On the Vibrational Damping of Structural Steel Beams

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Abstract

In this paper, experimental results and their considerations on the vibrational damping characteristics of model beams and of actual steel highway bridges are presented.

The fundamental characteristics of damping due to internal frictions of the steel beam and the friction of bearing are clarified by using three kinds of beams. Also, the damping characteristics of several actual bridges have been investigated and they are compared in this paper with the results obtained by the experiments on model beams.

These results are useful for analysis of the dynamic problems of bridges.

1. Introduction

In regard to the dynamic problems of a bridge, such as the problems of aerodynamic resistance of suspension bridge, the impact factor and the earthquake resistance, the damping characteristics of vibration must be fully investigated. The author has been carrying out for some time an experimental research on the damping characteristics of actual bridges and the results of these experiments are explained in Chapter 9 of this report. In some other chapters of this paper, experimental researches on model beams and some considerations of these experiments are described for the purpose of clarifying the fundamental characteristics of damping. The main purpose of the experiments on model beams are to investigate exactly the characteristics and the values of the vibrational damping resulting due from the different causes of damping. To achieve these purposes, three kinds of beam, welded, reveted and bolted beam, have been utilized.

2. Model Beams and Method of Experiments

The model beams used in this experiment are composed of two beams, set in

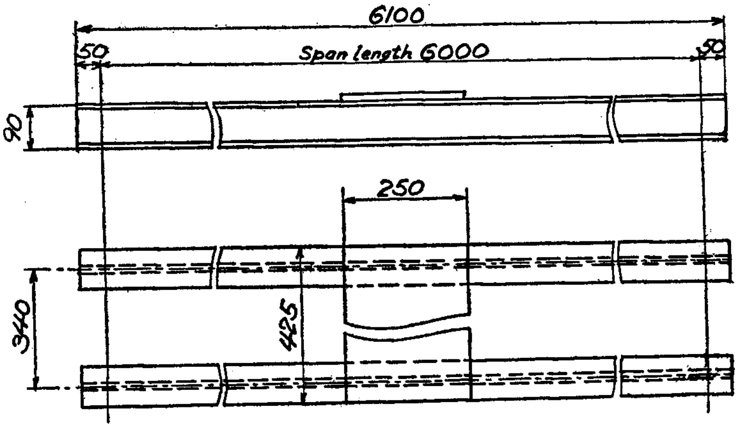
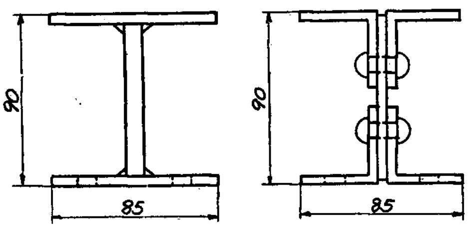


Fig. 1 Main Dimensions of Model Beam.

parallel, and they are of three kinds—welded, riveted and bolted beams. Figs. 1 and 2 show schematically the main dimensions and the cross sections of these beams. Table 1 shows the data of these beams. The natural frequency of each beam is determined as shown in Table 1. The reason for this is to approximately equalize the condition



(a) Welded Beam (b) Riveted Beam (Bolted Beam)

Fig. 2 Cross Sections of Model Beam.

Table 1.

	Welded Beam	Riveted Beam	Bolted Beam
Span Length (m)	6	6	6
Total Weight (kg)	196	207	210
Sectional Moment of Inertia (cm^4)	227	212	212
Free Vibrational Frequency (Theoretical) (sec^{-1})	7.52	7.05	6.91
Free Vibrational Frequency (Experimental) (sec^{-1})	7.67	7.18	6.95

which each beam has for vibration and to avoid a large difference between the vibration frequency of the model beams and that of the actual bridges.

For the support, ball-bearings are used as the hinge and rollers are used as the moving support. With these arrangements, the energy dissipated through the support can be disregarded almost completely. To investigate the effect of the supporting condition, sliding supports were used afterwards as the moving support.

Various different methods of experiment could be considered, but in this experiment, the damping free vibration was applied in order to obtain most exactly the damping characteristics. As the means of vibrating, a method of vibrating by hand was adopted and the damping free vibration was recorded in ASKANIA Vibrograph which was fixed on a supporting apparatus.

3. Damping Characteristics of the Welded Beam and the Riveted Beam

The object of the experiments described in Chapters 3, 4 and 5 in this paper is to investigate the energy which is dissipated by the internal friction of the beams. Therefore, a roller support has been used as the moving support.

Fig. 3 shows the amplitude decay curves for the riveted beam and the welded beam. It is clear from this figure that the condition of the decrement differs considerably depending upon the kinds of beams. The welded beam shows a smaller decrement than the riveted beam.

The relation between the logarithmic decrement and the vibrational amplitude is illustrated in Fig. 4.

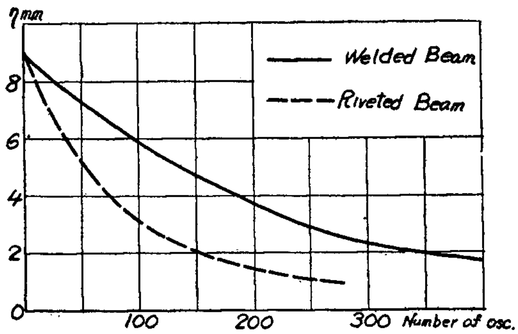


Fig 3 Amplitude Decay Curves for Welded and Riveted Beams.

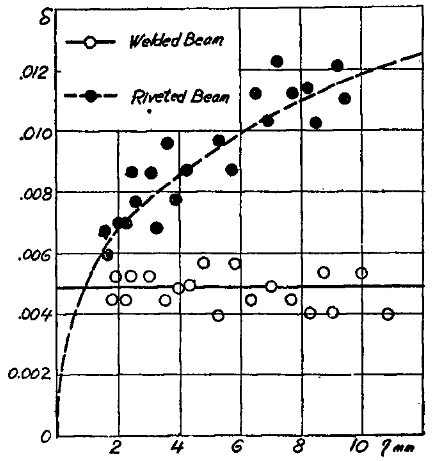


Fig. 4 η-δ Relations of Welded and Riveted Beams.

From Fig. 4 as far as this experiment is concerned, the following conclusion is obtained.

(1) The logarithmic decrement in the case of the welded beam is nearly constant and its value is $\delta \fallingdotseq 0.005$, and has no relation with the amplitude. This value is nearly equal to the logarithmic decrement of the internal friction of the steel.

(2) In the case of the riveted beam, the logarithmic decrement increases as the beam has a larger amplitude. The effect of this goes to show the fact that the damping characteristics cannot be shown in a linear form (this fact will be explained in Chapter 5) even within the stress range of this experiment in which the Hooke's Law can be applied. Expressing

the function the logarithmic decrement of the riveted beam as

$$\delta=\delta_0\eta^{\alpha} \tag{1}$$

and determining δ_0 and α from the experimental data by the method of least squares,

$$\delta=0.01198\eta^{0.344} \tag{2}$$

is obtained. The curve shown by Eq. (2) is shown with a dotted line in Fig. 4. The meaning of α will be explained in Chapter 5.

4. The Effect of Vibrational Velocity

For the damping term of the equation of vibration of beam, either the damping force proportional to the vibration velocity of deflection, i.e., $k\frac{\partial y}{\partial t}$, or that which is proportional to the vibration velocity of strain, i.e., $k\frac{\partial^5 y}{\partial t \partial x^4}$, is applied.

To investigate the relations between the damping constant and the vibrational velocity, the following experiment was made.

The beam used in this experiment was a welded beam which had a linear characteristics. By placing a weight on the welded beam, the free vibrational period was changed and thereby the relation between the vibrational velocity and the damping characteristics was investigated. The results of this experiment is shown in Table 2. From this experiment it is made clear that the logarithmic decrement has little relation to the vibrational periods and it shows the same value.

Table 2.

Case	A	B
Loaded Weight (kg)	no load	50.15
Free Vibrational Frequency (sec^{-1})	7.25	6.14
Logarithmic Decrement	0.0047	0.0048
Damping Coefficient (sec^{-1})	0.0355	0.0298

From these results, a damping, which is independent of the vibrational velocity, has to be chosen as the damping term of the equation of vibration of the beam. The effect of this characteristics of damping term is remarkable on the vibration of continuous beam and cantilever beam in which the vibration of higher order should be considered.[1] However in the case of a simple beam, the usual equation of vibration, which has a damping term proportional to the velocity, can be applied approximately by choosing the damping coefficient properly.

5. The Relation between the Vibrational Amplitude and the Logarithmic Decrement

As shown clearly by the experiment in Chapter 3, the logarithmic decrement of structure cannot be shown in a linear damping even in the range of an elastic limit of the riveted beam. In the case of steel structure, such as bridges, the logarithmic decrement is far small compared with the unity, even though the vibrational amplitude is considerably large. Therefore, the distribution of vibrational amplitude along the beam length is assumed to be same as that in the case of the linear vibration theory. The dissipated energy lost from the interior of the structure can be considered chiefly due to the friction of connection of the structural cross section. In order to simplify the problem, all effects of the dissipation of energy from the interior of the structure are considered to be exhibited by the stress-strain loop as shown in Fig. 5 and the area within the loop is assumed as

$$\Delta w = \oint \sigma d\varepsilon = \beta_n \varepsilon_0^n. \tag{3}$$

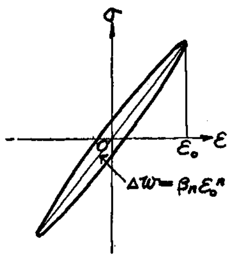


Fig. 5

Δw shows the dissipated energy lost from the unit volume during one cycle under the assumption mentioned above. β_n is the constant determined by the structural material, the type of structure, the method of the connection of structure, the basic stress (usually the dead load stress) and the value n.

Integrating Eq. (3) along the whole beam length and a sectional area, the total energy dissipated within one cycle is

$$\Delta W = \int_e \int_A \oint \sigma d\varepsilon dA dx = \beta_n I_n \int_l \left(\frac{\partial^2 y}{\partial x^2}\right)^n dx \tag{4}$$

where, $I_n = \int_A \xi^n dA$ (Sectional moment of n th order) and,
l: beam length, A: sectional area, y: deflection of the beam, x: coordinate along the beam length. ξ: length from the center of gravity of the section to dA. The maximum value of the potential energy of the beam during the vibration is

$$W = \frac{EI}{2} \int_l \left(\frac{\partial^2 y}{\partial x^2}\right)^2 dx \propto \eta^2. \tag{5}$$

In the case of the free vibration, when the logarithmic decrement is very small compared with the unity, the terms of higher order of the series expansion of δ are neglected and relation

$$\frac{\Delta W}{W}=2\delta \tag{6}$$

is obtained. From Eqs. (4) (5) and (6)

$$\delta=\frac{\beta_n I_n}{EI}\left\{\int_l\left(\frac{\partial^2 y}{\partial x^2}\right)^n dx \Big/ \int_l\left(\frac{\partial^2 y}{\partial x^2}\right)^2 dx\right\}. \tag{7}$$

Now considering a simple beam, since its distribution of the vibrational amplitude is approximately assumed as

$$y=\eta\,\sin\frac{\pi x}{l},$$

then δ is obtained by

$$\delta=\frac{\beta_n I_n}{EI}\left(\frac{\pi^2}{l^2}\eta\right)^{n-2}\left\{\Gamma\left(\frac{n+2}{2}\right)\Big/\Gamma\left(\frac{n}{2}+1\right)\right\}. \tag{8}$$

Consequently, δ is proportional to $(n-2)$th power of η, and has no relation with the velocity of vibration. The physical meaning of δ_0 and α in Eq. (1) can be clarified by comparing Eq. (8) with Eq. (1).

When $n=2$, β_n is shown by $\beta_2=E\delta$ using the logarithmic decrement δ, and in this case δ is a constant. When $n\neq2$, the value δ is not only the function of the deflection η but also it is the value decided by the form and the dimensions of the structure.[2)]

6. The Relations between Damping Characteristics and Fastening of Bolt on the Bolted Beam

The fact that the riveted beam shows, as in Chapter 3, a larger damping and a non-linear damping characteristic compared with thewelded beam is chiefly due to the difference of their cross connections.

Therefore, the effect of fastening the connection of cross sections of a beam upon the damping characteristics is investigated by using the bolted beam.

The relation between the torque T for fastening the bolts and the tension P of the bolts is shown as follows

$$T=CDP \tag{9}$$

where, D is the diameter of the bolt, C is a constant and $C=0.2$ (which is given by the experiment of H. Lenzen).

If the constant of C is chosen as $C=0.2$, the relations between the torque and the stress of bolt σ are shown as follows for the bolt used in this beam. $\left(D=\frac{3''}{8}\right)$

$T=50$ kg-mm : $\sigma=370$ kg/cm^2
$T=100$ kg-mm : $\sigma=740$ kg/cm^2

$T = 400$ kg-mm : $\sigma = 2960$ kg/cm^2

As the yield point of the bolt is $\sigma_y = 2600 \sim 3000$ kg/cm^2, the torque T adopted in this experiment is $0 \sim 400$ kg-mm. A torque-wrench obtained in the market was used to fasten the bolts.

The amplitude decay curves of the free vibration are shown in Fig. 6 for several values of torque. And the relations between the amplitude and the logarithmic decrement become as shown in Fig. 7. From Figs. 6 and 7 the following conclusions are obtained.

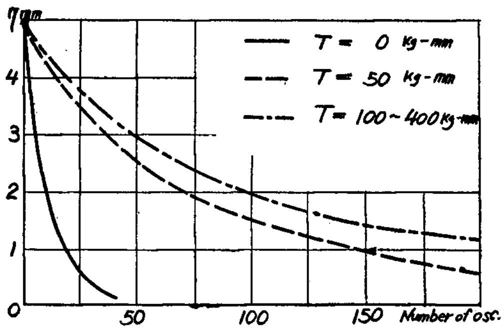


Fig. 6 Amplitude Decay Curves of Bolted Beam.

(1) When $T=0$ (in this case all the bolts were loosened as the torque wrench showed no indications), the logarithmic decrement is very large compared with that in the other cases, and it has a tendency to show a non-linear damping of higher order different from the Coulomb damping.

(2) The logarithmic decrement δ decreses if the torque increases. But the fastening of the bolts as tight as to exceed $T = 100$kg-mm has no effect on the damping.

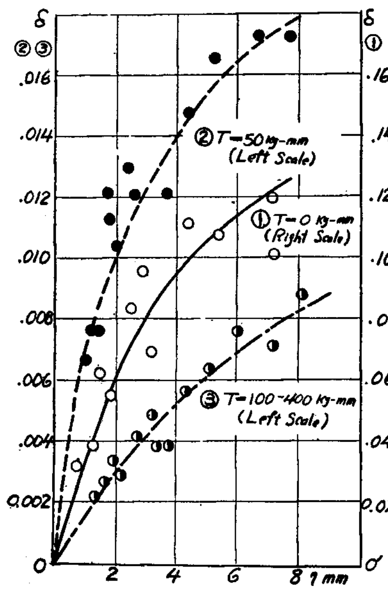


Fig. 7 η-δ Relations of Bolted Beam.

7. The effect of the Type of the Moving Support

In the experiments mentioned above, a roller support made of steel was used as the moving support. To investigate the effect of the material of the rollers, a wooden roller (made of pine) was used in place of the steel roller. Even in the case of the wooden roller, the amplitude decay curve was similar to that in the case of the steel roller as shown in Fig. 8. This result shows the

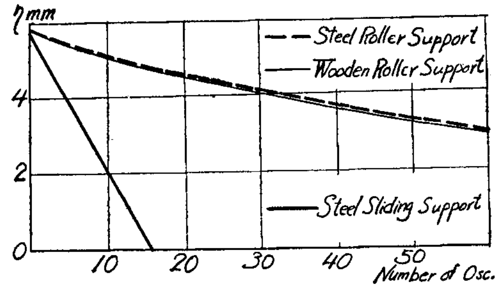


Fig. 8 Amplitude Decay Curve of Riveted Beam for Roller and Sliding Supports.

fact that the materials of the rollers has little effect on the damping compared with the effect of the internal frictions of the beams.

In order to investigate the effect of the types of the moving support, a sliding support was used. In this experiment the riveted beam was used. The amplitude decay curve in this experiment is as shown in Fig. 8.

It is clear from Fig. 8 that the effect of the Coulomb damping becames very remarkable when the moving support is changed from the roller support to the sliding support. The relation of the amplitude and the logarithmic decrement is shown in Fig. 9. The logarithmic decrement increases as the amplitude decreases, and this is nothing but a characteristic of the Coulomb damping.

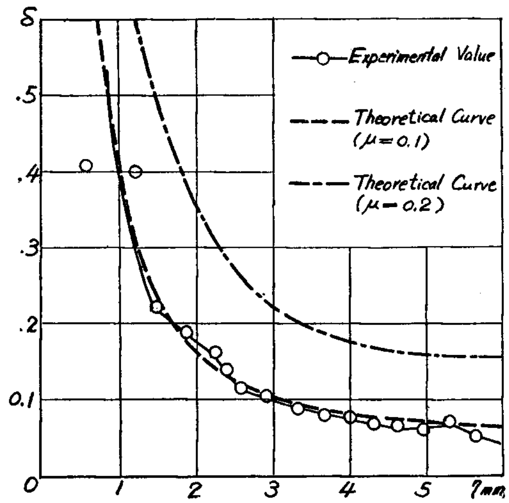


Fig. 9 η-δ Relation of Riveted Beam with Sliding Support.

8. The Energy Dissipated from the Support

It is found that the logarithmic decrement δ can be calculated by the following equation even when δ is comparatively large.

$$\delta=-\frac{1}{2}\log\left\{1-\frac{1}{W}(\varDelta W_1+\varDelta W_2\cdot\cdot\cdot\cdot)\right\} \tag{10}$$

where, $\varDelta W_1$, $\varDelta W_2 \ldots\ldots$ are the energy dissipated from different mechanisms of dissipation, and W is the energy of the beam during one cycle at a vibrating cycle. As it is clear from Eq. (10), the free vibration does not arise when

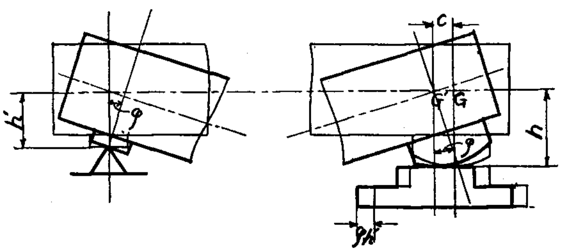
$$\varDelta W_1+\varDelta W_2+\cdot\cdot\cdot\cdot>W.$$

If $\varDelta W_1$ is assumed to be the energy dissipated from the internal friction of the beam and $\varDelta W_2$ is that dissipated from the support, Eq. (10) can be simplified as follows when only these two mechanisms of dissipation are considered.

$$\delta=-\frac{1}{2}\log\left\{1-\frac{1}{W}(\varDelta W_1+\varDelta W_2)\right\} \tag{11}$$

The value $\frac{\varDelta W_1}{W}$ in Eq. (11) is given in Eq. (6) in which only the effect of internal friction is considered.

ΔW_2 is shown by $\Delta W_2 = F\Delta s$ where the displacement of the support in one cycle is Δs and the frictional force of the support is F.



Hinged Support Sliding support

Fig. 10

Taking the displacement of G as c (Fig. 10) and the angle of rotation of the beam at the moving support as φ and taking into consideration the fact that an equal amount of rotation of the beam arises at the hinged support, the displacement of the supporting point in one cycle is shown as follows:

In case the beam displaces to the downward $\varphi h' + \varphi h - c$

In case the beam displaces to the upward $\varphi h' + \varphi h + c$

Therefore, the displacement of the supporting point in one cycle is

$$\Delta s = 4(\varphi h' + \varphi h).$$

Then, the energy dissipated through the support is

$$\Delta W_2 = 4\varphi(h+h')F. \tag{12}$$

As φ and F are given by

$$\varphi = \left(\frac{dy}{dx}\right)_{x=0} = \eta\frac{\pi}{l}$$

$$F = \mu\frac{G}{2}$$

where,

η: maximum deflection of the beam, l: span length of the beam,

μ: coefficient of friction of the sliding support, G: total weight of the beam,

the ΔW_2 is

$$\Delta W_2 = 2\frac{h+h'}{l}\eta\pi\mu G. \tag{13}$$

And as the W is shown in Eq. (5)

$$\frac{\Delta W_2}{W} = \frac{8(h+h')\mu G l^2}{\pi^3 E I \eta} \tag{14}$$

is obtained.

Substituting Eq. (14) and (6) for Eq. (11)

$$\delta = -\frac{1}{2}\log\left(1 - \frac{8(h+h')\mu G l^2}{\pi^3 E I \eta} - 2\delta_1\right) \tag{15}$$

Where, δ_1 is the logarithmic decrement when only the effect of internal friction is considered.

The η-δ relations calculated by using Eq. (15) are shown in Fig. 9 the dotted line for $\mu=0.1$ and the chain line for $\mu=0.2$.

In Fig. 9 the theoretical line for $\mu=0.1$ coincides well with the experimetal results. The effect of the internal friction shown by $2\delta_1$ in Eq. (15) can be disregarded in the riveted beam compared with the effect of the friction of the support.

9. Vibrational Damping of Actual Steel Highway Bridges

The vibration damping resulting from the internal friction of the steel beams and the friction at the support were experimented on the model beams and the results thereof were described in the preceeding chapters of this report.

The damping characteristics of actual bridges depend upon more complicated mechanisms than those of the model beams. In this chapter the experimental researches of the damping characteristics of actual bridges are described with some comparisons of the result obtained by the model beams.

(a) Methods of measurement of a damping constant

Various methods can be applied to measure the vibration damping of a bridge. The following methods were used in this experiment:

Method (1). Measure the amplitude of the damping free vibration after the moving load has passed.

Method (2). Measure the amplitude of the damping free vibration arisen in the following manner. First let some men jump on the bridge so as to be resonant with the natural vibration period of the bridge until the bridge attains a considerable amplitude, then let these men stop jumping suddenly. The vibrational amplitude thus obtained was larger than that obtained in the method (1).

Method (3). Vibrate by using an oscillator and calculate the logarithmic decrement using the following equation.

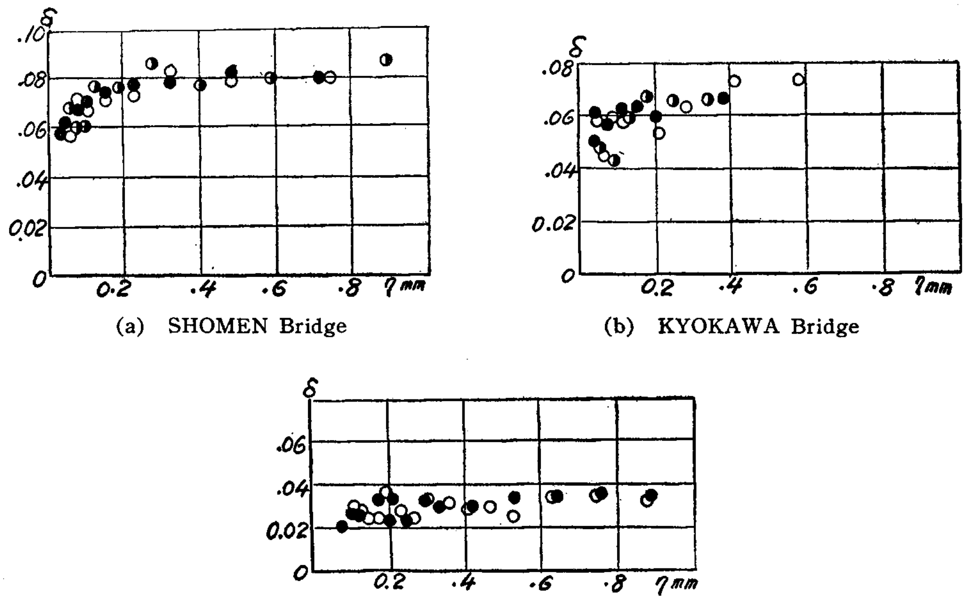
$$\delta=\frac{\pi}{2}\;\frac{\nu_2{}^2-\nu_1{}^2}{\nu_r{}^2}$$

where, ν_1 and ν_2 are the circular frequency corresponding to the amplitude of 0.707× (resonance amplitude) in resonance curve and ν_r is that of resonance amplitude.

A mechanical vibrograph has been used as the measuring instrument and its natural period is 1.8 sec and the geometric magnification is×15. Recently the ASKANIA Handvibrograph is used with mechanical vibrograph.

(b) Measured values of the damping characteristics

The amplitude used in this experiments was about 1mm. Analysing the record of the experiments, the effect of the Coulomb damping is hardly found on the bridges. The mean value of the logarithmic decrement obtained from the records of the damping free vibration is as shown in Table 3.



(a) SHOMEN Bridge (b) KYOKAWA Bridge

(c) MASUTANI Bridge

Fig. 11 Relations between Amplitude and Logarithmic Decrement.

Fig. 11 illustrates examples of the relation between the logarithmic decrement and the amplitude for three actual highway briges. It is noticed that there is a tendency that, if the amplitude increases, the logarthmic decrement also increases. However, as the amplitude of vibration is comparatively small in this case, the non-linear characteristics obtained by the model beams are not remarkable.

The logarithmic decrement calculated from the resonance curve obtained by using the oscillator are shown in parenthesis in Table 3.

It seems that, the value of δ obtained from the resonance curve in general is larger than the value of δ obtained from the damping free vibration. Although a sufficient investigation has not yet been carried out, it may be considered that an effect of vibration of higher order is included in the resonance curve.

The actual bridges show a fairly larger logarithmic decrement than that of the model beams. The causes of this result are that in the case of actual bridges, the roadway slabs are made of concrete and they have a more complicated mechanism

Table 3. Vibrational Periods and Logarithmic Decrement of Actual Bridges

Name of bridge	Type of bridges	Dimension of bridges*	Natural period (sec)	Logarithmic decrement
TAKAKURA	Cantilever beam with 2 spans	L=45.15m b=11.125m	0.290	0.0318
SHOMEN	Continuous beam with 3 spans	L=75.80m b=6.00m	0.304	0.0710 (0.183)
KYOKAWA	Cantilever beam with 3 spans	L=86.00m b=6.00m	0.284	0.0596 (0.102)
TAISHO	Two hinged arch	l=300ft b=21.92m	0.405	0.173
MASUTANI	Spandrel braced arch	l=77.2m b=6.00m	0.334	0.0294 (0.0406)
OKAWA	Warren truss	l=59.1m b=6.50m	0.323	0.0672
SAIJO	Box girder	l=36.00m b=5.5m	0.285	0.096
MOROTOMI	Warren truss	l=41.75m b=7.5m	0.251	(0.2–0.3)
YAMASU	Box girder	l=36.30m b=4.5m	0.368	0.066
HAKUUN	Box girder	l=28.0m b=6.0m	0.194	(0.079)

* L=Length of Bridge l=Length of Span b=Width of Bridge

of damping than the model beams. However, the damping constant of the bridge is considerably small compared with other structures, such as building structures. This means that, when as the resonance amplitude is large, the damping ratio has a considerable effect on the impact factor.

10. Conclusion

The experimental results and some considerations on the vibrational damping of model beams and actual bridges have been described in this report and the important conclusions obtained are as follows:

(1) In the case of the model beams, especially of the bolted beam and riveted beam, the non-linear characteristics of damping are remarkable even within the elastic limit of the steel.

(2) The logarithmic decrement of the welded beam is independent of the amplitude of vibration and is approximately equal to the logarithmic decrement caused by the internal friction of the steel itself.

(3) The logarithmic decrement of the bolted beam depends remarkably upon the degree of fastening of bolts, but the logarithmic decrement does not vary when the degree of fastening of bolts exceeds a certain critical values.

(4) When the bolts are fastened beyond the critical value, the logarithmic

decrement of the bolted beam is smaller than that of the riveted beam.

(5) The dissipated energy lost through the sliding support is so large that the effect of the internal friction can be disregarded. In this case, when the coefficient of friction of the sliding support is assumed as $\mu=0.1$, the theoretical result coincides well with the experimental result.

(6) The damping of actual bridges is large compared with that of the model beams because the mechanisms of damping of actual bridges are complicated, but it is smaller than the damping of other structures such as building structures.

Acknowledgment

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