The Theory of Creep of Visco-Plasto-Elastomer

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Abstract

Previously the authors proposed the concept of visco-plasto-elastomer[1], a dynamical model of which is shown in Fig. 1, because industrial materials used in practice usually have visco-plasto-elastic properties in their mechanical behaviors. In this paper various problems are discussed on the creep theory of visco-plasto-elastomer. Creep, recovery and repeated creep phenomena are analytically treated on the dynamical model, and as the result, various interesting phenomena and facts are found, for example, the existence of virgin state, dependence of the creep strain and the recovery strain on stress σ_0, existence of the permanent set, simple relation between the virgin creep strain and non-virgin creep strain, and so on.

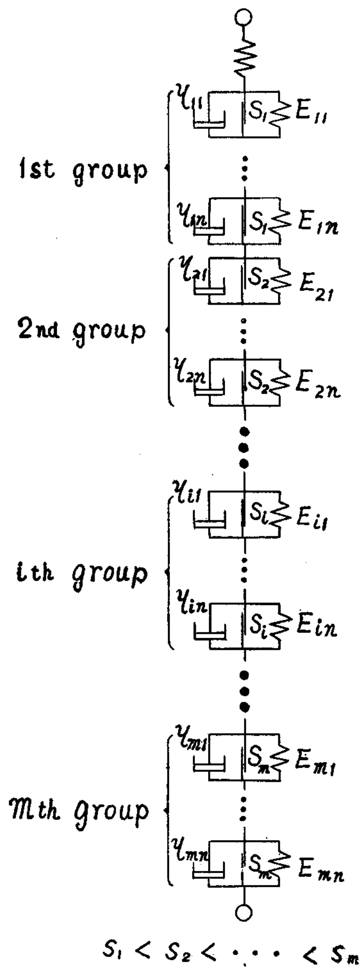

Fig. 1

1. Introduction

The phenomenological theory of visco-elasticity on the mechanical behaviors of materials has already been established in an elegant form. But in many cases, behaviors of materials used in industries do not agree with this theory, chiefly on account of their non-linearity. Therefore, we have previously proposed that these materials should be considered as visco-plasto-elastomer in their mechanical behaviors[1,2]. The visco-elastomer is, as is well known, represented by a dynamical model composed of two kinds of elements, spring and dashpot, while the visco-plasto-elastomer is represented by a model composed of three kinds of elements, spring, dashpot and slider (Coulomb frictional mechanism). According to the manner in which the elements are composed, many kinds of visco-plasto-elastomer are considered. Various creep phenomena of the dynamical model of strain retardation type shown in Fig. 1 are described in this paper.

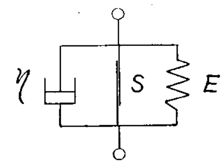

Fig. 2

2. Creep

Let us consider a case of the simplest model shown in Fig. 2 under a load $\sigma(t)$ which is a non-decreasing function of t. In this case, if we denote the deformation of the model by $\varepsilon(t)$, we obtain

$$\left.\begin{aligned} \varepsilon(t) &= 0, && \text{for } \sigma(t) \leq s \\ \sigma(t) &= E\varepsilon(t) + \eta\frac{d\varepsilon}{dt} + s, && \text{for } \sigma(t) > s \end{aligned}\right\} \tag{1}$$

where E, η and s are the spring constant of the spring element, the viscous coefficient of the dashpot element and the frictional force of the slider element of the model respectively. Integrating Eq. (1), we obtain

$$\left.\begin{aligned} \varepsilon(t) &= 0, && \text{for } \sigma \leq s \\ \varepsilon(t) &= \beta\int_{t'}^{t} e^{-\nu(t-\tau)}\{\sigma(\tau)-s\}d\tau, && \text{for } \sigma > s \end{aligned}\right\} \tag{2}$$

where $\beta \equiv 1/\eta$, $\nu \equiv E/\eta$ and t' is the instant when the following relation is satisfied.

$$\sigma(t') = s.$$

When $\sigma(t)$ is a step function, i.e. $\sigma(t)=\sigma_0 1(t)$, Eq. (2) becomes

$$\left.\begin{aligned} \varepsilon(t, \sigma_0) &= 0, && \text{for } \sigma_0 \leq s, \\ \varepsilon(t, \sigma_0) &= \frac{\beta}{\nu}(\sigma_0 - s)(1-e^{-\nu t}), && \text{for } \sigma_0 > s. \end{aligned}\right\} \tag{3}$$

Because the deformation of models corresponds to the strain of materials, let us use the terminology strain and stress instead of deformation and load respectively, hereafter. From Eq. (3), the strain of the model shown in Fig. 1 is given as follows:

$$\varepsilon(t, \sigma_0) = \frac{1}{E}\sigma_0 + \sum_{i=1}^{k}\sum_{j=1}^{n}\frac{\beta_{ij}}{\nu_j}(\sigma_0 - s_i)(1-e^{-\nu_j t}), \tag{4}$$

where k is an integer satisfying

$$s_k < \sigma_0 \leq s_{k+1}.$$

If corresponding distributed system is to be treated, n and m in the model shown in Fig. 1 should tend to infinity. Then Eq. (4) becomes

$$\varepsilon(t, \sigma_0) = \frac{1}{E}\sigma_0 + \int_0^\infty d\nu \int_0^{\sigma_0} \frac{F(\nu, s)}{\nu}(\sigma_0 - s)(1-e^{-\nu t})ds, \tag{5}$$

where $F(\nu, s)$ is the distribution function which specifies the various mechanical behaviors of visco-plasto-elastic materials and the following relation is satisfied,

$$F(\nu_j, s_i) = \lim_{\substack{(\nu_{j+1}-\nu_j)\to 0 \\ (s_{i+1}-s_i)\to 0}} \frac{\beta_{ij}}{(\nu_{j+1}-\nu_j)(s_{i+1}-s_i)}.$$

The first term of the right hand side of Eq. (5) is the instantaneous strain and the creep strain is usually represented as follows, (omitting the instantaneous strain σ_0/E from Eq. (5)),

$$\varepsilon_c(t, \sigma_0) = \int_0^\infty d\nu \int_0^{\sigma_0} \frac{F(\nu, s)}{\nu}(\sigma_0 - s)(1-e^{-\nu t})ds. \tag{6}$$

Let t tend to infinity, we obtain the creep strain in equilibrium state as follows;

$$\varepsilon_e \equiv \varepsilon_c(\infty, \sigma_0) = \int_0^\infty d\nu \int_0^{\sigma_0} \frac{F(\nu, s)}{\nu}(\sigma_0 - s)ds. \tag{7}$$

From Eq. (6), the creep rate R is given as follows:

$$R = \frac{\partial \varepsilon_c}{\partial t} = \int_0^{\sigma_0}(\sigma_0 - s)ds \int_0^\infty F(\nu, s)e^{-\nu t}d\nu$$

$$\equiv \int_0^{\sigma_0} f(t, s)(\sigma_0 - s)ds > 0, \tag{8}$$

where $f(t, s)$ is the Laplace transformation of $F(\nu, s)$. Considering that $F(\nu, s)$ is always non-negative, we obtain from Eq. (8) and (6),

$$\frac{\partial R}{\partial t} = \frac{\partial^2 \varepsilon_c}{\partial t^2} = -\int_0^{\sigma_0}(\sigma_0 - s)ds \int_0^\infty \nu F(\nu, s)e^{-\nu t}d\nu < 0, \tag{9}$$

$$\frac{\partial^2 \varepsilon_c}{\partial \sigma_0^2} = \int_0^\infty \frac{F(\nu, \sigma_0)}{\nu}(1-e^{-\nu t})d\nu > 0. \tag{10}$$

From Eq. (9) we can see that the creep rate decreases with the increase of time t. Eq. (10) shows the nonlinear characteristic of creep and it is remarkable that the creep strain ε_c at any instant is not proportional to the magnitude of stress σ_0, but varies with the increase of σ_0 so that $d^2\varepsilon_c/d\sigma_0^2$ is always positive. Differentiating Eq. (6) twice with respect to $\log_{10} t$, we have

$$\frac{\partial^2 \varepsilon_c}{\partial(\log_{10} t)^2} = (2.303)^2 t \int_0^{\sigma_0}(\sigma_0 - s)\left\{f(t, s) + t\frac{\partial f(t, s)}{\partial t}\right\}ds. \tag{11}$$

Since $f(t, s)$ is always positive and $\partial f(t, s)/\partial t$ is always negative, $\log_{10} t - \varepsilon_c$ curve is concave upwards when t is small and convex upwards when t is large. These features of time dependence of creep strain of linear visco-elastomers.

3. Recovery

If the load is removed after the creep strain of the model shown in Fig. 2 reach the equlibrium state, the strain generally decreases with time t. Let us define the recovery strain as the difference between the final strain in the preceding creep ($\varepsilon_e+\sigma_0/E$ in this case) and the strain at recovery stage. Then the recovery strain $\Delta(t,\sigma_0)$ of the model shown in Fig. 2 is given as follows:

$$\left.\begin{aligned} \Delta(t,\sigma_0) &= 0, && \text{for} \quad \sigma_0 \leqq 2s, \\ \Delta(t,\sigma_0) &= \frac{\beta}{\nu}(\sigma_0-2s)(1-e^{-\nu t}), && \text{for} \quad \sigma_0 > 2s, \end{aligned}\right\} \tag{12}$$

where t is the time measured from the instant when the load is removed. Accordingly, the recovery strain of the distributed system is given as follows. (Omitting the instantaneous strain σ_0/E as in case of the creep strain):

$$\Delta(t,\sigma_0) = \int_0^\infty d\nu \int_0^{\sigma_0/2} \frac{F(\nu,s)}{\nu}(\sigma_0-2s)(1-e^{-\nu t})ds. \tag{13}$$

Comparing this equation with Eq. (6), we can find that the simple relation,

$$\varepsilon_c(t,\sigma_0) = \frac{1}{2}\Delta(t, 2\sigma_0) \tag{14}$$

holds between the creep strain and the recovery strain. Letting t tend to infinity in Eq. (13), we have

$$\Delta(\infty,\sigma_0) = \int_0^\infty d\nu \int_0^{\sigma_0/2} \frac{F(\nu,s)}{\nu}(\sigma_0-2s)ds. \tag{15}$$

This does not coincide with $\varepsilon_c(\infty,\sigma_0)$, therefore we recognize that the following parmament set remains.

$$\begin{aligned} r(\sigma_0) &= \varepsilon_c(\infty,\sigma_0) - \Delta(\infty,\sigma_0) \\ &= \int_0^\infty \left\{ \int_0^{\sigma_0} \frac{F(\nu,s)}{\nu}(\sigma_0-s)ds - \int_0^{\sigma_0/2} \frac{F(\nu,s)}{\nu}(\sigma_0-2s)ds \right\} d\nu. \end{aligned} \tag{16}$$

Now, calculating the second creep after the recovery of infinitely long time by the similar procedure mentioned above we obtain the second creep stain as follows:

$$\varepsilon_c(t,\sigma_0) = \int_0^\infty d\nu \int_0^{\sigma_0/2} \frac{F(\nu,s)}{\nu}(\sigma_0-2s)(1-e^{-\nu t})ds. \tag{17}$$

This equation is the same as Eq. (13). The second recovery strain, the third creep strain, the third recovery strain, and so on are all of the same form as Eq. (13) or Eq. (17).

From the above discussions we can point out the existence of virgin state concerning creep phenomena of such materials. It is remarkable that we can see the two categories of creep curves, the one of which is termed virgin creep curve specified by Eq. (7) and the other non-virgin creep curve specified by Eq. (17).

4. Non-ideal creep and recovery[3)]

In this section, we treat the creep (or the recovery) when the visco-plasto-elastomers are loaded (or unloaded) before the preceding recovery finishes (or the preceding creep reaches the equilibrium state). We term these creep or recovery the non-ideal creep or non-ideal recovery, and we call the creep or the recovery treated in the preceding sections the ideal creep or the ideal recovery.

Firstly, let us discuss the case where the stress σ_0, which has been loaded on the model in Fig. 2 for the period t_1, is removed. Since the strain rate in this recovery stage is either negative or zero, (if we suppose that a strain rate is positive, a contradiction arise), the differential equatlon which prescribes the strain in this stage becomes as follows:

$$E\varepsilon+\eta\frac{d\varepsilon}{dt}-s=0 \tag{18a}$$

or

$$\varepsilon=\text{const.} \tag{18b}$$

Shifting the origin of the time to the instant of commencement of the recovery and solving Eq. (18a) under the initial condition,

$$\varepsilon=\varepsilon_1 \quad \text{at} \quad t=0$$

we obtain

$$\varepsilon=\frac{\beta s}{\nu}+\left(\varepsilon_1-\frac{\beta s}{\nu}\right)e^{-\nu t} \tag{19}$$

where ε_1 is the strain given by substituting $t=t_1$ in Eq. (3), then we have

$$\left.\begin{aligned} \varepsilon_1 &= 0, && \text{for} \quad \sigma_0\leqq s \\ \varepsilon_1 &= \frac{\beta}{\nu}(\sigma_0-s)(1-e^{-\nu t_1}), && \text{for} \quad \sigma_0>s. \end{aligned}\right\} \tag{20}$$

Differentiating Eq. (19) with respect to t, we have

$$\frac{d\varepsilon}{dt}=-\nu\left(\varepsilon_1-\frac{\beta s}{\nu}\right)e^{-\nu t}. \tag{21}$$

Then, if $\varepsilon_1<\beta s/\nu$, $d\varepsilon/dt$ becomes positive. On the other hand, we cannot ignore the fact that $d\varepsilon/dt$ should be either negative or zero. Therefore, if $\varepsilon_1<\beta s/\nu$, we cannot use Eq. (18a) as the differential equation of creep recovery, hence we have to use Eq. (18b) in this case, thus $d\varepsilon/dt=0$ and $\varepsilon=\varepsilon_1$. However, in case $\varepsilon_1>\beta s/\nu$, Eq. (18a) and Eq. (19) are available for the calculation of the recovery strain. We have, then, from Eq. (19) and Eq. (20),

$$\left.\begin{aligned} \Delta(t,\sigma_0) &= \varepsilon_1-\varepsilon = (\varepsilon_1-\beta s/\nu)(1-e^{-\nu t})\,, && \text{for} \quad \varepsilon_1 > \frac{\beta s}{\nu}\,, \\ \Delta(t,\sigma_0) &= 0\,, && \text{for} \quad \varepsilon_1 \leqq \frac{\beta s}{\nu}\,. \end{aligned}\right\} \tag{22}$$

Secondly, if it is reloaded by σ_0 after a duration of recovery for the period t_2, the strain is given as follows:

$$\varepsilon = \frac{\beta(\sigma_0-s)}{\nu}+\left\{\varepsilon_2-\frac{\beta(\sigma_0-s)}{\nu}\right\}e^{-\nu t}\,, \qquad \text{for} \quad \varepsilon_1 > \frac{\beta s}{\nu} \tag{23a}$$

$$\varepsilon = \frac{\beta(\sigma_0-s)}{\nu}+\left\{\varepsilon_1-\frac{\beta(\sigma_0-s)}{\nu}\right\}e^{-\nu t}\,, \qquad \text{for} \quad \varepsilon_1 \leqq \frac{\beta s}{\nu} \tag{23b}$$

where ε_2 is the value of Eq. (19) at $t=t_2$. The creep strain in this case is given by subtracting the initial strain ε_2 and ε_1 from Eq. (23a) and Eq. (23b) respectively. Therefore we have the creep strain as follows:

$$\left.\begin{aligned} \varepsilon_c &= -\left\{\varepsilon_2-\frac{\beta(\sigma_0-s)}{\nu}\right\}(1-e^{-\nu t})\,, && \text{for} \quad \varepsilon_1 > \frac{\beta s}{\nu} \\ \varepsilon_c &= -\left\{\varepsilon_1-\frac{\beta(\sigma_0-s)}{\nu}\right\}(1-e^{-\nu t})\,, && \text{for} \quad \varepsilon_1 \leqq \frac{\beta s}{\nu} \end{aligned}\right\} \tag{24}$$

Continuing the same procedure as mentioned above, we can evaluate the successive creep and recovery strains. For the load changing with time in the way shown in Fig. 3 (a), we obtain three kinds of creep and recovery curves shown in Fig. 3 (b) ~ Fig. 3 (d) according to the values of σ_0 and t_1.[3] In case $s<\sigma_0<2s$, ε_1 is less than $\beta s/\nu$ for all t_1, therefore we can not recognize recovery phenomena as shown in Fig. 3 (d). In case $\sigma_0>2s$, for small t_1 the first recovery does not appear but the recovery in succeeding stages is possible as shown in Fig. 3 (c), and for large t_1, we recognize the first recovery as shown in Fig. 3 (b).

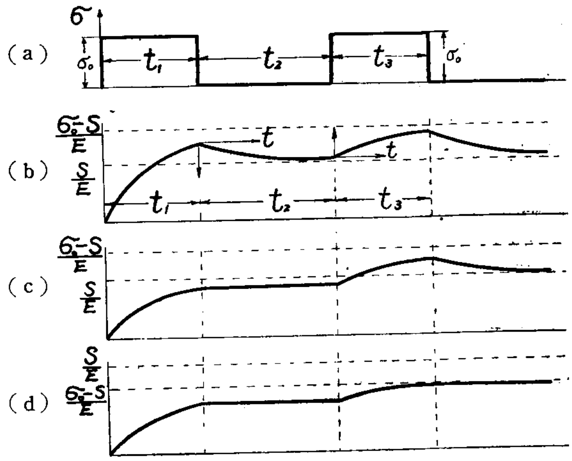

Fig. 3

Extending the above discussion, let us consider the first recovery of the model shown in Fig. 1 after the virgin creep of duration t_1. From Eq. (22) we obtain the recovery strain as follows,

$$\Delta(t,\sigma) = \sum_{i=1}^{k}\sum_{j=1}^{n}\left\{\frac{\beta_{ij}}{\nu_j}(\sigma_0-s_i)(1-e^{-\nu t_1})-\frac{\beta_{ij}s_i}{\nu_j}\right\}(1-e^{-\nu t})\,, \tag{25}$$

where k is the integer satisfying the following two conditions,

$$\left.\begin{aligned} &\frac{\beta_{kj}}{\nu_j}(\sigma_0 - s_k)(1-e^{-\nu_j t_1}) > \frac{\beta_{kj} s_k}{\nu_j}, \\ &\frac{\beta_{k+1,j}}{\nu_j}(\sigma_0 - s_{k+1})(1-e^{-\nu_j t_1}) \leqq \frac{\beta_{k+1,j} s_{k+1}}{\nu_j}. \end{aligned}\right\} \tag{26}$$

Eq. (26) is briefly rewritten as follows,

$$s_k < \frac{1-e^{-\nu_j t_1}}{2-e^{-\nu_j t_1}}\sigma_0 \leqq s_{k+1}. \tag{27}$$

Therefore, the first recovery strain of the distributed system after the virgin creep of duration t_1 is given as follows;

$$\Delta(t, \sigma_0) = \int_0^\infty d\nu \int_0^{\frac{1-e^{-\nu t_1}}{2-e^{-\nu t_1}}\sigma_0} \frac{F(\nu, s)}{\nu}\{(\sigma_0 - 2s) - (\sigma_0 - s)e^{-\nu t_1}\}(1-e^{-\nu t})ds. \tag{28}$$

By the same procedure as mentioned above, we obtain the creep strain in the second stage as follows:

$$\begin{aligned} \varepsilon_c(t, \sigma_0) = &\int_0^\infty d\nu \int_{\frac{1-e^{-\nu t_1}}{2-e^{-\nu t_1}}\sigma_0}^{\sigma_0} \frac{F(\nu, s)}{\nu}(\sigma_0 - s)e^{-\nu t_1}(1-e^{-\nu t})ds \\ &+ \int_0^\infty d\nu \int_0^{\frac{1-e^{-\nu t_1}}{2-e^{-\nu t_1}}\sigma_0} \frac{F(\nu, s)}{\nu}[(\sigma_0 - 2s) - \{(\sigma_0 - s)(1-e^{-\nu t_1}) - s\}e^{-\nu t_2}](1-e^{-\nu t})ds \end{aligned} \tag{29}$$

where t_2 is the duration period of first recovery. The first term of the right hand side of Eq. (29) is the creep strain due to the model elements which did not recover in the previous stage and corresponds to Eq. (23a), while the second term is the one due to the other elements which recovered and corresponds to Eq. (23b).

5. Repeated creep and recovery

If the creep of duration t_1 and the recovery of duration t_2 are alternately repeated infinitely many times, the oscillatory strain with the period t_1+t_2 will be expected as shown in Fig. 4. In the case when time t tends to infinity, we can easily see that the creep and recovery strain of the type shown in Fig. 3 (d) does not contribute to the amplitude ε_0 but only contribute to the mean strain ε_m of the oscillatory strain and the two types shown in Fig. 3 (b) and Fig. 3 (c) equally contribute to the amplitude ε_0 and mean strain ε_m.

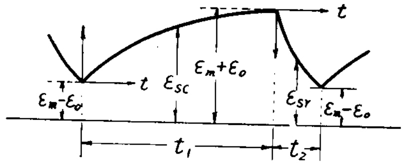

Fig. 4

Firstly, let us consider the model shown in Fig. 2. In the creep stage, we have the following equation and initial condition.

$$\left.\begin{aligned} &E\varepsilon_{sc}+\eta\dot{\varepsilon}_{sc}+s=\sigma_0\,, \\ &\varepsilon_{sc}=\varepsilon_m-\varepsilon_0\,, \qquad \text{at} \quad t=0\,. \end{aligned}\right\} \tag{30}$$

In the recovery stage, we have

$$\left.\begin{aligned} &E\varepsilon_{sr}+\eta\dot{\varepsilon}_{sr}-s=0\,, \\ &\varepsilon_{sr}=\varepsilon_m+\varepsilon_0\,, \qquad \text{at} \quad t=0\,. \end{aligned}\right\} \tag{31}$$

Solving the two equations (30) and (31), we obtain

$$\varepsilon_{sc}(t)=\frac{\beta(\sigma_0-s)}{\nu}+\left\{(\varepsilon_m-\varepsilon_0)-\frac{\beta(\sigma_0-s)}{\nu}\right\}e^{-\nu t} \tag{32}$$

$$\varepsilon_{sr}(t)=\frac{\beta s}{\nu}+\left\{(\varepsilon_m+\varepsilon_0)-\frac{\beta s}{\nu}\right\}e^{-\nu t}\,. \tag{33}$$

In the stationary state, the following relations must hold.

$$\left.\begin{aligned} \varepsilon_{sc}(t_1)&=\varepsilon_{sr}(0)=\varepsilon_m+\varepsilon_0 \\ \varepsilon_{sr}(t_2)&=\varepsilon_{sc}(0)=\varepsilon_m-\varepsilon_0\,. \end{aligned}\right\} \tag{34}$$

Then we have

$$\left.\begin{aligned} \varepsilon_0&=\frac{\beta(\sigma_0-2s)}{\nu}\cdot\frac{(1-e^{-\nu t_1})(1-e^{-\nu t_2})}{2(1-e^{-\nu t_1}e^{-\nu t_2})} \\ \varepsilon_m&=\frac{\beta(\sigma_0-s)}{\nu}\cdot\frac{(1-e^{-\nu t_1})(1+e^{-\nu t_2})}{2(1-e^{-\nu t_1}e^{-\nu t_2})} \\ &\qquad+\frac{\beta s}{\nu}\cdot\frac{(1+e^{-\nu t_1})(1-e^{-\nu t_2})}{2(1-e^{-\nu t_1}e^{-\nu t_2})}\,. \end{aligned}\right\} \tag{35}$$

Secondly, we treat the distributed system by similar procedures mentioned above and we obtain the following relations:

the creep strain;

$$\varepsilon_c=\int_0^\infty d\nu\int_0^{\sigma_0/2}\frac{F(\nu,s)}{\nu}(\sigma_0-2s)\frac{1-e^{-\nu t_2}}{1-e^{-\nu t_1}e^{-\nu t_2}}(1-e^{-\nu t})ds \tag{36}$$

the recovery strain;

$$\Delta=\int_0^\infty d\nu\int_0^{\sigma_0/2}\frac{F(\nu,s)}{\nu}(\sigma_0-2s)\frac{1-e^{-\nu t_1}}{1-e^{-\nu t_1}e^{-\nu t_2}}(1-e^{-\nu t})ds \tag{37}$$

the amplitude;

$$\varepsilon_0=\int_0^\infty d\nu\int_0^{\sigma_0/2}\frac{F(\nu,s)}{\nu}(\sigma_0-2s)\frac{(1-e^{-\nu t_1})(1-e^{-\nu t_2})}{2(1-e^{-\nu t_1}e^{-\nu t_2})}ds \tag{38}$$

the mean strain;

$$\begin{aligned} \varepsilon_m=&\frac{1}{E}\sigma_0+\int_0^\infty d\nu\int_{\sigma_0/2}^{\sigma_0}\frac{F(\nu,s)}{\nu}(\sigma_0-s)ds \\ &+\int_0^\infty d\nu\int_0^{\sigma_0/2}\frac{F(\nu,s)}{\nu}\left\{(\sigma_0-s)\frac{(1-e^{-\nu t_1})(1+e^{-ot_2})}{2(1-e^{-\nu t_1}e^{-\nu t_2})}+s\frac{(1+e^{-\nu t_1})(1-e^{-\nu t_2})}{2(1-e^{-\nu t_1}e^{-\nu t_2})}\right\}ds\,. \end{aligned} \tag{39}$$

Putting $t_1=t_2$ in these equations, we have

$$\left.\begin{aligned} \varepsilon_c &= \Delta = \int_0^\infty d\nu \int_0^{\sigma_0/2} \frac{F(\nu, s)}{\nu} (\sigma_0 - 2s) \frac{1-e^{-\nu t}}{1+e^{-\nu t_1}} ds \\ \varepsilon_0 &= \frac{1}{2}\int_0^\infty d\nu \int_0^{\sigma_0/2} \frac{F(\nu, s)}{\nu} (\sigma_0 - 2s) \tanh \frac{\nu t_1}{2} ds \\ \varepsilon_m &= \int_0^{\sigma_0}\left\{\int_0^\infty \frac{F(\nu, s)}{\nu} d\nu\right\}(\sigma_0 - s)ds - \frac{1}{2}\int_0^{\sigma_0/2}\left\{\int_0^\infty \frac{F(\nu, s)}{\nu} d\nu\right\}(\sigma_0 - 2s)ds \\ &\quad + \frac{1}{E}\sigma_0 . \end{aligned}\right\} \tag{40}$$

Let us consider the case when the loading of $\sigma_1 + \frac{\sigma_0}{2}$ on the visco-plasto-elastomer for a period t_1 and the loading of $\sigma_1 - \frac{\sigma_0}{2}$ (i.e. the visco-plasto-elastomer is loaded less than the previous stage by σ_0) for a period t_2 are alternately repeated infinitely many times. Then we find the interesting fact that the ε_c, Δ and the amplitude ε_0 are the same as those given by Eqs. (36), (37) and (38) respectively, that is, the mean load σ_1 does not affect these quantities.

6. Discussions on distribution function $F(\nu, s)$

It was previously described that the distribution function $F(\nu, s)$ is a very useful function for specifying mechanical behaviors of visco-plasto-elastic materials. This function is to be found from creep tests which cover large ranges of time t and load σ_0. The distribution function, which generally has a form of Fig. 5, is calculated from the results of creep tests by using Eq. (10)[5]. It is remarkable that the distribution function may also be used for the analysis of mechanical behaviors of pure elastic, pure plastic materials, Newtonian fluids, non-Newtonian fluids and linear visco-elastic materials by giving a special form to them.

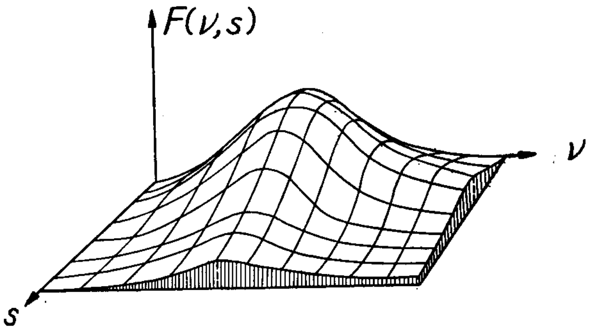

Fig. 5

Let us consider some examples of the distribution function $F(\nu, s)$, which is probable to be found in practical industrial materials.

(1) If the distribution function $F(\nu, s)$ is expressed as a product of two functions, one of which is a function of ν only, say $\Phi(\nu)$, and the other is a function of s only, say $\varphi(s)$, Eq. (17) becomes

$$\varepsilon_c(t, \sigma_0) = \int_0^{\sigma_0/2} \varphi(s)(\sigma_0 - 2s)ds \cdot \int_0^\infty \frac{\Phi(\nu)}{\nu}(1-e^{-\nu t})d\nu . \tag{41}$$

In this case, let us assume as the form of $\varphi(s)$ the one shown in Fig. 6 (a), (b), (c) and (d). Then we have the following expressions for the non-virgin creep strain.

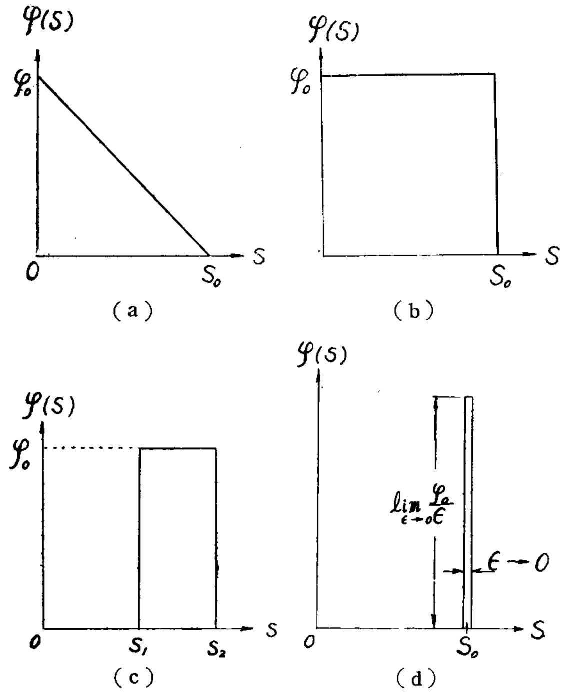

Fig. 6

In case of Fig. 6 (a):

$$\left.\begin{aligned} \varepsilon_c &= \frac{\varphi_0\sigma_0^2}{24}\left(6-\frac{\sigma_0}{s_0}\right)\int_0^\infty \frac{\Phi(\nu)}{\nu}(1-e^{-\nu t})d\nu\,, && \text{for} \quad \sigma_0 \leqq 2s_0 \\ \varepsilon_c &= \frac{\varphi_0 s_0}{6}(3\sigma_0-2s_0)\int_0^\infty \frac{\Phi(\nu)}{\nu}(1-e^{-\nu t})d\nu\,, && \text{for} \quad \sigma_0 > 2s_0 \end{aligned}\right\} \tag{42}$$

In case of Fig. 6 (b):

$$\left.\begin{aligned} \varepsilon_c &= \frac{\varphi_0\sigma_0^2}{4}\int_0^\infty \frac{\Phi(\nu)}{\nu}(1-e^{-\nu t})d\nu\,, && \text{for} \quad \sigma_0 \leqq s_0 \\ \varepsilon_c &= \frac{\varphi_0 s_0}{4}(\sigma_0-s_0)\int_0^\infty \frac{\Phi(\nu)}{\nu}(1-e^{-\nu t})d\nu\,, && \text{for} \quad \sigma_0 > s_0 \end{aligned}\right\} \tag{43}$$

In case of Fig. 6 (c):

$$\left.\begin{aligned} \varepsilon_c &= 0\,, && \text{for} \quad \sigma_0 \leqq s_1 \\ \varepsilon_c &= \varphi_0\left(\frac{\sigma_0}{2}-s_1\right)^2\int_0^\infty \frac{\Phi(\nu)}{\nu}(1-e^{-\nu t})d\nu\,, && \text{for} \quad s_1 < \sigma_0 \leqq s_2 \\ \varepsilon_c &= \varphi_0(s_2-s_1)(\sigma_0-s_1-s_2)\int_0^\infty \frac{\Phi(\nu)}{\nu}(1-e^{-\nu t})d\nu\,, && \text{for} \quad \sigma_0 > s_2 \end{aligned}\right\} \tag{44}$$

In case of Fig. 6 (d):

$$\left.\begin{aligned} \varepsilon_c &= 0\,, && \text{for} \quad \sigma_0 < s_0 \\ \varepsilon_c &= \varphi_0(\sigma_0-2s_0)\int_0^\infty \frac{\Phi(\nu)}{\nu}(1-e^{-\nu t})d\nu\,, && \text{for} \quad \sigma_0 > s_0 \end{aligned}\right\} \tag{45}$$

(2) In the case where the distribution function is represented by the expression

$$F(\nu, s) = \Phi(\nu)\,\delta(s) + \Phi'(\nu)\,\varphi(s)\,,$$

where $\delta(s)$ is the Dirac's δ-function.
Eq. (17) becomes

$$\varepsilon_c(t) = \int_0^\infty \frac{\Phi(\nu)}{\nu}(1-e^{-\nu t})d\nu + \int_0^{\sigma_0/2} \varphi(s)(\sigma_0 - 2s)ds \cdot \int_0^\infty \frac{\Phi'(\nu)}{\nu}(1-e^{-\nu t})d\nu\,.$$

7. Experimental results

In order to investigate the appropriateness of the creep theory we have experimented on vulcanized natural rubber used as vibration absorbers[3]. The compounds of the specimen are shown in Table 1. Results of this experiment are shown in Fig. 7, where the creep strain at 0.1 min. is taken for the standard. The curve (1) in this figure is the virgin creep curve obtained by applying the stress of 1.76 kg/cm² on the specimen which has never been stressed before. The curve (2) is the creep curve in

Table 1

Natural rubber	100
Zinc oxide	5
Stearic acid	1
Antioxident D	2
Sulfer	3
Accelerator DM	0.8
Micronex	50
Cure: $2.5\,kg/cm^2 \times 138°C \times 50\,min$	

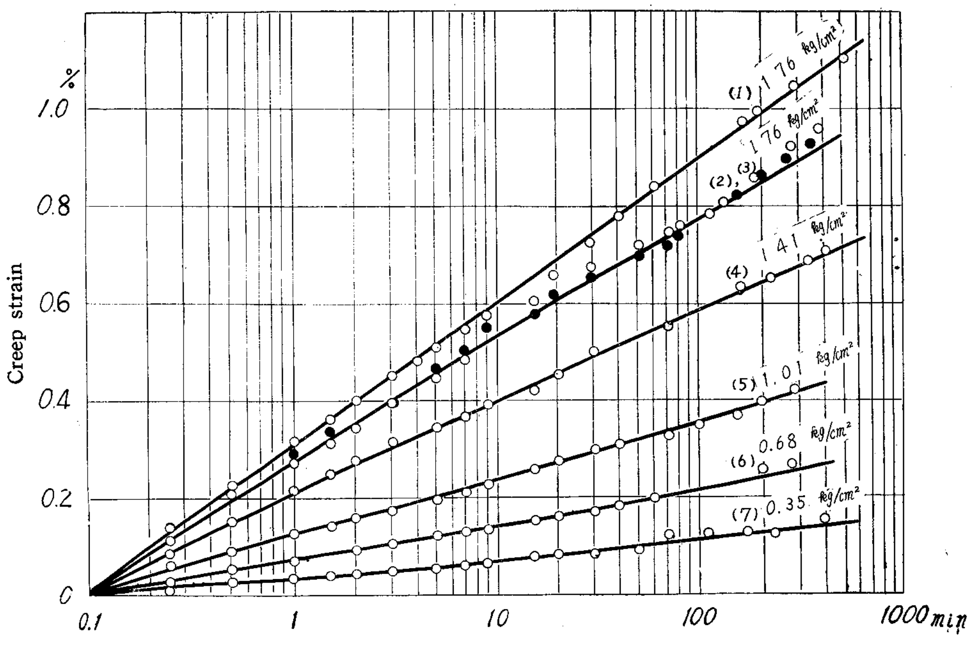

Fig. 7

second stage obtained by applying the same stress as in the virgin creep after the recovery of sufficiently long time. The curve (3) is the creep curve in third stage after the second recovery of sufficiently long time under the same stress as in the second stage. Curves (4), (5)··· and (7) are obtained in similar ways under the various stresses.

Curve (1) is the virgin creep curve and curves (2), (3)··· and (7) are non-virgin creep curves under various stresses. We can point out the remarkable fact that the existence of the virgin state was also confirmed experimentally in such material. If we illustrate the relation between the non-virgin creep strain and σ_0 from these data, we have Fig. 8. Curves in Fig. 8 are broken lines, all the break points of which have the same value $\sigma_0 = 2s_1 = 0.85$ kg/cm² as abscissa. Accordingly, it seems that $F(\nu, s)$ of this material takes the form $\Phi(\nu)\delta(s) + \Phi'(\nu)\delta(s-s_1)$. It will be interesting to relate

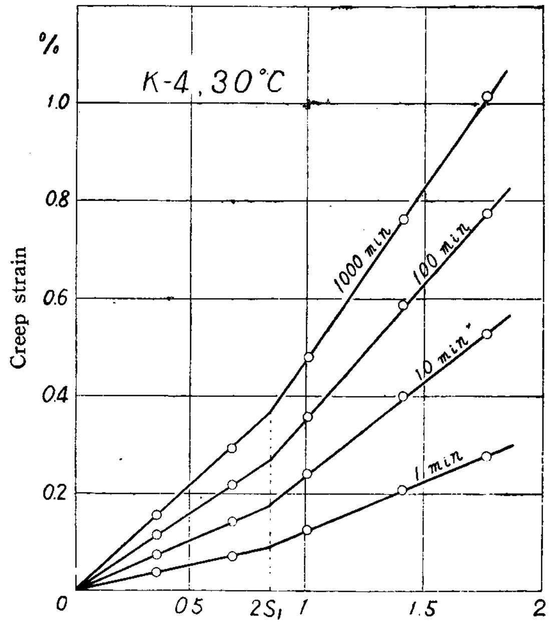

Fig. 8

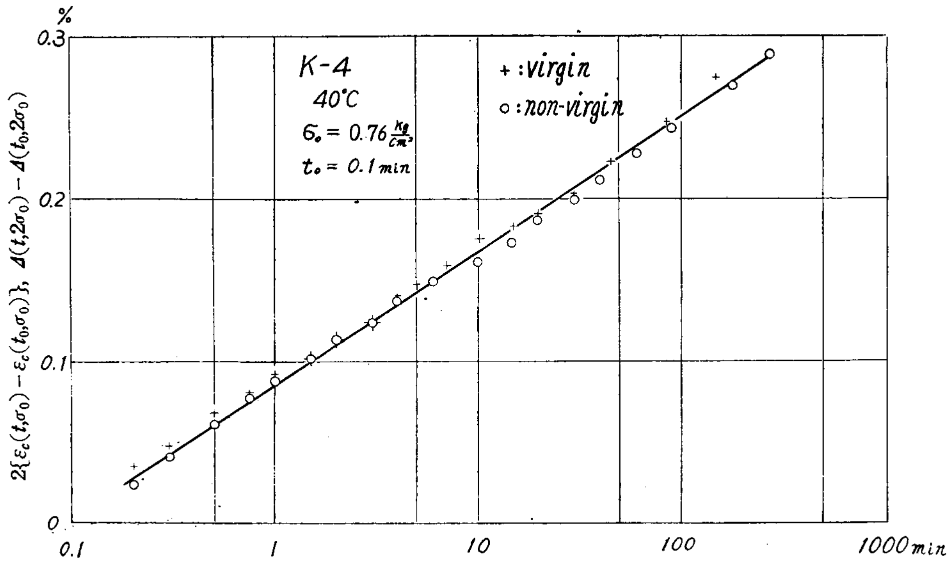

Fig. 9

this yield stress s_1 to micro structure of the material.

We also examined experimentally whether or not the relation indicated in Eq. (14) holds in this material, and we found that the equation holds exactly as shown in Fig. 9.

8. Concluding remarks

We have established the creep theory of visco-plasto-elastomer, which is very useful for the analysis of the phenomena of creep, recovery and permanent set of industrial materials. It must be noticed that the above discussions are restricted to small deformation. In the case of large deformation this theory will be applied under some appropriate modifications. The authors previously applied this theory to the problems of large deformation in the vibration tests of rubber vibration absorbers[4].

Reference

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5) This calculation can be performed using the approximate method for calcuration of distribution function of linear visco-etastomer. This method is described in the following paper: R. D. Andrews: Industrial and Engineering Chemistry, Vol. 44, No. 4.