tau}\right)^{j+1} \qquad \cdots\cdots(19)$$

$Q_j^{(i)}$, the probability of additional i vehicles entering and j vehicles leaving the parking space during Δt when r vehicles are parked, is given as follows from Eq. (4) and (19).

$$Q_j^{(i)} = {}_{N-r}C_i(\alpha\Delta t)^i\, {}_rC_j\left(\frac{\Delta t}{\beta\tau}\right)^j + O(\Delta t)^{i+j+1}$$

If the time interval t for the distribution of the probability for the occurrence of parking is equated to the time interval τ for the distribution of parking durations, the above equation is written as follows.

$$Q_j^{(i)} = {}_{N-r}C_i(\alpha\Delta t)^i\, {}_rC_j\left(\frac{\Delta t}{\beta}\right)^j + O(\Delta t)^{i+j+1} \qquad \cdots\cdots(20)$$

4. Statistical Equilibrium of the Two Probabilities of the Parkings to Occur and to End.

$P(r)$, the probability of a parked vehicle leaving and a traveling vehicle entering the parking space to occur at the same time can be obtained by means of statistical equilibrium.

Consider first an instant when r vehicles are parking. The probability of the number of parked vehicles changing from r due to additional vehicles entering or parked vehicles leaving during the consecutive time period is assumed to be equal to the probability of the number of vehicles which initially was more or less than r to become r due to the occurrence or the completion of parking during the same time period. The probability of the parking demands of k vehicles to occur or end during the same time period is considered to be the magnitude of $(\Delta t)^k$ order if we choose a sufficiently short time interval Δt. Thus as the chance of the parking demands of more than two vehicles to occur or end during the short time interval can be made negligibly small, we can get the next equilibrium condition from Eq. (4) and (19).

$$P(r)\left[(N-r)\alpha+\frac{r}{\beta}\right]\Delta t = P(r-1)[N-(r-1)]\,\alpha\Delta t+\frac{r+1}{\beta}\,P(r+1)\,\Delta t+O((\Delta t)^2) \quad \cdots(21)$$

The left side of the above equation represents the probability for the number of vehicles parked changing from r to $(r-1)$ or $(r+1)$ due to one vehicle leaving or a new one entering. The first term of the right side is the probability of $(r-1)$ parked vehicles increasing to r vehicles, and the second term of $(r+1)$ vehicles decreasing to r vehicles. When Δt tends to zero in Eq. (21), it becomes

$$(N-r+1)\,\alpha P(r-1)-\left[(N-r)\,\alpha+\frac{r}{\beta}\right]P(r)+\frac{r+1}{\beta}\,P(r+1)=0 \qquad \cdots\cdots(21')$$

If $r=0$, the first term of the above equation disappears, because $(r-1)$ vehicles can not exist. Thus

$$-N\alpha P(0)+\frac{1}{\beta}\,P(1)=0$$

Similarly when $r=N$, the third term of the above equation disappears because $P(r+1)$ can not exist.

$$(N-N+1)\,\alpha P(N-1)-\frac{N}{\beta}\,P(N)=0$$

Eq. (21′) exists for $r=1, 2, 3, \cdots, N-1$. Among the $(N+1)$ equations thus obtained, arbitrary one equation, for example the last equation, can be reduced from the other N equations. Thus excluding the last one, we have the following.

$$(N-r)\,\alpha P(r)-\frac{r+1}{\beta}\,P(r+1)=0$$

$$\therefore \quad P(r)=\frac{(r+1)\,P(r+1)}{(N-r)\,\alpha\beta} \qquad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(22)$$

From Eq. (22),

$$P(0)=\frac{1}{N\alpha\beta}P(1)$$

$$P(1)=\frac{2}{(N-1)\,\alpha\beta}P(2)$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$P(r-1)=\frac{r}{(N-r+1)\,\alpha\beta}P(r)$$

$$\therefore\quad P(0)=\frac{1}{N\alpha\beta}P(1)=\frac{1}{N\alpha\beta}\cdot\frac{2}{(N-1)\,\alpha\beta}P(2)=\cdots\cdots$$

$$=\frac{r\,!}{(\alpha\beta)^r\dfrac{N!}{(N-r)\,!}}P(r)=\frac{1}{(\alpha\beta)^r\,{}_NC_r}P(r)$$

It is obvious from the definition of probability that $\sum P(r)=1$. The above obtained relation always exists for $r=0, 1, 2, \cdots, N$. When we consider each parking space included within the survey district, the total parking capacity N should be substituted by n, the parking capacity of each parking space, and the following relation exists.

$$\sum_{r=0}^{n}P(r)=P(0)\,[1+\alpha\beta_N C_1+(\alpha\beta)^2\,{}_NC_2+\cdots+(\alpha\beta)^n\,{}_NC_n]=1$$

$$\therefore\quad P(r)=\frac{(\alpha\beta)^r\,{}_NC_r}{\sum_{r=0}^{n}(\alpha\beta)^r\,{}_NC_r}\qquad\cdots\cdots\cdots\cdots(23)$$

We have the next relation from Eqs. (2) and (3)

$$\alpha=\frac{a}{[N-a(1-L)]}.$$

If $L\ll 1$,

$$\alpha\fallingdotseq a/(N-a)\,,$$

therefore

$$P(r)=\frac{{}_NC_r\left(\dfrac{a\beta}{N-a}\right)^r}{\sum_{r=0}^{n}{}_NC_r\left(\dfrac{a\beta}{N-a}\right)^r}\qquad\cdots\cdots\cdots\cdots(23')$$

thus we can express the relation of $P(r)$ by the parking demand a.

Eqs. (23) and (23′) give the probability for the occurrence of r vehicles entering and leaving at the same time in a certain parking space. Also, as α is the probability of an arbitrary vehicle intending to park in an arbitrary unoccupied parking lot within the whole parking space to actually occupy the lot during a unit time, we have the following for each parking space.

$$\sum_{r=0}^{n}(N-r)\,\alpha P(r)=a\qquad\cdots\cdots\cdots\cdots(24)$$

As $P(r)$ in Eq. (23) has to satisfy Eq. (24), we have the following relation by substituting it.

$$a=\frac{\alpha\sum_{r=0}^{n}(\alpha\beta)^r\,{}_NC_r(N-r)}{\sum_{r=0}^{n}(\alpha\beta)^r\,{}_NC_r}$$

$$=\frac{N\alpha\sum_{r=0}^{n}{}_{N-1}C_r(\alpha\beta)^r}{\sum_{r=0}^{n}(\alpha\beta)^r\,{}_NC_r}\qquad\cdots\cdots(25)$$

For the parking volume a_c, we have the next equation.

$$\sum_{r=0}^{n} r\frac{1}{\beta}P(r)=a_c\qquad\cdots\cdots(26)$$

Eq. (23) must also satisfy this equation, and thus

$$a_c=\frac{\frac{1}{\beta}\sum_{r=0}^{n}r(\alpha\beta)^r\,{}_NC_r}{\sum_{r=0}^{n}(\alpha\beta)^r\,{}_NC_r}$$

$$=\frac{\frac{1}{\beta}(\alpha\beta)\,N\sum_{r=0}^{n-1}{}_{N-1}C_r}{\sum_{r=0}^{n}{}_NC_r(\alpha\beta)^r}=\frac{N\alpha\sum_{r=0}^{n-1}{}_{N-1}C_r(\alpha\beta)^r}{\sum_{r=0}^{n}{}_NC_r(\alpha\beta)^r}\qquad\cdots\cdots(27)$$

The probability $R(r)$ for a vehicle intending to enter a parking space to encounter the case of the parking space being occupied by r vehicles is given as follows,

$$R(r)=\frac{(N-r)\,\alpha P(r)}{a}=\frac{{}_{N-1}C_r(\alpha\beta)^r}{\sum_{r=0}^{n}{}_{N-1}C_r(\alpha\beta)^r}$$

5. Factor of Parking Impossibility and Capacity of Parking Space

In determining the capacity of a parking space, if we choose its volume so that the above mentioned factor of parking impossibility corresponds to a certain value, for instance, $L=0.01$ or 0.02, we can determine the capacity of the parking space in the following way. Namely, let the probability for a vehicle intending to park which has just arrived at the parking space and has found no vacant parking lot be equal to L. The factor of parking impossibility L can be obtained by substituting Eqs. (25) and (27) into Eq. (1),

$$L=\frac{{}_{N-1}C_n(\alpha\beta)^n}{\sum_{r=0}^{n}{}_{N-1}C_r(\alpha\beta)^r}\qquad\cdots\cdots(28)$$

In Eq. (28) it is considered that both α and β are given as the results of observations, but in general the parking demand a is observed more frequently than α the probability of a vacant parking lot being occupied within a certain time interval. So, using a relation $\alpha\fallingdotseq a/(N-a)$,

$$L=\frac{{}_{N-1}C_n\left(\frac{a\beta}{N-a}\right)^n}{\sum_{r=0}^{n}{}_{N-1}C_r\left(\frac{a\beta}{N-a}\right)^r}\qquad\cdots\cdots(28')$$

Generally we can compute the capacity by this Eq. (28′). Illustrating Eq. (28′) as in Fig. 3, we can find the capacity of a parking space n, if we can observe the total parking capacity N, parking demand a and the average value $\beta\tau$ of the distribution of the parking duration.

6. Approximate Computation and Numerical Example

The relation of determining the capacity of a parking space obtained above is very complicated to solve and, as shown in Fig. 3, n is less affected by the change of N when N becomes a considerably large value. It indicates that the total parking capacity of the whole district has little influence upon the determination of the capacity of a parking space. When we deal with the parking problem in the central business district of cities in practice, probably $N>100$, so in many cases it is sufficient to use an approximate formula reduced by putting $N\to\infty$. If we put in Eq. (23′) $N\gg n$ and let n tend to infinity, we will find $P(r)$ in the type of Poisson's distribution as follows.

$$P(r) = R(r) = e^{-a\beta}\frac{(a\beta)^r}{r\,!} \qquad \cdots\cdots(29)$$

Next, out of an infinite parking space capacity, if only n lots exist and the others considered as imaginary lots, a vehicle intending to park enters the imaginary lot if the parking space has no vacant lot at the time. If we suppose this imaginary parking corresponds to the volume of parking impossibility,

$$L = \sum_{r=n}^{\infty} R(r) = e^{-a\beta}\sum_{r=n}^{\infty}\frac{(a\beta)^r}{r\,!} \qquad \cdots\cdots(30)$$

Selecting a curve similar to that of Eq. (30), we can find the next equation,

$$n = a\beta + k\sqrt{a\beta} \qquad \cdots\cdots(31)$$

The constant k in Eq. (31) is given in Table 3, but a good approximation cannot be obtained by this equation except for the case of $a\beta>10$.

Table 3.

L	k
0.1	1.28
0.02	2.05
0.01	2.33
0.005	2.58

If we can know the distribution of the parking duration and the probability distribution for the occurrence of parking during a unit time period concerning the vehicles parking near the projected parking space, we can compute the most effective capacity of the parking space from Eqs. (28), (28′), (30) and (31) by assuming that the factor of parking impossibility as 0.01 or 0.02. In this case Figs. 3, 4 and 5 can be used to compute the approximate value. Furthermore, these are the equations to determine the capacity of all sorts of parking spaces no matter how large or small the spaces are, the large one might be a district having a considerably large area such as Marunouti or Hibiya District in Tôkyô (about 1 km²) and the small one a garage attached to a building and used exclusively.

For example, we can obtain the capacity of a parking space from the parking analysis diagram in the neighbourhood of the Kyôto City Hall as shown in Fig. 2. It is known that $a = 2.72$, $\beta = 3$,

$$\therefore \quad a\beta = 2.72 \times 3 = 8.16\,,$$

we can read $n = 14$ from the value $a\beta = 8.16$ in Fig. 3 assuming $L = 0.01$. Authors have made similar observations though of a small scale in some other districts in Kyôto City such as Sizyô-Kawaramati and others, and could determine their capacities. But in these studies the required capacities were computed with the consideration that all vehicles parking for more than two or three minutes, that is periods which can not be considered as mere stopping, have to enter an off-street parking space. It is illogical and impractical to accomodate all vehicles parking for such a short time, and only the vehicles which park over a certain period are taken into consideration in the off-street parking space. Therefore, it is necessary to pick up only the cars parked longer than a definite period of time from the observation records, and the probability distribution for the occurrence of parking and the distribution of parking duration should be analyzed only for the chosen vehicles and the thus obtained a and β should be the basis for computing the capacity. For instance, when the probability distribution for the occurrence of parking and the distribution of parking duration are obtained as is illustrated in Fig. 6 (a), (b), then if we consider only the vehicles parking over a certain period of time, the distribution curves become as shown by the chain-line. Therefore the average value a for the occurrence of parking and β the cardinal number for the parking duration will also change and the method described here should be applied to the corrected results. In

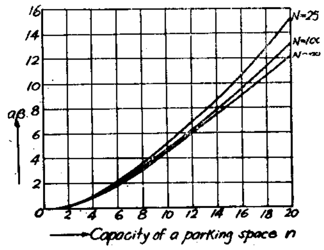

Fig. 3. Diagram showing the relation of parking volume a, parking duration β and the capacity of a parking space n, (In case of L=0.01).

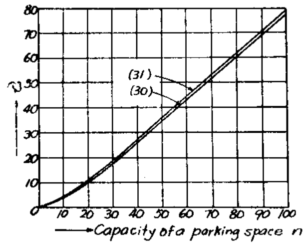

Fig. 4. Comparison of the approximate calculations, (In case of L= 0.01).

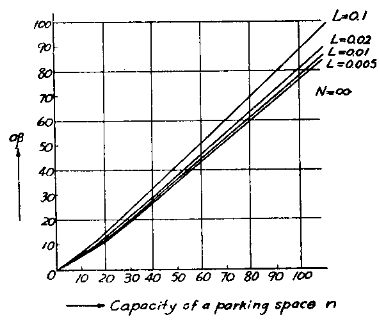

Fig. 5. Diagram to compute the capacity of a parking space n, (In case of $N=\infty$).

this case if the new ordinate axis is drawn through the minimum parking duration, for example twenty minutes, and β' is the cardinal number of the time interval containing the obtained average parking duration, β is given by the following.

$$\beta=\beta'+\beta_0$$

where β_0 is a cardinal number of the time interval corresponding to the shift of the ordinate axis.

The theoretical value itself computed from the above mentioned process may be insufficient for a smooth operation, and it may be necessary to give some allowance by considering the time and space required for cars to enter or leave.

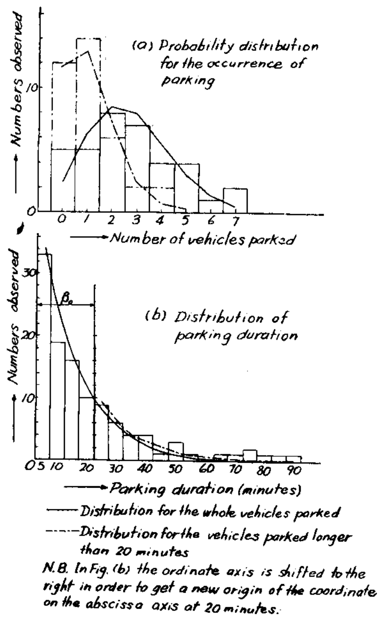

Fig. 6. Parking analysis diagram for the cars parked longer than twenty minutes.

7. Conclusion

The above mentioned method of computing the capacity of an off-street parking space made it possible to calculate the theoretical capacity of a parking space with the factor of parking impossibility adequately chosen, by means of statistical equilibrium as the result of introducing the probability theory to the parking problem.

As described above, this theoretical solution is not only applicable to determine the parking capacity required to meet the parking demand of a pretty wide district, but also is effective for determining the capacity of a small parking space attached to a building. As a method of determining the capacity of an off-street parking space, concerning which almost no theoretical analysis has been given so far, a theoretical solution applicable to a wide range was introduced. It is believed that this solution will provide rational and scientific basis for the parking projects in future.

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